Buda-Lund hydro model and the elliptic flow at RHIC

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Abstract

The ellipsoidally symmetric Buda-Lund hydrodynamic model describes naturally the transverse momentum and the pseudorapidity dependence of the elliptic flow in Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV. The result confirms the indication of quark deconfinement in Au+Au collisions at RHIC, obtained from Buda-Lund hydro model fits to combined spectra and HBT radii of BRAHMS, PHOBOS, PHENIX and STAR.

Introduction. PHENIX, PHOBOS and STAR experiments at RHIC produced a wealth of information on the asymmetry of the particle spectra with respect to the reaction plane [1, 2, 3, 4, 5, 6], characterized by the second harmonic moment of the transverse momentum distribution, denoted by v_2. It is measured as a function of the transverse mass and particle type at mid-rapidity as well as a function of the pseudo-rapidity $\eta = 0.5\log(\frac{|p|+p_z}{|p|-p_z})$.

The PHOBOS collaboration found [3], that $v_2(\eta)$ is a strongly decreasing function of $|\eta|$, which implies that the concept of boost-invariance, suggested by Bjorken in ref. [7], cannot be applied to characterize the hadronic final state of Au+Au collisions at RHIC.

We summarize here a successful attempt to describe the pseudo-rapidity dependence of the elliptic flow $v_2(\eta)$ at RHIC, for more details see ref. [8]. Our tool is the Buda-Lund hydrodynamic model [9, 10], which we extended in ref. [8] from axial to ellipsoidal symmetry.

Buda-Lund hydro for ellipsoidal expansions. Based on the success of the Buda-Lund hydro model to describe $Au+Au$ collisions at RHIC [12, 22], $Pb+Pb$ collisions at CERN SPS [13] and $h+p$ reactions at CERN SPS [14, 15], we describe the emission function in the core-halo picture, and assume that the core evolves in a hydrodynamical manner:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3}\frac{p^\mu d^4\Sigma_\mu(x)}{B(x,p)+s_q}, \quad (1)$$

where g is the degeneracy factor ($g = 1$ for identified pseudoscalar mesons, $g = 2$ for identified spin=1/2 baryons), and $p^\mu d^4\Sigma_\mu(x)$ is a generalized Cooper-Frye term, describing the flux of particles through a distribution of layers of freeze-out hypersurfaces, $B(x, p)$ is the (inverse) Boltzmann phase-space distribution, and the term s_q is determined by quantum statistics, $s_q = 0$, -1, and $+1$ for Boltzmann, Bose-Einstein and Fermi-Dirac distributions, respectively.

For a hydrodynamically expanding system, the (inverse) Boltzmann phase-space distribution is

$$B(x,p) = \exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right). \tag{2}$$

We will utilize some ansatz for the shape of the flow four-velocity, $u_\nu(x)$, chemical potential, $\mu(x)$, and temperature, $T(x)$ distributions. Their form is determined with the help of recently found exact solutions of hydrodynamics, both in the relativistic [16, 17] and in the non-relativistic cases [18, 19, 20].

The generalized Cooper-Frye prefactor is determined from the assumption that the freeze-out happens, with probability $H(\tau)d\tau$, at a hypersurface characterized by $\tau = const$ and that the proper-time measures the time elapsed in a fluid element that moves together with the fluid, $d\tau = u^\mu(x)dx_\mu$. We parameterize this hypersurface with the coordinates (r_x, r_y, r_z) and find that $d^3\Sigma^\mu(x|\tau) = u^\mu(x)d^3x/u^0(x)$. Using $\partial_t\tau|_r = u^0(x)$ we find that in this case the generalized Cooper-Frye prefactor is

$$p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x)H(\tau)d^4x, \tag{3}$$

This finding generalizes a result of ref. [21] from the case of a spherically symmetric Hubble flow to anisotropic, direction dependent Hubble flow distributions.

From the analysis of CERN SPS and RHIC data [13, 12, 22], we find that the proper-time distribution in heavy ion collisions is rather narrow, and $H(\tau)$ can be well approximated with a Gaussian representation of the Dirac-delta distribution,

$$H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}}\exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right), \tag{4}$$

with $\Delta\tau \ll \tau_0$.

We specify a fully scale invariant, relativistic form, which reproduces known non-relativistic hydrodynamic solutions too, in the limit when the expansion is non-relativistic. Both in the relativistic and the non-relativistic cases, the ellipsoidally symmetric, self-similarly expanding hydrodynamical solutions can be formulated in a simple manner, using a scaling variable s and a corresponding four-velocity distribution u^μ, that satisfy

$$u^\mu\partial_\mu s = 0, \tag{5}$$

which means that s is a good scaling variable if its co-moving derivative vanishes [17, 16].

