

A LATTICE SIMULATION OF THE SU(2) VACUUM STRUCTURE [†]

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Abstract

In this article we analyze the vacuum structure of pure SU(2) Yang-Mills using non-perturbative techniques. Monte Carlo simulations are performed for the lattice gauge theory with external sources to obtain the effective potential. Evidence from the lattice gauge theory indicating the presence of the unstable mode in the effective potential is reported.

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I. INTRODUCTION

Despite considerable progress a complete solution of non-abelian gauge theories has yet to be found. The ultraviolet properties of such theories have been well analyzed, but even in the simplest case, i.e. $SU(2)$ without matter, little is known about their infrared properties and vacuum structures. In order to gain a better understanding of these theories a necessary first step is the study of the vacuum. For a general review of the vacuum structures see [1]. In this article we will show some lattice techniques and simulations that can be important to reveal some aspects of the vacuum.

Several lattice Monte Carlo simulations [2] have been done to examine the vacuum structures. These simulations are usually very difficult because of the weakness of clear signals. The lattice simulation that we are proposing in this article use a method that remove the zero modes, and increase the possibilities of extracting information of the vacuum structures.

During the last few years many authors [3] have also explored the 3-dimensional case. It is advantageous to use 3-dimensional lattices because of the possibility to obtain cleaner data and larger volumes.

The $SU(2)$ case should not be fundamentally different from other non-abelian gauge theories. In fact, there is no reason why the QCD vacuum should differ qualitatively from the $SU(2)$ case. However, since the $SU(2)$ case is easier to implement in lattice calculation, it was chosen for our analysis.

II. EFFECTIVE POTENTIAL FOR NON-ABELIAN YANG-MILLS THEORIES

A powerful method to investigate the properties of Yang-Mills theories is to compute the effective potential in the background gauge [4]. This manifestly gauge invariant scheme is based on the observation that the loop expansion corresponds to an expansion in the parameter \hbar which multiplies the entire action. Hence a shift of the fields or a redefinition of the division of the Lagrangian into free and interacting parts can be performed at any

finite order of the loop expansion without violating the manifest gauge invariance.

Let us split the gauge field into a background, \mathcal{A}_μ^b , and a quantized field, η_μ^b , as $A_\mu^b = \mathcal{A}_\mu^b + \eta_\mu^b$. Therefore the effective action will also be a functional of \mathcal{A}_μ^b . The gauge fixing condition is that the covariant four-divergence of the quantum field computed on the background vanishes,

$$D_\mu \eta_\mu^b \equiv (\partial^\mu \delta^{bc} - ig \mathcal{A}_\mu^d (T^d)_{bc}) \eta_\mu^c = 0. \quad (2.1)$$

The generating functional of connected Green's functions is consequently defined as:

$$e^{\frac{i}{\hbar} \tilde{W}[J, \mathcal{A}]} = \int (d\eta) \Delta(\mathcal{A}, \eta) e^{\frac{i}{\hbar} \{S[\mathcal{A}+\eta] + (J, \eta) - \frac{1}{2\alpha} (G^a)^2\}}, \quad (2.2)$$

where $\Delta(\mathcal{A}_\mu^b, \eta_\nu^b)$ is the Faddeev-Popov determinant, and G^a is the gauge-fixing term. As pointed out by [5] several subtleties arise from the gauge fixing condition, due to the choosing of the α gauge fixing parameter in the presence of a non-trivial background. A complete discussion of this point is beyond the scope of this article.

Following Abbot's notation [4], let us define $\tilde{Q} = \delta \tilde{W}[J, \mathcal{A}] / \delta J$. Therefore the Legendre transform of \tilde{W} is

$$\tilde{\Gamma}[\tilde{Q}, \mathcal{A}] = \min_{\{J\}} \left[\tilde{W}[J, \mathcal{A}] - (J, \tilde{Q}) \right], \quad (2.3)$$

and the inverse relation is: $J = -\delta \Gamma[\tilde{Q}, \mathcal{A}] / \delta \tilde{Q}$.

