Intrinsic parton transverse momentum in next-to-leading-order pion production

Gábor Papp,[1,*] Gergely G. Barnaföldi,[2,†] Péter Lévai,[2,‡] and George Fai[3,§]
[1] Department for Theoretical Physics, Eötvös University, Pázmány P. 1/A, Budapest 1117, Hungary
[2] RMKI KFKI, P.O. Box 49, Budapest 1595, Hungary
[3] CNR, Kent State University, Kent OH 44242, USA
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We study pion production in proton-proton collisions within a pQCD-improved parton model in next-to-leading order augmented by intrinsic transverse momentum ($k_\perp$) of the partons. We find the introduction of intrinsic transverse momentum necessary to reproduce the experimental data in the CERN SPS to RHIC energy range, and we study its influence on the so-called K factor, the ratio of the NLO cross section to the Born term. A strong p_T dependence is seen, especially in the $3-6$ GeV transverse momentum region of the outgoing pion, where nuclear effects (e.g. the Cronin effect) play an important role.

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I. INTRODUCTION

Today's collider facilities raise the interest in testing perturbative QCD (pQCD) at work, and in searching for phenonema beyond its capability. However – at least for the experimentally available transverse momentum region – pQCD parton models underestimate the production of mesons in proton-proton (pp) collisions [1, 2], even at next-to-leading order (NLO) [3]. In order to restore the consistency with the data, two methods were proposed at leading order (LO): inclusion of the intrinsic transverse momentum of partons [1, 2], and/or an effective correction factor (K factor), accounting for higher order contributions [4, 5]. The physical background is to account for both, the missing higher order perturbative corrections and radiation effects. The latter is not present in electron-proton processes, however, it becomes important in pp collisions [6].

In this paper we present the first results on pion production in pp collisions applying a NLO pQCD parton model *with* intrinsic transverse momentum, taken from a Gaussian distribution with width $\langle k_\perp^2 \rangle$. In Section II the intrinsic transverse momentum is introduced into the formalism, and the appropriate NLO expressions are presented. The necessity of such an extension is demonstrated at $\sqrt{s} = 27.4$ GeV in Section III. Next, we display the best fit values of the width of the intrinsic transverse momentum distribution at NLO level for available $pp \to \pi + X$ experiments, in the energy range 20 GeV $\lesssim \sqrt{s} \lesssim 200$ GeV. Similarly to Ref. [1], we study the pion production in the 2 GeV $< p_T <$ 6 GeV transverse momentum region, where nuclear effects are considered to be important.

Finally, we extract the ratio of the NLO cross section to the Born term (K factor) at different energies and transverse momenta, and study its dependence on the amount of the included intrinsic transverse momentum. This way we provide a numerical foundation to the correction factors used in LO calculation [7]. We demonstrate, that a leading order calculation with a fitted, c.m. energy, transverse momentum and scale dependent K factor and additional intrinsic transverse momentum reproduces well the full NLO results at higher energies and momenta, and can be used as a fast method to get a reasonable estimate of a full NLO calculation.

II. MODEL

In order to extend the applicability of the original, infinite momentum frame parton model [8] to smaller transverse momenta, we introduce the intrinsic transverse momentum of the partons [9]. We write the four-momenta of the interacting partons (a and b) as [7]

$$
\begin{aligned}
p_a &= (x_a \frac{\sqrt{s}}{2} + \frac{k_{\perp,a}^2}{2 x_a \sqrt{s}},\ \vec{k}_{\perp,a},\ x_a \frac{\sqrt{s}}{2} - \frac{k_{\perp,a}^2}{2 x_a \sqrt{s}})\ , \quad (1)\\
p_b &= (x_b \frac{\sqrt{s}}{2} + \frac{k_{\perp,b}^2}{2 x_b \sqrt{s}},\ \vec{k}_{\perp,b},\ -x_b \frac{\sqrt{s}}{2} + \frac{k_{\perp,b}^2}{2 x_b \sqrt{s}})\ .
\end{aligned}
$$

In this notation x, the momentum fraction carried by the parton, becomes a parameter. The apparent fraction is $x - k_\perp^2/(xs)$, however, for practical applications at high energy ($p_T \gtrsim 3$ GeV; $\sqrt{s} \gtrsim 40$ GeV; $\langle k_\perp^2 \rangle \lesssim 2$ GeV^2), the distinction has a negligible ($\lesssim 5\%$) effect. Furthermore, we require that the longitudinal direction of the partons does not change sign due to the transverse momentum, i.e. $x > k_\perp/\sqrt{s}$.

