

## CLASSICAL ORDER PARAMETER DYNAMICS AND THE DECAY OF A METASTABLE VACUUM STATE<sup>a</sup>

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Transition of the ground state of a classical  $\Phi^4$  theory in  $2 + 1$  dimensions is studied from a metastable state into the stable equilibrium. The transition occurs in the broken  $Z_2$  symmetry phase and is triggered by a vanishingly small amplitude homogeneous external field  $h$ . A phenomenological theory is proposed in form of an effective equation of the order parameter which quantitatively accounts for the decay of the false vacuum. The large amplitude transition of the order parameter between the two minima displays characteristics reflecting dynamical aspects of the Maxwell construction.

The range of interest of the irreversible decay of a metastable vacuum state of finite energy density covers effects from cosmological phase transitions to instabilities observed in the mixed phase of first order phase transitions of condensed matter systems<sup>1</sup>.

Whether the relevant mechanism for a first order phase transition is the formation of bubbles of the new phase, as described by thermal nucleation theory, or the gradual change of a large region of the sample, due to small amplitude spinodal instabilities described by spinodal decomposition is also an intriguing question in heavy ion physics where the actual expansion rate of the plasma may favour one or the other scenario<sup>2</sup>.

The conventional treatment of the decay of metastable states is based on the nucleation theory but concurrent small amplitude spinodal instabilities are also present in the system. They are responsible for the flattening of the static effective potential (Maxwell cut)<sup>3</sup>. The clarification of their role in the metastable  $\rightarrow$  stable transition is the main theme of the present investigation.

### 1 The model and the time-history of the order parameter (OP)

We study numerically the dynamics of a classical  $\Phi^4$  theory in  $2 + 1$  dimensions governed by the discretized field equation of motion  $\Phi_{\hat{n}}(t + a_t) + \Phi_{\hat{n}}(t - a_t) - 2\Phi_{\hat{n}}(t) + a_t^2(-\Phi_{\hat{n}} + \Phi_{\hat{n}}^3 - h) - \frac{a_t^2}{a^2} \sum_i (\Phi_{\hat{n}+\hat{i}}(t) + \Phi_{\hat{n}-\hat{i}}(t) - 2\Phi_{\hat{n}}(t)) = 0$ , with initial conditions :  $\dot{\Phi}_{\hat{n}}(t = 0) = 0$ ,  $\Phi_{\hat{n}}(t = 0) = \Phi_0 + \xi_{\hat{n}}\Phi_1$  where  $\xi_{\hat{n}}$  is

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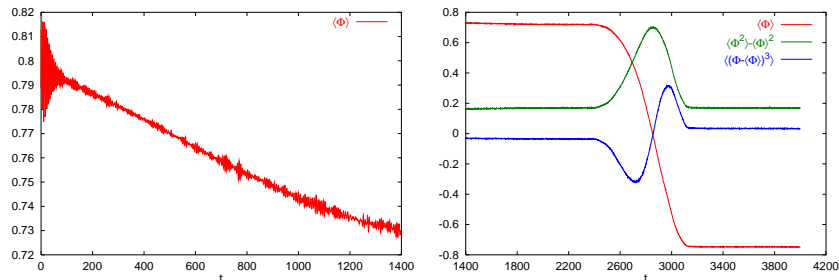


Figure 1: Main qualitative features of the  $\Phi(t)$ -trajectory for a  $512 \times 512$  lattice and the external field  $h = -0.04/\sqrt{6}$ .

an evenly distributed white noise in the range  $(-1/2, 1/2)$  (all quantities are expressed in units of mass). The corresponding initial kinetic power spectrum is  $E_k(\mathbf{k}) \sim \omega^2(\mathbf{k}) = -1 + 4(\sin^2(k_x a/2) + \sin^2(k_y a/2))/a^2$ . We have chosen  $\Phi_0 = 0.815$  and  $\Phi_1 = 4/\sqrt{6}$  which corresponds to a temperature value  $T_i = 0.57$  assuring that the system is in the broken symmetry phase. Our goal is to describe the evolution of the system only in terms of an effective equation for the OP  $\Phi(t) = \frac{1}{V} \sum_{\mathbf{n}} \Phi_{\mathbf{n}}(t)$ .

Fig. 1 shows a typical OP-history together with the later time history of the OP mean square (MS)-fluctuation ( $\langle \Phi^2 \rangle - \langle \Phi \rangle^2$ ) and of its third moment ( $\langle (\Phi - \langle \Phi \rangle)^3 \rangle$ ). In general one can distinguish five qualitatively distinct parts of the OP-trajectory that starts with large amplitude damped oscillations corresponding to the excitation of resonating modes, followed by a rather slow relaxation to a metastable state characterised by  $\langle \Phi \rangle \simeq 0.72$ . The (quasi)thermal motion in the metastable state is followed by the metastable  $\rightarrow$  stable transition induced by the external field  $h$ , during which we can observe characteristic variations of the second and third moments. Quantitative interpretation of this variation will be presented in the following section. The last portion of the trajectory represents thermal motion in the true ground state.

