

Detailed description of accelerating, simple solutions of relativistic perfect fluid hydrodynamics

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Abstract

In this paper we describe in full details a new family of recently found exact solutions of relativistic, perfect fluid dynamics. With an ansatz, which generalizes the well-known Hwa-Bjorken solution, we obtain a wide class of new exact, explicit and simple solutions, which have a remarkable advantage as compared to presently known exact and explicit solutions: they do not lack acceleration. They can be utilized for the description of the evolution of the matter created in high energy heavy ion collisions. Because these solutions are accelerating, they provide a more realistic picture than the well-known Hwa-Bjorken solution, and give more insight into the dynamics of the matter. We exploit this by giving an advanced simple estimation of the initial energy density of the produced matter in high energy collisions, which takes acceleration effects (i.e. the work done by the pressure and the modified change of the volume elements) into account. We also give an advanced estimation of the life-time of the reaction. Our new solutions can also be used to test numerical hydrodynamical codes reliably. In the end, we also give an exact, 1+1 dimensional, relativistic hydrodynamical solution, where the initial pressure and velocity profile is arbitrary, and we show that this general solution is stable for perturbations.

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INTRODUCTION

Hydrodynamics is in a unique position among the different branches of theoretical physics: it relies only on the assumption of local charge and energy-momentum conservation, and on the concept of local thermal equilibrium. The hydrodynamical equations do not have an internal scale, in contrast to other theories. Consequently, among the possible applications of hydrodynamics we find the largest known physical systems, systems of intermediate size and the smallest and shortest events of nature as well. For example, it is well known, that the evolution of the whole Universe (the Hubble flow) can be treated in a hydrodynamical manner. The applications in the ordinary sizes and temporal extents are known since the birth of hydrodynamics. Considering the smallest temporal and spatial scales: as a surprising new result, it has been understood recently, that the new form of nuclear matter created in high energy heavy ion collisions can be considered as an (almost) perfect fluid — because its properties (like the single-particle spectra, elliptic flow data and two-particle correlation functions) are best explained in terms of models based on perfect fluid hydrodynamics. From this point of view, we can say that hydrodynamics is simple and efficient, and it has applications from the very small to the very large scales.

The hydrodynamical equations, however, are highly nonlinear even in the nonrelativistic case, this is the reason for the appearance of chaos, instability and other beautiful flow patterns in realistic flows. Recent interest in hydrodynamics emerged mostly from the challenge to explain the properties of the nuclear matter in high energy collisions, it is clear that in this context one has to deal with not only hydrodynamics, but relativistic hydrodynamics. Relativity makes the case even worse: relativistic terms (such as speed-addition and proper-time factors) make the equations much more complicated as compared to the nonrelativistic case. From this point of view, hydrodynamics is a horrendously complicated area of physics, which can be seen also from the intensity of recent efforts — both analytical and numerical — to find relevant solutions for the relativistic hydrodynamical equations.

There are only a few exact solutions for these equations. One (and historically the first) is the famous Landau-Khalatnikov solution discovered more than 50 years ago [1–3]. This is a 1+1 dimensional solution, and has realistic properties: it describes a 1+1 dimensional expansion, and does not lack acceleration. Landau was able to calculate approximately an important observable, the rapidity distribution of the flowing particles from this solution,

and it was found to be an (approximately) Gaussian distribution. Recent measurements at the RHIC particle accelerator (BNL, USA) suggest that this prediction is not at big variance with the observations made even at the highest presently attainable collision energies. However, considering its formulation, the Landau-Khalatnikov solution is a very unpleasant one: it is an implicit solution, the independent variables (the space and and the time coordinate) are given by extremely complicated integral formulas, involving the fluid rapidity and temperature.

Another renowned relativistic hydrodynamical solution is the Hwa-Bjorken solution [4, 5], which is a simple, explicit and exact, but accelerationless solution. This solution (in its original form, where no conserved baryonic charges are present) is boost-invariant, and from this it follows that the rapidity distribution should be constant. This prediction would be valid for the limiting case of infinitely high center of mass collision energy. Although such a flat „plateaux” (a narrow domain of constant rapidity distribution around mid-rapidity) is indeed seen in recent rapidity measurements at RHIC, the observed distribution is far from constant, so Bjorken’s approximation fails to describe the data. However, the solution allowed Bjorken to obtain a simple estimate of the initial energy density reached in high energy reactions from final state hadronic observables. (This is why mostly only Bjorken is credited with this solution, though Hwa discovered it independently almost 10 years before.) The Bjorken-estimation is widely used, but it is well known, that the acceleration of the matter influences the energy density estimation. In order to obtain more realistic estimation, one should use explicit and exact, but accelerating solutions of relativistic fluid dynamics.

In this paper we present a detailed description of such new simple, exact and explicit solutions of relativistic fluid dynamics. The essential features of these solutions were first presented in two short Letters in refs. [6, 7]. To our best knowledge, these solutions were the first explicit solutions with relativistic acceleration, following the implicit and accelerating Landau-Khalatnikov solution. (There are other interesting solutions to relativistic hydrodynamics, see e.g. refs. [8–11], but these solutions are all accelerationless.)

Apart from the purely theoretical point of view of finding new exact solutions for nonlinear differential equations, our new solutions are important due to other reasons, too. First, they provide an advanced estimate of the initial energy density and life-time of high energy reactions, we focus on these applications in this paper. Second, they can be applied to test numerical solutions of relativistic hydrodynamics: no finite, accelerating relativistic

solutions were available before for 1+3 dimensional tests.

The organization of the paper is as follows. We discuss the equations of relativistic hydrodynamics, then present the newly found solutions. We then calculate the rapidity distribution, and use it to estimate the initial energy density and the life-time. In the four Appendices, details of calculations are summarized. In Appendix A we derive the solutions themselves, and in Appendix B we prove their uniqueness (in some sense discussed there). Appendix C presents the calculation of the rapidity distribution, and finally, in Appendix D we present a general solution and investigate the stability of the solutions for a specific choice of the equation of state.

NOTATION AND BASIC EQUATIONS

In this section we specify the notations used throughout the paper, and derive the equations of relativistic hydrodynamics.

Notation

We use three-dimensional as well as four-dimensional notations depending on which one is more convenient. In four-dimensional notation the independent variables are the components of the x^μ coordinate four-vector: $x^\mu = (t, \mathbf{r})$, with t being the time and \mathbf{r} the coordinate three-vector, $\mathbf{r} = (r_x, r_y, r_z)$. The metric tensor is denoted by $g_{\mu\nu}$, we use the $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ sign convention. The four-velocity field is denoted by u^μ , normalized to unity, $u^\mu u_\mu = 1$, that is, we treat the speed of light as 1. The three-velocity \mathbf{v} is defined as $u^\mu = \gamma(1, \mathbf{v})$, with $\gamma = (1 - v^2)^{-1/2}$. We denote the so-called comoving derivative by $\frac{d}{dt}$: $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}\nabla$.

The thermodynamical quantities are the following: ε is the energy density (including rest energy contribution), p is the pressure, $w = \varepsilon + p$ is the enthalpy density, T is the temperature, σ is the entropy density. When there are some (conserved or non-conserved) charges present, we denote them by n_i , and the corresponding chemical potentials by μ_i . All the above densities are defined in the local rest frame, so these densities are scalar quantities, while for example the entropy density in the laboratory frame is defined as the 0-th component of the entropy current vector, σu^0 .

The equations of relativistic hydrodynamics

Assuming perfect fluid (that is, neglecting heat conductivity, bulk and shear viscosity) the equations of relativistic hydrodynamics are obtained by Landau's heuristic argumentation: we guess the form of the energy-momentum tensor $T_{\mu\nu}$ in the local rest frame: $T_{\mu\nu} = \text{diag}(\varepsilon, p, p, p)$, and from this we have in arbitrary frame

$$T_{\mu\nu} = wu_\mu u_\nu - pg_{\mu\nu}. \quad (1)$$

The equations then follow from the energy-momentum conservation law $\partial_\nu T^{\mu\nu} = 0$. Substituting eq. (1) and projecting orthogonal and parallel to u^μ , we obtain the relativistic Euler equation and the energy conservation equation as

$$wu^\nu \partial_\nu u^\mu = (g^{\mu\rho} - u^\mu u^\rho) \partial_\rho p, \quad (2)$$

$$w\partial_\mu u^\mu = -u^\mu \partial_\mu \varepsilon. \quad (3)$$

The general form of the charge conservation equations (for one type of charge) is as follows:

$$\sum_i \mu_i \partial_\mu (n_i u^\mu) = 0, \quad (4)$$

this is valid for the case of non-conserved charges, as their chemical potentials vanish, for one conserved charge n with non-vanishing chemical potential, as the conservation law is $\partial_\mu (nu^\mu) = 0$, and also for various mixtures. (For instance, in case of baryonic or electric charge, particles and antiparticles carry opposite charges, and they chemical potentials are the same but of opposite sign, so eq. (4) is valid also for this case.)

For latter usage we write down these equations also in a three-dimensional notation. The relativistic Euler equation, the energy conservation equation and the continuity equation (for one conserved charge) are

$$\frac{w}{1-v^2} \frac{d\mathbf{v}}{dt} = - \left(\nabla p + \mathbf{v} \frac{\partial p}{\partial t} \right), \quad (5)$$

$$\frac{1}{w} \frac{d\varepsilon}{dt} = - (\nabla \mathbf{v}) - \frac{1}{1-v^2} \frac{d}{dt} \frac{v^2}{2}, \quad (6)$$

$$\frac{d}{dt} \ln \frac{n}{\sqrt{1-v^2}} = - (\nabla \mathbf{v}). \quad (7)$$

The thermodynamical quantities obey general rules, which follow directly from thermody-

namics:

$$d\varepsilon = Td\sigma + \sum_i \mu_i dn_i, \quad (8)$$

$$w = T\sigma + \sum_i \mu_i n_i, \quad (9)$$

$$dp = \sigma dT + \sum_i n_i d\mu_i. \quad (10)$$

The last equation is a consequence of the first two ones. From eqs. (8) and (10) and eq. (3) one can derive the entropy conservation equation:

$$\partial_\mu (\sigma u^\mu) = 0. \quad (11)$$

This is a consequence of the special choice of the energy-momentum tensor. The perfectness of the fluid can be defined alternatively by requiring that eq. (11) holds.