It is convenient to introduce the dimensionless, generalized space-time rapidity variables (η_x, η_y, η_z), defined by the identification of

$$\sinh \eta_x = r_x \frac{\dot{X}}{X}, \tag{6}$$

similar equations hold for y and z. The characteristic sizes (for example, the lengths of the major axis of the expanding ellipsoid) are (X, Y, Z) that depend on proper-time τ and their derivatives with respect to proper-time are denoted by $(\dot{X}, \dot{Y}, \dot{Z})$. Eq. (5) is satisfied by the choice of

$$s = \frac{\cosh \eta_x - 1}{\dot{X}_f^2} + \frac{\cosh \eta_y - 1}{\dot{Y}_f^2} + \frac{\cosh \eta_z - 1}{\dot{Z}_f^2}, \tag{7}$$

$$u^\mu = (\gamma,\, \sinh \eta_x,\, \sinh \eta_y,\, \sinh \eta_z), \tag{8}$$

and from here on $(\dot{X}_f, \dot{Y}_f, \dot{Z}_f) = (\dot{X}(\tau_0), \dot{Y}(\tau_0), \dot{Z}(\tau_0)) = (\dot{X}_1, \dot{X}_2, \dot{X}_3)$, assuming that the rate of expansion is constant in the narrow proper-time interval of the freeze-out process. The above form has the desired non-relativistic limit,

$$s \to \frac{r_x^2}{2X_f^2} + \frac{r_y^2}{2Y_f^2} + \frac{r_z^2}{2Z_f^2}, \tag{9}$$

where again $(X_f, Y_f, Z_f) = (X(\tau_0), Y(\tau_0), Z(\tau_0)) = (X_1, X_2, X_3)$. From now on, we drop subscript $_f$. The normalization condition of $u^\mu(x)u_\mu(x) = 1$ yields the value of γ. For the fugacity distribution we assume a shape, that leads to Gaussian profile in the non-relativistic limit,

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s, \tag{10}$$

corresponding to the solution discussed in refs. [18, 19, 23]. We assume that the temperature may depend on the position as well as on proper-time. We characterize the inverse temperature distribution similarly to the shape used in the axially symmetric model of refs. [9, 10], and discussed in the exact hydro solutions of refs. [18, 19],

$$\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s}\, s\right)\left(1 + \frac{T_0 - T_e}{T_e}\, \frac{(\tau - \tau_0)^2}{2\Delta\tau^2}\right) \tag{11}$$

where T_0, T_s and T_e are the temperatures of the center, and the surface at the mean freeze-out time τ_0, while T_e corresponds to the temperature of the center after most of the particle emission is over (cooling due to evaporation and expansion). Sudden emission implies $T_e = T_0$ and $\Delta\tau \to 0$.

The observables can be calculated analytically from the Buda-Lund hydro model, using a saddle-point approximation in the integration. This approximation is exact both in the Gaussian and the non-relativistic limit, and if $p^\nu u_\nu / T \gg 1$ at the point of maximal emittivity.

The results are summarized in Figs. 1 and 2. We find that a small asymmetry between the two transverse Hubble constants gives a natural explanation of the transverse momentum dependence of v_2. The parameters are taken from Buda-Lund hydro model fits in refs. [12, 22], where the axially symmetric version of the model was utilized, although the mean transverse flow is reduced due to the less central nature of collisions studied in this study, which is a summary of ref. [8]. Note that we did not yet fine-tune the ellipsoidal Buda-Lund hydro model to describe these v_2 data, instead we searched the parameter set by hand. So at the moment we are not yet ready to report the best fit parameters and the error bars on the extracted parameter values. However, as indicated by Figs. 1 and 2, even this fit by eye method is successful in reproducing the data on elliptic flow at RHIC.

First we tuned the model to describe the p_t dependent elliptic flow of identified particles at midrapidity, as shown in Fig. 1. Then we calculated the value of the transverse momentum integrated $v_2(\eta = 0)$ and found [8], that this value is below the published PHOBOS data point at mid-rapidity. We attribute the difference of 0.02 to a non-flow contribution [24, 5]. The PHOBOS collaboration pointed out the possible existence of such a non-flow contribution in their data in ref. [3], as they did not utilize the fourth order cumulant measure of v_2.

