New Family of Simple Solutions of Relativistic Perfect Fluid Hydrodynamics *

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Abstract

A new class of accelerating, exact and explicit solutions of relativistic hydrodynamics is found — more than 50 years after the previous similar result, the Landau-Khalatnikov solution. Surprisingly, the new solutions have a simple form, that generalizes the renowned, but accelerationless, Hwa-Bjorken solution. These new solutions take into account the work done by the fluid elements on each other, and work not only in one temporal and one spatial dimensions, but also in arbitrary number of spatial dimensions. They are applied here for an advanced estimation of initial energy density and life-time of the reaction in ultra-relativistic heavy ion collisions.

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* Dedicated to Y. Hama on the occasion of his 70th birthday.

Fluid dynamics is beautifully simple: it is based only on local conservation of charge, momentum and energy as well as on the additional assumption of local thermal equilibrium. Consequently, the hydrodynamical equations do not have internal scale, and their applications range from the smallest experimentally accessible scales of physics, such as the perfect fluid, a new form of matter created in high energy heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) located in Brookhaven National Laboratory (BNL, USA), through a glass of wine and through astrophysical and stellar objects, like stellar nebulae, to the largest known object, the evolution of our Universe. Fluid dynamics is beautifully and sometimes horrendously complicated: non-linear terms lead to instability, chaos, complexity, formation of eddies and other beautiful flow patterns, that are observable from the smallest to the largest scales of physics, like elliptic flow in high energy heavy ion collisions, hurricanes, super-nova explosion, or, the Hubble flow of our Universe. Fluid dynamics is beautifully relevant, too: presently this theory yields the best description of the single particle momentum distributions, elliptic flow patterns, and two-particle correlation data of the thousands of elementary particles created in the Little Bangs of relativistic heavy ion collisions at RHIC.

It is a matter of fact, that all but one of the presently known exact relativistic hydro solutions lack an important feature: the acceleration of matter. The only exception is the famous Landau-Khalatnikov (LK) solution, discovered more than 50 years ago [1–3]. This solution is a 1+1 dimensional, implicitly formulated but fully analytic solution of relativistic hydrodynamics. It predicts a realistic, approximately Gaussian rapidity distribution. However, due to its extremely complicated nature, the LK solution does not allow for an estimation of the initial energy density. Another renowned and exact solution is the 1+1 dimensional, boost-invariant, accelerationless Hwa-Bjorken (HB) solution [4, 5]. This solution allowed Bjorken to give a simple estimate of the initial energy density reached in heavy ion collisions from final state hadronic observables. It is well known, that the HB solution (in its original form, for $\mu_B = 0$) leads to a flat rapidity distribution, which is at variance with present observations at RHIC, except perhaps when observations are limited to a narrow region around mid-rapidity. Acceleration effects are, however, important in the estimation of the initial energy density even at mid-rapidity, if the expanding system is finite: even the most central fluid element performs work on the volume elements closer to the surface, and this work reduces the internal energy of cells even at mid-rapidity. We present such a new, accelerating family of solutions below, and apply it to data analysis in Au+Au collisions at RHIC. It can also be applied to test numerical solutions of relativistic hydrodynamics - no finite, accelerating, exact solution was available before for such tests in 1+3 dimensions.

Notation and basic equations: The metric tensor is $g^{\mu\nu} = diag(1,-1,-1,-1)$, $u^\mu = \gamma(1,\mathbf{v})$ is the four-velocity field, $\mathbf{v} = v\mathbf{n}$ is the three-velocity. The pressure is denoted by p, the energy density by ε, the temperature by T, the charged particle density by n, the chemical potential by μ and the entropy density by σ.

