Renormalized spin coefficients in the accumulated orbital phase for unequal mass black hole binaries

László Á. Gergely[1,2⋆], **Peter L. Biermann**[3,4,5,6,7†], **Balázs Mikóczi**[8¶], **Zoltán Keresztes**[1,2‡]

Abstract. We analyze galactic black hole mergers and their emitted gravitational waves. Such mergers have typically unequal masses with mass ratio of the order 1/10. The emitted gravitational waves carry the imprint of spins and mass quadrupoles of the binary components. Among these contributions, we consider here the quasi-precessional evolution of the spins. A method of taking into account these third post-Newtonian (3PN) effects by renormalizing (redefining) the 1.5 PN and 2PN accurate spin contributions to the accumulated orbital phase is developed.

[1] Department of Theoretical Physics, University of Szeged, Tisza Lajos krt 84-86, Szeged 6720, Hungary
[2] Department of Experimental Physics, University of Szeged, Dóm tér 9, Szeged 6720, Hungary
[3] Max Planck Institute for Radioastronomy, Bonn, Germany
[4] Department of Physics and Astronomy, University of Bonn, Germany
[5] Department of Physics and Astronomy, University of Alabama, Tuscaloosa, AL, USA
[6] Department of Physics, University of Alabama at Huntsville, AL, USA
[7] FZ Karlsruhe and Physics Department, University of Karlsruhe, Germany
[8] KFKI Research Institute for Particle and Nuclear Physics, Budapest 114, P.O.Box 49, H-1525 Hungary
⋆gergely@physx.u-szeged.hu †plbiermann@mpifr-bonn.mpg.de
¶mikoczi@rmki.kfki.hu ‡zkeresztes@titan.physx.u-szeged.hu

1. Introduction

Mankind has admired the sky with its Milky Way, stars, planets, moons and the Sun, the meteors and the Northern Light. In the age of satellites we map the sky through all electromagnetic frequencies and also through high energy cosmic particles. Gravitational waves are the last great frontier to be reached and passed for the search for our physical understanding of the universe via the messengers reaching us with the speed of light or very close to it:

Gravitational waves, these fleeting distortions of space, the signature of accelerating masses in the universe, remain to be discovered. Therefore it is of paramount importance to understand, which objects are about to generate strong gravitational waves.

When the black holes residing in the centers of almost all galaxies merge, following the merger of their host galaxies, relatively strong gravitation waves are emitted and various aspects of this process were already investigated [1]-[6]. A recent review of the generic aspects of these galaxies nuclei as sources for ultra high energy cosmic rays is in [7].

Starting from the broken powerlaw mass distribution of galactic central black holes [8]-[11], with simple assumptions on the cross section and relative velocity of the two galaxies, we have shown [6] that the most likely mass ratio $\nu = m_2/m_1 \leq 1$ of the merging black holes is between 1/3 and 1/30, thus a typical value will be at about 1/10. Within this range of mass ratios the key stages of the evolution leading to and following the merger can all be treated with post-Newtonian (PN) approximations. A numerical analysis of such mass ratios was recently presented in [12].

In the next section we describe the sequence of events which will accompany the merger of two supermassive galactic black holes. Then from Section 3 we concentrate on the gravitational radiation dominated dissipative regime. An important quantity in gravitational wave detection is the *number of cycles spent in the frequency range of the detector*. This quantity is proportional to the *accumulated orbital phase* (the total angle covered by the quasi-circular orbital evolution). The gravitational waves emerging from this process carry the imprint of the physical characteristics (finite size) of the black holes, like their spins and quadrupole moments. Spin precessions allow for increased accuracy and degeneracy breaking among the parameters characterizing the source [13].

We remark that according to [14], the mass quadrupole - mass monopole coupling for maximally rotating black holes (with quadrupole moment originating entirely in their rotation) gives rise to contributions in the accumulated orbital phase larger by a half to one order of magnitude than the spin-spin contribution, at least for the typical unequal mass ratios. Therefore for a successful detection this qualifies as the second most important finite mass contribution to be considered, after the spin-orbit effect.

In Section 3 we discuss the effect of the quasi-precessional evolution of the spins on the accumulated phase of the gravitational waves and we present estimates of the PN order in which various finite size contributions to the change in the relative configurations of the spins and orbital angular momentum contribute. The cumbersome equations leading to these estimates are presented in the Appendix.