In order to get a closed set of equations, we have to specify the Equation of State (EoS) of the flowing matter. In the following we shall investigate a case, which resembles to that of ideal gas, that is

$$\varepsilon = \kappa p. \quad (12)$$

The κ constant is 3 for a three-dimensional, ultra-relativistic ideal gas, but now we retain the more general possibility of arbitrary κ . With the choice of eq. (12) the charge conservation equation decouples from the Euler and the energy conservation equations. That is, in order to find new solutions, one has to deal with \mathbf{v} and p only, because eqs. (5) and (6) contains only these variables, and charge densities occur in the calculation only after one found proper solutions for $p(t, \mathbf{r})$ and $\mathbf{v}(t, \mathbf{r})$.

An important property of this specific EoS is, that if one finds a solution with this EoS, then these solutions can be generalized for a bag-model type of EoS in a straight-forward manner, as the following replacements leave the hydrodynamical equations invariant:

$$\varepsilon \rightarrow \varepsilon + B, \quad (13)$$

$$p \rightarrow p - B. \quad (14)$$

The value of the bag constant, B cancels from eqs. (5) and (6), that is, one can introduce an arbitrary value of the bag constant. This does not change the time evolution of the entropy density or the flow field as long as it is a constant, independent of position and time. Because this possibility is simple, we do not explicitly write down the bag constant in the following calculations, but emphasize, that this can be done without difficulty.

Rindler coordinates

We discuss below exact solutions of relativistic perfect fluid hydrodynamics in 1+1 dimensions and spherical solutions in 1+d dimensions as well (the 1+1 dimensional case is a special case of this latter one). The number of spatial dimensions is denoted by d . The notation r stands for the r_z spatial coordinate in 1+1 dimensions, and for the radial coordinate in 1+d dimensions. All quantities depend only on t and r , and the velocity field is radial. We use the well-known Rindler coordinates τ and η as independent variables, which naturally fit to the Hwa-Bjorken solution.

These coordinates have different definitions for $t > r$ (we will refer to this case as „inside the forward lightcone”) and for $t < r$ (that is, outside the lightcone). We have

$$t = \tau \cosh \eta, \quad r = \tau \sinh \eta \quad (15)$$

inside the lightcone and

$$t = \tau \sinh \eta, \quad r = \tau \cosh \eta \quad (16)$$

outside the lightcone.

In the next section we use these coordinates in order to find new solutions of the hydrodynamical equations. We investigate an ansatz for the flow and pressure field, which leads to a new and interesting class of exact, explicit and simple solutions. Among these solutions we find ones that overcome the shortcoming of almost all presently known exact solutions: the lack of acceleration.

DERIVING NEW SOLUTIONS

We parametrize the velocity with the $\Omega(\tau, \eta)$ rapidity as $v = \tanh \Omega(\tau, \eta)$. In this section we take $t > r$, that is we are inside the forward lightcone. The Rindler variables τ and η are defined by eq. (15). The Bjorken-solution is easily written down in terms of these variables saying $\Omega = \eta$, and $\frac{\sigma}{\sigma_0} = \frac{\tau_0}{\tau}$. Our new ansatz is the following generalization of the Bjorken flow:

$$\Omega = \lambda \eta, \quad \lambda \in \mathbb{R}. \quad (17)$$

Here λ is an (up to now) arbitrary constant, a parameter of the solution. Substituting (17) into the hydrodynamical equations (the Euler and energy conservation equations with the

$\varepsilon = \kappa p$ EoS) after some calculations detailed in Appendix A we obtain that p must have the following form:

$$p = p_0 \left(\frac{\tau_0}{\tau} \right)^{-K(\kappa+1)} \frac{1}{\cosh^{\frac{K+\lambda}{\lambda-1}(\kappa+1)}((\lambda-1)\eta)}. \quad (18)$$

with K being a constant. The value of λ and K are constrained by the equation

$$\begin{aligned} & \kappa K + (d-1) \frac{\sinh \lambda \eta}{\sinh \eta} \cosh((\lambda-1)\eta) + \\ & + \lambda (\cosh^2((\lambda-1)\eta) - \kappa \sinh^2((\lambda-1)\eta)) = 0. \end{aligned} \quad (19)$$

The detailed calculations which lead to these equations are found in Appendix A.

Let us emphasize, that eq. (19) is derived from the hydrodynamical equations including the equations of state within lightcone, $|t| > |r|$, using Rindler coordinates. Outside the lightcone, for $|t| < |r|$, this equation is modified to eq. (69), that governs the “external” hydrodynamical solutions.

In the next subsection we summarize and investigate the possible solutions.

NEW EXACT SOLUTIONS

There are 4 different sets of the parameters λ , d , κ and K which solve eq. (19), as discussed below. A fifth solution in this class is the $\lambda = 1$ particular case, which is known as the Hwa-Bjorken solution in 1+1 dimensions, and whose multi-dimensional generalizations, often called as Buda-Lund type of relativistic solutions, were discovered in the past years [9, 10].

In Appendix B, we present the proof that these five solutions are the only possible non-trivial solutions in this class (where $\Omega = \lambda\eta$.) In Appendix B, we also prove the uniqueness of these solutions in a broader sense: If one allows λ to be not only a constant but a proper-time dependent function (i.e. if $\Omega = \lambda(\tau)\eta$), no other exact solutions exist, except the five solutions a.) - e.) listed in Table I. The properties of these solutions are detailed below.

Flow profile and pressure

In all cases, the velocity field and the pressure is expressed as

$$v = \tanh \lambda \eta, \quad (20)$$

$$p = p_0 \left(\frac{\tau_0}{\tau} \right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh \frac{\eta}{2} \right)^{-(d-1)\phi_\lambda}. \quad (21)$$

Case	λ	d	κ	ϕ_λ
a.)	2	$\in \mathbb{R}$	d	0
b.)	$\frac{1}{2}$	$\in \mathbb{R}$	1	$\frac{\kappa+1}{\kappa}$
c.)	$\frac{3}{2}$	$\in \mathbb{R}$	$\frac{4d-1}{3}$	$\frac{\kappa+1}{\kappa}$
d.)	1	$\in \mathbb{R}$	$\in \mathbb{R}$	0
e.)	$\in \mathbb{R}$	1	1	0

TABLE I: The new family of solutions.

Table I shows the possible cases: every row represents one solution. In what follows, we discuss the various cases separately.

- Case *a.*): In this case, $\lambda = 2$, and the solution is obtained in arbitrary number of spatial dimensions, (d is arbitrary), but then κ should be equal to d . For instance, in three dimensions κ should be 3, which is the EoS of an ultrarelativistic ideal gas or a photon gas. The corresponding flow profile has a non-vanishing relativistic acceleration: $u_\nu \partial^\nu u^\mu \neq 0$. We will see that this solution in the same form solves the hydrodynamical equations outside the lightcone, too. If we write down the velocity field in Minkowskian coordinates, it has the form both for $t < r$ and $t > r$

$$v = \frac{2tr}{t^2 + r^2}, \quad (22)$$

and we can solve the equation of motion for an individual particle, that is, we can determine the flow trajectories $r(t)$. For a fluid element that is at position r_0 at t_0 time-point, we have

$$r(t) = \frac{1}{a_0} (\sqrt{1 + (a_0 t)^2} + 1) \quad (23)$$

with $a_0 = \frac{2r_0}{|r_0^2 - t_0^2|}$. These trajectories have constant a_0 acceleration in the local rest frame. A possible interesting application of this is mentioned briefly later. The trajectories of this solution are shown in Fig. 1. It has a realistic equation of state, corresponding to a massless ultra-relativistic gas in d dimensions.

Note that we have found this solution first, using other methods. This solution can also be found using the criteria that the pressure depends only on proper-time and the solution should work in any d number of dimensions. These criteria select this case and the well known the $\lambda = 1$ Hwa-Bjorken solution, detailed as case *d.*)

- Case *b.*): In this case $\lambda = \frac{1}{2}$, the number of spatial dimensions is arbitrary, but $\kappa = 1$. The flow is accelerating. This case together with case *c.*) was found first in $d = 3$ dimensions by T. S. Biró [12], and we generalized it for arbitrary d . An important point is that this case and case *d.*) are the only ones where the pressure field can have explicit η -dependence, and p can be finite in η (actually, it is finite if and only if $d \neq 1$).
- Case *c.*): This case is characterized with $\lambda = \frac{3}{2}$, d is arbitrary, but $\kappa = \frac{4d-1}{3}$, that is, for example for $d = 1$ we get $\kappa = 1$, and for $d = 3$ it gives $\kappa = \frac{11}{3}$. This case also was found in $d = 3$ by T. S. Biró first, and we have generalized it. The pressure field is also finite in η for $d \neq 1$.
- Case *d.*): In this case $\lambda = 1$, d and κ can be arbitrary. For $d = 1$ this case is known as the Hwa-Bjorken solution, and for $d > 1$ these solutions were found in refs. [9, 10]. The flow profile in the $d > 1$ case is called Hubble flow. Thus these solutions were already known inside the light cone, but we quoted them for the sake of completeness. They are also the basis of new solutions, the the extensions of these solutions to the region outside the lightcone. These solutions are discussed in the next subsection. Inside the light-cone, this flow is not accelerating, $u_\nu \partial^\nu u^\mu = 0$.
- Case *e.*): In this case, the value of the parameter λ is arbitrary, but d and κ must be equal to 1. The flow is accelerating, $u_\nu \partial^\nu u^\mu \neq 0$ if $\lambda \neq 1$. The velocity field is pretty general, but the EoS is very particular. Nevertheless, this solution can be considered as a „smooth extrapolation” between the previous cases, although this kind of generalization — the arbitrary λ case, which can be expected knowing the previous cases — works only with this special choice of d and κ . In the forthcoming we shall use this solution to calculate the rapidity distribution. We show in Appendix C, that the width of the observable rapidity distribution is controlled by the parameter λ . Hence in principle, the value of the parameter λ of the hydrodynamical solution can be obtained from measurements.