In high energy collisions, the entropy density is large, but net charge density is small. In perfect fluids, entropy and four-momentum are locally conserved,

$$\partial_\nu(\sigma u^\nu) = 0, \tag{1}$$

$$\partial_\nu T^{\mu\nu} = 0, \tag{2}$$

where the energy-momentum tensor is

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}. \tag{3}$$

The relativistic Euler equation and the energy conservation law are projections of Eq. (2):

$$(\varepsilon + p)u^\nu \partial_\nu u^\mu = (g^{\mu\rho} - u^\mu u^\rho)\,\partial_\rho p, \tag{4}$$

$$(\varepsilon + p)\partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon = 0. \tag{5}$$

For simplicity, let us consider the case, when all the conserved charges c_i have $\mu_i = 0$. Such an approximation is common in high energy physics at RHIC, and is assumed both in the LK and in the HB solutions. The thermodynamics of the flowing matter is characterized by the Equations of State (EoS). Let us consider

$$\varepsilon - B = \kappa(p + B), \tag{6}$$

where B stands for the bag constant and $\kappa = 1/c_s^2$, where c_s stands for the speed of sound, $c_s^2 = dp/d\varepsilon$. The bag constant B may have either a vanishing or a non-vanishing value, characteristic for a hadronic or, for a pre-hadronic state, respectively. In what follows, d stands for the number of spatial dimensions. In a heavy ion collision, $d = 3$, irrespective of the characteristics of the flow pattern. The $\kappa = d$ EoS corresponds to a gas of massless particles (e.g photons) or an ultra-relativistic ideal gas of massive particles. In this case,

$\sigma \propto T^d$ is also true. Observe, that Eq. (6) closes Eqs. (4,5) for the pressure and the three independent components of velocity.

We discuss below exact solutions of relativistic perfect fluid hydrodynamics in 1+1 dimensions and spherical solutions in 1+d dimensions as well. The notation r stands for the r_z spatial coordinate in 1+1 dimensions, and for the radial coordinate in 1+d dimensions. We use the well-known Rindler coordinates (τ and η), which naturally fit to the Hwa-Bjorken solution in the forward light-cone:

$$r = \tau \sinh \eta \quad , \quad t = \tau \cosh \eta. \tag{7}$$

We rewrite the equations of hydrodynamics in Rindler coordinates for a special case, when $v = \tanh \Omega(\eta)$, i.e. the Ω fluid rapidity depends only on η. Eqs. (4-6) the yield the following equations for $p(\tau, \eta)$ and $\Omega(\eta)$:

$$(\kappa + 1)\frac{\mathrm{d}\Omega}{\mathrm{d}\eta} = -\frac{\tau}{p}\frac{\partial p}{\partial \tau} - \coth(\Omega - \eta)\frac{1}{p}\frac{\partial p}{\partial \eta}, \tag{8}$$

$$\frac{\kappa+1}{\kappa}\frac{\mathrm{d}\Omega}{\mathrm{d}\eta} = -\frac{\tau}{p}\frac{\partial p}{\partial \tau} - \tanh(\Omega - \eta)\frac{1}{p}\frac{\partial p}{\partial \eta} - \frac{\kappa+1}{\kappa}\frac{d-1}{\sinh\eta}\frac{\sinh\Omega}{\cosh(\Omega-\eta)}. \tag{9}$$

In what follows, we specify full solutions of relativistic hydrodynamics, that are valid in the complete forward light-cone.

The new class of accelerating solutions is given by the following velocity and pressure fields:

$$v = \tanh \lambda\eta, \tag{10}$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh \frac{\eta}{2}\right)^{-(d-1)\phi_\lambda} - B. \tag{11}$$

The constants λ, d, κ and ϕ_λ are constrained, lines (a)–(e) of Table I show the cases that satisfy the hydrodynamical equations. Line (a) stands for the well known Hwa-Bjorken solution [4, 5], also called as Hubble solution when $d = 3$. Our new, $\lambda \neq 1$ solutions are listed in lines (b)–(e) of Table I. They describe accelerating flow, indicated by the curvature of the fluid world lines and expressed mathematically by $u^\mu \partial_\mu u^\nu \neq 0$. Case (b), where $\lambda = 2$ is shown in Fig. 1. Its fluid world lines, $r(t)$ evolve as

$$r(t) = \frac{1}{a_0}(\sqrt{1 + (a_0 t)^2} + 1) \quad , \quad a_0 = \frac{2r_0}{|r_0^2 - t_0^2|}, \tag{12}$$

where r_0 and t_0 specify the initial condition. These are trajectories with constant a_0 acceleration in the local rest frame. Both the $\lambda = 1$, and $\lambda = 2$ cases can be extended to external