We note that our presently best choice of parameter set correspond to a high, $T_0 > T_c = 170$ MeV central temperature, with a cold surface temperature of $T_s \approx 105$ MeV, see Figs. 1 and 2.

Summary and conclusions. We have generalized the Buda-Lund hydro model to the case of ellipsoidally symmetric expanding fireballs. We kept the parameters as determined from fits to the single particle spectra and the two-particle Bose-Einstein correlation functions (HBT radii) [12, 22], and interpreted them as angular averages over the direction of the reaction plane. Then we observed that a small splitting between the expansion rates parallel and transverse to the direction of the impact parameter, as well as a small tilt of the particle emitting source is sufficient to describe simultaneously the transverse momentum dependence of the collective flow of identified particles [1] as well as the pseudorapidity dependence of the collective flow [3, 4] at RHIC.

The results confirm the indication for quark deconfinement at RHIC found in refs. [12, 22], based on the observation, that some of the particles are emitted from a region with higher than the critical temperature, $T > T_c = 170$ MeV. The size of this volume is about 750 fm^3, corresponding to 1/8-th of the total volume measured on the $\tau = \tau_0$ main freeze-out hypersurface [8]. At the same time, this analysis indicates that the surface temperature is rather cold, $T_s \approx 105$ MeV, so approximately 7/8 of the particles are emitted from a rather cold hadron gas. So the picture is similar to a fireball, which is heated from inside.

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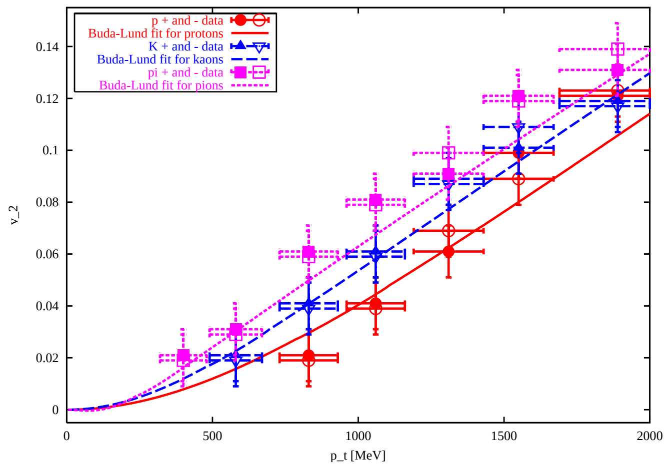

Figure 1: Buda-Lund fit to the $v_2(p_t)$ data

Here we see the fit to the PHENIX $v_2(p_t)$ data of identified particles [1]. The parameter set is: $T_0 = 210$ MeV, $\dot{X} = 0.57$, $\dot{Y} = 0.45$, $\dot{Z} = 2.4$, $T_s = 105$ MeV, $\tau_0 = 7$ fm/c, $\vartheta = 0.09$, $X_f = 8.6$ fm, $Y_f = 10.5$ fm, $Z_f = 17.5$ fm, $\mu_{0,\pi} = 70$ MeV, $\mu_{0,K} = 210$ MeV and $\mu_{0,p} = 315$ MeV, and the masses are taken as their physical value.

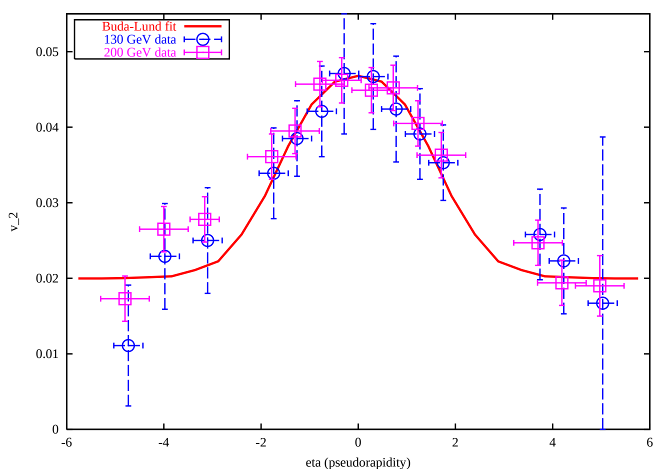

Figure 2: Buda-Lund fit to the $v_2(\eta)$ data

This image shows the fit to the 130 GeV Au+Au and 200 GeV Au+Au $v_2(\eta)$ data of PHOBOS [3, 4], with the ellipsoidal generalization of the Buda-Lund hydro model. Here we used the same parameter set as at fig. 1, with pion mass and chemical potential, and a constant non-flow parameter of 0.02.

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