The Method for Ranking Quasi-Optimal Alternatives in Interval Game Models Against Nature

Yuri Romanenkov[1[0000-0002-3526-7237]], Viktor Kosenko[2[0000-0002-4905-8508]], Olena Lobach[3[0000-0001-7494-9997]], Evgen Grinchenko[4[0000-0002-3973-9078]], Marina Grinchenko[5[0000-0002-8383-2675]]

[1] N. Ye. Zhukovsky National Aerospace University "Kharkiv Aviation Institute", Chkalov. 17, 61070 Kharkov, Ukraine
[2] State Enterprise "Kharkiv Research Institute of Mechanical Engineering", Krivokonevskaya. 30, 61016 Kharkiv, Ukraine
[3]National Technical University "Kharkiv Polytechnic Institute", Kyrpychova. 2, 61002, Kharkiv, Ukraine
[4]Kharkiv National University of Internal Affairs, L. Landau avenue. 27, 61080 Kharkiv, Ukraine
[5]National Technical University "Kharkiv Polytechnic Institute", Kyrpychova. 2, 61002, Kharkiv, Ukraine
KhAI.management@ukr.net, kosv.v@ukr.ua, e.v.lobach@gmail.com, gengrinchenko@gmail.com, marinagrunchenko@gmail.com

Abstract. The task of selecting the optimal strategy in the interval game with nature is considered; in particular, the situation when in the interactive dialogue of an analyst and decision support system there are cases of objective ambiguity caused, on the one hand, by interval uncertainty of data, and on the other hand – by the chosen model of the task formalization. The method for ranking quasi-optimal alternatives in interval game models against nature is proposed, which enables comparing interval alternatives in cases of classical interval ambiguity. In this case, the function of the analyst preferences is used with respect to the values of the criterion that help determine the indicators for the quantitative ranking of alternatives. By selecting a specific type of the preference function, the researcher artificially converts the primary uncertainty of the data into the uncertainty of the preference function form, which nevertheless enables avoiding the ambiguity in the “fuzzy” areas of quasi-optimal alternatives.

Keywords: Playing Against Nature, Optimal Strategy, Interval Data.

1 Introduction

The formalization of the decision-making task is one of the key stages of the management cycle and the efficient control of organizational and technical systems largely depends on the relevance and correctness of the management cycle. [1]. In the case of an antagonistic situation, the section of applied mathematics known as "game theory" is used to solve such tasks. [2, 3]. Numerous methods of classical game theory are successfully implemented in modern decision support system (DSS) [4], in particular, decision-making techniques under complete ambiguity and risk. And classical and derived criteria [5], as well as modified ones [6–8] are used for selecting alternatives.

The efficiency of these criteria is ensured when the initial data of the decision-making task is absolutely correct. However, when there are various kinds of uncertainties in the initial data, the problem of adapting the criteria arises as well as organizing their final values for the pool of alternatives.

There are different approaches to solve similar tasks in the context of various data, e.g. interval [9, 10], fuzzy [11], stochastic [12]. In this case, with some sets of initial data, a situation may arise when alternatives are considered incomparable [13], that is, there are "fuzzy" areas of quasi-optimal alternatives and the only best one cannot be selected within them.

This leads to the situation when within the interactive dialogue between an analyst and DSS there are cases of the objective ambiguity that is caused on the one hand, by interval uncertainty of data, and on the other hand – by the chosen model of the task formalization.

Thus, the topical scientific and practical task is to develop techniques and means for avoiding the ambiguity in the "fuzzy" areas of quasi-optimal alternatives according to the analyst request.

2 Problem statement

Consider the classical deterministic decision-making task under complete uncertainty according to [8], which can be presented as the matrix whose lines correspond to decision variants and columns – to factors. At the intersection of the columns and lines, gains e_{ij} are located, they correspond to decisions E_i under appropriate conditions F_j (see Table 1).