Sections 4 and 5 explore a convenient analytical way to encompass the effect of spin and quadrupolar quasi-precessions even at a lower level accuracy. The leading order contribution to the spin-spin (Section 4) and spin-orbit (Section 5) contributions to the accumulated orbital phase are discussed both for the equal mass and unequal mass cases in distinct subsections.

We summarize our findings in the Concluding Remarks.

2. The merger of two supermassive galactic black holes: an odyssey

First the dance: Two galaxies with central black holes approach each other to within a distance where dynamical friction keeps them bound, spiraling into each other. In the case when both galaxies with their central black holes were radio galaxies, their jets get distorted and form the Z-shape [15]: the radio emitting blobs and tails produce the appearance of dancing veils.

Second the meeting of the eyes: The central regions in each galaxy begin to act as one unit, in a sea of stars and dark matter of the other galaxy. During this phase, the central region can be stirred up, and produce a nuclear starburst.

Third the lock: The black holes begin to lose orbital angular momentum due to the interaction with the nearby stars [16], and next by gravitational radiation.

The spin axes tumble and precess. This phase can be identified with the apparent superdisk, as the rapidly precessing jet produces the hydrodynamic equivalent of a powerful wind, by entraining the ambient hot gas, pushing the two radio lobes apart and so giving rise the a broad separation [17]. Due to the combined effect of precessions and orbital angular momentum dissipation by gravitational waves the spin (and hence jet) direction of the dominant black hole is reoriented approximately in the direction of the original orbital angular momentum, leading to a spin-flip [6].

Fourth the plunge: The two black holes actually merge, with not much angular orbital momentum left to be radiated away, thus the direction of the spin remained basically unchanged (in the case of the typical mass ratio range we discuss; for other setups the final spin may vary [18]-[23]). The final stage in this merger leads to a rapid increase in the frequency of the gravitational waves, called "chirping", but this chirping will depend on the angles involved.

Fifth the rejuvenation: Now the newly oriented more massive merged black hole starts its accretion disk and jet anew, boring a new hole for the jets through its environment (the GigaHertz peaked sources). In this stage the newly active jet is boring through the interstellar clouds with a gigantic system of shockwaves, that accelerate protons and other charged particles in a relativistic tennis-game. These very energetic particles then interact in those same clouds, that slow down the progression of the relativistic jet, and can so give rise to an abundance of neutrinos, TeV photons from pion decay, and yet other particles and photons. These particles and their interactions reach energies far beyond any Earth-bound accelerator.

Sixth the phoenix: The newly oriented jets begin to show up over some kpc, and this corresponds to the X-shaped radio galaxies, while the old jets are fading but still visible [24]. This also explains many of the compact steep spectrum sources, with disjoint directions for the inner and outer jets.

Seventh the cocoon: The old jets have faded, and are at most visible in the low radio frequency bubbly structures, such as seen for the Virgo cluster region around M87. The feeding is slowing down, while a powerful jet is still there, fed from the spin of the black hole.

Eighth the wait: The feeding of the black hole is down to catching some gas out of a common red giant star wind as presumably is happening in our Galactic center. This stage seems to exist for all black holes, even at very low levels of activity. This black hole and its galaxy are ready for the next merger, with the next black hole suitor, which may or may not come.

We predict that the super-disk radio galaxies should have large outer distortions in their radio images, and should be visible in low keV X-rays. A detection of the precessing jet, and its sudden weakening, would then immediately precede the actual merger of the two black holes, and so may be a predictor of the gravitational wave signal.

3. The contribution of the spin precessions to the accumulated orbital phase

In the rest of the paper we concentrate on the gravitational radiation dominated dissipative regime, when the supermassive black hole binary radiates away gravitational waves.

If the binary system evolves unperturbed, the orbit circularizes faster than it shrinks in radius. Therefore we consider circular orbits for which the wave frequency is twice the orbital frequency.

Besides the Keplerian contribution to the accumulated orbital phase there are general relativistic contributions, which scale with the post-Newtonian (PN) parameter $\varepsilon \equiv m/r \approx v^2$ (with m the total mass and r the radius of the reduced mass orbit about the total mass; we take $G = 1 = c$). It is generally agreed from comparison with numerical evolution, that all contributions up to 3.5 PN orders have to be taken into account. To this order, beside the general relativistic contributions starting at 1PN, there are various contributions originating in the finite size of the binary components. These are related either to their rotation (spin) or to their irregularities in shape (like a mass quadrupole moment).