Case:	λ	d	κ	ϕ_λ
(a)	1	$\in \mathbb{R}$	$\in \mathbb{R}$	0
(b)	2	$\in \mathbb{R}$	d	0
(c)	$\in \mathbb{R}$	1	1	0
(d)	$\frac{1}{2}$	$\in \mathbb{R}$	1	$\frac{\kappa+1}{\kappa}$
(e)	$\frac{3}{2}$	$\in \mathbb{R}$	$\frac{4d-1}{3}$	$\frac{\kappa+1}{\kappa}$

TABLE I: The new family of solutions is given by lines (b)-(e), while case (a) is the Hwa-Bjorken-Hubble solution.

($|\mathbf{r}| > t$) solutions that are uniformly accelerating, hence they contain event horizons. This property of the uniform acceleration was utilized recently by Kharzeev and Tuchin [11] to describe thermalization in heavy ion reactions via the Unruh effect. Case (c) describes a one dimensional fluid, with a special EoS of $\kappa = 1$, but its parameter of acceleration, λ can be chosen arbitrarily. After our derivation of cases (b) and (c), T. S. Biró pointed out [6], that the $\lambda = 1/2$, $\kappa = 1$ and the $\lambda = 3/2$, $\kappa = 11/3$, $d = 3$ cases are also solutions. We have generalized them for any $d \in \mathbb{R}$, as shown in lines (d) and (e). The exponent ϕ_λ is introduced to indicate, that the pressure is explicitly dependent on space-time rapidity η in these cases, and the pressure tends to zero for large values of $|\eta|$. Thus cases (d) and (e) are finite solutions. Note that in $d = 3$ dimensions, case (e) has $\varepsilon - 3p > 0$, similarly to the lattice QCD EoS.

Even when one considers the case, that λ can be a $\lambda(\tau)$ function, and the pressure may have a form of $p = H(\tau)U(\eta) - B$, that conditions generalize the actual solutions of eqs. (10,11), we have proven that no non-trivial, additional to Table I solutions exist in this more general class. Our proof was easily obtained from a second order Taylor expansion of the hydrodynamical equations, but is not detailed here.

During the time evolution of heavy ion collisions at RHIC, and during the quark-hadron transition in the early Universe, the chemical potentials of conserved charges are very small, $p \simeq p(T, \mu_i = 0)$. In this case, $(1+\kappa)p = \sigma T$. The entropy conservation, Eq. (1) can be solved for σ, and comparing with $(1+\kappa)p = \sigma T$ yields

$$\sigma = \sigma_0 \nu_\sigma(s) \left(\frac{p+B}{p_0}\right)^{\frac{\kappa}{\kappa+1}}, \quad T = \frac{T_0}{\nu_\sigma(s)} \left(\frac{p+B}{p_0}\right)^{\frac{1}{\kappa+1}}, \tag{13}$$

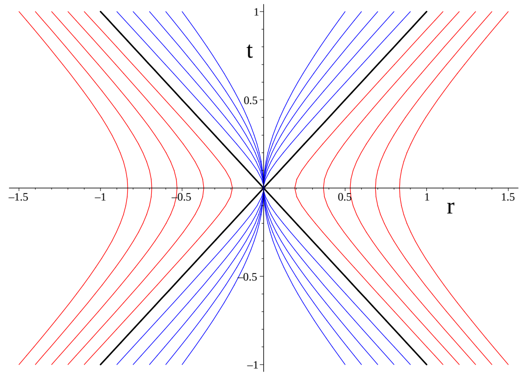

FIG. 1: (Color online) Fluid trajectories of the $\lambda = 2$ solution.

where $(1+\kappa)p_0 = T_0\sigma_0$, and $p(\tau,\eta)$ is given by Eq. (11). Here we have introduced a scaling function $\nu_\sigma(s)$, that can be any positive function that satisfies $\nu_\sigma(0) = 1$. The scaling variable s has, by definition, a vanishing comoving derivative: $\frac{\partial s}{\partial t} + (\mathbf{v}\nabla)s = 0$. For $\lambda = 1$ we find that $s(\tau,\eta) = \eta$. For $\lambda \neq 1$ the scaling variable is

$$s(\tau,\eta) = \left(\frac{\tau_0}{\tau}\right)^{\lambda-1} \sinh\left((\lambda-1)\,\eta\right). \tag{14}$$

The scaling function $\nu_\sigma(s) > 0$ appears similarly how it shows up also in accelerationless solutions [7–10].