Outside the lightcone

The hydrodynamical equations can also be rewritten in Rindler-coordinates outside the lightcone (that is, for $|t| < |r|$). We investigate here how the solutions presented in the

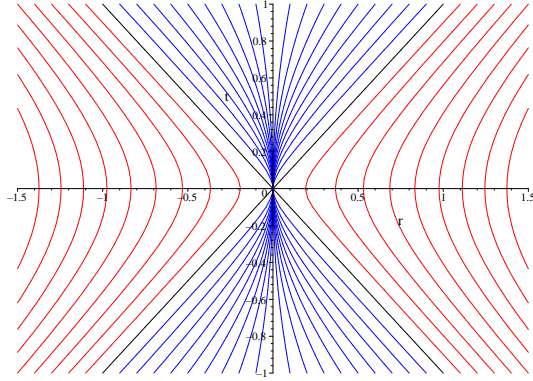


FIG. 1: (Color online) Fluid trajectories of the $\lambda = 2$ solution.

previous subsection can be extended to this domain. The equations are found in Appendix A, here we only go through the possibilities. The statements below can be directly checked if one substitutes these expressions into eq. (69) in Appendix A, which is derived from the hydrodynamical equations in the $|t| < |r|$ region.

- Case *a.*): This solution can be extended outside the lightcone in a straightforward manner. The properties are the same: it works in arbitrary d with $\kappa = d$, the velocity field has the same expression in Minkowskian coordinates as inside the lightcone (see eq. (22)), hence it is also uniformly accelerating. Fig. 1 shows the trajectories outside the lightcone, too. These trajectories are given by the same form, eq. (23), outside as well as inside the lightcone.
- Case *b.*) and *c.*): These solutions can be extended to $t < r$ only in the $d = 1$ case, but these extensions are special cases of the extension of case *e.*) discussed below. These extensions do not depend explicitly on η . (Such a dependence was only present for $t > r$ only if $d \neq 1$.)
- Case *d.*): An interesting new result is that there are solutions outside the lightcone which can be viewed as the extensions of the $\lambda = 1$, or Hwa-Bjorken internal solutions. The velocity and the pressure have the form

$$v = \tanh \eta = \frac{t}{r}, \quad (24)$$

$$p = p_0 \left(\frac{\tau_0}{\tau} \right)^{\kappa+1} \cosh^{-\frac{d-1}{\kappa}(\kappa+1)} \eta. \quad (25)$$

Moreover, this extension is an accelerating flow. The equation of the trajectories is

$$r(t) = \frac{1}{a_0}(\sqrt{1 + (a_0 t)^2} + 1) \quad (26)$$

for $r_0 > t_0$. These trajectories are also of constant a_0 acceleration, as the a.) ($\lambda = 2$) case. However, the expression of a_0 is different from that of case a.), we have now $a_0 = \frac{r_0}{r_0^2 - t_0^2}$.

It is interesting to note at this point the following. As both the $\lambda = 2$ and the $\lambda = 1$ external solutions are uniformly accelerating (in local rest frame), they both contain event horizons. This property of the uniform acceleration was utilized recently by Kharzeev and Tuchin [13] to describe thermalization in heavy ion reactions via the Unruh effect.

- Case e.): The case when λ is arbitrary and $d = \kappa = 1$ can be extended to $t < r$ without difficulties, the outside solution is also an accelerating one. The velocity and the pressure is given by

$$v = \tanh \lambda \eta, \quad (27)$$

$$p = p_0 \left(\frac{\tau_0}{\tau} \right)^{2\lambda}, \quad (28)$$

but remember, that the definition of τ and η are given by eq. (16), which differs from the case of $|t| > |r|$.

Temperatures and densities

The previously presented pressure and velocity fields solve eqs. (5) and (6), the Euler and the energy conservation equations. It is specific to the $\varepsilon = \kappa p$ EoS utilized here, that the continuity equation can be treated separately. But if we investigate the temperature and perhaps other charge densities, the relation between ε and p does not fully determine the EoS. In this subsection we solve the continuity equations for some cases.

Before doing this, we introduce the scaling variable $S(\tau, \eta)$ with the definition that its comoving derivative vanishes, that is (for one-dimensional or spherical flows)

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + v \frac{\partial S}{\partial r} = 0. \quad (29)$$

$S(\tau, \eta)$	$\lambda \neq 1$	$\lambda = 1$
$t > r$	$(\tau_0/\tau)^{\lambda-1} \sinh((\lambda-1)\eta)$	η
$t < r$	$(\tau_0/\tau)^{\lambda-1} \cosh((\lambda-1)\eta)$	τ/τ_0

TABLE II: The $S(\tau, \eta)$ scaling functions inside and outside the light-cone. Note, that the $\lambda = 1$ case has to be handled separately from the other cases.

Every specific flow profile has a corresponding S function, which can be determined by solving eq. (29). Table II contains the expressions of S for the investigated class of solutions. (A remark: S is not uniquely determined by eq. (29), if S is a good scaling function, then any function of S is also a good scaling function. But we shall see, that this ambiguity does not really matter, as it cancels from the observables.)

After this preparation, let us describe the solutions to the entropy (or in general, continuity type of) equations.

First, if there are no charges present at all, then from the (9) Gibbs-Duhem relation we have $T\sigma = (\kappa + 1)p$, and it can be verified, that the solution of the entropy conservation , eq. (11) is

$$\sigma = \sigma_0 \left(\frac{p}{p_0} \right)^{\frac{\kappa}{\kappa+1}} \nu_\sigma(S), \quad (30)$$

$$T = T_0 \left(\frac{p}{p_0} \right)^{\frac{1}{\kappa+1}} \frac{1}{\nu_\sigma(S)} \quad (31)$$

where $p(\tau, \eta)$ is given by eq. (21), and of course $(\kappa + 1)p_0 = T_0\sigma_0$. The function $\nu_\sigma(S) > 0$ can be chosen arbitrarily. This approximation (that is, the baryonic charges are negligibly small) is often used for the description of high energy heavy ion collisions and of the quark-hadron transition of the early Universe.

Second, if we have one conserved charge n , and as an ideal gas EoS we have $p = nT$, then the solution of the conservation equation is very similar:

$$n = n_0 \left(\frac{p}{p_0} \right)^{\frac{\kappa}{\kappa+1}} \nu_n(S), \quad (32)$$

$$T = T_0 \left(\frac{p}{p_0} \right)^{\frac{1}{\kappa+1}} \frac{1}{\nu_n(S)}. \quad (33)$$

The constants are chosen so that $p_0 = n_0 T_0$.

The case of more than one charges can be treated similarly: we have $(\kappa + 1)p = T\sigma + \sum_i n_i \mu_i$, and the solution is

$$\sigma = \sigma_0 \left(\frac{p}{p_0} \right)^{\frac{\kappa}{\kappa+1}} \nu_\sigma(S), \quad (34)$$

$$T = T_0 \left(\frac{p}{p_0} \right)^{\frac{1}{\kappa+1}} \frac{1}{\nu_\sigma(S)}, \quad (35)$$

$$n_i = n_0^{(i)} \left(\frac{p}{p_0} \right)^{\frac{\kappa}{\kappa+1}} \nu_i(S), \quad (36)$$

$$\mu_i = \mu_0^{(i)} \left(\frac{p}{p_0} \right)^{\frac{\kappa}{\kappa+1}} \mathcal{M}_i(S), \quad (37)$$

where the functions and the constants are constrained as $\nu_i(S)\mathcal{M}_i(S) = 1$ and $(\kappa + 1)p_0 = T_0\sigma_0 + \sum_i n_0^{(i)}\mu_0^{(i)}$. These forms are solutions to the hydrodynamical and continuity equations simultaneously.

Let us emphasize, that the scaling functions of the scaling variable S are quite arbitrary, they are only constrained by using a physical normalization, which implies $\nu_i(0) = \mathcal{M}_i(0) = 1$. As we mentioned earlier, any function S' of the scaling variable S is again a good scaling variable, as long as $S'(0) = 0$, because the same solutions are obtained when new scaling functions are chosen, that correspond to the new scaling variable as $\nu'_i(S'(S)) = \nu_i(S)$. Thus it is possible to use the most natural form of the scaling variable S , when describing these solutions.

THE RAPIDITY DISTRIBUTION

If we want to apply our solutions to the description of some physical phenomena, we have to calculate the final state observables from these solutions. We have chosen the rapidity distribution, and in order to be able to fit measured data, we apply here our new solution described as case *e.*) in the previous section. Here the parameter λ (which we can call „acceleration parameter”, because it somehow influences the acceleration of the flow) can be arbitrary, but the dimension is restricted to $d = 1$, and what is more inconvenient, κ is also equal to 1. But the λ parameter can be fitted to measured datasets, and this compensates the drawback of the specific EoS.

The detailed calculation of the rapidity distribution is presented in Appendix C, here we briefly summarize the main steps. We use Boltzmann approximation, and neglect baryonic

charges and chemical potentials, so T and σ are given by eqs. (30) and (31). Although the solution is valid only for the EoS of massless particles, we assume that at the freeze-out particles of rest mass m appear (e.g. pions with $m = 140\text{MeV}$). We have chosen the freeze-out condition as follows: the freeze-out hypersurface is pseudo-orthogonal to the four-velocity field u^μ , and the temperature at $\eta = 0$ reaches a given T_f value. The equation of this hypersurface is

$$\left(\frac{\tau_f}{\tau}\right)^{\lambda-1} \cosh((\lambda-1)\eta) = 1, \quad (38)$$

where τ_f is the proper-time coordinate when the temperature reaches T_f at $\eta = 0$ (see Appendix C for details).

