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Singular vectors of the WA_2 algebra

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Abstract

The null vectors of an arbitrary highest weight representation of the WA_2 algebra are constructed. Using an extension of the enveloping algebra by allowing complex powers of one of the generators, analysed by Kent for the Virasoro theory, we generate all the singular vectors indicated by the Kac determinant. We prove that the singular vectors with given weights are unique up to normalisation and consider the case when W_0 is not diagonalisable among the singular vectors.

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The highest weight representations of the Virasoro algebra or its extensions play an important role in the analysis of conformal field theories. In fact, the degenerate representations are crucial since they contain null vectors, which restrict the possible fusions of the fields, give differential equations for the correlation functions and determine the field content of the theory. A careful analysis of their embedding pattern makes it possible to construct the characters of the irreducible representations, from which the modular invariant partition function of the model is built up. For this reason knowledge of these null vectors and their embedding pattern is essential.

Basically there are two approaches to give explicit expressions for these null vectors. Much of the effort is based on one of these, the fusion procedure of Bauer et al [1]. The other one uses complex powers of the generators and was introduced by Malikov, Feigin and Fuchs (MFF) for the modules over KM algebras [2]. Later Kent extended the method for the Virasoro algebra [3]. Ganchev and Petkova discovered a relationship between them by considering the reduction of the MFF singular vectors [5]. Analysing the reduction of the $\hat{sl}_3$ singular vectors they proposed a similar extension for the WA_2 algebra [6].

The aim of the paper is to set up consistently this extended space, i.e. following the earlier works we generalise the method of complex powers for the WA_2 algebra. Contrary to the naive hope, instead of using complex powers of the mode L_{-1} we consider complex powers of the generator W_{-1}. Using this extension which allows us to define an analytic continuation of the null vectors, found explicitly by Bowcock and Watts [4], we generate singular vectors in arbitrary h.w. representations. In particular we show that the h.w. singular vectors with given weights are unique up to normalisation. We construct singular vectors that are not eigenstates of the W_0, but are annihilated by the positive modes. Including these non h.w. type singular vectors we produce all the singular vectors given by the Kac formula [8].

First we recall the results of the degenerate representations of the algebra. The WA_2 algebra is defined by the following commutation relations:

$$\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m} \\
[L_n, W_m] &= (2n-m)W_{n+m} \\
[W_n, W_m] &= \frac{22+5c}{48}\frac{c}{3\cdot 5!}(n^2-4)(n^2-1)n\delta_{n+m} \\
&\quad +\frac{1}{3}(n-m)\Lambda_{n+m} + \frac{22+5c}{48}\frac{1}{30}(n-m)(2n^2-nm+2m^2-8)L_{n+m}
\end{aligned} \tag{1}$$

here the Λ field is related to the $(L_{-2}L_{-2} - \frac{3}{5}L_{-4})|0\rangle$ state in the sense of the meromorphic conformal field theory [7], and we used the normalisation of [4].

A highest weight representation is characterised by its h, w, and c values and contains a h.w. state which satisfies:

$$L_n|hw\rangle = \delta_{n,0}h|hw\rangle \quad ; \quad W_n|hw\rangle = \delta_{n,0}w|hw\rangle \quad ; \quad n \geq 0 \tag{2}$$

The Verma module, for which we write $V(h,w,c)$, is generated by acting with the negative modes L_{-n}, W_{-m} on the h.w. state. (Sometimes we write W^2 for L and W^3 for W). A basis of the space is defined by using the following ordering:

$$W_{-I}|hw\rangle = W^{j_1}_{-i_1}W^{j_2}_{-i_2}\dots W^{j_n}_{-i_n}|hw\rangle = W_{-i_1}\dots W_{-i_k}L_{-i_{k+1}}\dots L_{-i_n}|hw\rangle \tag{3}$$

where $i_l > 0$, $j_l \geq j_{l+1}$, and if $j_l = j_{l+1}$ for some l then $i_l \leq i_{l+1}$. Here I denotes the collection of the j_l and i_l numbers ($I = \{i_1, \ldots, i_k; i_{k+1}, \ldots, i_n\}$) and $|I| = \sum_{l=1}^{n} i_l$ is the level of the corresponding state. Using the natural grading on this representation space given by the eigenvalues of L_0 we can write:

$$V(h,w,c) = \bigoplus_{0,1,2\ldots} V_n(h,w,c) \tag{4}$$

where $V_n(h,w,c)$ is the eigenspace of L_0 with eigenvalue $h+n$.