Using the background field as discussed above, the Lagrangian for the SU(2) Yang-Mills theory is written as:

$$\begin{aligned} \mathcal{L}(A_\mu^b = \mathcal{A}_\mu^b + \eta_\mu^b) = & - \frac{1}{4} F_{\mu\nu}^b F_b^{\mu\nu} = - \frac{1}{4} F(\mathcal{A})_{\mu\nu}^b F(\mathcal{A})_b^{\mu\nu} + \\ & - \frac{1}{2} \eta_\mu^b (-D^2 \delta^{\mu\nu} + D^\mu D^\nu)^{bc} \eta_\nu^c + g \varepsilon^{bcd} \eta_\mu^b F(\mathcal{A})_c^{\mu\nu} \eta_\nu^d + \\ & - g \varepsilon^{bcd} (D^\mu \eta^\nu)^b \eta_\mu^c \eta_\nu^d - \frac{1}{4} g^2 \varepsilon^{bcd} \varepsilon^{bef} \eta_\mu^c \eta_\nu^d \eta_\rho^e \eta_\sigma^f g^{\mu\rho} g^{\nu\sigma}. \end{aligned} \quad (2.4)$$

As shown in [6] there are only two possible backgrounds that yield a static chromomagnetic field that is the interesting field configuration to study the vacuum properties. One is the so-called "non-abelian background" (see [7] and [5]), the other is called "Abelian", and is the one that we will discuss in this article.

Without loss of generality we can always choose the chromomagnetic field along the z direction, the abelian background can be written as:

$$\mathcal{A}_\mu^b = \frac{1}{2} H \delta^{b3} (\delta_{\mu 2} x - \delta_{\mu 1} y), \quad (2.5)$$

where H is constant. We have $F(\mathcal{A})F(\mathcal{A}) = \frac{1}{2}H^2$. As discussed in [8] and [9] after several manipulation the effective potential can be evaluated using:

$$-\frac{V(H)}{\Omega T} = \frac{1}{2}H^2 + \frac{gH}{8\pi^2} \left(\int_{-\infty}^{\infty} dk_z \sum_{n=0}^{\infty} \sum_{s=\pm 1} \sqrt{\nu_{k,n,s}} - 2 \int_{-\infty}^{\infty} dk_z \sum_{n=0}^{\infty} \sqrt{\tilde{\nu}_{k,n}} \right), \quad (2.6)$$

where ΩT is the 4D volume, and $\nu_{k,n,s} = (2n + 1 + 2S_z)gH + k_z^2$ are the eigenvalue of the quadratic part of the Lagrangian. To derive this expression, the multiplicity of each eigenvalue, the overall factor 2 of charge degeneracy, and the ghost contribution has been taken in account. The last term is the contribution of the ghosts, that is the eigenvalue of the operator $(-D^2)^{bc}$ with the same boundary conditions as for the gluon sector. One finds $\tilde{\nu}_{k,n} = k_z^2 + (2n + 1)gH$. To compute the expression (2.6) we have to regularize using the Salam and Strathdee method.

This expression yields to the famous Savvidy [10] result, which is the one-loop effective energy density for SU(2) in the presence of a static chromomagnetic field with abelian background:

$$\frac{V(H)}{\Omega T} = \frac{1}{2}H^2 + \frac{11}{48\pi^2}g^2H^2 \left(\ln \frac{gH}{\mu^2} - \frac{1}{2} \right) + \dots \quad (2.7)$$

The remarkable feature of this expression is that it exhibits a minimum for H different from zero, namely at $gH_{min} = \mu^2 \exp(-\frac{24\pi^2}{11g^2})$. As Nielsen and Olesen [9] realized, this minimum has unstable modes. This instability can be seen from the fact that there is an imaginary part of the effective energy density. The imaginary part comes solely from the $n = 0$ and $S_z = -1$ contribution and in fact can be calculated directly,

$$Im \left\{ \frac{V(H)}{\Omega T} \right\} = -\frac{c}{2} Im \left\{ \int_{-\infty}^{\infty} dk \sqrt{k^2 - gH} \right\} = -\frac{1}{8\pi} g^2 H^2. \quad (2.8)$$

Note that the existence of the imaginary part is essential to obtain asymptotic freedom. This is because in the absence of the imaginary part the ultraviolet limit of (2.6) implies

that the beta function will be the same as the one of a scalar particle of mass $m^2 = 2gH$. It is just the imaginary part which prevents us, after regulation, from rotating the integration contour in the complex plane and thereby spoiling the asymptotic freedom.

Although H_{min} is not a classical solution there is a possibility that non-perturbative effects might cause this configuration to dominate the vacuum. Since the above mentioned preliminary studies were done, the property of this non-trivial vacuum has been thoroughly investigated. In particular a scenario, the so called ‘‘Copenhagen vacuum’’ [9] was proposed, in which quantum fluctuations and gluons condensations might create domains of constant chromomagnetic configurations.

In reference [8] this scenario was criticized arguing that in a strong field configuration a perturbative analysis is unreliable, and unstable configurations can be analyzed only by non-perturbative methods. Therefore the possible technique presently available to tackle this problem is the lattice regularization. Monte Carlo simulation can be used to generate the typical vacuum configurations, and thus, to obtain direct information about the vacuum structure.