Table 1. Decision efficiency matrix $|e_{ij}|$

	F_1	F_2	F_3	$\ldots$	F_m
E_1	e_{11}	e_{12}	e_{13}	$\ldots$	e_{1m}
E_2	e_{21}	e_{22}	e_{23}	$\ldots$	e_{2m}
E_3	e_{31}	e_{32}	e_{33}	$\ldots$	e_{3m}
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
E_n	e_{n1}	e_{n2}	e_{n3}	$\ldots$	e_{nm}

A set of optimal variants E_0 consists of the variants E_{i0}, which belongs to the set of all variants E and the value of the Z_{i0} criterion which is maximal among all its values Z_i:

$$E_0 = \left\{ E_{i0} \mid E_{i0} \in E \wedge Z_{i0} = \max_i Z_i \right\}. \quad (1)$$

Let one of the classical or derived criteria be used as Z_{i0} criterion [8]:

- maximin (Wald)

$$Z_{MM} = \max_i \left(\min_j \left(e_{ij} \right) \right) \quad (2)$$

- Bayes-Laplace

$$Z_{BL} = \max_i \left(\sum_{j=1}^{m} e_{ij} q_j \right); \quad (3)$$

- Savage

$$Z_S = \min_i \left(\max_j \left(\max_i \left(e_{ij} \right) - e_{ij} \right) \right); \quad (4)$$

- extended maximin

$$Z_{ME} = \max_p \left(\min_q \left(\sum_{i=1}^{n} \sum_{j=1}^{m} e_{ij} p_i q_j \right) \right); \quad (5)$$

- gambler

$$Z_{AG} = \max_i \left(\max_j \left(e_{ij} \right) \right); \quad (6)$$

- Hurwitz

$$Z_{HW} = \max_i \left(\alpha \cdot \min_j \left(e_{ij} \right) + \left(1 - \alpha \right) \cdot \max_j \left(e_{ij} \right) \right),\ \alpha \in \left[0,1 \right]; \quad (7)$$

- Hodge-Lehman

$$Z_{HL} = \max_i \left(\nu \cdot \sum_{j=1}^{n} e_{ij} q_j + \left(1 - \nu \right) \cdot \min_j \left(e_{ij} \right) \right),\ \nu \in \left[0,1 \right]; \quad (8)$$

- Germier

$$Z_G = \max_i \left(\min_j \left(e_{ij} q_j \right) \right); \quad (9)$$

- BL(MM)-criterion

$$
\begin{aligned}
& I_1 := \left\{ i \,\middle|\, i \left\{1,\ldots,m\right\} \,\&\, e_{i0j0} - \min_j\left(e_{ij0}\right) \le \varepsilon_{\partial on} \right\}, \\
& I_2 := \left\{ i \,\middle|\, i \left\{1,\ldots,m\right\} \,\&\, \max_j\left(e_{ij}\right) - \max_j\left(e_{i0j}\right) \ge e_{i0j0} - \min_j\left(e_{ij0}\right) \right\}, \\
& Z_{BL(MM)} = \max_{I_1 \cap I_2}\left(\sum_{j=1}^{m} e_{ij} q_j\right);
\end{aligned}
\tag{10}
$$

- product-criterion

$$
Z_P = \max_i\left(\prod_{j=1}^{m} e_{ij} q_j\right). \tag{11}
$$

When gains are presented as an interval

$$
\left[e_{ij}\right] = \left[\underline{e_{ij}},\ \overline{e_{ij}}\right], \tag{12}
$$

the values of the selected criterion for each alternative can be calculated according to (2)–(12) as intervals

$$
\left[Z_i\right] = \left[\underline{Z_i},\ \overline{Z_i}\right]. \tag{13}
$$

The basis for considering the estimates in the interval form is formed by the following circumstances [14]:

1. in the process of short-term prediction, estimates in the interval form can be synthesized in a natural way, that is, as a result of fulfilling a prediction task;
2. results of measuring the parameters of the system, direct or indirect, performed with errors (strictly speaking, results of all measurements), can be represented in the interval form;
3. if there is at least one model parameter in the interval form in a model, all parameters of the model must be reduced to the interval form as the least complex form of description of parametric uncertainty in order to observe data homogeneity;
4. interval models are more preferable than the probabilistic-statistical ones in the case of making one-moment single decisions;
5. the apparatus of interval analysis proved its effectiveness in solving different scientific and practical tasks;
6. interval algorithms typically do not require specialized tools for software implementation.