With a saddle-point integration in η , for $\lambda > 1/2$, $m/T_f \gg 1$ and $\nu_\sigma(s) = 1$ we got

$$\frac{dn}{dy} \approx \frac{dn}{dy}\Big|_{y=0} \cosh^{\pm\frac{\alpha}{2}-1} \left(\frac{y}{\alpha}\right) e^{-\frac{m}{T_f}[\cosh^\alpha(\frac{y}{\alpha})-1]}, \quad (39)$$

with $\alpha = \frac{2\lambda-1}{\lambda-1}$. In an actual fit to data, the parameter α can be determined more directly, than λ . From the value of the shape parameter α , the parameter of the acceleration λ can thus be determined from the inverse formula as

$$\lambda = \frac{\alpha-1}{\alpha-2}. \quad (40)$$

This equation also indicates, that the flat rapidity distribution corresponds to $\lambda \rightarrow 1$ and $\alpha \rightarrow \infty$.

In the formula eq. (39) and in the subsequent discussion of this distribution the upper sign always means the true 1 + 1 dimensional case (that is, not only the flow, but the produced particles have only one degree of freedom), while the lower sign is for the case when the 1 + 1 dimensional solution is embedded in the 1+3 dimensional space-time, and the produced particles have transverse degrees of freedom as well. The embedding means that the fluid occupies infinite volume, and it is homogeneous (it has no velocity, no pressure gradient components) in transverse directions.

The Gaussian „width” of the eq. eq. (39) distribution (obtained from the second derivative at $y = 0$) is

$$\Delta y^2 = \frac{\alpha}{m/T_f \mp 1/2 + 1/\alpha}. \quad (41)$$

We see from the expression of Δy^2 that it changes sign if λ (and thus α) varies. It has two roots in λ : $\Delta y^2 = 0$ if $\lambda = 1$, or $\lambda = \frac{1}{2} \left(1 + \frac{T_f}{2m \mp T_f + T_f}\right)$. If $\lambda = 1$ (this is the Hwa-Bjorken

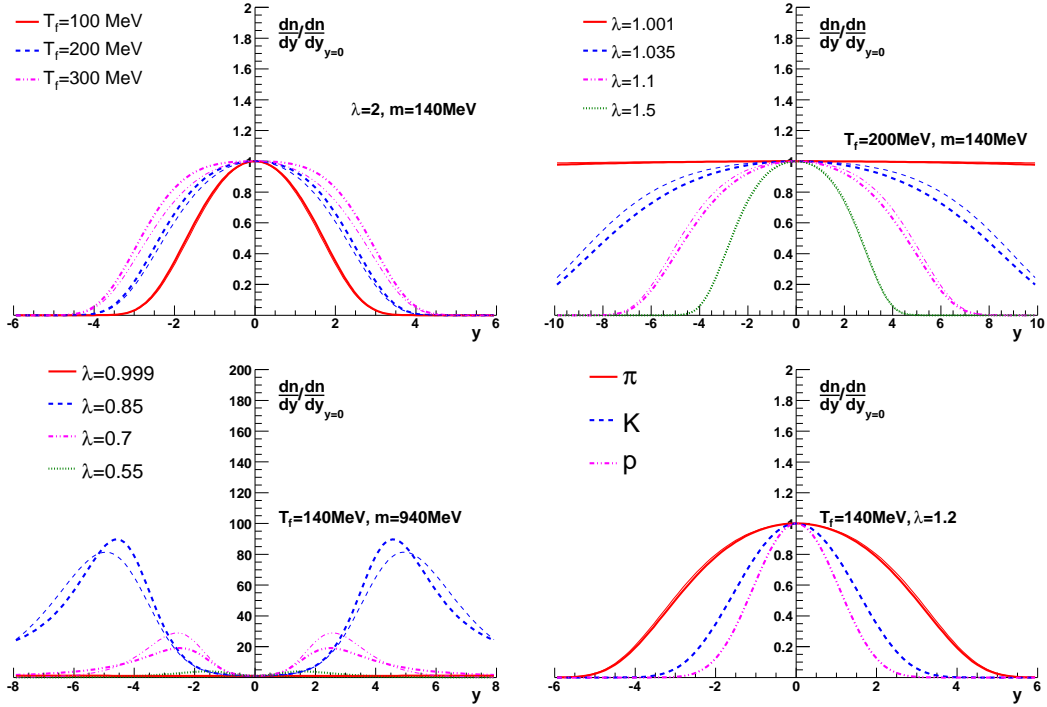


FIG. 2: (Color online) Normalized rapidity distributions from the new solutions in 1+1 dimensions for various λ , T_f and m values. Thick lines show the result of numerical integration, thin lines the analytic approximation from eq. (39). For $\lambda > 1$ and not too big T_f it can be used within about 10 % error.

case), the distribution is entirely flat, and if λ equals the other root, it is not flat but has only higher order non-vanishing derivatives at $y = 0$. In other cases, the sign of Δy^2 determines the behavior of the rapidity distribution (around $y = 0$): it has a minimum at $y = 0$, if $\Delta y^2 < 0$, and it has a maximum (that is, it is approximately Gaussian) if $\Delta y^2 > 0$. The typical cases are plotted in Fig. 2. This figure is just an illustration, the parameter values on this figure are not realistic ones, they were chosen in order to make the behavior of the distribution transparent in the different cases.

APPLICATIONS

In the previous section we have calculated the rapidity distribution from our new solution. We are now able to fit real rapidity measurements with our calculation, and we can extract the λ parameter from the data. (Fig. 2 shows that the analytic approximation for not too

high temperatures has less than 10% error, so it can be used reliably.) We can also see on Fig. 2 that in the $\lambda \rightarrow 1$ limiting case the distribution approximates the Bjorken-like flat one, and the more λ differs from 1, the more the distribution deviates from the flat shape. That is, the acceleration effects make the distribution a finite one, and since our calculation takes acceleration into account, it provides a realistic description of the real distributions, which have finite widths. We present two applications: the improvement of the Bjorken energy density estimation, and an advanced reaction life-time estimation.

Energy density estimation

We follow Bjorken's method [5] and deviate from it at the point when acceleration effects (which are neglected by the Bjorken estimate) become important. Let us focus on a thin transverse piece of the produced matter at mid-rapidity, illustrated by Fig. 2 of ref. [5]. The radius R of this slab is estimated by the radius of the colliding hadrons or nuclei: $R = 1.18A^{1/3}\text{fm}$. Its volume is $dV = (R^2\pi)\tau d\eta$, where τ is the proper time of observation and $d\eta$ is the space-time rapidity element corresponding to the slab. The energy content in this slab is $dE = \langle m_t \rangle dn$, where $\langle m_t \rangle$ is the average transverse mass at mid-rapidity, so similarly to Bjorken, the initial energy density is

$$\varepsilon_0 = \frac{\langle m_t \rangle}{(R^2\pi)\tau_0} \frac{dn}{d\eta_0}. \quad (42)$$

Here τ_0 is the proper-time of thermalization. (Bjorken's estimate was $\tau_0 \approx 1\text{fm}$.) For accelerationless, boost-invariant Hwa-Bjorken flows $\eta_0 = \eta_f = y$, however, for our accelerating solution we have to apply a correction factor of $\frac{\partial y}{\partial \eta_f} \frac{\partial \eta_f}{\partial \eta_0}$. From our $\lambda \in \mathbb{R}$ solutions the shift of the point of maximum emittivity is $\frac{\partial y}{\partial \eta_f} = (2\lambda - 1)$, while the volume element change is $\frac{\partial \eta_f}{\partial \eta_0} = (\tau_f/\tau_0)^{\lambda-1}$. These two factors contain the acceleration effects on the energy density estimation: the first one is the effect of work done by the pressure, and the second one characterizes how acceleration influences the expansion of the initial volume element. For an accelerating flow both factors should be greater than 1, which is indeed the case if $\lambda > 1$.

Thus the initial energy density ε_0 can be accessed by an advanced estimation ε_c as

$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1) \left(\frac{\tau_f}{\tau_0} \right)^{\lambda-1}, \quad \varepsilon_{Bj} = \frac{\langle m_t \rangle}{(R^2\pi)\tau_0} \frac{dn}{dy}. \quad (43)$$

Here ε_{Bj} is the Bjorken estimation, which is recovered if $\frac{dn}{dy}$ is flat (i.e. $\lambda = 1$), but if $\lambda > 1$, ε_0 is *under-estimated* by the Bjorken formula. Fig. 4 shows our fits to BRAHMS dn/dy

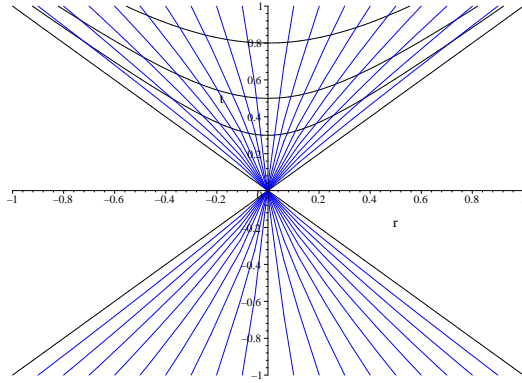


FIG. 3: (Color online) Fluid trajectories of the $\lambda = 1.2$ solution, which fits to BRAHMS $\frac{dn}{dy}$ data. We illustrated the possible hypersurfaces (the hypersurfaces pseudo-orthogonal to the flow trajectories), among these ones there is the proper freeze-out hypersurface, selected by the criterion $T(\eta = 0) = T_f$.

data [14]. From these fits we have found $\lambda = 1.18 \pm 0.01$. On Fig. 3 we plotted the fluid trajectories and illustrated the ensemble of possible freeze-out hypersurfaces, as given by eq. (38) inside the lightcone for this specific λ value. For illustration we also calculated the rapidity distribution for $\lambda = 1.18$: the bottom panel on Fig. 2 shows it with this λ value for pions, kaons and protons.