III. GENERAL RULES FOR IMPLEMENTING THE BACKGROUND FIELD METHOD ON LATTICE

There are different ways to implement the background field method for lattice gauge theory [11]. The procedure presented here is quite general and has the advantage that it can be applied with different kinds of sources. Alternative methods are discussed in [2].

Instead of the usual variable, $U_n^\mu = e^{iag\eta_b^\mu(n)T^b}$, we introduce a new link variable in the presence of a background field, $U(\mathcal{A})_n^\mu = f(a, \eta_\mu^{(\mathcal{A})a}(n))$, in such a way that the continuum limit gives the expected continuum expressions,

$$\lim_{a \rightarrow 0} \eta_\mu^{(\mathcal{A})b}(n) = \eta_\mu^b(x) + \mathcal{A}_\mu^b(x) \quad (3.1)$$

$$\lim_{a \rightarrow 0} \left(S[U(\mathcal{A})_n^\mu] - S[U_n^\mu] \right) = S[\eta_\mu^a + \mathcal{A}_\mu^b] - S[\eta_\mu^a], \quad (3.2)$$

where S is the usual action for lattice gauge theories, and can eventually be substituted by an improved action.

The Euclidean generator functional is defined as:

$$e^{-\bar{W}[j,\mathcal{A}]} = \frac{\int_{bc(\eta(\mathcal{A}))} (dU) e^{-S_W[U(\mathcal{A})] - j \kappa[\eta(\mathcal{A})]}}{\int_{bc(\eta)} (dU) e^{-S_W[U]}} , \quad (3.3)$$

where κ is a functional of the source that must be chosen to recover the continuum limit in the scaling region. The normalization here is chosen to be the same as the one without the background field. This choice is not the only possible one since the potential is defined only up to an arbitrary additive constant.

The important point is that in the presence of a background field we must take the boundary condition which makes $\eta(\mathcal{A})$ periodic, $U(\mathcal{A})_n^\mu = U(\mathcal{A})_{n+La}^\mu$, where L is the lattice size. However the normalization integral is calculated with periodic boundary conditions for the links without the constant chromomagnetic field, $U_n^\mu = U_{n+La}^\mu$.

Amongst the possible ways to define the link in the presence of a background field, a natural choice is

$$U(\mathcal{A})_n^\mu = e^{\frac{i}{2}agA_\mu^b(n)T^b} e^{iag\eta_\mu^b(n)T^b} e^{\frac{i}{2}agA_\mu^b(n)T^b} . \quad (3.4)$$

We used three kinds of different sources in our computations. For simplicity in the following, we shall always choose the z direction as the spatial direction of the constant chromomagnetic field. The first possibility is the so called ‘‘abelian source’’ because it points in the third color direction. Such a source explores the Cartan subalgebra of $SU(2)$, and thus is called abelian. The natural counterpart is the ‘‘diagonal source’’, when the source is a combination of the generators in the color directions 1 and 2. For simplicity we choose the combination which is proportional to $T_\pm = T^1 \pm iT^2$. This source explores the non-diagonal matrix elements of $SU(2)$. A third possibility is the so called ‘‘quadratic source’’ where a combination which is parallel to the external chromomagnetic field is used.

These three sources have advantages and disadvantages when used in lattice Monte Carlo simulations. We now describe the abelian source in more detail. For the abelian source the term in equation (3.3) read as

$$j \kappa[\eta(\mathcal{A})] = \sum_{n,b,\mu} j \frac{a^4}{2} \delta^{b3} (\delta_{\mu 2} - \delta_{\mu 1}) \eta_{\mu}^{(\mathcal{A})b}(n), \quad (3.5)$$

with j as an arbitrary source strength. Using the prescription given by (3.4), the link variable is

$$\begin{aligned} U(\mathcal{A})_n^{\mu} &= U_n^{\mu} \quad (\text{for } \mu = 0, 3) \\ U(\mathcal{A})_n^1 &= e^{-\frac{i}{4}a^2gHn_2T^3} U_n^1 e^{-\frac{i}{4}a^2gHn_2T^3} \\ U(\mathcal{A})_n^2 &= e^{\frac{i}{4}a^2gHn_1T^3} U_n^2 e^{\frac{i}{4}a^2gHn_1T^3}, \end{aligned} \quad (3.6)$$

in the presence of a background field (2.5). Then equation (2.3) becomes

$$\tilde{\Gamma}[\tilde{q}, \mathcal{A}] = \min_{\{j\}} \left[\tilde{W}[j, \mathcal{A}] - (j, \tilde{q}) \right], \quad (3.7)$$

which can be obtained explicitly by (3.3).