Due to the continued accretion of surrounding matter into the supermassive galactic black holes they spin up considerably. By taking into account only the angular momentum transfer from accreting matter, the dimensionless spin parameters $\chi_i = S_i/m_i^2$ (with $i = 1, 2$) grow to the maximally allowed value 1 even for initially non-rotating black holes [25]. By taking into account the torque produced by the energy input of the in-falling (horizon-crossing) photons emitted from the steady-state thin accretion disk, which counteracts the torque due to mass accretion, the limiting value of the dimensionless spin parameter is slightly reduced to 0.9982 [26]. Various refinements of this process, with the inclusion of open or closed magnetic field lines [27]-[35], also jets in the magnetosphere of the hole [36]-[37] have not changed essentially this prediction. Therefore we assume for the present considerations maximal rotation $\chi_i \approx 1$. Due to this rotation the black hole is centrifugally flattened (it becomes an oblate spheroid), a deformation which can be characterized by a mass quadrupole scalar Q_i [38], or its dimensionless counterpart $p_i = Q_i/m_i m^2 \approx -\left(m_i/m\right)^2$.

The accumulated orbital phase can be formally given as the PN expansion [14], [39], [40]

$$\phi = \phi_c + \phi_N + \phi_{1PN} + \phi_{1.5PN} + \phi_{2PN} + \phi_{2.5PN} + \phi_{3PN} + \phi_{3.5PN} \ . \quad (1)$$

Spin-orbit (SO) contributions appear at 1.5 PN, spin-spin (SS; composed of proper S_1S_2 and self-contributions S_1^2 and S_2^2) and mass quadrupole - mass monopole coupling (QM) contributions at 2PN. They come with various PN corrections (PNSO at 2.5 PN; PNSS, PNQM and SO^2 at 3PN; finally 2PNSO at 3.5 PN; these were not computed yet with the exception of PNSO [41]-[42]). At 3PN there are additional spin and quadrupole contributions, originating in the quasi-precessional evolution of the spin and orbital angular momentum vectors. The number of gravitational wave cycles $\mathcal{N}$ can be computed from the accumulated orbital phase as $\mathcal{N} = \left(\phi_c - \phi\right)/\pi$ (the gravitational wave frequency being twice the orbital frequency for quasi-circular motion).

The spin precession equations with SO, SS and QM contributions were given by Barker and O'Connell [43]. The relative geometry of the two spins $\mathbf{S_i}$ and orbital angular momentum $\mathbf{L}$ can be best described by the angles $\gamma = \arccos(\widehat{\mathbf{S}}_\mathbf{1} \cdot \widehat{\mathbf{S}}_\mathbf{2})$ and $\kappa_i = \arccos(\widehat{\mathbf{S}}_\mathbf{i} \cdot \hat{\mathbf{L}}_\mathbf{N})$ (an overhat denotes the direction of the respective vector). These angles are related by the the spherical cosine identity

$$\cos\gamma = \cos\kappa_1 \cos\kappa_2 + \cos\Delta\psi \sin\kappa_1 \sin\kappa_2 \ , \quad (2)$$

with $\Delta\psi$ the relative azimuthal angle of the spins. Due to quasi-precessions, κ_i and γ evolve. Supplementing the SO and SS contributions already given [44] with the QM contributions (with the quadrupole moment arising from pure rotation), we give these expressions to second order accuracy:

$$(\cos\kappa_i)^{\bullet} = (\cos\kappa_i)^{\bullet}_{SO} + (\cos\kappa_i)^{\bullet}_{SS} + (\cos\kappa_i)^{\bullet}_{QM} \ , \tag{3}$$

$$(\cos\gamma)^{\bullet} = (\cos\gamma)^{\bullet}_{SO} + (\cos\gamma)^{\bullet}_{SS} + (\cos\gamma)^{\bullet}_{QM} \ , \tag{4}$$

with the detailed contributions enlisted in the Appendix.