Using the Bjorken estimate of $\varepsilon_{Bj} = 5 \text{ GeV}/\text{fm}^3$ as given in ref. [15], and $\tau_f/\tau_0 = 8 \pm 2 \text{ fm}/c$, we find an initial energy density of $\varepsilon_c = (2.0 \pm 0.1)\varepsilon_{Bj} = 10.0 \pm 0.5 \text{ GeV}/\text{fm}^3$. If the evolution deviates from a $1+1$ dimensional perfect flow, then our estimation is only a lower limit for the initial energy density.

Life-time estimation

For a Hwa-Bjorken type of accelerationless, coasting longitudinal flow, Sinyukov and Makhlin [16] determined the longitudinal length of homogeneity as

$$R_{long} = \sqrt{\frac{T_f}{m_t}} \tau_{Bj}. \quad (44)$$

Here m_t is the transverse mass and τ_{Bj} is the (Bjorken) freeze-out time. This result provides a means to determine the life-time of the reaction: it can be simply identified with τ_{Bj} . However, if the flow is accelerating, the estimated origin of the trajectories is shifted back in

α	7.4 ± 0.13
$\frac{dn}{dy} _{y=0}$	294 ± 1
χ^2/NDF	30.6/14
CL	0.6 %
T_f (MeV)	200 (fixed)
m (MeV)	140 (fixed)
λ	1.18 ± 0.01 (derived)

TABLE III: The parameters of the best fit with eq. (39) (1+3 dimensional case, lower signs) to dn/dy data of negative pions, as measured by the BRAHMS collaboration [14] in central (0-5%) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The free fit parameters were α and $\frac{dn}{dy}|_{y=0}$. The fit range was $-3 < y < 3$. The confidence level of the fit, CL was calculated from χ^2/NDF . The temperature parameter T_f was fixed to 200 MeV, corresponding to the slope parameter of the single particle spectra, as in the fitted 1+3 dimensional case a 1+1 dimensional exact hydro solution was embedded to a 1+3 dimensional space. For the pion mass, a fixed 140 MeV value was used. The parameter of the acceleration, λ was calculated from the fitted parameter α using eq. (40).

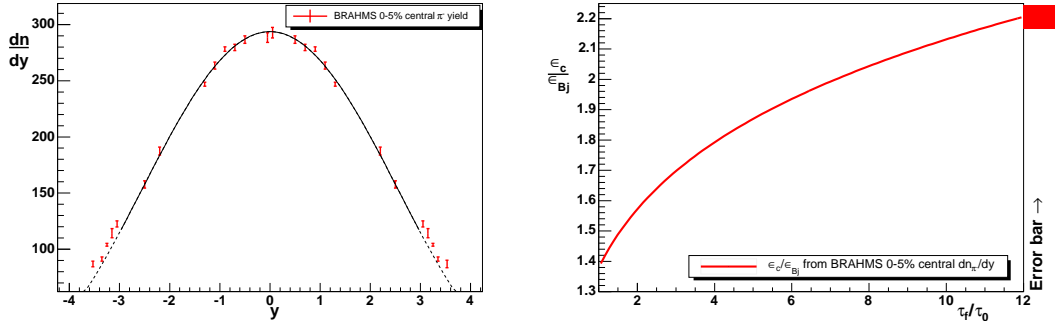


FIG. 4: (Color online) Panel (a): dn/dy data of negative pions, as measured by the BRAHMS collaboration [14] in central (0-5%) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, fitted with eq. (39) (1+3 dimensional case, lower signs). The fit range was $-3 < y < 3$, to exclude target and projectile rapidity region, CL = 0.6 %. The parameters of the fit are summarized in Table III. Panel (b): ϵ_c/ϵ_{Bj} ratio as a function of τ_f/τ_0 .

proper-time, so the life-time of the reaction is under-estimated by τ_{Bj} . (This was pointed out also in refs. [17–20].) From our solutions we have (for a broad but finite rapidity distribution, where the saddle-point approximation is valid):

$$R_{long} = \sqrt{\frac{T_f}{m_t}} \frac{\tau_c}{\lambda} \quad \Rightarrow \quad \tau_c = \lambda \tau_{Bj}. \quad (45)$$

So the new estimation of the life-time, τ_c contains a λ multiplication factor. As mentioned in the previous subsection, BRAHMS rapidity distributions in Fig. 4 yield $\lambda = 1.18 \pm 0.01$, so they imply a 18 ± 1 % increase in the estimated life-time of the reaction.

SUMMARY

In this paper, we presented new simple, explicit analytic solutions to the relativistic hydrodynamical equations. These new solutions are characterized by a non-vanishing relativistic acceleration, which is a significant development compared to the already known solutions. Many of these new solutions work not only inside the lightcone, but outside of it as well. As a by-product, we have demonstrated that the well-known Hwa-Bjorken solution also has such an extension, and this extension turns out to have uniformly accelerating trajectories.

We have calculated the rapidity distribution of the particles from our new solutions numerically, and presented a simple and reliable analytic approximation as well. These distributions have finite widths, hence they can be used to fit real experimental rapidity distribution data. We have fitted BRAHMS data, and obtained a value of $\lambda = 1.18 \pm 0.01$ for the acceleration parameter of the solution.

The famous Bjorken estimate of the initial energy density could be refined, taking acceleration effects into account. Using the Bjorken estimate given in the BRAHMS White Paper [15] ($\varepsilon_{Bj} = 5 \text{ GeV/fm}^3$) and $\tau_f/\tau_0 = 8 \pm 2 \text{ fm/c}$, we find an initial energy density of $\varepsilon_c = (2.0 \pm 0.1)\varepsilon_{Bj} = 10.0 \pm 0.5 \text{ GeV/fm}^3$, which is a significant increase. We have also given an improved estimation for the life-time of the reaction: a 18 ± 1 % increase is due to the presence of acceleration, based on our model of the initial accelerating period.

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Note that during the process of finalizing this manuscript, we have encountered two interesting, new family of relativistic hydrodynamical solutions, in ref. [23] and also in ref. [24], both of which look to be very interesting and deserve more detailed, further investigations.

APPENDIX A: HYDRODYNAMICAL EQUATIONS IN RINDLER COORDINATES

In this Appendix we present the derivation of eqs. (18) and (19) in more detail. First we rewrite the hydrodynamical equations in terms of the Rindler variables, then investigate the assumption for the velocity field made in eq. (17). In this Appendix, as in the body of the paper, Ω stands for the rapidity of the flow, $v = \tanh \Omega$, and here we use the notation $\frac{d}{dr} = \frac{\partial}{\partial r} + v \frac{\partial}{\partial t}$, this appears on the r.h.s. of the Euler equation. We treat the two different domains of the variables — inside and outside the lightcone — carefully, because the Rindler coordinates have different definitions. We shall see that the $\Omega = \eta$ case is a particular one. (We refer to it as Hwa-Bjorken case, but emphasize that for $d > 1$ such solutions were found only recently.) So we investigate it in a separate subsection, in the following two subsections — where we derive the general equations inside and outside the lightcone — we assume $\lambda \neq 1$.

Inside the forward lightcone

First, we deal with the $t > r$, $t > 0$ case. The definition of the Rindler coordinates, the inverse formulas and the derivation rules are as follows:

$$t = \tau \cosh \eta \quad , \quad r = \tau \sinh \eta, \quad (46)$$

$$\tau = \sqrt{t^2 - r^2} \quad , \quad \eta = \operatorname{arctanh} \frac{r}{t}, \quad (47)$$

$$\frac{\partial}{\partial t} = \cosh \eta \frac{\partial}{\partial \tau} - \frac{\sinh \eta}{\tau} \frac{\partial}{\partial \eta}, \quad (48)$$

$$\frac{\partial}{\partial r} = -\sinh \eta \frac{\partial}{\partial \tau} + \frac{\cosh \eta}{\tau} \frac{\partial}{\partial \eta}, \quad (49)$$

$$\frac{d}{dt} = \frac{\cosh(\Omega - \eta)}{\cosh \Omega} \frac{\partial}{\partial \tau} + \frac{\sinh(\Omega - \eta)}{\tau \cosh \Omega} \frac{\partial}{\partial \eta}, \quad (50)$$

$$\frac{d}{dr} = \frac{\sinh(\Omega - \eta)}{\cosh \Omega} \frac{\partial}{\partial \tau} + \frac{\cosh(\Omega - \eta)}{\tau \cosh \Omega} \frac{\partial}{\partial \eta}. \quad (51)$$

The domain of the variables is $-\infty < \eta < +\infty$, and $0 \leq \tau < \infty$. We introduce the notation $Q = \frac{1}{\kappa+1} \ln p$. (The convenience of this notation is due to the special EoS we utilize here, the $\varepsilon = \kappa p$ one.) Rewriting and rearranging the Euler and energy conservation equations we obtain the following:

$$\frac{\partial \Omega}{\partial \tau} + \frac{1}{\tau} \frac{\partial Q}{\partial \eta} + \tanh(\Omega - \eta) \left(\frac{\partial Q}{\partial \tau} + \frac{1}{\tau} \frac{\partial \Omega}{\partial \eta} \right) = 0, \quad (52)$$

$$\begin{aligned} \kappa \frac{\partial Q}{\partial \tau} + \frac{1}{\tau} \frac{\partial \Omega}{\partial \eta} + \frac{d-1}{\cosh(\Omega - \eta)} \frac{\sinh \Omega}{\tau \sinh \eta} + \\ + \tanh(\Omega - \eta) \left(\frac{\partial \Omega}{\partial \tau} + \frac{\kappa}{\tau} \frac{\partial Q}{\partial \eta} \right) = 0. \end{aligned} \quad (53)$$

Here we used the expression for the divergence of the velocity field: $(\nabla \mathbf{v}) = \frac{\partial v}{\partial r} + \frac{d-1}{r} v$.