IV. THE PRESENCE OF THE UNSTABLE MODE ON LATTICE

We detect the presence of unstable modes in the lattice is by analyzing the energy density. Let us consider the contributions to the effective potential from equation (2.6). As discussed above, the quantity,

$$\sqrt{\nu_{k,n,s}} = \sqrt{gH (2n + 1 + 2S_z) + k_z^2} \quad (4.1)$$

becomes imaginary for $S_z = -1$, $n = 0$ and sufficiently small k_z^2 . Due to the finiteness of the lattice k_z is quantized as well, as $k_z = 2m\pi/L_z$, where m is an integer. The lowest inhomogeneous z -mode, $m = 1$, becomes stable for lattices whose extent in the z -direction is smaller than

$$L_z^{critical} = \frac{2\pi}{\sqrt{gH}}. \quad (4.2)$$

The homogeneous mode, $m = 0$, which is always unstable, is eliminated by imposing the condition

$$\prod_{j=1}^{L_z} U_{z=j}^3 = 1 , \quad (4.3)$$

in the path integral. This enables us to search for the critical size, $L_z^{critical}$, by changing H and looking for a sign of instability.

In these Monte Carlo simulations we generated a background field H in the z direction and measured the expectation value of the plaquette in the 1-2 plane F_{12} as a function of β and j . We used a heat bath updating procedure with periodic boundary conditions in presences of sources. The special feature of this simulation was that we forced the Polyakov line in the z direction (4.3) to take a fixed value. We made Monte Carlo simulations in a lattice volume $L^3 * L_z$, where L is the size of the x, y, t directions. We bypassed the difficulty of needing for a huge lattice because the directions x, y and t do not have to be large. In fact, they are irrelevant for the homogeneous mode, $m=0$, and the dependence on these directions shows up only through the excitation of higher n -modes which represent higher order perturbative effects. Lattice artifacts might induce some small dependence on t, x, y , but lattice simulations with different $L_t * L_x * L_y$, but fixed L_z , show insignificant dependence on these parameters, as predicted theoretically. We have varied L_z from 4 to 50, and L_t, L_x, L_y from 4 to 16 reaching a maximum lattice of $10^3 * 50$. Unfortunately our programs were designed only to handle even lattice sizes because of the checkerboard addresses.

We monitored the quantity

$$X = \frac{P_z[F_{12}(\beta, 0)] - P_z[F_{12}(\beta, j)]}{j P_z[F_{12}(\beta, 0)]}, \quad (4.4)$$

where P_z is the plaquette in the z direction. This quantity is proportional to the contribution of the plaquette in the z direction of $\Delta E(F_{12}^{ext}/(F_{12}^{ext})^2)$ and therefore is very sensitive to the presence of the unstable modes. We measured X for different values of β and j performing, for large j 4500 sweeps after discharging 500 for thermalization. For smaller j we increase the number of sweeps until a significant amount of data was collected. 4 updating sweeps were made between subsequent measurements.

Our data show no sign of the unstable mode away from the critical β region ($\beta = 2.1 - 2.5$). The situation changes dramatically in the critical region where the instability appears as a decrease of the vacuum energy contribution to the plaquette in the z direction. This effect becomes more evident in the presence of strong sources.

Thus far, we analyzed systematically the critical region of β from 2.10 to 2.50 with steps of 0.05 for $j = 1.00, 0.75, 0.50, 0.25$; for some β we also explored $j = 0.125$ and $j = 0.1$. We computed the value of $L_z^{critical}$ by interpolating for X and taking the median value. $L_z^{critical}$ was found to be dependent on the sources strength j and β in this region.

To represent a physical quantity $L_z^{critical}$ must scale with j according to equation (4.2). The manifestation of the rescaling is evident for β near $\beta = 2.30$. Moreover the agreement between the theoretical effective potential predictions and the lattice simulation results is very good.