We imply by interval $\left[z\right] = \left[\underline{z},\ \overline{z}\right]$ a closed limited subset R of the form $\left[\underline{a},\ \overline{a}\right] = \left\{x \in R \mid \underline{a} \le x \le \overline{a}\right\}$, which can be described by the following characteristics:

$\underline{z}$, in f [z] is the left end of interval [z]; $\overline{z}$, sup[z] is the right end of interval [z]; $mid[z]=\frac{\underline{z}+\overline{z}}{2}$ is the middle (median) of interval [z]; $wid[z]=\overline{z}-\underline{z}$ is the width of interval [z].

For the two intervals $[z]=[\underline{z},\ \overline{z}]$ and $[y]=[\underline{y},\ \overline{y}]$ in classical interval arithmetic $([z],[y]\in IR)$, the following operations were assigned:

$$[z]+[y]=[\underline{z}+\underline{y},\ \overline{z}+\overline{y}]; \tag{14}$$

$$[z]-[y]=[\underline{z}-\overline{y},\ \overline{z}-\underline{y}]; \tag{15}$$

$$[z]\cdot[y]=\left[min\left\{\underline{z}\underline{y},\underline{z}\overline{y},\overline{z}\underline{y},\overline{z}\overline{y}\right\},\ max\left\{\underline{z}\underline{y},\underline{z}\overline{y},\overline{z}\underline{y},\overline{z}\overline{y}\right\}\right]; \tag{16}$$

$$[z]/[y]=[z]\cdot\left[1/\overline{y},\ 1/\underline{y}\right],\ \ 0\notin[y]. \tag{17}$$

Interval arithmetic operations have the following properties:

$$([z]+[y])+[x]=[z]+([y]+[x]); \tag{18}$$

$$([z]\cdot[y])\cdot[x]=[z]\cdot([y]\cdot[x]); \tag{19}$$

$$[z]+[y]=[z]+[y]; \tag{20}$$

$$[z]\cdot[y]=[z]\cdot[y]; \tag{21}$$

$$([z]+[y])\cdot[x]\subseteq[z]\cdot[x]+[y]\cdot[x]. \tag{22}$$

The distance between two intervals $[z],[y]\in IR$ is determined by magnitude

$$dist([z],[y])=max\left\{\left|\underline{z}-\underline{y}\right|,\left|\overline{z}-\overline{y}\right|\right\}=\rho([z],[y]) \tag{23}$$

and have the following properties:

$$dist([z],[y])\geq 0; \tag{24}$$

$$dist([z],[y])=0,\ \text{when}\ [z]=[y]; \tag{25}$$

$$dist([z],[y])=dist([z],[y]); \tag{26}$$

$$dist([z],[x]) \le dist([z],[y]) + dist([y],[x]). \tag{27}$$

The key difference between classical interval arithmetic and interval analysis is in the following. In classic interval arithmetic, the distribution law is not observed, there are no inverse elements, similar terms cannot be reduced within its frameworks. This leads to that the technique of symbol transformations is lost during formalization of operations with intervals.

The main objective of interval analysis, by contrast, is not automation of computing, but rather finding the region of possible result values, taking into consideration structures of functions and data, assigned in symbolic form.

Within this approach, interval magnitudes are considered at the intermediate stages of calculations and analysis. Only at the last stage of decision-making, if necessary, they are transformed into pointwise solutions. It will make it possible to give the possibility to save completeness of information on the set of possible solutions up to the last moment.