In order to analyse the reducibility of the representation space we introduce the following parametrisation of the W weights:

$$h = \frac{1}{3}(x^2 + xy + y^2) - (\alpha - \frac{1}{\alpha})^2 \qquad w = \frac{1}{27}(x-y)(2x+y)(x+2y) \tag{5}$$

where $c = 2 - 24(\alpha - \frac{1}{\alpha})^2$. (Sometimes we write t for $\frac{1}{\alpha^2}$). We associate with each h.w. representation of the WA_2 algebra a h.w. representation of sl_3 described by the $\Lambda(x,y) = x\lambda_1 + y\lambda_2$ weight. Here the λ_is are the two fundamental weights of sl_3. We remark that the parametrisation of the W weights is redundant i.e. the Λ weight is well defined up to a Weyl reflection. In other words not only $\Lambda(x,y)$ but also $\Lambda(-x, x+y)$, $\Lambda(-y, x+y)$, $\Lambda(-x-y, x)$, $\Lambda(y, -x-y)$ and $\Lambda(-y,-x)$ correspond to the same h and w.

We know from the determinant formula that if $x = a\alpha - \frac{c}{\alpha}$ or $y = b\alpha - \frac{d}{\alpha}$, with positive integers (a,c) or (b,d), then the representation of the W algebra is reducible (degenerate) and there is a h.w. null state at level ac or bd. If both hold the representation is called doubly degenerate. We call a vector singular or null if it is annihilated by the positive modes. A null vector is not always a h.w. vector since sometimes W_0 is not diagonalisable. Clearly null vectors define invariant subspaces. In the case of degenerate representation we denote the h.w. vector of the Verma module by (ab, cd) or $|ab, cd\rangle$.

Now we summarise the well-known results of the representation theory. In the case of degenerate representation (say a, c are positive integers) there is a null state at level ac. This is a h.w. state, which can be parametrised by $(-a, a+b; cd)$, i.e. it corresponds to the weight $\Lambda' = \Lambda - a\alpha e_1$ of sl_3, where e_1 is its first simple root, and is an eigenvector of W_0 with the appropriate weight. Unfortunately the general form of the operator, which generates the null state from the h.w. state, is not known in terms of the natural variables of the W algebra. We have the following partial result: in the $(pq, 11)$ representation the following operator generates a singular vector at level p:

$$\mathcal{O}_{p,1} = \sum_{\{n_i\}:\sum_{i=1}^{r} n_i = p} c_{\{n_i\}}(p,q,\alpha) \prod_{i=1}^{r} \Big(W_{-n_i} - \alpha[\frac{1}{6}(n_i(t-3) + 2(t-p-2q)) + N_i] L_{-n_i} \Big) \tag{6}$$

Where

$$c_{\{n_i\}}(p,q,\alpha) = (-\alpha)^{p-r} \prod_{\substack{i=1 \\ i \neq N_k; k=1,\ldots r-1}}^{p-1} i(i-p)(i+t-p-q)\alpha^2 \quad ; \quad N_k = \sum_{j=1}^{k} n_j \tag{7}$$

Furthermore $\mathcal{O}_{p,1}|pq, 11\rangle$ defines a h.w. singular vector with weight $(-p, p+q; 11)$ [4, 6]. The operators generating null vectors at level p in the representation $|qp, 11\rangle$, $|11, pq\rangle$, $|11, qp\rangle$ can be obtained from (6) by $\alpha \to -\alpha$, $\alpha \to -\frac{1}{\alpha}$ and $\alpha \to \frac{1}{\alpha}$, respectively.