Our special assumption was that $\Omega = \lambda \eta$, so substituting $\frac{\partial Q}{\partial \eta}$ from eq. (52) into (53) we have

$$\tanh((\lambda - 1)\eta) \left(\tau \frac{\partial Q}{\partial \tau} + \lambda \right) + \frac{\partial Q}{\partial \eta} = 0, \quad (54)$$

$$\begin{aligned} \kappa \tau \frac{\partial Q}{\partial \tau} + (d-1) \frac{\sinh \lambda \eta}{\sinh \eta} \cosh((\lambda - 1)\eta) + \\ + \lambda (\cosh^2((\lambda - 1)\eta) - \kappa \sinh^2((\lambda - 1)\eta)) = 0. \end{aligned} \quad (55)$$

From eq. (55) we see that $\tau \frac{\partial Q}{\partial \tau}$ must be τ -independent, that is Q must have the form $Q = H(\eta) + K(\eta) \ln \tau$. Then from eq. (54) we have that $K(\eta)$ is also constant. Again with eq. (55) we have then

$$Q = K \ln \tau + H(\eta), \quad (56)$$

where the $H(\eta)$ function is determined by eq. (54). The solution is easily obtained as

$$H(\eta) = -\frac{K + \lambda}{\lambda - 1} \ln \cosh((\lambda - 1)\eta) + \text{const.} \quad (57)$$

for $\lambda \neq 1$, so p has the form

$$p = p_0 \left(\frac{\tau_0}{\tau} \right)^{-K(\kappa+1)} \frac{1}{\cosh^{\frac{K+\lambda}{\lambda-1}(\kappa+1)}((\lambda - 1)\eta)}. \quad (58)$$

The integration constant was absorbed into the definitions of the p_0 and τ_0 initial values. For the K constant we have from eq. (55)

$$\begin{aligned} & \kappa K + (d-1) \frac{\sinh \lambda \eta}{\sinh \eta} \cosh((\lambda-1)\eta) + \\ & + \lambda \left(\cosh^2((\lambda-1)\eta) - \kappa \sinh^2((\lambda-1)\eta) \right) = 0. \end{aligned} \quad (59)$$

Essentially this is eq. (19), so what we need is to find such λ and K constants, which solve this equation, then we have the pressure and the velocity field. The solutions were found, they are presented in the body of the paper.

Outside the forward lightcone

In this case the calculation goes almost in the same way as inside the lightcone. The definitions and the derivation rules are:

$$t = \tau \sinh \eta, \quad r = \tau \cosh \eta, \quad (60)$$

$$\tau = \sqrt{r^2 - t^2}, \quad \eta = \operatorname{arctanh} \frac{t}{r}, \quad (61)$$

$$\frac{\partial}{\partial t} = -\sinh \eta \frac{\partial}{\partial \tau} + \frac{\cosh \eta}{\tau} \frac{\partial}{\partial \eta}, \quad (62)$$

$$\frac{\partial}{\partial r} = \cosh \eta \frac{\partial}{\partial \tau} - \frac{\sinh \eta}{\tau} \frac{\partial}{\partial \eta}, \quad (63)$$

$$\frac{d}{dt} = \frac{\sinh(\Omega - \eta)}{\cosh \Omega} \frac{\partial}{\partial \tau} + \frac{\cosh(\Omega - \eta)}{\tau \cosh \Omega} \frac{\partial}{\partial \eta}, \quad (64)$$

$$\frac{d}{dr} = \frac{\cosh(\Omega - \eta)}{\cosh \Omega} \frac{\partial}{\partial \tau} + \frac{\sinh(\Omega - \eta)}{\tau \cosh \Omega} \frac{\partial}{\partial \eta}. \quad (65)$$

We have in this case for the hydrodynamical equations

$$\frac{\partial Q}{\partial \tau} + \frac{1}{\tau} \frac{\partial \Omega}{\partial \eta} + \tanh(\Omega - \eta) \left(\frac{\partial \Omega}{\partial \tau} + \frac{1}{\tau} \frac{\partial Q}{\partial \eta} \right) = 0, \quad (66)$$

$$\begin{aligned} & \frac{\partial \Omega}{\partial \tau} + \frac{\kappa}{\tau} \frac{\partial Q}{\partial \eta} + \frac{d-1}{\cosh(\Omega - \eta)} \frac{\sinh \Omega}{\tau \cosh \eta} + \\ & + \tanh(\Omega - \eta) \left(\kappa \frac{\partial Q}{\partial \tau} + \frac{1}{\tau} \frac{\partial \Omega}{\partial \eta} \right) = 0. \end{aligned} \quad (67)$$

If we now assume $\Omega = \lambda \eta$, then going through the same steps as inside the lightcone we have (for $\lambda \neq 1$)

$$p = p_0 \left(\frac{\tau_0}{\tau} \right)^{-K(\kappa+1)} \sinh^{-\frac{K+\lambda}{\lambda-1}(\kappa+1)}((\lambda-1)\eta), \quad (68)$$

and the equation which constrains K and λ is

$$\begin{aligned} \frac{\kappa (K + \lambda)}{\sinh((\lambda - 1)\eta)} - (d - 1) \frac{\sinh \lambda \eta}{\cosh \eta} + \\ + (\kappa - 1) \lambda \sinh((\lambda - 1)\eta) = 0. \end{aligned} \quad (69)$$

This equation is analogous to eq. (59), but it is slightly different. We used this equation to investigate the solutions outside the lightcone.

The Hwa-Bjorken case

The $\lambda = 1$ case is the Hwa-Bjorken solution in 1+1 dimensions, and the Buda-Lund case (refs. [9, 10]) in 1+d dimensions. which is simply obtained from eqs. (52) and (53). We have

$$\frac{\partial Q}{\partial \eta} = 0, \quad \kappa \frac{\partial Q}{\partial \tau} = -\frac{d}{\tau}, \quad (70)$$

and the solution is

$$\Omega = \eta, \quad p = p_0 \left(\frac{\tau_0}{\tau} \right)^{\frac{d}{\kappa}(\kappa+1)}. \quad (71)$$

What is perhaps more interesting, is that this solution can be extended to outside the lightcone, as we see it from eqs. (66) and (67), if we substitute $\lambda = 1$:

$$\frac{\partial Q}{\partial \tau} + \frac{1}{\tau} = 0, \quad \frac{\partial Q}{\partial \eta} = -\frac{d-1}{\kappa} \tanh \eta, \quad (72)$$

so the „outside Hwa-Bjorken-Buda-Lund solution” is

$$\Omega = \eta, \quad p = p_0 \left(\frac{\tau_0}{\tau} \right)^{(\kappa+1)} \cosh^{-\frac{d-1}{\kappa}(\kappa+1)} \eta. \quad (73)$$

This is an accelerating solution, its properties are discussed in the body of the paper.

APPENDIX B: TAYLOR EXPANSION AND UNIQUENESS

In this Appendix we sketch the proof of uniqueness of the presented solutions. We deal only with the $t > r$ case. We investigate a less restrictive assumption for the velocity field than in the body of the paper, namely allow the λ factor to be proper-time dependent:

$$\Omega = \lambda(\tau) \eta. \quad (74)$$

We denote the $\tau \frac{\partial}{\partial \tau}$ derivation operator by a comma, and if possible, we suppress the τ argument of the functions in the notation. The $\lambda(\tau) = 1$ case was investigated in Appendix A as well as in the body of the paper, so we assume $\lambda(\tau) \neq 1$ throughout this Appendix.

We will use a Taylor-expansion around $\eta = 0$ for Ω and the pressure (that is, for $Q = \frac{1}{\kappa+1} \ln p$):

$$Q = A(\tau) + B(\tau)\eta + \frac{C(\tau)}{2}\eta^2 + \frac{D(\tau)}{3}\eta^3 + \frac{E(\tau)}{4}\eta^4 + \dots, \quad (75)$$

We have nothing more to do than to put this into eqs. (52) and (53), and make the l.h.s. vanish order by order in powers of η . It can be done without any essential difficulty, but it is a very long calculation. Here in this Appendix we only consider the case when one makes an additional assumption for the pressure field, namely, that it factorizes in terms of τ and η , as in the solutions presented in the body of the paper:

$$p = F(\tau) H(\eta) \quad (76)$$

That is, for the Taylor expansion we have

$$Q = A(\tau) + B\eta + \frac{C}{2}\eta^2 + \frac{D}{3}\eta^3 + \frac{E}{4}\eta^4 + \dots, \quad (77)$$

where $A(\tau) = \frac{1}{\kappa+1} \ln F(\tau)$ and the other functions are constants (denoted by the same letter), the expansion coefficients of $\frac{1}{\kappa+1} \ln H(\eta)$. We will need only the second order terms for the present calculation: we expand eq. (52) to $\mathcal{O}(\eta^3)$, and eq. (53) to $\mathcal{O}(\eta^4)$. The following auxiliary formulas are needed:

$$\begin{aligned} \tanh((\lambda-1)\eta) &= (\lambda-1)\eta - \frac{1}{3}(\lambda-1)^3\eta^3 + \dots, \\ \frac{\sinh \lambda\eta}{\cosh((\lambda-1)\eta) \sinh \eta} \frac{1}{\sinh \eta} &= \lambda - \frac{1}{3}\lambda(\lambda-1)(\lambda-2)\eta^2 + \\ &+ \frac{1}{45}\lambda(\lambda-1)(\lambda-2)(6\lambda^2 - 12\lambda + 7)\eta^4 + \dots \end{aligned}$$

