SENSITIVITY OF THE CHIRAL PHASE TRANSITION OF QCD TO THE SCALAR MESON SECTOR *

T. HERPAY

Department of Physics of Complex Systems,
Eötvös University, H-1117 Budapest, Hungary
E-mail: herpay@complex.elte.hu

A. PATKÓS†

Department of Atomic Physics, Eötvös University,
H-1117 Budapest, Hungary
E-mail:patkos@ludens.elte.hu

ZS. SZÉP

Research Group for Statistical Physics of the Hungarian Academy of Sciences,
H-1117 Budapest, Hungary
E-mail:szepzs@achilles.elte.hu

Gribov's theory of light quark confinement implies the existence of two kinds of scalar bound states. The phase diagram of the three-flavor QCD is mapped out in the $(m_\pi - m_K)$–plane with help of the $SU_L(3) \times SU_R(3)$ linear sigma model supplemented with the assumption that the masses of the so-called superbound scalars do not change under the variation of the pion and kaon mass. The phase boundary along the $m_\pi = m_K$ line is found in the interval 15 MeV $<$ $\mathrm{m_{crit}}$ $<$ 25 MeV, irrespective which $f_0 - \sigma$ linear combination is identified with the pure superbound state.

1. Introduction

Volodia Gribov repeatedly has expressed in the early nineteen-eighties his view that deconfinement in QCD is going to turn out to be a complete analogue of atomic ionisation, which one would not call a phase transition. There is increasing evidence that the temperature driven transition from the hadronic phase to quark-gluon plasma is a smooth crossover indeed [1].

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†Speaker

He based his confinement theory also on an analogy with a phenomenon of atomic physics, e.g. supercritical binding. This would occur for light quarks irrespective to the accurate value of their mass [2]. The role of chiral symmetry is not evidently important in this theory, and the consequences on its spontaneous breakdown were not yet fully clarified [3].

In this contribution to the commemoration of the 75th anniversary of V.N. Gribov, we exploit semi-quantitatively his ideas on the nature of the scalar meson sector when discussing the chiral symmetry restoring quark-hadron transition in different points of the $m_\pi - m_K$-plane with help of a variant of the linear sigma model ($L\sigma M$). In a recent publication [4] we constructed a continuation of the parameters of this model from the physical point to an arbitrary mass-point m_π, m_K requiring agreement with results of Chiral Perturbation Theory (ChPT) for the decay constants f_π, f_K and the trace of the squared mass matrix in the $\eta - \eta'$ sector, M_η^2, at the tree level. In the $\eta - \eta'$ sector the predictions of the two models for the separate mass eigenvalues nicely coincide. It was emphasised that for a complete specification of the parameters of the model one needs a single extra information on the dependence of scalar spectra on the masses of the pseudoscalar nonet. This information is beyond ChPT, therefore in general only *ad hoc* assumptions can be made and tested through the consequences.

According to Gribov's confinement theory in addition to normal $\bar{q}q$ states also superbound states do exist, which contain quarks with *negative kinetic energy*. Some repulsive interaction between such quarks increases their energy above zero and makes these associations physically observable [5]. These states are of smaller size and higher energy than the normal scalar meson states, therefore it was suggested to associate them with f_0 in the isoscalar and with a_0 in the isovector channel. In the $SU(2) \times SU(2)$ model they form another $O(4)$ quartet in addition to σ, π. It is rather natural to assume that the masses of the superbound states are not sensitive to the variation of m_π.

In the three flavor case a similar "doubling" of the multiplet structure can be assumed, if the strange quark is light enough. In this paper we shall explore the boundary of the first order transition region around the chiral point in the three-flavor QCD [6] taking into account the extra requirement to have scalars in the spectra whose mass does not vary with m_π, m_K. The consequences of identifying the physically observable scalars with some mixture of the pure normal and pure superbound states will be investigated.

Numerical investigations of the chiral symmetry restoration were done in the framework of lattice QCD and systematically improved for the 3-flavor degenerate case $m_u = m_d = m_s \neq 0$. The initial estimate for the critical pseudoscalar meson mass, $m_{\rm crit}({\rm diag}) \approx 290$ MeV [7] was seen to be reduced to $60 - 70$ MeV [8] or may be to even further down [9] when finer lattices and improved lattice actions are used. Very recently de Forcrand and Philipsen reported an estimate $m_{crit} \approx (0.1-0.2)T_c$, which very conservatively means $m_{crit} \approx 15 - 30$MeV [10]. One should be conscious of the fact, that it is extremely difficult to reach continuum results in this mass regime.