The starting point of our calculation is factorization: the hadronic cross sections up to a power correction may be written as a convolution over hard partonic (pQCD)

*Electronic address: pg@ludens.elte.hu
†Electronic address: bgergely@rmki.kfki.hu
‡Electronic address: plevai@rmki.kfki.hu
§Electronic address: fai@cnr4.physics.kent.edu

processes,

$$\frac{d\sigma}{dy d^2p_T} = \sum_{abc} \int dx_a dx_b d^2k_{\perp,a} d^2k_{\perp,b} \frac{dz_c}{\pi z_c^2} \quad (2)$$
$$f_{a/p}(x_a, Q; k_{\perp,a}) f_{b/p}(x_b, Q; k_{\perp,b}) \frac{d\sigma}{d\hat{t}} D_{\pi/c}(z_c, Q_f) \ ,$$

where $d\sigma/d\hat{t}$ is the partonic cross section of the reaction $a + b \to c + d$ (LO with condition $\delta(1 + (\hat{t} + \hat{u})/\hat{s})$), or $a + b \to c + d + e$ (NLO with fixed z_c) and (at fixed scales) is the function of the partonic Mandelstam variables only. In order to avoid singularities due to the intrinsic transverse momentum we use regularization

$$\hat{s} \to \hat{s} + M^2, \quad \hat{t} \to \hat{t} - M^2/2, \quad \hat{u} \to \hat{u} - M^2/2 \ , \quad (3)$$

with $M = 1.8$ GeV. The factorization is done at the *factorization* scale Q, where the parton content of the initial proton is determined by the parton distribution function $f_{a/p}$ (PDF). For simplicity, we assume a Gaussian dependence of the PDFs on the intrinsic transverse momentum, with a width $\langle k_\perp^2 \rangle$,

$$f(x, Q, k_\perp^2) = f(x, Q) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}. \quad (4)$$

We note, that such a separation may be viewed as a first approximation to the *unintegrated* PDF, where recent studies indeed found a shape close to Gaussian [10]. Finally, the hadrons are created collinearly with the outgoing parton c with momentum fraction z_c at *fragmentation* scale Q_f. The partonic cross section explicitly depends on three scales, Q, Q_f and the *renormalization* scale Q_r.

In principle, the scales can be determined such that the final result has the minimum sensitivity to them [3], and usually are set to be equal. However, in this paper we use the results of a previous study [11] of *pp* and *pA* data, where the reproduction the Cronin effect in *pA* reactions [12] imposed such scales, that the corresponding $\langle k_\perp^2 \rangle \approx 2$ GeV^2.

We note that there is some ambiguity in the choice of scales. In the literature typical scales are fixed to hadronic or partonic variables, κp_T or $\kappa p_T/z_c$, respectively, where κ is an $\mathcal{O}(1)$ number. Other choices are also possible, e.g. an invariant scale, $Q^2 = \kappa^2 \, \hat{s}\hat{t}\hat{u}/(\hat{s}^2+\hat{t}^2+\hat{u}^2)$ as proposed in Ref. [8]. However, in our case we found that this choice is equivalent to $\kappa p_T/z_c$.

At NLO level and no intrinsic transverse momentum Eq. (2) is usually rewritten with variable change $(x_a,x_b,z_c) \to (\hat{v},\hat{w},z_c)$, where $\hat{t} = -(1-\hat{v})\hat{s}$ and $\hat{u} = -\hat{v}\hat{w}\hat{s}$, as

$$\frac{d\sigma}{dy d^2p_T} = \frac{1}{\hat{s}} \sum_{abc} \int d\hat{v} d\hat{w} \frac{dz_c}{\pi z_c^2} \ J \quad (5)$$
$$d^2k_{\perp,a} d^2k_{\perp,b} \ f_{a/p} f_{b/p} D_{\pi/c}(z_c, Q_f) \frac{d\sigma^{NLO}}{d\hat{v}} \ ,$$

with the proper kinematical boundaries, J being the Jacobian of the transformation ($1/J = \hat{v}(1-\hat{v})\hat{w}$ for $\langle k_\perp^2 \rangle$=0), and $d\sigma^{NLO}/d\hat{v}$ is the sum of $2 \to 2$ and $2 \to 3$ cross sections [13],

$$\frac{d\sigma^{Born}}{d\hat{v}} \delta(1-\hat{w}) + \frac{\alpha_s(Q_r)}{\pi} \mathcal{K}^{ab\to cd}(\hat{s}, \hat{v}, \hat{w}, Q, Q_r, Q_f) \ ,(6)$$

with *renormalization* scale Q_r (chosen to be equal to the factorization scale Q). In this paper, however, we use Eq. (2) directly, since for $\langle k_\perp^2 \rangle \neq 0$ the momentum fraction x cannot be expressed analytically from $\hat{v}$ and $\hat{w}$, while the inverse transformation can be done.