Generalising the method of ref. [2, 3] we extend the enveloping algebra by allowing complex powers of the generator W_{-1}. The commutation relations with the new generator are:

$$[(W_{-1})^a, W_n^k] = \sum_{j=1}^{\infty} \binom{a}{j} \Big[W_{-1}, ...\Big[W_{-1}, [W_{-1}, W_n^k]\Big]...\Big] (W_{-1})^{a-j} \tag{8}$$

or

$$[W_n^k, (W_{-1})^a] = \sum_{j=1}^{\infty} \binom{a}{j} (W_{-1})^{a-j} \Big[...\Big[[W_n^k, W_{-1}], W_{-1}\Big]...W_{-1}\Big] \tag{9}$$

where $k = 2, 3$ and $\binom{a}{j} = \frac{a(a-1)...(a-j+1)}{j!}$. In both cases we have the commutator of W_n^k with W_{-1} j times. This formula is valid not only for W_n^k but for arbitrary W_{-I}.

We define the generalised Verma module, $(\tilde{V}(h, w, c))$, that can be obtained by acting with $(W_{-1})^a$ on the elements of the original Verma module, for arbitrary complex number a. We use the same ordering as before but now allowing complex powers of W_{-1}. Due to the grading given by L_0 we can split the representation space into eigenspaces of this operator. We call an operator well defined if it has the following form:

$$a_0(W_{-1})^a + \sum_I a_I (W_{-1})^{a-|I|} W_{-I} \tag{10}$$

Here W_{-I} contains negative modes only and we allow the sum to be infinite, but all the coefficients have to be finite. From now on we use a new terminology: we call the term $a_I(W_{-1})^{a-|I|}W_{-I}$ in the above sum an operator at level $|I|$.

This framework we have just established is a consistent framework in the sense that if any well defined operator acts on a state of the generalised Verma module, which was created from the h.w. state by some other well defined operator, then the resulting state is also created by some well defined operator. To see this one has to consider the product of two well defined operators acting on $|hw\rangle$ and reorder it into the usual basis.

$$\begin{aligned}\Big(a_0(W_{-1})^a + \textstyle\sum_I a_I (W_{-1})^{a-|I|} W_{-I}\Big)\Big(b_0(W_{-1})^b + \textstyle\sum_{I'} b_{I'} (W_{-1})^{b-|I|'} W_{-I'}\Big)|hw\rangle \\ = \Big(c_0(W_{-1})^c + \textstyle\sum_{I''} c_{I''} (W_{-1})^{c-|I|''} W_{-I''}\Big)|hw\rangle \quad ,\end{aligned} \tag{11}$$

where $c = a + b$. Clearly the only thing we have to show is that all the $c_{I''}$ coefficients are finite. Considering the commutation relations we can see that the operators which contribute at level $|I''|$ have level less than or equal to $\frac{4}{3}|I''|$. The number of these levels are finite and furthermore each level gives only a finite contribution for the level in question so this shows that the contributions are finite. (In order to see the factor $\frac{4}{3}$ we remark that we can obtain terms, which contain W_{-1} from the reordering of W_{-I} and $W_{-I'}$. However from the commutation relation it follows that we need three W modes to produce one W_{-1}. Considering in a similar manner the nested commutators of W_{-I} with W_{-1} we can say that the number of W_{-1} obtained in this way is at most one quarter of the level, say for example $([[W_{-2}, W_{-1}], W_{-1}] \sim W_{-1})$.)