The order of magnitude estimates of the terms in Eqs. (A.1)-(A.9) (cf. a footnote in Ref. [45], according to which $O(\delta x) = O(\dot{x}) r \varepsilon^{-1/2}$), and assuming maximal rotation lead to

$$\begin{aligned}
\mathcal{O}\left(\delta_{SO}\kappa_1\right) &\approx \varepsilon^{3/2}\left(1+\nu^{-1}\right)^{-2}\left(2+\nu^{-1}\right) \approx \varepsilon^{3/2}\nu \ , \\
\mathcal{O}\left(\delta_{SS}\kappa_1\right) &\approx \varepsilon^{3/2}\left(1+\nu^{-1}\right)^{-2}\left(1+\varepsilon^{1/2}\nu^{-1}\right) \approx \varepsilon^{3/2}\nu(\nu+\varepsilon^{1/2}) \ , \\
\mathcal{O}\left(\delta_{QM}\kappa_1\right) &\approx \varepsilon^{3/2}(\nu+\varepsilon^{1/2}\nu^2) \approx \varepsilon^{3/2}\nu \ , \\
\\
\mathcal{O}\left(\delta_{SO}\kappa_2\right) &\approx \varepsilon^{3/2}\left(1+\nu\right)^{-2}\left(2+\nu\right) \approx \varepsilon^{3/2} \ , \\
\mathcal{O}\left(\delta_{SS}\kappa_2\right) &\approx \varepsilon^{3/2}\left(1+\nu\right)^{-2}\left(1-\varepsilon^{1/2}\nu\right) \approx \varepsilon^{3/2} \ , \\
\mathcal{O}\left(\delta_{QM}\kappa_2\right) &\approx \varepsilon^{3/2}(\nu+\varepsilon^{1/2}) \approx \varepsilon^{3/2}(\nu+\varepsilon^{1/2}) \ , \\
\\
\mathcal{O}\left(\delta_{SO}\gamma\right) &\approx \varepsilon\left(2+\nu+\nu^{-1}\right)^{-1}\left(\nu-\nu^{-1}\right) \approx \varepsilon \ , \\
\mathcal{O}\left(\delta_{SS}\gamma\right) &\approx \varepsilon^{3/2}\left[\left(1+\nu^{-1}\right)^{-2}-\left(1+\nu\right)^{-2}\right] \approx \varepsilon^{3/2} \ , \\
\mathcal{O}\left(\delta_{QM}\gamma\right) &\approx \varepsilon^{3/2}\left(-\nu+\nu\right) \approx \varepsilon^{3/2}\nu \ .
\end{aligned} \tag{5}$$

Thus the angle γ varies at 1PN, while the angles κ_i only at 1.5 PN *during one orbital period*. However γ appears only in the 2PN order S_1S_2 contribution to the accumulated orbital phase as opposed to the angles κ_i which are present in all finite-size contributions, in particular in the 1.5 PN order SO contribution.

Therefore 3PN order quasi-precessional contributions to the accumulated orbital phase in the comparable mass case $\nu \lesssim 1$ arise from $\delta_{SO}\kappa_i$, $\delta_{SS}\kappa_i$, $\delta_{QM}\kappa_i$ and $\delta_{SO}\gamma$; while in the unequal mass case, found typical for supermassive galactic black hole binaries only from $\delta_{SO}\kappa_2$, $\delta_{SS}\kappa_2$ and $\delta_{SO}\gamma$.

4. Leading order evolution of the spin-spin coefficient

From among the quasi-precessional evolutions, we have recently integrated [14] the SO evolution (averaged over one orbit) of the angle $\Delta\psi$, finding

$$\Delta\psi = \left(\Delta\psi\right)_0 + \frac{3\mu n\left(\nu^{-1}-\nu\right)}{2a}t \ . \tag{6}$$

Here $\mu = m_1m_2/m$ the reduced mass, a the radius and $n = 2\pi/T_{orbit} = \pi/T_{wave}$. The time-dependent expression for γ is then given by Eq. (2), with $\Delta\psi$ from Eq. (6). The variation of both $\Delta\psi$ and γ thus have a periodicity with period

$$T_{3PNS} = F\left(\nu\right)\varepsilon^{-1}T_{wave} \ , \tag{7}$$

where

$$F\left(\nu\right) = \frac{4\left(2+\nu+\nu^{-1}\right)}{3\left(\nu^{-1}-\nu\right)} \ . \tag{8}$$

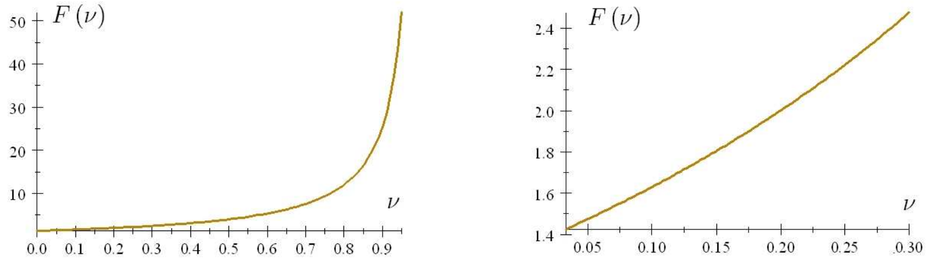

Figure 1. (Color online) (a) The factor $F(y)$, shown for the whole allowed range of mass ratios, diverges in the equal mass case. (b) The factor $F(y)$ in the typical mass ratio range $(1/30,\ 1/3)$ found for merging supermassive galactic black hole binaries is of order unity.