Effective models (linear or non-linear sigma models, Nambu–Jona-Lasinio model) represent another, in a sense complementary, approach to the study of the phase structure, which one expects to work the better the lighter quark masses are used [11,12,13]. It is surprising that only moderate effort was invested to date to improve the pioneering studies of the $SU(3) \times SU(3)$ linear sigma model by Meyer-Ortmanns and Schaefer [11] which used a saddle point approximation valid in the limit of infinite number of flavors, and derived $m_{\rm crit}({\rm diag}) \lesssim 51$ MeV. An extension of their work to unequal pion and kaon masses was achieved by C. Schmidt [14]. He found $m_{\rm crit}({\rm diag}) = 47$ MeV and a phase boundary approaching the m_K-axis rather sharply. The phase boundary was calculated also by Lenaghan [15] using the Hartree-approximation to the effective potential derived in CJT-formalism. For the complete determination of the couplings of the three-flavor chiral meson model he fixed the $T = 0$ mass of the σ particle in addition to the experimental mass spectra of the pseudoscalar sector. The emerging phase boundary is rather sensitive to this mass. The estimate for $m_{\rm crit}({\rm diag})$ which one can extract from Fig. 3 of [15] for $m_\sigma = 900$ MeV is compatible with [11,14].

Our method of parametrisation and solution of $L\sigma M$ was described in detail in [4]. Therefore here we shall only review the set of the equations to be solved for the determination of the transition point. The changes arising from the implementation of the insensitivity of Gribov's superbound scalars to the variation of the fundamental masses will be emphasised. We conclude by giving the most characteristic features of the phase boundary.

2. $L\sigma M$ parametrisation consistent with ChPT

We outline first, how one obtains with ChPT the m_π, m_K-dependence of the masses and decay constants in the pseudoscalar sector. The dependence of the pion and kaon masses as well as of their decay constants on the quark masses were determined in [16] for the $SU(3) \times SU(3)$ nonlinear sigma model:

$$m_\pi^2 = 2A\left[1+\frac{1}{f^2}\left(\mu_\pi - \frac{1}{3}\mu_\eta + 16A(2L_8-L_5)+\right.\right.$$
$$\left.\left.+16A(2+q)(2L_6-L_4)\right)\right], \quad (1)$$
$$m_K^2 = A(1+q)\left[1+\frac{1}{f^2}\left(\frac{2}{3}\mu_\eta + 8A(1+q)(2L_8-L_5)\right.\right.$$
$$\left.\left.+16A(2+q)(2L_6-L_4)\right)\right], \quad (2)$$
$$f_\pi = f\left[1+\frac{1}{f^2}\left(-2\mu_\pi - \mu_K + 8AL_5 + 8A(2+q)L_4\right)\right], \quad (3)$$
$$f_K = f\left[1-\frac{1}{f^2}\left(\frac{3}{4}(\mu_\pi+\mu_\eta+2\mu_K) - 4A(1+q)L_5 - 8A(2+q)L_4\right)\right]. \quad (4)$$

Here A, q are related to the quark masses, f is the coupling of the nonlinear sigma model. μ_{PS} defines the so-called chiral logarithm for each pseudoscalar meson (PS) proportional to $\ln(m_{PS}/M_0)$ with $M_0 = 4\pi f$. The determination of the low energy chiral constants L_i is discussed in depth in [4].

One inverts the first two equations with $O(f^{-2})$ accuracy and finds the following m_π, m_K-dependence for the decay constants:

$$f_\pi = f\left[1-\frac{1}{f^2}(2\mu_\pi + \mu_K - 4m_\pi^2(L_4+L_5) - 8m_K^2 L_4)\right], \quad (5)$$
$$f_K = f\left[1-\frac{1}{f^2}\left(\frac{3}{4}(\mu_\pi+\mu_\eta+2\mu_K) - 4m_\pi^2 L_4 - 4m_K^2(L_5+2L_4)\right)\right].$$