For later applications we have to show that all singular vectors at level zero are proportional to $|hw\rangle$. The proof is rather lengthy so we just sketch it here. First we suppose that this is not the case i.e.

$$\sum_{I, |I| \neq 0} a_I (W_{-1})^{-|I|} W_{-I} |hw\rangle \tag{12}$$

is a h.w. singular vector, with W_0 eigenvalue w. Then we define an ordering among the modes, we say that $I = \{i_1, \ldots, i_k; i_{k+1}, \ldots, i_n\}$ precedes $J = \{j_1, \ldots, j_l; j_{l+1}, \ldots, j_m\}$ if $|I| < |J|$ or if $|I| = |J|$ then there exists an r such that $i_r < j_r$. Considering the commutations relations we can see that acting on W_{-I} with an L (W) mode we can get at most $[\frac{k+2}{3}]$ ($[\frac{k}{3}] + 1$) W_{-1}s, respectively, where $[x]$ denotes the integer part of x. The idea of the proof is that we suppose that $a_I = 0$ if $|I| < k$ or if W_{-I} can give contributions to a smaller level then $k-1$. From this hypothesis we shall show that the coefficients at level k and the coefficients which can contribute at level $k-1$ have to vanish. However these terms at level $k+l$ contain precisely $3l, 3l+1$ or $3l+2$ W modes. Since the coefficients which contain more Ws are zero due to the induction hypothesis it is not hard to see that all the coefficients have to vanish. First we consider the highest level which contribute and has non zero terms. Then we take the term which contains the most W and the smallest in the ordering defined above with $I = \{i_1, \ldots, i_k; i_{k+1}, \ldots, i_n\}$. Acting on it with L_{i_1-1} (W_{i_1-1} if $k = 0$) we find that the coefficient of $(W_{-1})^{-|I|+1} W_{-I'}$ is not zero. (Here we obtained I' deleting i_1 from I). Since this was the only term which contributes we conclude that a_I vanishes. Following the same procedure from level to level we can show that all the coefficients have to vanish. The only non trivial thing is that if we consider a term at level $k+l$ which contains $3l$ W modes then we can get contributions from the terms which contain $3l-1$ W modes at level $k+l-1$. However taking under investigation these terms first then later the other this problem can be solved easily. We remark that I may contain the mode L_{-1} so we really have to use the fact that the vector is an eigenvector of W_0. Unfortunately this proof can not be applied to the case when the algebra is extended with complex powers of L_{-1}, although the commutation relations are much more simple in this case.

Now we are ready to give a generalisation of the null vectors (6). First we reorder the expression into our basis:

$$\mathcal{O}_{p,1}|pq, 11\rangle = \Big((W_{-1})^p + \sum_{I, |I| \leq p} a_I(p, q, \alpha)(W_{-1})^{p-|I|} W_{-I}\Big)|pq, 11\rangle \tag{13}$$

Then we show that the $a_I(p, q, \alpha)$ coefficients are polynomials in p for fixed I. Since $a_I(p, q, \alpha)$ contains only sums of products of polynomials of p, what we need is that these products are still polynomials in p. Considering the original expression, (6), only those terms can give contributions to $a_I(p, q, \alpha)$ whose W_{-1}s are between $(W_{-1})^p$ and $(W_{-1})^{p-\frac{4}{3}|I|}$. They correspond to the partitions which have at least $p - \frac{4}{3}|I|$ ones. However the number of these partitions can be estimated as a polynomial in p with a power which is a function of $|I|$. If we study the contributions of each partition we see that the product in (7) and the coefficient of the L modes contain less factors than $\frac{4}{3}|I|$. Moreover if we observe that the number of commutations are functions of $|I|$, collecting all the powers, we can conclude that the coefficients after the ordering are polynomials in p. However this means that we can make a unique analytic continuation of this operator and so we have:

$$\mathcal{O}_{a,1}|ab, 11\rangle = \Big((W_{-1})^a + \sum_{I} a_I(a, b, \alpha)(W_{-1})^{a-|I|} W_{-I}\Big)|ab, 11\rangle \tag{14}$$

Here a and b are arbitrary complex numbers and the operator which creates this state is a well defined operator. In the notation we suppress the dependence on q and α if it is clear from context.