4.1. Equal masses

For equal mass binaries this time-scale becomes infinite $\lim_{\nu\to 1} F(\nu) = \infty$ (see Fig1a) expressing the fact that the time-dependent part of γ goes to zero. Thus γ is a constant and *there is no 3PN quasi-precessional contribution to the accumulated orbital phase for equal mass binaries.*

4.2. Unequal masses

For the typical mass ratio the factor $F(\nu)$ is of the order of unity (see Fig 1b), thus the variation time-scale of γ is $\varepsilon^{-1} \in (10, 1000)$ times larger than the wave period. The contribution of the quasi-precessions can be taken into account then by keeping only the first term in Eq. (2) when computing the 2PN order S_1S_2 contribution to the accumulated orbital phase. e. g. by introducing a *renormalized spin coefficient.* This coefficient arises by modifying the 2PN contribution with the average of some of the 3PN contributions. By using this renormalized coefficient in the context of the 2PN accurate dynamics, one can bring closer the 2PN analytical prediction to the numerical results [14].‡

Thus, in the unequal mass case we propose to replace

$$\sigma_{S_1S_2} = \frac{S_1S_2}{48\eta m^4}(-247\cos\gamma + 721\cos\kappa_1\cos\kappa_2)\ , \tag{9}$$

with

$$\overline{\sigma_{S_1S_2}} = \frac{79S_1S_2}{8\eta m^4}\cos\kappa_1\cos\kappa_2\ , \tag{10}$$

in the respective 2PN contribution to the accumulated orbital phase

$$\begin{aligned} \phi_{2PN} &= -\frac{1}{\eta}\left(\frac{9275495}{14450688} + \frac{284875}{258048}\eta + \frac{1855}{2048}\eta^2 - \frac{15}{64}\sigma\right)\tau^{1/8}\ , \\ \sigma &= \sigma_{S_1S_2} + \sigma_{SS-self} + \sigma_{QM}\ . \end{aligned} \tag{11}$$

(Here $\eta = \mu/m$ is the symmetric mass ratio and τ a dimensionless time parameter.)

‡ However we stress that there are other finite size 3PN order contributions to the phase, like PNSS, PNQM and SO^2, which are still not computed.

5. Leading order evolution of the spin-orbit coefficient

The 1.5 PN contribution to the accumulated orbital phase is

$$\phi_{1.5PN} = -\frac{3}{4\eta}\left(\frac{1}{4}\beta - \pi\right)\tau^{1/4} , \tag{12}$$

with the SO contribution given by

$$\beta = \frac{1}{12}\sum_{i=1}^{2}\chi_i \cos\kappa_i \left(113\frac{m_i^2}{m^2} + 75\eta\right) . \tag{13}$$

5.1. Equal masses

The quasi-precessional equations with the SO, SS and QM contributions included were recently integrated by Racine for the equal mass case [46]. According to his analysis, there is a previously unknown constant of the motion

$$\lambda = \frac{2}{L_N}\left(S_1 \cos\kappa_1 + S_2 \cos\kappa_2\right) . \tag{14}$$

As for equal masses $\beta = (47L_N/24)\,\lambda =$const., *the quasi-precessional evolution at 3PN does not affect the coefficient β up to the 3PN accuracy.*

5.2. Unequal masses

In the unequal mass case the leading order evolutions of the angles κ_i are $\delta_{SO}\kappa_2$ and $\delta_{SS}\kappa_2$. Averaged over one orbit they give [44]

$$\dot{\kappa}_2 = \frac{3\left(1+\nu\right)m_1^2}{2a^3}\sin\kappa_1 \sin\Delta\psi . \tag{15}$$