The extension to the $U(3)\times U(3)$ ChPT is somewhat more complicated. It was worked out in [17,18,19,20] and allows the determination of $m_\eta(m_\pi, m_K), m_{\eta'}(m_\pi, m_K)$. For the parametrisation of $L\sigma M$ one more independent relation can be obtained from the mixing $\eta-\eta'$ sector, for which in [4] we have chosen the trace of the 2×2 squared mass matrix, denoted by $M_\eta^2 \equiv m_\eta^2 + m_{\eta'}^2$:

$$M_\eta^2 = 2m_K^2 - 3v_0^{(2)} + 2(2m_K^2+m_\pi^2)(3v_2^{(2)} - v_3^{(1)}) \quad (6)$$
$$+\frac{1}{f^2}\Big[8v_0^{(2)}(2m_K^2+m_\pi^2)(L_5+3L_4) + m_\pi^2(\mu_\eta - 3\mu_\pi) - 4m_K^2\mu_\eta$$
$$+\frac{16}{3}(6L_8 - 3L_5 + 8L_7)(m_\pi^2 - m_K^2)^2$$
$$+\frac{32}{3}L_6(m_\pi^4 - 2m_K^4 + m_K^2 m_\pi^2) + \frac{16}{3}L_7(m_\pi^2 + 2m_K^2)^2\Big].$$

Also the constants $v_i^{(j)}$ were determined in [4] from the $T = 0$ properties of the Goldstone particles.

Next, we turn to the problem of parameterising $L\sigma M$ to provide spectra the closest possible to ChPT. The Lagrangian of the $SU_L(3) \times SU_R(3)$ symmetric linear sigma model with explicit symmetry breaking terms is given [21] by

$$L(M) = \frac{1}{2}\mathrm{tr}(\partial_\mu M^\dagger \partial^\mu M + \mu_0^2 M^\dagger M) - f_1 \left(\mathrm{tr}(M^\dagger M)\right)^2 \\ - f_2 \mathrm{tr}(M^\dagger M)^2 - g\left(\det(M) + \det(M^\dagger)\right) + \epsilon_0\sigma_0 + \epsilon_8\sigma_8, \tag{7}$$

where M is a complex 3×3 matrix, defined by the σ_i scalar and π_i pseudoscalar fields $M := \frac{1}{\sqrt{2}}\sum_{i=0}^{8}(\sigma_i + i\pi_i)\lambda_i$, with λ_i : $i = 1\ldots 8$ the Gell-Mann matrices and $\lambda_0 := \sqrt{\frac{2}{3}}\mathbf{1}$. The last two terms of (8) break the symmetry explicitly, the possible isospin breaking term $\epsilon_3\sigma_3$ is not considered.

A detailed analysis of the symmetry breaking patterns which might occur in the system described by this Lagrangian can be found in [12]. The field expectation values $\langle\sigma_0\rangle, \langle\sigma_8\rangle$ both contain strange ($\equiv y$) and non-strange ($\equiv x$) components:

$$x = (\sqrt{2}\langle\sigma_0\rangle + \langle\sigma_8\rangle)/\sqrt{3}, \qquad y = (\langle\sigma_0\rangle - \sqrt{2}\langle\sigma_8\rangle)/\sqrt{3}. \tag{8}$$

With help of the PCAC relations and the tree level mass formulae (see Table 1, where the $x - y$ basis is used instead of $0 - 8$) the following expressions can be derived for the couplings of $L\sigma M$:

$$\begin{aligned} x &= f_\pi, \qquad y = (2f_K - f_\pi)/\sqrt{2}\,, \\ f_2 &= \frac{(6f_K - 3f_\pi)m_K^2 - (2f_K + f_\pi)m_\pi^2 - 2(f_K - f_\pi)M_\eta^2}{4(f_K - f_\pi)(8f_K^2 - 8f_K f_\pi + 3f_\pi^2)}, \\ g &= \frac{2f_K m_K^2 + 2(f_K - f_\pi)m_\pi^2 - (2f_K - f_\pi)M_\eta^2}{\sqrt{2}(8f_K^2 - 8f_K f_\pi + 3f_\pi^2)}, \\ M^2 &\equiv -\mu_0^2 + 4f_1(x^2 + y^2) \\ &= \frac{1}{2}M_\eta^2 + \frac{g}{\sqrt{2}}(2f_K - f_\pi) - 2f_2[(f_\pi - f_K)^2 + f_K^2]. \end{aligned} \tag{9}$$

The sources $\epsilon_x = (\sqrt{2}\epsilon_0 + \epsilon_8)/\sqrt{3}, \epsilon_y = (\epsilon_0 - \sqrt{2}\epsilon_8)/\sqrt{3}$, which explicitly break chiral symmetry are determined with help of the Gell-Mann–Oakes–Renner relations:

$$\epsilon_x = m_\pi^2 x, \qquad \epsilon_y = \frac{\sqrt{2}}{2}(m_K^2 - m_\pi^2)x + m_K^2 y. \tag{10}$$