## 2 The effective OP-theory

For the description of (quasi)thermal motion near a (meta)stable point we assume the validity of the ergodicity hypothesis for a system which consist of a single degree of freedom, the OP of the system. We describe its local time evolution by an effective Newton type equation:  $\ddot{\Phi}(t) + \eta(\Phi)\dot{\Phi}(t) + f(\Phi) = \zeta(t)$ .  $\eta(\Phi)$ ,  $f(\Phi)$ ,  $\zeta(t)$  are obtained by a fitting procedure attesting in this way also the presence of a term violating time-reversal invariance  $\eta(\Phi)$ .  $\zeta(t)$  is the

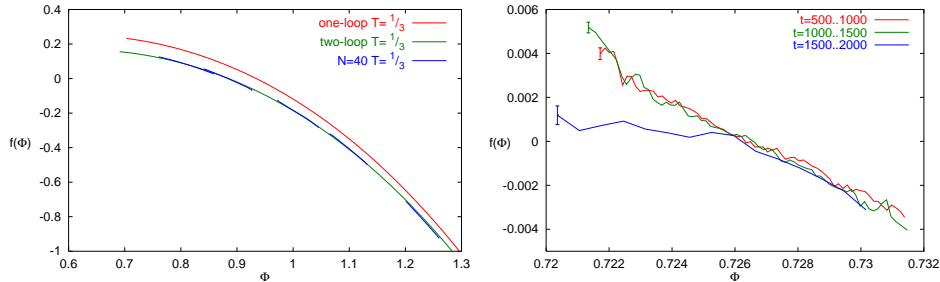


Figure 2: Comparison of the force measured on a  $40 \times 40$  lattice with the one calculated from a one-loop and a two-loop effective potential. On the right hand one can see the bending down of the force prior the transition time for a  $100 \times 100$  lattice and  $h = -0.04/\sqrt{6}$ .

“error” of the best global fit to the homogeneous equation at time  $t$ . The force felt by the OP,  $f(\Phi)$  agrees with the force derived from the equilibrium two-loop effective potential as shown in Fig. 2. To probe this agreement in a relatively wide region one has to measure the force by shifting the center of motion to different values of  $\Phi$  by applying appropriate  $h$  fields to the system. The coefficient  $\eta$  is well-defined and positive, but its value depends quite substantially on the time resolution. The analysis of the relationship of time averaging to the nonzero value of  $\eta$  is left for future investigations.

The thermalization “history” of the system is shown in Fig. 3. The displayed temperature variations correspond to the kinetic energies of the soft and hard modes. During the time evolution they approach each other, manifesting relaxation to a local thermal equilibrium. Because of our “white noise” initial condition the equilibration process is characterized by an energy transfer towards the low  $\mathbf{k}$  modes.

The fluctuation moments depicted in Fig. 1 along with the measured force shown on the right of Fig. 2 tell about how the transition proceeds. The increased values of the moments indicate the enhanced importance of the soft interactions. Preceding directly the transition towards the direction of negative  $\Phi$  values, the fitted force bends down and its average becomes a small positive constant. The vanishing of the force implies the flatness of the effective potential along the motion of the OP (the mode  $\mathbf{k} = 0$  in momentum space) indicating the dynamical realization of the Maxwell-cut.

Taking into account the existence of a mixed phase during the transition period we can construct a model that reproduces exactly the shape of the moments and shows the vanishing of the force felt by OP. Upon space averaging

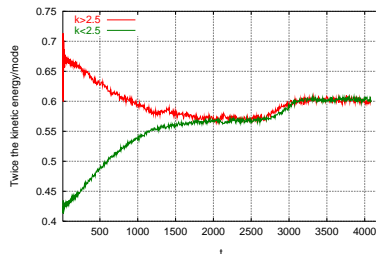


Figure 3: The time evolution of the kinetic energy content of the  $|\mathbf{k}| > 2.5$  and  $|\mathbf{k}| < 2.5$  regions averaged over the corresponding  $|\mathbf{k}|$ -intervals.

the microscopic field equation of motion of section 1 we find the following equation of motion for the OP:  $0 = \ddot{\Phi} - \Phi + \Phi^3 + 3\overline{\varphi^2}^V \Phi + \overline{\varphi^3}^V - h \equiv \ddot{\Phi} + dV_{inst}/d\Phi$  where the symbol  $\overline{\varphi^n}^V$  means the space average of  $\varphi^n$ , ( $\overline{\varphi}^V = 0, \Phi(t, \mathbf{x}) = \Phi(t) + \varphi(\mathbf{x}, t)$ ). The instant potential  $V_{inst}$ , which is a fluctuating quantity, contains as a *deterministic* piece the sum of the tree-level potential and of the slowly varying part of the second and third moments  $\overline{\varphi^n}^V(t) = (\overline{\varphi^n}^V)_{det} + \zeta_n(t)$ ,  $n = 2, 3$ . We assume that the space can be splitted into sharp domains (neglecting the thickness of the walls in between), where the field is the sum of the constant background values  $\Phi_{0\pm}$  and the fluctuations  $\varphi_{\pm}$  around it,  $\Phi_{\pm}(\mathbf{x}, t) = \Phi_{0\pm} + \varphi_{\pm}(\mathbf{x}, t)$ . Based on the smooth evolution of the temperature as displayed in Fig. 3 we assume also local equilibrium in both phases.