In case of mixtures, where various non-vanishing conserved charges n_i with $\mu_i \neq 0$ are present and contribute to the pressure, the more general form of the thermodynamic potential, $p = p(T,\mu_i) = (T\sigma + \sum_i \mu_i n_i)/(1+\kappa) - B$ leads to similar exact solutions of relativistic hydrodynamics, but new, arbitrary scaling functions $\nu_i(s) > 0$ appear, with $n_i \propto \nu_i(s)$ and $\mu_i \propto \frac{1}{\nu_i(s)}$. These forms solve the continuity equations for n_i and Eqs. (4-5).

The rapidity distribution, $\frac{\mathrm{d}n}{\mathrm{d}y}$ is given below for case (c) of Table I in a Boltzmann approximation. We consider the $\nu_\sigma(s) = 1$ case, when our solutions also solve the Landau-Khalatnikov equation, $\partial^\nu T u_\mu = \partial_\mu T u^\nu$. The freeze-out temperature is $T(\eta = 0, \tau = \tau_f) = T_f$, where subscript $_f$ stands for *f*reeze-out. We assume, that the freeze-out hypersurface is pseudo-orthogonal to u^μ. With a saddle-point integration in η, for $m/T_f \gg 1$, where m is the particle mass, $\lambda > 0.5$, $\mu_i = 0$ and $\nu_\sigma(s) = 1$ we got

$$\frac{\mathrm{d}n}{\mathrm{d}y} \approx \left.\frac{\mathrm{d}n}{\mathrm{d}y}\right|_{y=0} \cosh^{\pm\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) e^{-\frac{m}{T_f}\left[\cosh^\alpha\left(\frac{y}{\alpha}\right)-1\right]}, \tag{15}$$

with $\alpha = \frac{2\lambda-1}{\lambda-1}$. The "Gaussian width" of this distribution is $\Delta y^2 = \frac{\alpha}{m/T_f \mp 1/2 + 1/\alpha}$. The upper sign is for the $1+1$ dimensional case, the lower sign is for the case when the $1+1$

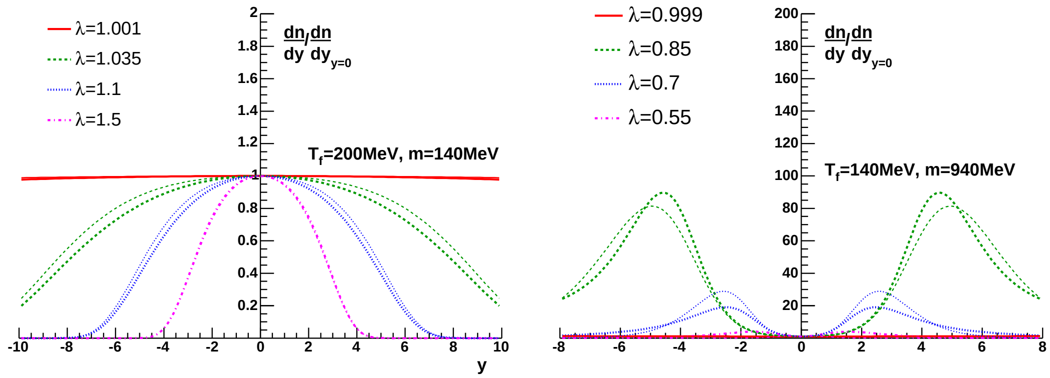

FIG. 2: (Color online) Normalized rapidity distributions from the new solutions in 1+1 dimensions for various λ, T_f and m values. Thick lines show the result of numerical integration, thin lines the analytic approximation from Eq. (15). For $\lambda > 1$ and not too big T_f it can be used within about 10 % error.

dimensional solution is embedded in the $1 + 3$ dimensional space-time. In this latter case, the transverse mass distribution is integrated in a saddle-point approximation from m to infinity. The resulting rapidity distribution has a minimum at $y = 0$, if $\Delta y^2 < 0$, it is flat if $\Delta y^2 = 0$, i.e. $\lambda = 1$, or $\lambda = \frac{1}{2}\left(1 + \frac{T_f}{2m \mp T_f + T_f}\right)$, otherwise it is nearly Gaussian, as illustrated in Fig. 2.