Table 1. Squared masses of the pseudoscalar boson nonet and their parity partners

m_π^2	$= -\mu_0^2 + 2(2f_1 + f_2)x^2 + 4f_1y^2 + 2gy$
$m_{a_0}^2$	$= -\mu_0^2 + 2(2f_1 + 3f_2)x^2 + 4f_1y^2 - 2gy$
m_K^2	$= -\mu_0^2 + 2(2f_1 + f_2)(x^2 + y^2) + 2f_2y^2 - \sqrt{2}x(2f_2y - g)$
m_κ^2	$= -\mu_0^2 + 2(2f_1 + f_2)(x^2 + y^2) + 2f_2y^2 + \sqrt{2}x(2f_2y - g)$
$m_{\eta_{xx}}^2$	$= -\mu_0^2 + 2(2f_1 + f_2)x^2 + 4f_1y^2 - 2gy$
$m_{\eta_{yy}}^2$	$= -\mu_0^2 + 4f_1x^2 + 4(f_1 + f_2)y^2$
$m_{\eta_{xy}}^2$	$= -2gx$
$m_{\sigma_{xx}}^2$	$= -\mu_0^2 + 6(2f_1 + f_2)x^2 + 4f_1y^2 + 2gy$
$m_{\sigma_{yy}}^2$	$= -\mu_0^2 + 4f_1x^2 + 12(f_1 + f_2)y^2$
$m_{\sigma_{xy}}^2$	$= 8f_1xy + 2gx$

Note: The expressions of the squared masses of parity partners having the same isospin and hypercharge appear in one block. Different isomultiplets are separated by double lines. In the lowest big block the matrix elements of the mixing in the $\eta-\eta'$ and $\sigma-f_0$ sectors are given in the $x-y$ base.

The requirement of the agreement of $L\sigma M$ with ChPT is fulfilled when the (m_π, m_K)-dependence of the couplings x, y, f_2, g, M^2 is determined with help of Eqs.(5,6). Before the procedure just described was first proposed in Ref.[4] $m_\pi - m_K$ mass tuning was taken into account only in (10). The quality of this not fully complete parametrisation (e.g. only the combination M^2 of f_1 and μ_0^2 is determined at this stage) can be assessed by comparing the separate the $m_\pi - m_K$ mass dependence of m_η and $m_{\eta'}$ obtained in the present parametrisation of $L\sigma M$ with the predictions of ChPT. In Fig. 1 the comparison is done for $m_\pi = 0$ and the agreement is fairly good up to $m_K \approx 800$MeV.

The combination M^2 of f_1 and μ_0^2 can be split up only by making use of the expression of the admixed scalars, therefore the use of one characteristics of the mixed scalar spectra is unavoidable [22].

For the separate determination of μ_0^2 and f_1 we have fixed first the mass of the f_0 meson. This meson mass is identified with the heavier eigenvalue calculated in the mixing scalar subspace. Since there is the possibility of mixing between the normal and the superbound multiplets we have chosen as a second possibility also fixing the trace $m_\sigma^2 + m_{f_0}^2 = m_{\sigma_{xx}}^2 + m_{\sigma_{yy}}^2$ (see Table 1) in the squared mass subspace of admixed scalars. In order to test the sensitivity of the results to the scalar masses (which are not very accurately known) we have performed the calculations for both alternatives with several numerical values.

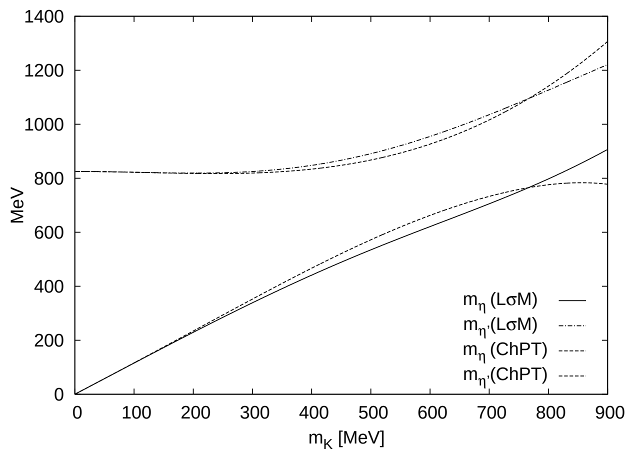

Figure 1. The tree–level kaon mass dependence of m_η and $m_{\eta'}$ for $m_\pi = 0$. The labels refer to the results of ChPT and the predictions of linear sigma model (LσM), respectively.