Now we show that this vector is a singular vector in the $(ab, 11)$ representation at level a. Let us act first with the operator L_1:

$$L_1 \mathcal{O}_{a,1}|ab, 11\rangle = \sum_I Q_I(a, b, \alpha)(W_{-1})^{a-1-|I|} W_{-I}|ab, 11\rangle \tag{15}$$

Here $Q_I(a, b, \alpha)$ is a linear combination of $a_{I'}(a, b, \alpha)$, where $|I'| \leq \frac{4}{3}|I|+2$ and this way clearly is a polynomial in a. If we take a to be an integer larger than $\frac{4}{3}|I| + 2$ then $Q_I(N, b, \alpha)$ is the coefficient of an operator which is an element of the enveloping algebra and which we would find in $L_1 \mathcal{O}_{N,1}|Nb, 11\rangle$. Since this coefficient really has to vanish we conclude that $Q_I(a, b, \alpha)$ is a polynomial in a which vanishes for every integer larger than $\frac{4}{3}|I| + 2$, so it is identically zero. Similarly we can prove that this vector is a h.w. vector, i.e. it is annihilated by the positive modes and can be described by the $(-a, a + b)$ h.w. state.

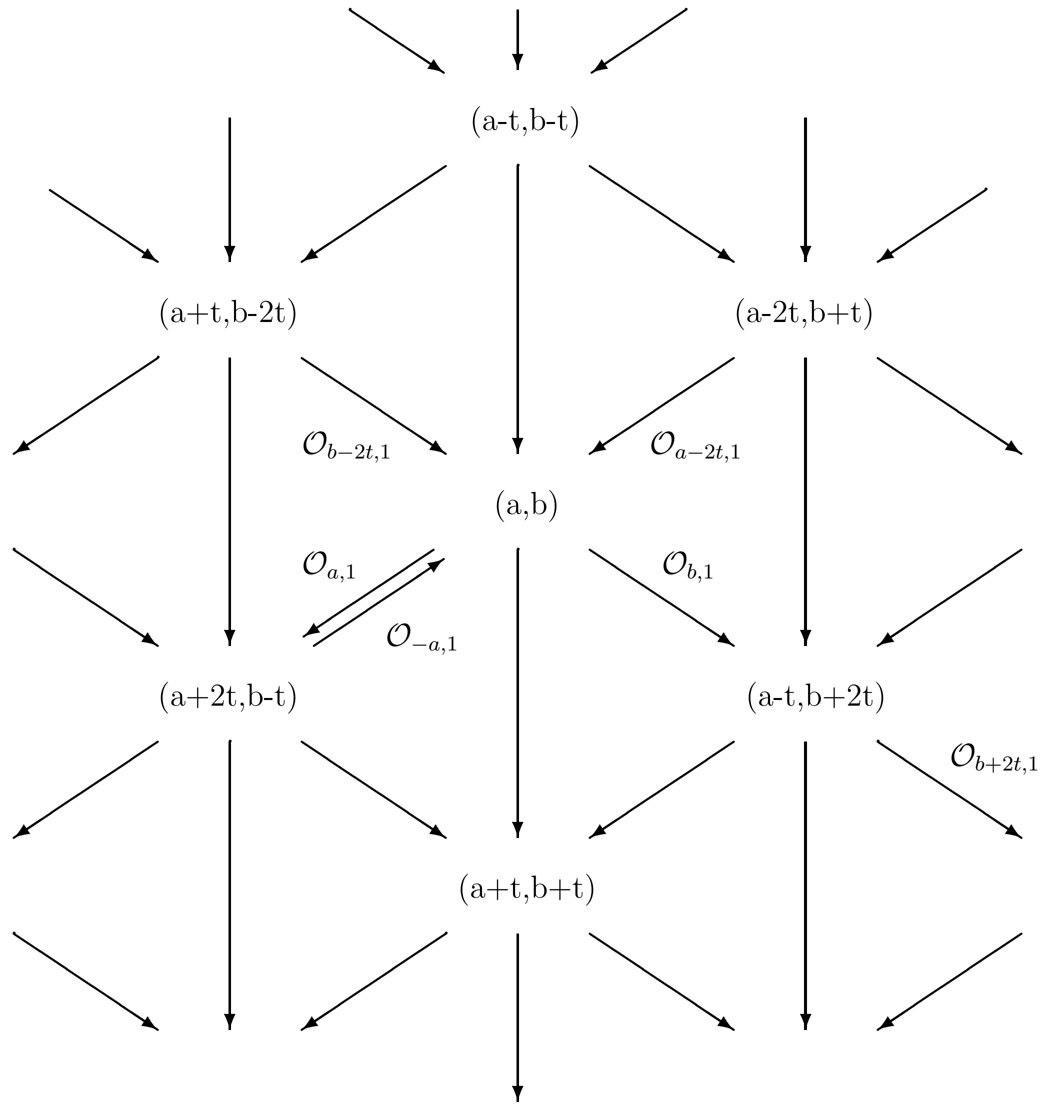
From now on we write (a, b) for the $(ab, 11)$ h.w. vector. The remnants of the parametrisation ambiguity mentioned earlier in the language of (a, b) are the shifted Weyl reflections i.e. the equivalent parametrisations are: $(b, -a - b + 3t)$, $(-b + 2t, -a + 2t)$, $(-a - b + 3t, a)$, $(a + b - t, -b + 2t)$ and $(-a + 2t, a + b + t)$. This means that there are at least two ways to generate null vectors in a certain representation.