Eq. (52) thus gives

$$\begin{aligned} B + \{\lambda' + C + (\lambda-1)(A' + \lambda)\}\eta + D\eta^2 + \\ + \left(E - \frac{1}{3}(\lambda-1)^3(A' + \lambda)\right)\eta^3 = 0, \end{aligned} \quad (78)$$

and eq. (53) is (to second order in η)

$$\begin{aligned} \{\kappa A' + d\lambda\} + (\lambda-1)\kappa B\eta + \\ + (\lambda-1)\left\{\frac{d-1}{3}\lambda(2-\lambda) + \lambda' + \kappa C\right\}\eta^2 = 0. \end{aligned} \quad (79)$$

We immediately see that $B = D = 0$, and we obtain the following equations:

$$\kappa A' + d\lambda = 0, \quad (80)$$

$$\lambda' + C + (\lambda - 1)(A' + \lambda) = 0, \quad (81)$$

$$\frac{d-1}{3}\lambda(2-\lambda) + \lambda' + \kappa C = 0, \quad (82)$$

$$\frac{1}{3}(\lambda-1)^3(A'+\lambda) - E = 0. \quad (83)$$

For brevity we did not write the $\mathcal{O}(\eta^4)$ term in eq. (79), but we will use the equation, which follows from the condition that the $\mathcal{O}(\eta^4)$ term vanish:

$$\begin{aligned} & \frac{d-1}{45}\lambda(\lambda-1)(\lambda-2)(6\lambda^2-12\lambda+7) + \\ & + (\lambda-1)\kappa E - (\kappa C + \lambda')\frac{(\lambda-1)^3}{3} = 0. \end{aligned} \quad (84)$$

Now substituting A' from eq. (80) into eq. (81), and then λ' from eq. (81) in eq. (82), this last equation will contain only λ (not its derivative), and the C constant. So eq. (82) will yield an *algebraic* equation for λ , that is, λ must be a constant, thus λ' vanishes. It follows that A' is also constant. So the constants are

$$A' = -\frac{d}{\kappa}\lambda, \quad (85)$$

$$C = -\lambda(\lambda-1)\left(1 - \frac{d}{\kappa}\right), \quad (86)$$

$$E = \frac{1}{3}\lambda(\lambda-1)^3\left(1 - \frac{d}{\kappa}\right), \quad (87)$$

and from eqs. (82) and (84) we have two equations containing only λ :

$$\kappa - d = \frac{(d-1)(2-\lambda)}{3(\lambda-1)}, \quad (88)$$

$$2(\lambda-1)^3(\kappa-d) + \frac{d-1}{15}(\lambda-2)(6\lambda^2-12\lambda+7) = 0. \quad (89)$$

We excluded the trivial $\lambda = 0$ case, and as mentioned before, the $\lambda = 1$ Hwa-Bjorken case was treated separately. Substituting eq. (88) into eq. (89) we obtain

$$(d-1)(\lambda-2)(4\lambda^2-8\lambda+3) = 0. \quad (90)$$

It is now easy to see that there are only those solutions which we presented in the body of the paper: if $d = 1$, then eq. (88) implies that $\kappa = 1$, but then eq. (90) is satisfied by

any value of the parameter λ . Second, if $\lambda = 2$, then eq. (90) is satisfied in any number of dimensions d , but eq. (88) implies a condition for the equation of state, $\kappa = d$. Third, the other two roots of eq. (90) are $\lambda = \frac{3}{2}$, and $\lambda = \frac{1}{2}$, and it is clear, that there are no other solutions of eq. (88). So the only possibilities are those listed in Table I. They are indeed solutions, as can be verified straightforwardly by substituting these solutions to the hydrodynamical equations in Rindler coordinates. The conclusion of this Appendix is that we cannot extend the investigated class of solutions in an easy way even if we allow λ to be τ -dependent.

APPENDIX C: RAPIDITY DISTRIBUTIONS

In this Appendix we derive eq. (39), the formula which gives the rapidity distribution of the produced particles for our new 1+1 dimensional solutions, which are given by eqs. (20) and (21), and the constants are those in case *e.*) in Table I. We assumed $\mu_B = 0$, that is $\varepsilon + p = T\sigma$. We utilized the following expressions for σ and T , similarly to eqs. (30) and (31):

$$T = T_f \left(\frac{p}{p_f} \right)^{\frac{1}{1+\kappa}}, \quad \sigma = \sigma_f \left(\frac{p}{p_f} \right)^{\frac{\kappa}{1+\kappa}}, \quad (91)$$

so we assumed $\nu_\sigma(S) \equiv 1$ for the scaling function, and $p(\tau, \eta)$ is given by eq. (21). Of course, $p_f = \sigma_f T_f$. The subscript f means freeze-out. (The constants in eqs. (30) and (31) can always be re-scaled, so we denoted the proper-time coordinate of the freeze-out hypersurface at $\eta = 0$ by τ_f , and p_f , T_f and σ_f are the values of the thermodynamical quantities in this space-time point.) Here, as in the body of the paper, m is the mass of the produced particle, and we parametrize the momentum of it as $k^\mu = (E, p) = m(\cosh y, \sinh y)$, with y being the rapidity.

We calculate the rapidity distribution in two cases. First, in the fully 1 + 1 dimensional case, when not only the flow but also the produced particles have only 1 degrees of freedom, as they propagate along the z axis. Second, in the case, when this 1 + 1 dimensional solution is embedded in a 1 + 3 dimensional space, and the produced particles can have transverse momentum as well.

The 1 + 1 dimensional case

The rapidity distribution is given by

$$\frac{dn}{dy} = E \frac{dn}{dp} = \int d^2x S(x^\mu, k^\mu), \quad (92)$$

where $S(x^\mu, k^\mu)$ is the source function, and the notation d^2x refers to the fact that we are in an 1 + 1 dimensional space-time. The explicit form of $S(x^\mu, k^\mu)$ is

$$S(x, k) = \frac{g}{2\pi\hbar} \exp\left(\frac{\mu(x)}{T(x)} - \frac{k_\mu u^\mu(x)}{T(x)}\right) k_\mu d\Sigma^\mu(x) \delta(x - x_f), \quad (93)$$

where the notations are: u^μ is the four-velocity of the flow, μ is the chemical potential, g is the spin-degeneracy factor, and $d\Sigma_\mu(x)$ is the Cooper-Frye flux term. (The Cooper-Frye flux term, together with the delta function, which selects the freeze-out hypersurface, is essentially the Lebesgue vector-measure of the freeze-out hypersurface).

In our particular solution, $\mu(x) = 0$, and $k_\mu u^\mu(x) = m \cosh(\lambda\eta - y)$. We selected the freeze-out condition in a way to take advantage of the simplicity of this relation: our condition is that the temperature at $\eta = 0$ reaches a given T_f value, and the hyper-surface is pseudo-orthogonal to the four-velocity field, or in other words, $u^\mu(x)$ is parallel to $d\Sigma^\mu(x)$. (This hypersurface is *not* of constant temperature nor of constant τ . This choice is also motivated by the success of the Buda-Lund model in describing particle spectra, see e.g. [21, 22].) Another formulation of this condition is that the freeze-out hypersurface is „locally synchronized”: an observer which moves together with the flow sees this hypersurface as a constant time surface in its neighborhood. The equation of this hypersurface is

$$\left(\frac{\tau_f}{\tau}\right)^{\lambda-1} \cosh((\lambda-1)\eta) = 1. \quad (94)$$

We can now calculate the Cooper-Frye term. We have for the integration

$$\begin{aligned} & \int d^2x \delta(x - x_f) k_\mu d\Sigma^\mu(x) \rightarrow \\ & \rightarrow \int_{-\infty}^{\infty} d\eta m \tau_f \cosh(\lambda\eta - y) \cosh^{\frac{1}{\lambda-1}-1}((\lambda-1)\eta). \end{aligned} \quad (95)$$

So the rapidity distribution is

$$\frac{dn}{dy} = \int_{-\infty}^{\infty} d\eta \frac{m\tau_f}{2\pi\hbar} \cosh(\lambda\eta - y) \cosh^{\frac{1}{\lambda-1}-1}((\lambda-1)\eta) \times$$

$$\times \exp\left(-\frac{m}{T_f} \cosh(\lambda\eta - y) \cosh^{\frac{\lambda}{\lambda-1}}((\lambda-1)\eta)\right). \quad (96)$$

This integration can be calculated numerically, but an analytic approximation is also available. The formula of the saddle-point integration is

$$\int_{-\infty}^{\infty} dx f(x)g(x) \approx g(x_0) \sqrt{\frac{2\pi f^3(x_0)}{-f''(x_0)}}, \quad (97)$$

for an $f(x)$ function, that has a sharp maximum peak at x_0 , and a $g(x)$, which changes smoothly around x_0 . Essentially this is an approximation where one treats the smooth function as a constant $g(x_0)$, and the sharp function as a Gaussian peak with parameters obtained from $f(x_0)$ and $f''(x_0)$. This method can be applied very effectively when the peaked function has a parameter, which rules its half-width, so this parameter can be changed in such a way that $f(x)$ approximates a Dirac-delta. The more $f(x)$ is like a Dirac-delta, the better the saddle-point approximation is (and in the limiting case it becomes exact).