3. Quasiparticle thermodynamics and phase diagram

The renormalised set of the equations of state and of the selfconsistent pion propagator determine the temperature dependence of the vacuum expectation values x and y. The scheme of the perturbation theory applied here agrees with the Optimal Perturbation Theory of Chiku and Hatsuda [23], which was renormalised using the approach of [24].

The tree level mass of π involves now the thermal mass parameter:

$$m_\pi^2 = M^2(T) + 2(2f_1 + f_2)x^2 + 4f_1 y^2 + 2gy, \tag{11}$$

and all other meson masses to be used in the tadpole integrals below agree with the formulas appearing in Table 1 with the replacement $-\mu_0^2 \to M^2(T)$. If all quantum corrections are condensed into $M^2(T)$, then the tree–level masses of other mesons are expressible through the mass of the pion. One might expect that the pion has the lowest mass and therefore for $M^2(T) > 0$ these squared masses are all positive, which is not the case when $-\mu_0^2 < 0$ is used in the propagators. We define a physical region of x and y where all tree-level mass squares are positive, and thus the one-loop contribution of the meson fluctuations to EoS is real. This region is most severely restricted by the mass of σ, which strongly decreases near the phase transition. We will restrict our attention to the solution of the EoS's in the physical region.

For the determination of the thermal mass we use the Schwinger–Dyson equation for the inverse pion propagator at zero external momentum. At one-loop it receives the contribution $\Pi(M(T), p = 0)$, which is the self-energy function of the pion at zero external momentum, plus the counterterm contribution $-\mu_0^2 - M^2(T)$. We apply the principle of minimal sensitivity (PMS) [23], that is we require that the pion mass be given by its tree-level expression:

$$\Pi(M(T), p = 0) - \mu_0^2 - M^2(T) = 0. \tag{12}$$

$\Pi(M(T), p)$ itself is a linear combination of the tadpole and bubble diagrams (the latter not included in the treatment of [12]), with coefficients derived with help of the 4-point and 3-point couplings among mass eigenvalue fields.

The self-energy can be represented as a linear combination of tadpole integrals, which gives when substituted into Eq. (12):

$$0 = -M^2(T) - \mu_0^2 + \sum_{i=\pi,\,K,\,\eta,\eta'}^{\alpha=\sigma,\,\pi} c^{\pi}_{\alpha_i} I(m_{\alpha_i}(T), T)\,. \tag{13}$$

Here $c^{\pi}_{\alpha_i}$ are the weights of the renormalised tadpole contributions evaluated with different mass eigenstate mesons $\alpha_i = \sigma_i, \pi_i$. The integrals over the corresponding propagators are evaluated with effective tree-level masses where $M^2(T)$ replaces $-\mu_0^2$. In this way (13) is actually a gap equation which determines the thermal mass parameter, $M^2(T)$. With help of Eq. (11) this equation can be also understood as a gap equation for the pion mass (the pion mass is present also in the expressions of $I(m_{\alpha_i}, T)$ through m_{α_i}!):

$$m_\pi^2 = -\mu_0^2 + 2(2f_1 + f_2)x^2 + 4f_1 y^2 + 2gy + \sum_{i=\pi,\,K,\,\eta,\eta'}^{\alpha=\sigma,\,\pi} c^{\pi}_{\alpha_i} I(m_{\alpha_i}(T), T)\,. \tag{14}$$

For the determination of the order parameters x, y we solved the two renormalised equations of state using the solution of the gap equation in the propagator masses:

$$-\epsilon_x - \mu_0^2 x + 2gxy + 4f_1 xy^2 + 2(2f_1 + f_2)x^3 + \sum_{i=\pi,\,K,\,\eta,\eta'}^{\alpha=\sigma,\pi} J_i t^x_{\alpha_i} I(m_{\alpha_i}(T), T) = 0\,, \tag{15}$$

$$-\epsilon_y - \mu_0^2 y + gx^2 + 4f_1 x^2 y + 4(f_1 + f_2)y^3 + \sum_{i=\pi,\,K,\,\eta,\eta'}^{\alpha=\sigma,\pi} J_i t^y_{\alpha_i} I(m_{\alpha_i}(T), T) = 0\,, \tag{16}$$

The quantities $t^x_{\alpha_i}$ and $t^y_{\alpha_i}$ give the corresponding weights. J_i is the isospin multiplicity factor: $J_\pi = 3$, $J_K = 4$, and $J_{\eta,\eta'} = 1$. The coefficients c and t were listed in [4]. One finds $c^\pi_{\alpha_i} = J_i t^x_{\alpha_i}/x$, which ensures that the solution for the mass of the pion obeys Goldstone's theorem.