For unequal masses $\nu \approx 10^{-1}$ the estimates (5) allow to consider κ_1 as a constant (in a first approximation). The leading-order time-dependence of $\Delta\psi$ is given by Eq. (6). Integration gives

$$\kappa_2 = \left(\kappa_2\right)_0 - \frac{\varepsilon^{1/2}}{(1-\nu)}\sin\kappa_1 \cos\left[\left(\Delta\psi\right)_0 + \frac{3\mu n\left(\nu^{-1}-\nu\right)}{2a}t\right] . \tag{16}$$

(We have employed $\varepsilon \approx m/a$.) The time-varying part has the same period T_{3PNS} given by Eq. (7), which is of the scale of ε^{-1} the orbital period. In agreement with this and the estimate (5)

$$\frac{\mathcal{O}\left(\kappa_2 - \left(\kappa_2\right)_0\right)}{\mathcal{O}\left(\delta_{SO,SS}\kappa_2\right)} = \frac{\varepsilon^{1/2}}{\varepsilon^{3/2}} = \varepsilon^{-1} . \tag{17}$$

As the time varying part averages out, to leading order accuracy we can simply identify the angle κ_2 with its constant initial value $\left(\kappa_2\right)_0$.

In conclusion, in the unequal mass case and in the leading order approximation $\kappa_1 =$const. the time variation due to quasi-precessions renders κ_2 to another constant at 3PN accuracy. *There is no need to renormalize β, when taking into account the leading order quasi-precessional contributions.*

6. Concluding Remarks

The process of the merger of two supermassive galactic black holes gives rise to exciting signals in all the messengers accessible now and in the near future, and will allow to probe some of the most profound secrets of Nature, occurring at energies and circumstances far beyond anything on Earth. We have described the sequence of events during the process of merging of such supermassive galactic black holes with reference to related processes involving accretion, jets, energetic particles and electromagnetic signatures. We argued that the class of radio galaxies with a super-disk are particularly promising candidates for future powerful low frequency gravitational wave emission.

The accumulated orbital phase is a quantity to be known precisely for successful detection of gravitational waves. Besides Keplerian, first and second PN order general relativistic contributions, the finite size of the black holes also contributes, first at 1.5 PN orders via the spin-orbit interaction, then at 2PN by the spin-spin and quadrupole-monopole coupling. These contributions are further modulated by quasi-precessional evolutions of the spins. However this is but a subset of all possible finite size contributions at 3PN, as stressed earlier. The way we propose to take into account the precessional 3PN contributions is to add their average to the lower order terms, a procedure we call *renormalization* of the lower order coefficients.

We have proven here that there are no such quasi-precessional contributions in the equal mass case. Furthermore we have proved for the unequal mass case that renormalizing the 1.5 PN spin-orbit coefficient will not change its value. At 2PN there are quadrupolar and spin-spin contributions to consider. As the former can be regarded as constant up to 3PN, the only modification due to quasi-precessions comes from the spin-spin interaction. The renormalized spin-spin coefficient at 2PN thus is adjusted by the average of the 3PN quasi-precessional contribution, taken over the period of change induced by these quasi-precessions (a time-scale one PN order lower than the orbital period). We have checked that the renormalized spin-spin coefficient gives closer results to the numerical estimates for compact binaries with the mass ratio of order 1/10 [14].

Our results related to the renormalization of the spin coefficients are significant for supermassive black hole binaries, which are typically LISA sources. However as we have employed nothing but the mass ratio of the binary in our considerations, they also apply for binaries composed of an intermediate mass and a stellar mass black hole, which qualify as LIGO sources.

Acknowledgments

Participation of LÁG at the GWDAW13 meeting was supported by the Polányi Program of the Hungarian National Office for Research and Technology (NKTH). LÁG was also successively supported by a ROF Award of LSBU and by Collegium Budapest. Support for work with PLB has come from the AUGER membership and theory grant 05 CU 5PD 1/2 via DESY/BMBF and VIHKOS. The work of BM was supported by the Hungarian Scientific Research Fund (OTKA) grant no. 68228; and of ZK by the OTKA grant no. 69036.