Now we shall show that we have only one singular vector at a certain level with a given W_0 eigenvalue. To see this first we remark that the null vectors, which are generated by some $\mathcal{O}_{a,1}$, have always $(W_{-1})^a$ leading coefficient. Suppose that we generated a singular vector at level a: $\mathcal{O}_{a,1}|ab\rangle$. Since this h.w. state can be described by the $(-a, a + b)$ parameters, we have a singular vector at level zero, namely:

$$\mathcal{O}_{-a,1}\mathcal{O}_{a,1}|ab, 11\rangle = (1 + (W_{-1})^{-1}(\ldots))|ab, 11\rangle = |ab, 11\rangle \tag{16}$$

In the second equality we used the fact that the only singular vector at level zero is the h.w. state itself. If there is another singular vector at this level with the same W_0 eigenvalue then it may have leading coefficient $(W_{-1})^a$. Since the null vectors are not the same, as we supposed, taking their difference we have a singular vector at level a with leading coefficient at most $(W_{-1})^{a-1}$. But acting now with $\mathcal{O}_{-a,1}$ on this h.w. state we obtain a singular vector at level zero, which does not contain $|ab, 11\rangle$, i.e. it is zero. However this implies that the vector is zero itself. Summarising we found that the only singular vector at level a with weight $(-a, a + b)$ in the (a, b) representation is the same as that which we can generate, and so it is unique.

Now we take a representation described by (a, b). Acting on it with one of the operators, say with $\mathcal{O}_{b1}$, the representation obtained can be described by $(a+b, -b)$. Clearly this weight is connected to the original h.w. by a Weyl reflection. We can use now the shifted Weyl reflections and reparametrise it as $(a - t, b + 2t)$. So we can act again and obtain the $(a - 2t, b + 4t)$ representation. Using the six directions, given by the reparametrisation invariance, acting with the appropriate operators we obtain the following picture, which looks like the weight diagram of sl_3:



Each arrow indicates that we have an operator that generates a singular vector in the representation. If we have a singular vector in (a, b) at level a then we have a singular vector in $(a + 2t, b - t)$ at level $-a$. This shows that if there is an arrow which goes in one way then there is another one which goes in the opposite way. The diagram is commutative since the singular vectors generated are unique.

