Chiral phase transition in an extended linear sigma model: initial results *

Gy. Wolf, P. Kovács

Institute for Particle and Nuclear Physics, Wigner Research Center for Physics, Hungarian Academy of Sciences, POB 49, H-1525 Budapest, Hungary

Zs. Szép

MTA-ELTE Statistical and Biological Physics Research Group, H-1117 Budapest, Hungary

We investigate the scalar meson mass dependence on the chiral phase transition in the framework of an SU(3), (axial)vector meson extended linear sigma model with additional constituent quarks and Polyakov loops. We determine the parameters of the Lagrangian at zero temperature in a hybrid approach, where we treat the mesons at tree-level, while the constituent quarks at 1-loop level. We assume two nonzero scalar condensates and together with the Polyakov-loop variables we determine their temperature dependence according to the 1-loop level field equations.

PACS numbers: 12.39.Fe,12.40.Yx, 14.40.Be, 14.40.Df, 21.65.Qr, 25.75.Nq

1. Introduction

The investigation of the QCD phase diagram is a very important subject both theoretically and experimentally nowadays. The ongoing and future heavy ion experiments such as RHIC, and CERN/LHC study the low density part of the phase diagram which can also be investigated theoretically by lattice QCD, at CBM/FAIR the high density part will be studied, which is still not settled theoretically, so it is worth to investigate the phase diagram thoroughly.

Our starting point is the (axial)vector meson extended linear sigma model with additional constituent quarks and Polyakov-loop variables. The

---

* Presented at the Workshop on Unquenched Hadron Spectroscopy: Non-Perturbative Models and Methods of QCD vs. Experiment, At the occasion of Eef van Beveren's 70th birthday

previous version of the model, without constituent quarks and Polyakov-loops, was exhaustively analyzed at zero temperature in [1, 2, 3][1]. The Lagrangian of the model is given by,

$$\begin{aligned}
\mathcal{L} = &\operatorname{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2 \operatorname{Tr}(\Phi^\dagger\Phi) - \lambda_1[\operatorname{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2 \operatorname{Tr}(\Phi^\dagger\Phi)^2 \\
&- \frac{1}{4}\operatorname{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \operatorname{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \operatorname{Tr}[H(\Phi + \Phi^\dagger)] \\
&+ c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\operatorname{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \operatorname{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
&+ \frac{h_1}{2}\operatorname{Tr}(\Phi^\dagger\Phi)\operatorname{Tr}(L_\mu^2 + R_\mu^2) + h_2\operatorname{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\operatorname{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \qquad (1)\\
&+ g_3[\operatorname{Tr}(L_\mu L_\nu L^\mu L^\nu) + \operatorname{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\operatorname{Tr}(L_\mu L^\mu L_\nu L^\nu) \\
&+ \operatorname{Tr}(R_\mu R^\mu R_\nu R^\nu)] + g_5\operatorname{Tr}(L_\mu L^\mu)\operatorname{Tr}(R_\nu R^\nu) + g_6[\operatorname{Tr}(L_\mu L^\mu)\operatorname{Tr}(L_\nu L^\nu) \\
&+ \operatorname{Tr}(R_\mu R^\mu)\operatorname{Tr}(R_\nu R^\nu)] + \bar{\Psi} i \partial\!\!\!/ \Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi,
\end{aligned}$$

where

$$\begin{aligned}
D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}.
\end{aligned}$$

Here Φ stands for the scalar and pseudoscalar fields, L^μ and R^μ for the left and right handed vector fields, $\Psi = (u, d, s)^{\mathrm{T}}$ for the constituent quark fields, while H for the external field.

2. Parametrization

In order to go to finite temperature/chemical potential, parameters of the Lagrangian have to be determined, which is done at $T = \mu = 0$. For this we calculate tree-level masses and decay widths of the model and compare them with the experimental data taken from the PDG [4]. For the comparison we use a χ^2 minimalization method [5] to fit our parameters (for more details see [1]). It is important to note that in the present work we also included in the scalar and pseudoscalar masses the contributions coming from the fermion vacuum fluctuations by adapting the method of [6].

We have 14 unknown parameters, namely m_0, λ_1, λ_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, δ_S, Φ_N, Φ_S, and g_F. Here g_F is the coupling of the additionally introduced Yukawa term, which can be determined from the constituent quark masses through the equations $m_{u/d} = g_F\phi_N/2$, $m_s = g_F\phi_s/\sqrt{2}$.

[1] In the present work we use a different anomaly term (c_1 term). This, however, does not influence the results much.