In our case, $f(\eta)$ is the exponential, $g(\eta)$ is the remaining, and $\frac{m}{T_f}$ is the parameter, which governs the half-width of $f(\eta)$. For convenience, we introduce the $\alpha = \frac{2\lambda-1}{\lambda-1}$ notation. We have

$$\eta_0 = \frac{y}{2\lambda-1}, \quad f(\eta_0) = \exp\left\{-\frac{m}{T_f} \cosh^{\frac{1}{\alpha}}\left(\frac{y}{\alpha}\right)\right\}, \quad (98)$$

$$f''(\eta_0) = -f(\eta_0) \frac{m}{T_f} \lambda (2\lambda-1) \cosh^{\frac{\lambda}{\lambda-1}-1}\left(\frac{y}{\alpha}\right), \quad (99)$$

$$g(\eta_0) = \frac{m\tau_f}{2\pi\hbar} \cosh^{\frac{1}{\lambda-1}}\left(\frac{y}{\alpha}\right). \quad (100)$$

Putting all this together, we obtain (approximately)

$$\frac{dn}{dy} = \mathcal{N} \cosh^{\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) \exp\left\{-\frac{m}{T_f} \cosh^{\alpha}\left(\frac{y}{\alpha}\right) + \frac{m}{T_f}\right\}, \quad (101)$$

$$\mathcal{N} = \sqrt{\frac{2\pi T_f}{m\lambda(2\lambda-1)} \frac{m\tau_f}{2\pi\hbar}} \exp\left(-\frac{m}{T_f}\right). \quad (102)$$

This is the expression of $\frac{dn}{dy}$ in the true 1 + 1 dimensional case. The normalization constant \mathcal{N} is $\left.\frac{dn}{dy}\right|_{y=0}$, the value of $\frac{dn}{dy}$ at mid-rapidity.

The 1 + 3 dimensional, embedded case

If we take the transverse dynamics into account, that is, we embed the 1 + 1 dimensional solution in a 3 dimensional space (with $v_x = v_y = 0$ and no pressure gradients in transverse directions), the previous (101)–(102) formulas need to be modified.

We have (in 3 spatial dimensions) $\frac{1}{(2\pi\hbar)^3}$ instead of $\frac{1}{2\pi\hbar}$, and m_T instead of m , where $m_T = \sqrt{E^2 - p_z^2} = \sqrt{m^2 + p_x^2 + p_y^2}$ is the transverse mass. With these modifications, the above (101)–(102) expressions have the following meaning:

$$E \frac{dn}{d^3\mathbf{k}} = \frac{dn}{dy 2\pi m_T dm_T} = \mathcal{N} \cosh^{\frac{\alpha}{2}-1} \left(\frac{y}{\alpha} \right) \times \exp \left\{ -\frac{m_T}{T_f} \cosh^\alpha \left(\frac{y}{\alpha} \right) + \frac{m_T}{T_f} \right\}, \quad (103)$$

$$\mathcal{N} = \sqrt{\frac{2\pi T_f}{m_T \lambda (2\lambda - 1)} \frac{m_T \tau_f}{(2\pi\hbar)^3}} \exp \left(-\frac{m_T}{T_f} \right). \quad (104)$$

In order to obtain the rapidity distribution, we have to integrate over the transverse mass:

$$\frac{dn}{dy} = 2\pi \int_m^\infty dm_T m_T \sqrt{\frac{2\pi T_f}{m_T \lambda (2\lambda - 1)} \frac{m_T \tau_f}{(2\pi\hbar)^3}} \times \exp \left\{ -\frac{m_T}{T_f} \cosh^\alpha \left(\frac{y}{\alpha} \right) \right\} \cosh^{\frac{\alpha}{2}-1} \left(\frac{y}{\alpha} \right). \quad (105)$$

Here m is the real mass of the produced particles, e.g. $m = 140$ MeV for pions. The integral over m_T can be also calculated in the limiting case of $\frac{m}{T} \rightarrow \infty$. (This is the same condition as that for the saddle-point integration to be exact.) In this case, we can apply the simple approximation

$$\int_a^\infty h(x) \exp(-\beta x) dx \approx \frac{h(a)}{\beta} \quad (106)$$

for $a \gg \frac{1}{\beta}$, and for any $h(x)$, that varies smoothly around a . Our case is $h(m_T) = m_T^{3/2}$. So we have

$$\frac{dn}{dy} = \mathcal{N}' \cosh^{-\frac{\alpha}{2}-1} \left(\frac{y}{\alpha} \right) \exp \left\{ -\frac{m}{T_f} \cosh^\alpha \left(\frac{y}{\alpha} \right) \right\}, \quad (107)$$

$$\mathcal{N}' = \sqrt{\frac{2\pi m T_f^3}{\lambda (2\lambda - 1)} \frac{A m \tau_f}{2\pi\hbar}}, \quad (108)$$

with A being the transverse cross section of the fluid. These are the results quoted in eq. (39).

APPENDIX D: GENERAL SOLUTION FOR THE $d = 1$, $\kappa = 1$ CASE, INVESTIGATION OF STABILITY

If $\varepsilon = \kappa p$, and we take the choice of $\kappa = 1$, then the hydrodynamical equations (that is, eqs. (5) and (6)) have an intrinsic simplicity. Let us write them down in this case! In this

Appendix we denote the derivation with respect to t by dotting, and the derivation with respect to r by comma. So the Euler equation is

$$\frac{1}{1-v^2} (\dot{v} + vv') = -\frac{1}{2p} (p' + vp\dot{)}. \quad (109)$$

We have $\varepsilon + p = 2p$ for the enthalpy density. The energy conservation is

$$\frac{1}{2p} (\dot{p} + vp') = -\frac{1}{2(1-v^2)} ((v^2)\dot{)} + v(v^2)') - v', \quad (110)$$

or in another form

$$\frac{1}{1-v^2} (v' + vv\dot{)} = -\frac{1}{2p} (\dot{p} + vp'). \quad (111)$$

We used the fact that in $d = 1$ dimensions $(\nabla\mathbf{v}) = v'$ simply. Taking a proper linear combination of eqs. (109) and (111) we have the following:

$$\frac{1}{1-v^2} v' = \Omega' = -\frac{1}{2p} \dot{p}, \quad (112)$$

$$\frac{1}{1-v^2} \dot{v} = \dot{\Omega} = -\frac{1}{2p} p'. \quad (113)$$

Here we again used the $\Omega = \operatorname{arctanh}v$ notation. These equations are analogous to the Cauchy-Riemann equations known in complex analysis, and the general solution of them is easily written down: it contains two arbitrary functions, $F(t+r)$ and $G(t-r)$, one for each direction of wave-propagation:

$$\Omega(t, r) = \frac{1}{2} \{G(t-r) + F(t+r)\}, \quad (114)$$

$$\ln(p(t, r)/p_0) = \{G(t-r) - F(t+r)\}. \quad (115)$$

Thus this case is a particular one: the *general* solution of the relativistic hydrodynamical equations is found, and then it can be easily fitted to a given initial condition. If we prescribe $p(t(r), r)$ and $\Omega(t(r), r)$ on some $t = t(r)$ Cauchy hypersurface, then the proper F and G functions can be found as detailed below, and then the solution of the initial value problem is that the two wave-shapes propagate in the opposite direction. As a first example, let us consider the $\Omega = \lambda\eta$ solution presented in the body of the paper. It can be easily verified, that the following F and G functions indeed yield it:

$$F(t+r) = \lambda \ln \left(\frac{t+r}{\tau_0 p_0^{-1/2\lambda}} \right), \quad (116)$$

$$G(t-r) = -\lambda \ln \left(\frac{t-r}{\tau_0 p_0^{-1/2\lambda}} \right). \quad (117)$$

To simplify the presentation of the general case, let us introduce

$$\Pi(t, r) = \frac{1}{2} \ln(p(t, r) / p_0), \quad (118)$$

and give the initial boundary conditions in the form of three functions, the Cauchy hypersurface $t_0(r)$, the initial velocity field $v = \tanh(\Omega)$ given by the form of $\Omega(t_0(r), r)$ and the initial pressure distribution is given by $p = p_0 \exp(2\Pi(t_0(r), r))$.

Let us introduce the positive and the negative light-cone coordinates on the initial Cauchy hypersurface as

$$x^\pm(r) = t_0(r) \pm r. \quad (119)$$

It is important to assume, that on the Cauchy hypersurface, these relations can be inverted, and r can be expressed as a function of both the positive and the negative light-cone coordinates:

$$r = f(x^+), \quad (120)$$

$$r = g(x^-). \quad (121)$$

For example, if the initial Cauchy hypersurface is given by $\tau = \tau_0 = \text{const}$, then the equation of this hypersurface in time and coordinate is $t_0(r) = \sqrt{\tau_0^2 + r^2}$ and the relation between the coordinate r and the light-cone coordinates,

$$x^\pm(r) = t_0(r) \pm r = \sqrt{\tau_0^2 + r^2} \pm r, \quad (122)$$

which relations can be inverted as

$$r = f(x^+) = \frac{x^+}{2} - \frac{\tau_0^2}{2x^+}, \quad (123)$$

$$r = g(x^-) = -\frac{x^-}{2} + \frac{\tau_0^2}{2x^-}. \quad (124)$$

Then the general solution of the hydrodynamical equations in this case simplifies in terms of Ω and Π to the following relations:

$$\Omega(t, r) = \frac{1}{2} [\Omega(t_0(f(t+r)), f(t+r)) + \Omega(t_0(g(t-r)), g(t-r))] + \quad (125)$$

$$+ \frac{1}{2} [\Pi(t_0(f(t+r)), f(t+r)) - \Pi(t_0(g(t-r)), g(t-r))], \quad (126)$$

$$\Pi(t, r) = \frac{1}{2} [-\Omega(t_0(f(t+r)), f(t+r)) + \Omega(t_0(g(t-r)), g(t-r))] - \quad (127)$$

$$- \frac{1}{2} [\Pi(t_0(f(t+r)), f(t+r)) + \Pi(t_0(g(t-r)), g(t-r))]. \quad (128)$$

A by-product of the treatment of this Appendix is, that any 1 + 1 dimensional relativistic hydrodynamical solution with $\varepsilon = p$ EoS (including our one presented in the body of the paper, with $\Omega = \lambda\eta$) is always stable in terms of Ω and Π : if there is a perturbation in the initial condition for Π and Ω , then this perturbation propagates in both directions, but it does not grow exponentially. It is interesting to contrast this behaviour, the stability of our solutions in terms of rapidity and logarithmic pressure, to the numerically found instability of the famous Landau-Khalatnikov solution for the perturbation of their initial conditions. Such a stability of our new, general solutions makes them particularly attractive for testing various algorithms that integrate the solutions of relativistic hydrodynamics numerically.

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