We are now ready to generate null vectors in the generic case, i.e. in the (rp, sq) representation space. First we remark that this representation is equivalent to the $(ab, 11)$ representation if $a = r - (s - 1)t$ and $b = p - (q - 1)t$. The null state in this representation has parameters $(-r, r + p, sq)$ or equivalently parameters $(a^{'}b^{'}, 11)$ where $a^{'} = -r - (s - 1)t$ and $b^{'} = r + p - (q - 1)t$. Of course we can use the reparametrisation and write $a' = r + (s + 1)t$ and $b' = p - (s + q - 1)t$ for this state. However this state can be obtained in the following form:

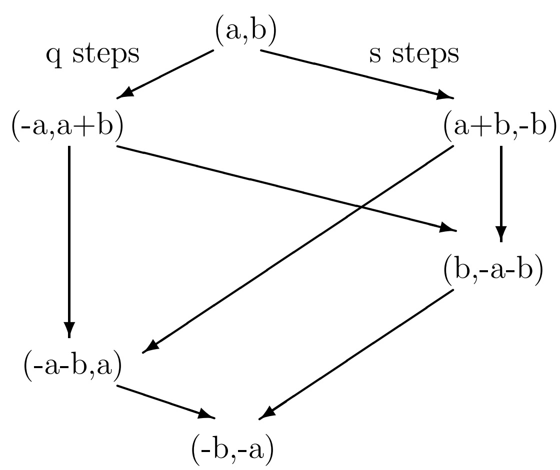
$$\mathcal{O}_{r,s}|rp, sq\rangle = \mathcal{O}_{r+(s-1)t,1}\mathcal{O}_{r+(s-3)t,1} \dots \mathcal{O}_{r-(s-3)t,1}\mathcal{O}_{r-(s-1)t,1}|rp, sq\rangle \tag{17}$$

Since this singular vector has the same weight as that which is given by the Kac formula and the singular vectors are unique we conclude that the state (17) is an element of the Verma module and defines a null state at level rs.

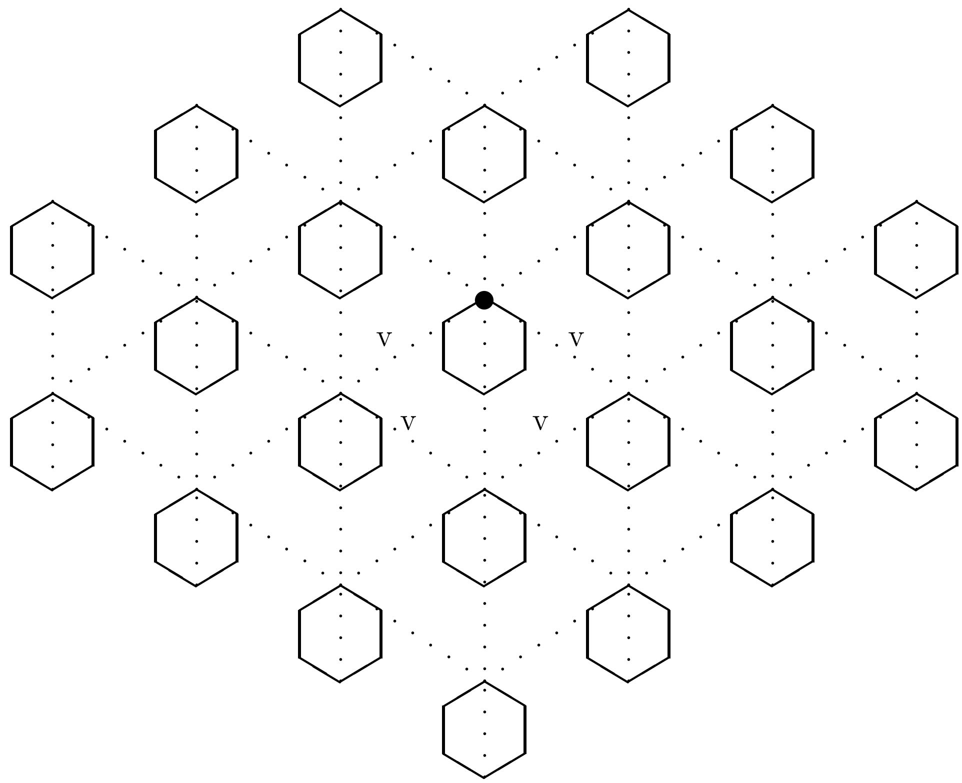
We remark that there are a lot of other ways to generate the same singular vector. We can reparametrise the $|rp, sq\rangle$ h.w. vector as $|11, ab\rangle$ and use the other type of operator $\mathcal{O}_{1,a}$, which have similar properties as the $\mathcal{O}_{a,1}$ operators. Since they do not give any new information about the theory and each singular vector is unique (so it is enough for us to generate it one way) we are not concerned with the other type of operators.