Table 1. Parameters determined by χ^2 minimalization

Parameter	Value	Parameter	Value
ϕ_N [GeV]	0.1622	h_2	11.6586
ϕ_S [GeV]	0.1262	h_3	4.7028
C_1 [GeV^2]	-0.7537	δ_S [GeV^2]	0.1534
C_2 [GeV^2]	0.3953	c_1 [GeV]	1.12
λ_1	undetermined	g_1	-5.8943
λ_2	65.3221	g_2	-2.9960
h_1	undetermined	g_F	4.9429

It is worth to note that if we do not consider the very uncertain scalar-isoscalar sector m_0, and λ_1 always appear in the same combination $C_1 = m_0^2 + \lambda_1 \left(\phi_N^2 + \phi_S^2\right)$ in all the expressions, thus we can not determine them separately. Additionally a similar combination appears for m_1 and h_1 in the vector sector as $C_2 = m_1^2 + \frac{h_1}{2}\left(\phi_N^2 + \phi_S^2\right)$ (see details in [1]). The parameter values of the fit without scalars are given in Table 1. Since λ_1 and h_1 are undetermined they can be tuned to select the f_0^L (a.k.a. σ) from the scalar spectrum (by its mass and decay widths) and its mass has, as we will see, a huge effect on the thermal properties of the model.

3. Field equations

In our approach we have four order parameters, which are the ϕ_N non-strange and ϕ_S strange condensates, and the Φ and $\bar{\Phi}$ Polyakov-loop variables. The condensates arise due to the spontaneous symmetry breaking[2], while the Polyakov-loop variables naturally emerge in mean field approximation, if one calculates free fermion grand canonical potential on a constant gluon background. The effect of fermions propagating on a constant gluon background in the temporal direction formally amounts to the appearance of imaginary color dependent chemical potentials (for details see [7, 8]).

At finite temperature/baryochemical potential we can set up four coupled field equations for the four fields, which are just the requirements that the first derivatives of the grand canonical potential according to the fields must vanish. As a first approximation we apply a hybrid approach in which we only consider vacuum and thermal fluctuations for the fermions, but not for the bosons. We use a mean field Polyakov-loop potential $U(\Phi, \bar{\Phi})$ of a polynomial form with coefficients determined in [9]. Within this simplified

[2] Since isospin symmetry is assumed, we have only two condensates: ϕ_N and ϕ_S.

treatment the equations are the following:

$$-\frac{d}{d\Phi}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right)+\frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)}+\frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)}\right)=0, \tag{2}$$

$$-\frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right)+\frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)}+\frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)}\right)=0, \tag{3}$$

$$m_0^2\phi_N+\left(\lambda_1+\frac{1}{2}\lambda_2\right)\phi_N^3+\lambda_1\phi_N\phi_S^2-h_N+\frac{g_F}{2}N_c\left(\langle u\bar{u}\rangle_T+\langle d\bar{d}\rangle_T\right)=0, \tag{4}$$

$$m_0^2\phi_S+(\lambda_1+\lambda_2)\,\phi_S^3+\lambda_1\phi_N^2\phi_S-h_S+\frac{g_F}{\sqrt{2}}N_c\langle s\bar{s}\rangle_T=0, \tag{5}$$

where

$$\begin{aligned}
g_q^+(p)&=1+3\left(\bar{\Phi}+\Phi e^{-\beta E_q^+(p)}\right)e^{-\beta E_q^+(p)}+e^{-3\beta E_q^+(p)},\\
g_q^-(p)&=1+3\left(\Phi+\bar{\Phi} e^{-\beta E_q^-(p)}\right)e^{-\beta E_q^-(p)}+e^{-3\beta E_q^-(p)},\\
E_q^{\pm}(p)&=E_q(p)\mp\mu_B/3,\ E_{u/d}(p)=\sqrt{p^2+m_{u/d}^2},\ E_s(p)=\sqrt{p^2+m_s^2},
\end{aligned}$$

and

$$\langle q\bar{q}\rangle_T=-4m_q\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_q(p)}\left(1-f_\Phi^-(E_q(p))-f_\Phi^+(E_q(p))\right), \tag{6}$$

with the modified distribution functions

$$\begin{aligned}
f_\Phi^+(E_p)&=\frac{\left(\bar{\Phi}+2\Phi e^{-\beta(E_p-\mu_q)}\right)e^{-\beta(E_p-\mu_q)}+e^{-3\beta(E_p-\mu_q)}}{1+3\left(\bar{\Phi}+\Phi e^{-\beta(E_p-\mu_q)}\right)e^{-\beta(E_p-\mu_q)}+e^{-3\beta(E_p-\mu_q)}},\\
f_\Phi^-(E_p)&=\frac{\left(\Phi+2\bar{\Phi} e^{-\beta(E_p+\mu_q)}\right)e^{-\beta(E_p+\mu_q)}+e^{-3\beta(E_p+\mu_q)}}{1+3\left(\Phi+\bar{\Phi} e^{-\beta(E_p+\mu_q)}\right)e^{-\beta(E_p+\mu_q)}+e^{-3\beta(E_p+\mu_q)}}.
\end{aligned}$$

4. Results

Solving Eqs. 2-5 we get the temperature dependence of the order parameters, which can be seen in Fig. 1. In [1] it was shown that the $q\bar{q}$ scalar