Appendix A. Quasi-precessional evolutions of the relative spin angles

The SO and SS contributions of the instantaneous variations of the relative spin angles [44] supplemented with the respective QM contributions (assuming the quadrupole moment arises from pure rotation), to second order accuracy are:

$$(\cos\kappa_1)^{\bullet}_{SO} = \frac{3S_2}{2r^3}\left(2+\nu^{-1}\right)\mathbf{\hat{L}_N}\cdot\left(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2}\right) , \tag{A.1}$$

$$(\cos\kappa_1)^{\bullet}_{SS} = \frac{3S_2}{r^3}\left[(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_2})\mathbf{\hat{L}_N}\cdot\left(\mathbf{\hat{r}}\times\mathbf{\hat{S}_1}\right) + \frac{S_1}{L_N}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_1})\mathbf{\hat{r}}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2})\right] , \tag{A.2}$$

$$(\cos\kappa_1)^{\bullet}_{QM} = -\frac{3\mu m^3}{r^3}\left[\frac{p_1}{S_1}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_1})\mathbf{\hat{L}_N}\cdot(\mathbf{\hat{r}}\times\mathbf{\hat{S}_1}) + \frac{p_2}{L_N}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_2})\mathbf{\hat{r}}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2})\right] , \tag{A.3}$$

$$(\cos\kappa_2)^{\bullet}_{SO} = -\frac{3S_1}{2r^3}(2+\nu)\mathbf{\hat{L}_N}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2}) , \tag{A.4}$$

$$(\cos\kappa_2)^{\bullet}_{SS} = \frac{3S_1}{r^3}\left[(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_1})\mathbf{\hat{L}_N}\cdot(\mathbf{\hat{r}}\times\mathbf{\hat{S}_2}) - \frac{S_2}{L_N}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_2})\mathbf{\hat{r}}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2})\right] , \tag{A.5}$$

$$(\cos\kappa_2)^{\bullet}_{QM} = -\frac{3\mu m^3}{r^3}\left[\frac{p_2}{S_2}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_2})\mathbf{\hat{L}_N}\cdot(\mathbf{\hat{r}}\times\mathbf{\hat{S}_2}) + \frac{p_1}{L_N}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_1})\mathbf{\hat{r}}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2})\right] , \tag{A.6}$$

$$(\cos\gamma)^{\bullet}_{SO} = \frac{3L_N}{2r^3}\left(\nu-\nu^{-1}\right)\mathbf{\hat{L}_N}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2}) , \tag{A.7}$$

$$(\cos\gamma)^{\bullet}_{SS} = \frac{3}{r^3}\left[S_2(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_2}) - S_1(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_1})\right]\mathbf{\hat{r}}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2}) , \tag{A.8}$$

$$(\cos\gamma)^{\bullet}_{QM} = \frac{3\mu m^3}{r^3}\left[\frac{p_2}{S_2}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_2}) - \frac{p_1}{S_1}(\mathbf{\hat{r}}\cdot\mathbf{\hat{S}_1})\right]\mathbf{\hat{r}}\cdot(\mathbf{\hat{S}_1}\times\mathbf{\hat{S}_2}) . \tag{A.9}$$

References

[1] Peters P C 1964 *Phys. Rev.* **136** B1224
[2] Peters P C , Mathews S 1963 *Phys. Rev.* **131** 435
[3] Thorne K S 1979 *Proc. Royal Soc. London* **A 368** 9
[4] Biermann, P.L., Chirvasa, M., Falcke, H., Markoff, S., & Zier, Ch., invited review at the Paris Conference on Cosmology, June 2000, in Proceedings, Eds. N. Sanchez, H. de Vega, p. 148 - 164 (2005); astro-ph/0211503
[5] Merritt D, Ekers R 2002 *Science* **297** 1310-1313
[6] Gergely L Á, Biermann P L 2009 *Astrophys. J.* **697** 1621
[7] Biermann P L, Becker J K, Caramete A, Curutiu L, Engel R, Falcke H, Gergely L Á, Isar P G, Maris I C, Meli A, Kampert K-H, Stanev T, Tascau O, Zier C 2009 in Proceedings of the CRIS 2008 – Cosmic Ray International Seminar: Origin, Mass, Composition and Acceleration Mechanisms of UHECRs, Malfa, Italy 2008, Ed. A Insolia, *Nucl. Phys. B Proc. Suppl.* **190** p. 61-78; arXiv: 0811.1848v3 [astro-ph]
[8] Press W H, Schechter P 1974 *Astrophys. J.* **187** 425