The specific algorithmic implementation of operations with interval values $\left[e_{ij}\right]$ does not play a decisive role in this case, although it can be the subject of the specific studies to narrow final intervals artificially.

According to the rules of classical interval analysis [15], a set (13) can be unambiguously ranked only when intervals $\left[Z_i\right]$ do not intersect

$$[Z_k] \le [Z_l] \Leftrightarrow \left((\forall Z_k \in [Z_k])(\forall Z_l \in [Z_l])(Z_k \le Z_k) \right). \tag{28}$$

Otherwise, there is a "weak" inequality:

$$[Z_k] \le [Z_l] \Leftrightarrow \left((\exists Z_k \in [Z_k])(\exists Z_l \in [Z_l])(Z_k \le Z_k) \right), \tag{29}$$

that is, the intervals are considered incomparable in the context of the classical paradigm of interval analysis.

The formulation of the research task. In the situation described by formula (29), i.e. when a group of intersecting interval values appears, among which it is impossible to choose a larger value, it is necessary to develop a method for overcoming the uncertainty that can be used by a direct request from the analyst.

3 Solution

The variant of formalizing interval comparison proposed in [15], which is reduced to determining the reliability of hypotheses about the actual location of real numbers within the corresponding intervals, cannot be used as a quantitative measure of the ratio between these numbers [14]. The other way is proposed in [13] and is linked to the correction of the interval logic which, however, fails in some particular cases [14].

Another option for lax formalization of the problem of comparing interval numbers is to use the magnitudes of the distance between interval numbers as a comparison measure (23). In this case, it becomes fundamentally possible to construct and analyze the graph with interval numbers in vertices, however lax compliance with distribution logic makes practical application of this approach difficult.

Let us use the method of formalizing the interval comparison proposed in [14], in particular, let us introduce a monotonically increasing function that is not negative on the whole real axis (see Fig. 1).

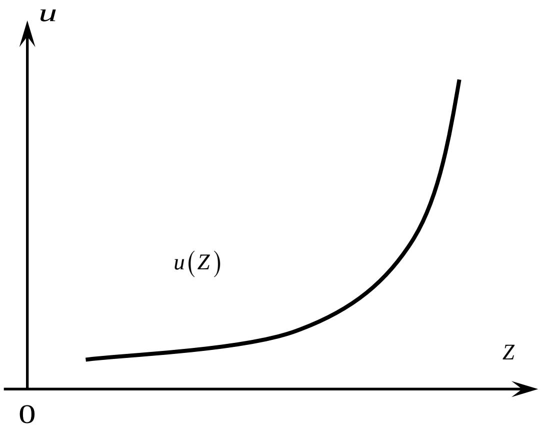

Fig. 1. An example of the function of the decision maker's preferences regarding the values of the criterion

When a specific type of function $u(Z)$ is selected, the indicator can be calculated for each alternative as follows:

$$u^* = \frac{1}{\overline{Z} - \underline{Z}} \int_{\underline{Z}}^{\overline{Z}} u(Z)\,dZ \,, \tag{30}$$

it is numerically equal to the height of a rectangle equivalent in area to a certain integral of the function $u(Z)$ within the interval of the criterion value (see Fig. 2).

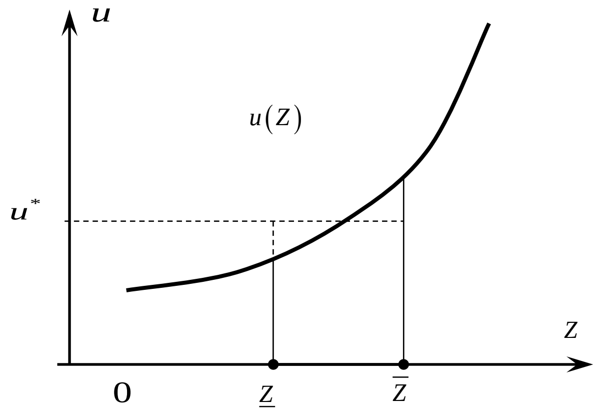

Fig. 2. Graphical interpretation of the characteristic indicator of the criterion interval value

Using indicators (30) calculated for every alternative, a set (13) can be ranked quantitatively.