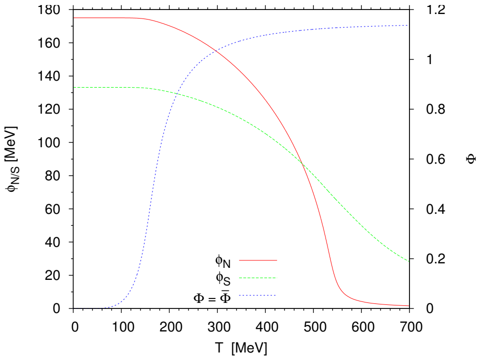

Fig. 1. Temperature dependence of the order parameters with $m_\sigma = 1.3$ GeV

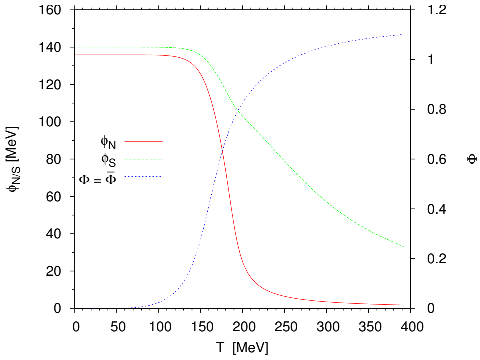

Fig. 2. Temperature dependence of the order parameters with $m_\sigma = 0.4$ GeV

nonet most probably contains f_0's with masses higher than 1 GeV. If we set $\lambda_1 = 0$ we get $m_{f_0^L} = 1.3$ GeV, which is in agreement with [1]. However in this case we get a very high pseudocritical temperature, $T_c \approx 550$ MeV, for ϕ_N, which is much larger than earlier results (e.g. on lattice $T_c \approx 150$ MeV [10]). Now, if we tune λ_1 to get $m_{f_0^L} = 400$ MeV (which corresponds to the physical particle $f_0(500)$), than T_c goes down to $150 - 200$ MeV, which can be seen in Fig. 2. This finding is in line with the results of [11], where they used a similar model, but without vector mesons. This suggests that in order to get a good pseudocritical temperature we would need a scalar-isoscalar particle with low mass (~ 400 MeV), which is probably not a $q\bar{q}$ state according to [1]. In Fig. 3 and 4 we show the temperature dependence

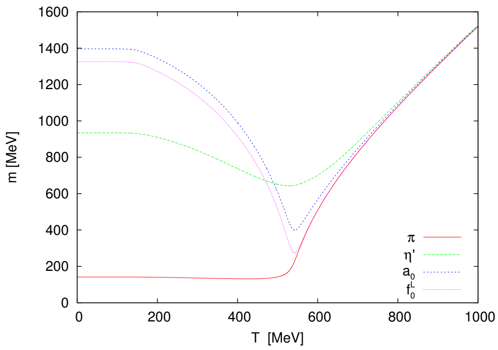

Fig. 3. Temperature dependence of the scalars with $m_\sigma = 1.3$ GeV

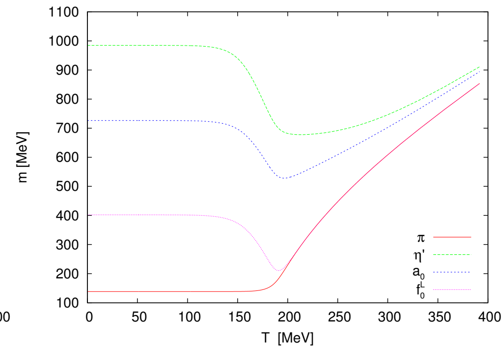

Fig. 4. Temperature dependence of the scalars with $m_\sigma = 0.4$ GeV

of the scalar meson masses. The mass of the parity partners (π and f_0^L) reaches the same value above the phase transition temperature.

5. Acknowledgement

Authors were supported by the Hungarian OTKA fund K109462 and by the HIC for FAIR Guest Funds of the Goethe University Frankfurt.

REFERENCES

[1] D. Parganlija, P. Kovács, Gy. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D **87**, 014011 (2013).

[2] P. Kovács and Gy. Wolf, Acta Phys.Polon.Supp. 6 (2013) 3, 853-858

[3] P. Kovács, A. Lukács, J. Váróczy, Gy. Wolf and M. Zétényi, Phys.Rev. D89 (2014) 054004

[4] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D**86**, 010001 (2012).

[5] F. James and M. Roos, Comput. Phys. Commun. **10**, 343 (1975).

[6] S. Chatterjee and K. A. Mohan, Phys. Rev. D **85**, 074018 (2012).

[7] C. P. Korthals Altes, R. D. Pisarski and A. Sinkovics, Phys. Rev. D **61**, 056007 (2000).

[8] G. Markó and Zs. Szép, Phys. Rev. D **82**, 065021 (2010).

[9] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73** (2006) 014019.

[10] Y. Aoki *et al.*, Phys. Lett. B **643**, 46 (2006).

[11] B.J. Schaefer and M. Wagner, Phys. Rev. D **79** (2009) 014018.