Let us now estimate the energy density reached in heavy ion reactions, just after thermalization ($\tau = \tau_0 \approx 1$ fm/c). Let us focus on a thin transverse piece of produced matter at mid-rapidity, illustrated by Fig. 2 of Ref. [5]. The radius R of this slab is estimated by the radius of the colliding hadrons or nuclei, its volume is $dV = (R^2\pi)\tau d\eta$. The energy content in this slab is $dE = \langle m_t \rangle dn$, where $\langle m_t \rangle$ is the average transverse mass at $y = 0$, so similarly to Bjorken, the initial energy density is

$$\varepsilon_0 = \frac{\langle m_t \rangle}{(R^2\pi)\tau_0} \frac{dn}{d\eta_0}. \tag{16}$$

For accelerationless, boost-invariant Hwa-Bjorken flows $\eta_0 = \eta_f = y$, however, for our accelerating solution we have to apply a correction factor of $\frac{\partial \eta_f}{\partial \eta_0} \frac{\partial y}{\partial \eta_f} = \left(\tau_f/\tau_0\right)^{\lambda-1}(2\lambda - 1)$. Thus the initial energy density ε_0 can be accessed by an advanced estimation ε_c as

$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}, \quad \varepsilon_{Bj} = \frac{\langle m_t \rangle}{(R^2\pi)\tau_0} \frac{dn}{dy}. \tag{17}$$

Here ε_{Bj} is the Bjorken estimation, which is recovered if $\frac{\mathrm{d}n}{\mathrm{d}y}$ is flat (i.e. $\lambda = 1$), but if $\lambda > 1$, ε_0 is *under-estimated* by the Bjorken formula. Fig. 3 shows our fits to BRAHMS

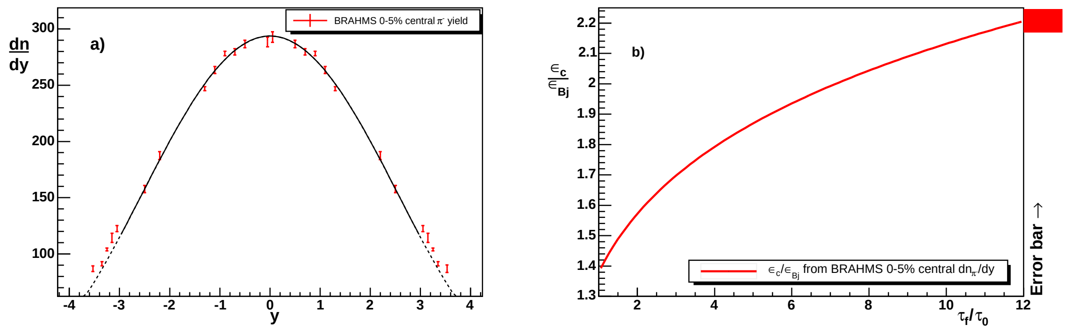

FIG. 3: (Color online) Panel (a): dn/dy data of negative pions, as measured by the BRAHMS collaboration [12] in central (0-5%) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, fitted with Eq. (15) (1+3 dimensional case). The fit range was $-3 < y < 3$, to exclude target and projectile rapidity region, CL = 0.6 %. Panel (b): $\varepsilon_c/\varepsilon_{Bj}$ ratio as a function of τ_f/τ_0.

dn/dy data [12]. Using the Bjorken estimate of $\varepsilon_{Bj} = 5$ GeV/fm^3 as given in Ref. [13], and $\tau_f/\tau_0 = 8 \pm 2$ fm/c, we find an initial energy density of $\varepsilon_c = (2.0 \pm 0.1)\varepsilon_{Bj} = 10.0 \pm 0.5$ GeV/fm^3. If the evolution deviates from a 1+1 dimensional perfect flow, then our estimation is only a lower limit for the initial energy density.