The actual value of the OP is determined by the surface ratio  $p(t)$  occupied by the stable phase:  $\Phi(t) = p(t)\Phi_{0-} + (1 - p(t))\Phi_{0+}$ . Simple calculation yields

$$\begin{aligned} (\overline{\varphi^2}^V)_{det}(t) &= \frac{\Phi_{0+} - \Phi(t)}{\Phi_{0+} - \Phi_{0-}} \left( \overline{\Phi_-^2(\mathbf{x}, t)}^V - \overline{\Phi_+^2(\mathbf{x}, t)}^V \right) + \Phi_{0+}^2 - \Phi^2(t) + \overline{\varphi_+^2}^V, \\ (\overline{\varphi^3}^V)_{det}(t) &= \frac{\Phi_{0+} - \Phi(t)}{\Phi_{0+} - \Phi_{0-}} \left( \overline{\Phi_-^3(\mathbf{x}, t)}^V - \overline{\Phi_+^3(\mathbf{x}, t)}^V \right) + \overline{\Phi_+^3(\mathbf{x}, t)}^V \\ &\quad - 3\Phi(t)(\overline{\varphi^2}^V)_{det}(t) - \Phi^3(t). \end{aligned}$$

If one takes the values of  $\Phi_{0\pm}, \overline{\varphi_{\pm}^n}^V$  from the respective equilibria a quite accurate description of the shape of the two fluctuation moments arises in the whole transition region and its close neighbourhood using the measured  $\Phi(t)$  to parametrize their  $t$ -dependence. Substituting the expressions of the moments into the equation of the OP one finds for the deterministic part of the force,  $f(\Phi) = \frac{\overline{\Phi_+^3(\mathbf{x}, t)}^V - \overline{\Phi_-^3(\mathbf{x}, t)}^V}{\Phi_{0+} - \Phi_{0-}} \Phi - \Phi + \frac{\Phi_{0+} \overline{\Phi_-^3(\mathbf{x}, t)}^V - \Phi_{0-} \overline{\Phi_+^3(\mathbf{x}, t)}^V}{\Phi_{0+} - \Phi_{0-}} - h$ . The average of the equations of motion in the respective equilibria,  $\langle \Phi_{\pm}^3(\mathbf{x}, t) \rangle - \Phi_{0\pm} - h = 0$  implies the vanishing of the deterministic force, when exploiting the equality of averaging over the volume and the statistical ensemble. The equation of motion

for OP goes into  $\ddot{\Phi}(t) + \zeta_3(t) + 3\zeta_2(t)\Phi(t) = 0$  which reflects the dynamical realization of the Maxwell construction when assuming the validity of local equilibrium in both phases.

### 3 The nucleation picture

The statistics of the release time, the time necessary for the system to escape from the metastable region by nucleating a growing bubble of the stable state, is of the form  $P(t) \sim \exp(-t\Gamma(h)L^2)$  where  $\Gamma(h) = A \exp(-S_2(h)/T)$  is the nucleation rate and  $L$  is the lattice size<sup>4</sup>. We extract the exponent  $S_2$  by fitting the measured rate to the expression of  $\Gamma(h)$  assuming the  $h$ -independence of  $A$ . Comparing with the bounce action of the nucleation theory we obtain:  $S_{\text{thin wall}} = \frac{4\pi}{9}hT \sim 15S_{\text{measured}}$ . Using the  $T \neq 0$  effective potential instead of the bare one one can achieve a much better agreement, but still a factor of 2 discrepancy remains with the mean field approach.

### 4 Nucleation vs. spinodal instabilities

Our results offer a “dualistic” resolution of the competition between the nucleation and the spinodal phase separation mechanisms in establishing the true equilibrium. We find that the statistical features of the decay of the false vacuum agree with the results of thermal nucleation. Alternatively, the effective OP-theory displays the presence of soft modes and produces dynamically a Maxwell cut when the time dependence of the transition trajectory is described in the effective OP theory. During the transition the OP travels through a narrow but flat valley around the  $\mathbf{k} = 0$  mode along which we expect the effective potential to be flat. This flatness is reflected also in the decrease of the kink-like action  $S_{\text{measured}}$  relative to  $S_{\text{thinwall}}$ . The larger is the system the smaller is the external field which is able to produce the instability.

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