The shape of the function $u(Z)$ (for example $u(Z)=Z$, $u(Z)=Z^2$, $u(Z)=Z^3$) determines the analyst's preferences for the interval value of the criterion. For example, when $u(Z)=Z$, interval alternatives with equal midpoints are interpreted as equivalent, while $u(Z)=Z^2$ preference will be given to a wider interval alternative.

By selecting a specific type of function $u(Z)$, the researcher artificially converts the initial ambiguity of data into the ambiguity of the preference function form, which, nevertheless, enables avoiding the ambiguity in "fuzzy" arear of quasi-optimal alternatives.

3.1 Example

The Table 2 represents an interval matrix of the game with nature.

Table 2. The decision efficiency interval matrix example

	F_1	F_2	F_3	F_4
E_1	$[5,5.5]$	$[15,16.5]$	$[1,1.1]$	$[5,5.5]$
E_2	$[6,6.6]$	$[12,13.2]$	$[19,20.9]$	$[2,2.2]$
E_3	$[10,11]$	$[14,15.4]$	$[0,0.5]$	$[6,6.6]$
E_4	$[1,1.1]$	$[15,16.5]$	$[4,4.4]$	$[6,6.6]$
E_5	$[12,13.2]$	$[1.9,2.3]$	$[5,5.5]$	$[16,17.6]$

The alternatives estimates obtained according to the maximin criterion are presented in Table 3.

Table 3. Simulation results

	Z_1	u_1^* $u(Z)=Z$	u_2^* $u(Z)=Z^2$	u_3^* $u(Z)=Z^3$
E_1	$[1,1.1]$	1,05	1,103	1,1603
E_2	$[2,2.2]$	2,1	4,413	9,282
E_3	$[0,0.5]$	0,25	0,083	0,0313
E_4	$[1,1.1]$	1,05	1,103	1,1603
E_5	$[1.9,2.3]$	2,1	4,423	9,345

Obviously, by the maximin criterion, two alternatives are quasioptimal – E_2 and E_5, whose estimates are incomparable in the paradigm of classical interval analysis. The calculation of the indicator u^* according to (30) for different forms of the preference function allows to make an unequivocal reasonable choice of the only optimal alternative – E_5.

3.2 The critical analysis of results

The proposed technique for ranking alternatives has the following features.

1. The developed method cannot and should not be considered as the only or “best” one within the given task. However, the fact that this technique is rather subjective (while selecting the function of preference) does not violate the logic of the decision-making process. The analyst can work with the uncertainty until he makes sure that the only optimal solution according to the selected criterion cannot be obtained.
2. The proposed technique is algebraically simple and does not contain operations that can lead to the artificial broadening of intervals. However, the researcher, that is formalizing the decision-making task and selecting nontrivial criterion, should take into consideration the fact that operations with interval data (especially with intervals containing zero) can dramatically extend the criterion final interval criterion. That is why the proposed technique (as the interval analysis as a whole) is efficient only for interval data of small width or for sparse interval matrices.
3. The interactive mode of operation of an analyst and DSS should remain dominant with respect to automatic modes in the context of decision-making tasks; the proposed technique should be used according to the analyst request.

4 Conclusions

1. The method is proposed for ranking quasi-optimal alternatives in interval game models against nature, which enables comparing interval alternatives in cases of classical interval ambiguity.

2. Recommendations on the practical implementation of the proposed method were compiled. Specifically, recommendations for parametric setting of preference functions depending on the location of interval estimates were formulated.
3. The algorithm that implements the proposed method is simple and its result is clear, which is important in the process of making managerial decisions.

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