Life-time determination: For a Hwa-Bjorken type of accelerationless, coasting longitudinal flow, Sinyukov and Makhlin [14] determined the longitudinal length of homogeneity as $R_{\text{long}} = \sqrt{\frac{T_f}{m_t}}\tau_{Bj}$. Here m_t is the transverse mass and τ_{Bj} is the (Bjorken) freeze-out time. However, if the flow is accelerating, the estimated origin of the trajectories is shifted, so the life-time of the reaction is under-estimated by τ_{Bj}. (This was pointed out also in refs. [15–18].) From our solution (c) we obtain

$$R_{\text{long}} = \sqrt{\frac{T_f}{m_t}}\frac{\tau_c}{\lambda} \quad \Rightarrow \quad \tau_c = \lambda\tau_{Bj}. \tag{18}$$

BRAHMS data of Fig. 3 yield $\lambda = 1.18 \pm 0.01$, and imply a 18 $\pm$1 % increase in the estimated τ_c.

Relation to earlier solutions: In our case, similarly to the Hwa-Bjorken case, the initial condition can be given on a $\tau = \tau_0$ hypersurface in the forward light-cone, or on any $\tau_0(\eta)$ continuous Cauchy-surface. Note, however, that we discuss smooth initial conditions on this initial hypersurface, hence the Landau solution, that starts from a step function, a finite box filled with a constant energy density, will not be part of the new family of solutions presented below: in our case, we solve the same dynamical equations as Landau and Khalatnikov, but with modified boundary conditions. Another similarity to the Landau-Khalatnikov solution is that in certain limiting cases, we obtain nearly Gaussian rapidity distributions. Our rapidity distributions are characterized by two parameters, the scale (rapidity density at mid-rapidity) and a shape parameter, which measures the acceleration effects. In the accelerationless case, the Hwa-Bjorken limit is recovered exactly, both for the flow profile in the forward light-cone, and for the flat rapidity density. We however find external solutions too, that are valid outside the light-cone. Similarly to the Landau-Khalatnikov solution, the initial condition outside the light-cone can be specified at $t = 0$, where the matter is at rest, $v(t = 0, r) = 0$. However, in our case, the initial energy density has an inhomogeneous distribution even in the case of these external solutions, hence $p(t = 0, r)$ is never a step function in our family of solutions, in contrast to the Landau initial conditions. It is also interesting to mention, that in the case of our solutions in the future light cone, the acceleration tends to zero for late times at any given location, but this limit is not uniform. For example, in case of the $\lambda = 2$ solutions, the acceleration vanishes for late times at any position, but it remains constant along the fluid lines.

In summary, we have presented a new family of accelerating, exact and simple solutions of relativistic hydrodynamics. These new solutions are simple, although their finding was a complicated process that lasted for decades. BRAHMS pointed out before [12], that the rapidity distribution of negative pions in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV is flatter than a Gaussian, but not completely flat, hence neither the Landau-Khalatnikov, nor the Hwa-Bjorken solution describes it. Our new exact solutions describe well these BRAHMS observations. We have found that at least 10 ± 0.5 GeV/fm^3 initial energy densities are reached at $\tau_0 = 1$ fm in Au+Au collisions at RHIC. We have also given an advanced estimate of the life-time of the reaction. Both estimates include work effects for the first time, and connect initial conditions and final hadronic observables with simple and explicit formulas.

As an outlook, the results presented here could be applied to advanced estimates of initial energy densities in relativistic heavy ion reactions from CERN SPS through RHIC to LHC. In the limit when the rapidity distribution is flat, the Bjorken energy density estimate is recovered. However, for rapidity distribution with a finite width, an advanced formula is found, which yields increased values of the initial energy density as compared to the Bjorken estimate.

Although we have proven, that our solutions are unique in the considered general class of parametric solutions of hydrodynamics, more work is necessary to investigate the stability and possible further generalizations of these solutions. In particular, exact solutions are with ellipsoidal symmetry and relativistic acceleration are yet to be found, but would be most interesting, as they could provide new insights to the connection between elliptic flow data and initial conditions. Also, exact solutions with more general equations of state would be most interesting. These could allow for the investigation of the dependencies on the speed of sound of the initial energy density estimates.

A more detailed and significantly longer description of the results summarized above is being prepared and will be submitted for a publication separately.

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