An important condition is that $L_z^{critical}$ should be greater than the deconfinement phase transition length, i.e. $L_z^{critical}$ must belong to the confinement phase. From the renormalization group considerations we have

$$L_z^{critical} \Lambda^{critical} = \left(\frac{11}{6\pi\beta}\right)^{-51/121} e^{\frac{-3\pi^2\beta}{11}} \left[1 + O\left(\frac{1}{\beta}\right)\right]. \quad (4.5)$$

Hence the ratio between the $\Lambda^{critical}$ and the renormalization scale parameter Λ is independent of β . Using the well known value of Λ given by [12], we have

$$\begin{aligned} \frac{\Lambda}{\Lambda^{critical}(j = 1.00)} &= 1.226 \pm 0.019 \\ \frac{\Lambda}{\Lambda^{critical}(j = 0.75)} &= 1.390 \pm 0.025 \\ \frac{\Lambda}{\Lambda^{critical}(j = 0.50)} &= 1.572 \pm 0.026 \\ \frac{\Lambda}{\Lambda^{critical}(j = 0.25)} &= 1.934 \pm 0.031. \end{aligned} \quad (4.6)$$

It is clear from these results that $L_z^{critical}$ belongs to the confinement phase and there is a good agreement with the renormalization group equation (see Fig. 2).

Using our data we also evaluated L_{deconf} as a function of β . These values are obtained by comparing the plaquette in the t and z directions. From L_{deconf} using the renormalization

equation we estimated the renormalization scale parameter Λ . This data are compatible with reference [12] where simulations were done specifically to analyze the deconfinement transition. Nevertheless the comparison is interesting because it shows that from our data we can extract L_{deconf} which is clearly distinct from $L_z^{critical}(j)$.

The limit $j \rightarrow 0$ is obtained by studying the ratio (4.6) for several j and then extrapolating to zero. The result is $\Lambda/\Lambda^{critical}(j = 0) = 2.5 \pm 0.2$, which corresponds to $L_z^{critical}(\beta = 2.3, j = 0) \sim 18$. The ratio of the characteristic lengths is found to be

$$\frac{L_z^{critical}}{L_{deconf}} = 2.5 \pm 0.2. \quad (4.7)$$

It is worthwhile noting that the energy density measured by (4.4) shows a small peak at $\tilde{L}_z \approx 2L_z^{critical}$, indicating the onset of the instability of the next $k = 4\pi/L_z$, mode. One may expect weaker singularities at $\approx mL_z^{critical}$ which correspond to higher momentum modes, too.

V. SUMMARY AND CONCLUSIONS

We support the analysis made by [8] showing that the vacuum structure of SU(2) is a non-perturbative effect, necessitating lattice regularization. In particular we analyzed the difference between stable and unstable configurations, and the origin of the instability for SU(2).

Near the critical value of the coupling constant (near $\beta = 2.3$) we detected the presence of the unstable mode for the first time by Monte Carlo simulations. In particular by changing the lattice volume and the source j we are able to turn the unstable mode on and off. This allowed us to analyze the behavior of the unstable mode, showing that it has the correct behavior in the limit $j \rightarrow 0$ and under the renormalization group equations.

We found that there is a new length scale, $L^{critical}$, in the theory given by (4.7) which is significantly larger than the confinement radius, L_{deconf} . This is rather puzzling since no correlations were expected to show up beyond the confinement radius, L_{deconf} . Another

surprising result was the appearance of the singularity driven by the instability of the higher momentum mode with momentum $k = 4\pi/L_z$. This suggests the presence of "resonances" corresponding to even longer length scales, $mL^{critical}$.

One may interpret our results by exchanging the z and the time directions. In this language we found singularities in the pressure at temperatures below the deconfinement transition driven by unstable electric condensate. In order to make more precise conclusions one has to delve in to the continuum and the $j \rightarrow 0$ limit.

We can establish two conditions that the lattice system should satisfy in order to reflect the richness of the non-perturbative vacuum of the continuum theory. One such condition is that a lattice size greater than $L_z(\beta, j = 0)$ is needed to display the instability. Another one comes from the observation that the instability disappears outside the scaling window. Thus this instability is a distinguishing feature of the continuum rather than the strong coupling vacuum. In order to study the continuum theory we should remain in the region where the instability is manifest. The disappearance of the instability as $\beta \rightarrow 0$ may provide a clue to understanding the difference between the physics of the continuum and the strong coupling lattice theory.

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FIGURE CAPTIONS

Figure 1.

The quantity $X = \frac{P_z(\beta,0) - P_z(\beta,j)}{jP_z(\beta,0)}$ as a function of the lattice size L_z for $\beta = 2.35$ and $j = 1.0$ (stars); $j = 0.75$ (squares); $j = 0.50$ (crosses); $j = 0.25$ (diamonds).

Figure 2.

$L_z^{critical}(j = 1.00)$ (stars), $L_z^{critical}(j = 0.75)$ (squares), $L_z^{critical}(j = 0.50)$ (crosses), $L_z^{critical}(j = 0.25)$ (diamonds), with their renormalization group dependence fit (dashed lines); and the deconfinement transition from reference [12] (full line).

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