[9] Lauer T R et al. 2007 *Astrophys. J.* **662** 808
[10] Wilson A S , Colbert E J M 1995 *Astrophys. J.* **438** 62 - 71
[11] Ferrarese L et al. 2006 *Astrophys. J. Suppl.* **164** 334
[12] Gonzalez J A, Sperhake U, Bruegmann B 2009 *Phys. Rev. D* **79** 124006
[13] Lang R N, Hughes S A 2006 *Phys. Rev. D* **74** 122001; Erratum-ibid. 2007 *D* **75** 089902(E)
Lang R N, Hughes S A 2008 *Astrophys. J.* **677** 1184
[14] Gergely L Á, Mikóczi B 2009 *Phys. Rev. D* **79** 064023
[15] Gopal-Krishna, Biermann P L, Wiita P 2003 *Astrophys. J. Lett.* **594** L103 - L106
[16] Zier Ch 2007 *Month. Not. Roy. Astr. Soc.* **378** 1309
[17] Gopal-Krishna, Wiita P J, Joshi S 2007 *Month. Not. Roy. Astr. Soc.* **380** 703
[18] Rezzolla L, Barausse E, Dorband E N, Pollney D, Reisswig C, Seiler J, Husa S 2008 *Phys. Rev. D* **78** 044002
[19] Washik M C, Healy J, Herrmann F, Hinder I, Shoemaker D M, Laguna P, Matzner R A 2008 *Phys. Rev. Lett.* **101** 061102
[20] Tichy W, Marronetti P 2008 *Phys. Rev. D* **78** 081501
[21] Barausse E, Rezzolla L 2009 *Predicting the direction of the final spin from the coalescence of two black holes*, e-print: arXiv:0904.2577
[22] Healy J, Laguna P, Richard A. Matzner R A, Shoemaker D M 2009 *Final Mass and Spin of Merged Black Holes and the Golden Black Hole*, e-print: arXiv:0905.3914
[23] Sperhake U, Cardoso V, Pretorius F, Berti E, Hinderer T, Yunes N 2009 *Cross section, final spin and zoom-whirl behavior in high-energy black hole collisions,* e-print: arXiv:0907.1252
[24] Rottmann H. 2001 PhD thesis: *Jet-Reorientation in X-shaped Radio Galaxies*, Bonn Univ.: (http://hss.ulb.uni-bonn.de/diss_online/math_nat_fak/2001/rottmann _helge/index.htm)
[25] Bardeen J M 1970 *Nature* **226** 64
[26] Page D N, Thorne K S 1974 *Astrophys. J.* **191** 499
[27] Blandford R D, Znajek R L 1977 *Month. Not. Roy. Astr. Soc.* **179** 433
[28] Camenzind M 1986 *Astron. & Astroph.* **156** 137
[29] Camenzind M 1986 *Astron. & Astroph.* **162** 32
[30] Camenzind M 1987 *Astron. & Astroph.* **184** 341
[31] Takahashi M, Nitta S, Tamematsu Y, Tomimatsu A 1990 *Astrophys. J.* **363** 206
[32] Nitta S Y, Takahashi M, Tomimatsu A 1991 *Phys. Rev. D* **44** 2295
[33] Hirotani K, Takahashi M, Nitta S, Tomimatsu A 1992 *Astrophys. J.* **386** 455
[34] Li L X 2000 *Astrophys. J.* **533** L115
[35] Wang D X, Xiao K, Lei W H 2002 *Month. Not. Roy. Astr. Soc.* **335** 655
[36] Falcke H, Biermann P L 1995 *Astron. & Astroph.* **293** 665
[37] Kovács Z, Biermann P L, Gergely L Á 2009 *Maximal spin and energy conversion efficiency in a symbiotic system of black hole, disk and jet*, in preparation
[38] Poisson E 1998 *Phys. Rev. D* **57** 5287
[39] Blanchet L, Faye G, Iyer B R, Joguet B 2002 *Phys. Rev. D* **65** 061501; Errata-ibid. 2005 *D* **71** 129902
[40] Mikóczi B, Vasúth M, Gergely L Á 2005 *Phys. Rev. D* **71** 124043
[41] Buonanno A, Cook G B, Pretorius F 2007 *Phys. Rev. D* **75** 124018
[42] Blanchet L, Buonanno A, Faye G 2006 *Phys. Rev. D* **74** 104034; Errata-ibid. 2007 *D* **75** 049903
[43] Barker B M, O'Connell R F 1975 *Phys. Rev. D* **12** 329
[44] Gergely L Á, Perjés Z, Vasúth M 1998 *Phys. Rev. D* **58** 124001
[45] Gergely L Á 2000 *Phys. Rev.* **D 62** 024007
[46] Racine E 2008 *Phys. Rev. D* **78** 044021