In order to obtain the embedding relations of the singular vectors we have to consider those h.w. states of the previous diagram which are at positive integer level. This condition can be formulated as follows: after m steps right and n steps left the level is a positive integer, i.e. $np + mr - t(n(q-n) - m(s+n-m))$ is a positive integer. We have to handle in a different way the case when t is irrational or rational.

First we consider the irrational case. Clearly in this case the coefficient of t has to be zero. This means that we have the following possibilities: $n = q, m = 0$; $n = 0, m = s$; $n = q + s, m = s$; $n = q, m = q + s$ or $n = q + s, m = q + s$. However these null vectors are exactly the same we would obtain from the character formula [9] or from the Kac determinant and the embedding picture looks like the Weyl reflections of the top h.w. vector.



Now we consider the case of rational t, say $t = \frac{u}{v}$. Furthermore we suppose that $p + q < r + s < u < v$. They correspond to the minimal models. It is not hard to see that moving v units in each direction as many times as we want we arrive always at a positive integer level. Moreover we will have the same embedding picture beginning with each point as we had in the irrational case. This means that in the following diagram the hexagons remind us of the earlier embedding diagram with size q and s and they build a v periodic lattice:



Clearly what we obtain is nothing but the levels given by the character formula where the singular vectors exist. Each vertex of the hexagons on the diagram denotes a singular vector which is embedded into the top module. (For a much more detailed analysis see our forthcoming paper [10]). Since the singular vectors are unique at a certain level we constructed all the singular vectors given by the Kac determinant.

Now we turn to the analysis of the case when W_0 is not diagonalisable among the singular vectors. The problem arises only when we have two singular vectors at the same level. This can be described by the weight (a,a) and it has $w=0$. We remark that if $w=0$ in $|hw\rangle$ and we have a singular vector at a certain level then we have another one on the same level changing every W mode to its negative. This shows that the eigenvalues of the singular vectors are inverse of each other. Without loss of generality we consider the case when the singular vectors are at an even level. We split the eigenvectors into even and odd part as: $(P_+ + xP_-)|hw\rangle$ and x is defined such a way that

$$W_0 P_- |hw\rangle = P_+|hw\rangle \ ; \quad W_0 P_+|hw\rangle = \gamma P_-|hw\rangle \tag{18}$$

Since this state is an eigenvector of W_0, with eigenvalue x, we have $\gamma = x^2$. There is another singular vector namely the $(P_+ - xP_-)|hw\rangle$ state with eigenvalue $-x$. The W_0 eigenvalues of the singular vectors are $\alpha(a-t)a$ and $-\alpha(a-t)a$, respectively. If the eigenvectors correspond to the same eigenvalues then $x=0$ and necessarily $t=a$. In this case the only h.w. singular vector is P_+ because the h.w. singular vector at level a always has $(W_{-1})^a$ leading term. This shows that $xP_- \to 0$ when $x \to 0$. However from this it follows that P_- is finite since if P_-

were not finite then necessarily $x^\beta P_-$, $0 < \beta < 1$, would be a finite non zero eigenvector of W_0 with zero eigenvalue (18), but this is excluded since the only eigenvector of W_0 with zero eigenvalue is P_+. Since P_- is obviously not zero (18), it is a singular vector. This singular vector can be obtained in the $x \to 0$ $t \to a$ limit from the singular vectors defined earlier and W_0 maps it into the h.w. singular vector. This shows that they are in the same Jordan cell. The case when the singular vectors are at odd level is the same except for changing even with odd.

In summary, we consistently established a framework for the WA_2 algebra containing complex powers of one of the generators. In this extended space we succeeded in generating singular vectors given by the Kac formula. The generalisation of the method for other W algebras is in progress [10].

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