The solution of Eqs. (14), (15), (16) for given m_π, m_K allows to establish the nature of the temperature driven transitions. First order transitions are signalled by multivaluedness in the temperature evolution of both the non-strange and strange condensates. For large values of the kaon mass, we claim that the phase transition is driven by the variation of the non-strange condensate, since each of the multiple solutions of the strange condensate are very close to each other, and all stay at high values. Subsequent decrease of the strange condensate at higher temperature displays only a crossover.

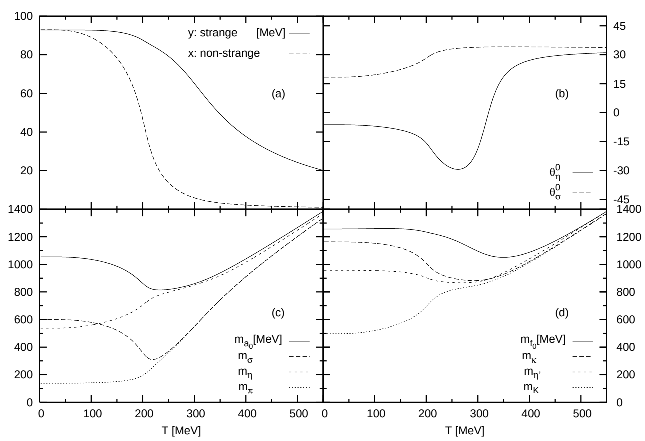

Figure 2. The temperature dependence in the physical point of: (a) the non-strange (x) and strange (y) condensates; (b) the pseudoscalar (θ_η) and scalar (θ_σ) mixing angles (in the (0-8) basis); (c) the mass of the chiral partners (π, σ) and (a_0, η); (d) the mass of f_0, κ, η', K mesons.

In the physical point the system exhibits clear crossover as can be seen in Fig.2. Let us discuss next what happens near the $m_\pi = m_K$ diagonal. In Fig.3 the gradual deformation of the boundary line is shown when four different (m_π, m_K)-independent conditions (listed on the figure) are imposed on the spectra of the admixed scalars. The boundary reaches the diagonal in the range $m_{crit} \approx 20$ MeV independently of the condition imposed. When moving away from the diagonal for $m_K > m_\pi$ the larger is the scalar mass scale the farther the boundary goes away from the $m_\pi = 0$

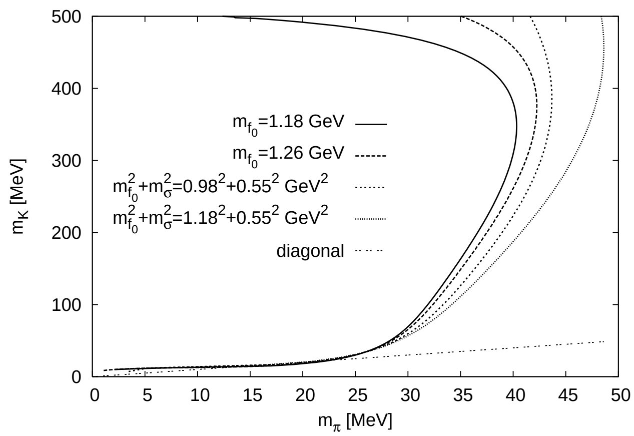

Figure 3. Variation of the boundary of first order transitions near and above the $m_\pi = m_K$ diagonal (note the rather different units on the two axes!)

axis. Therefore with this method one cannot make any definite statement on the location of the tricritical point on the m_K-axis defined as the point where the transition changes from a discontinuous nature into a continuous one.

In conclusion of this study we find that the assumption of the $m_\pi - m_K$ independence of the mass of the heavier isoscalar-scalar member of the nonet, suggested by Gribov's confinement picture leads to a rather unique conclusion, that the $m_\pi = m_K$ diagonal crosses the critical curve of the chiral phase transitions of the 3-flavor QCD at a rather low value: 15 MeV $\leq$ $\mathrm{m_{crit}} \leq$ 25 MeV.

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