Several codes are available for calculating jet cross sections at NLO level [13, 14]. Here we have chosen to extend the one by the Aversa group [13] calculating the partonic cross sections at next-to-leading-log level, with the intrinsic transverse momentum distribution. The calculations presented in the following sections were performed with the MRST-cg PDF [15] and KKP FF [16] parameterization.

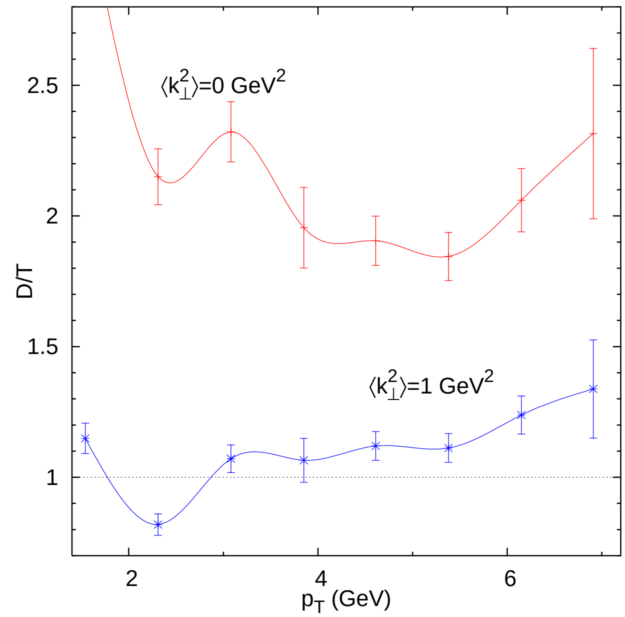

FIG. 1: Comparison of experimental data [17] to the NLO pQCD parton model result in $p + p \to \pi^+ + X$ reaction at $E_{lab} = 400$ GeV, with and without intrinsic transverse momentum.

III. RESULTS

A. Comparison to data

First, we demonstrate the importance of the intrinsic transverse momentum to reproduce experimental data in the transverse momentum region 3 GeV $< \sqrt{s} <$ 6 GeV. In this part, we use the scales proposed in Ref. [3] to make a direct comparison to the 400 GeV FNAL experiment [17]. Fig. 1. shows, that at $\sqrt{s} = 27.4$ GeV the NLO calculations of pion production in *pp* collision underpredict the experimental data by a factor of 2 using

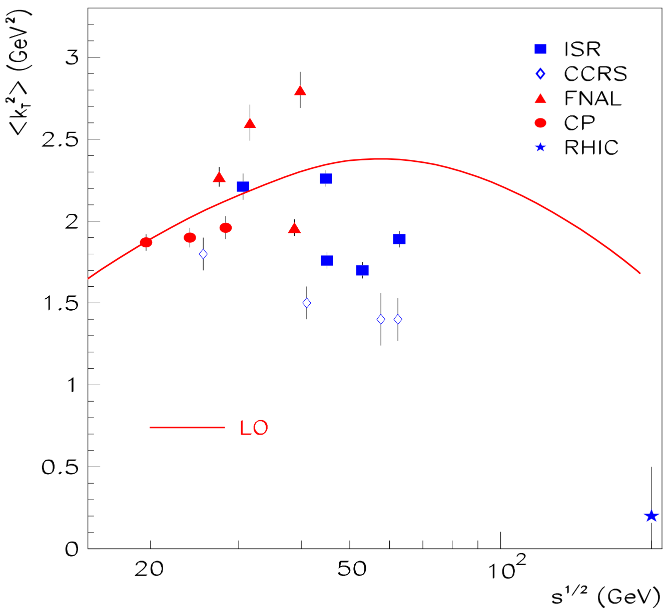

FIG. 2: Best fits of $\langle k_\perp^2 \rangle$ to experimental data in $pp \to \pi + X$ reactions from 19.5 to 200 GeV. Different symbols refer to different experiments, see [17, 20]. The solid line represents the average value of a LO calculation [1].

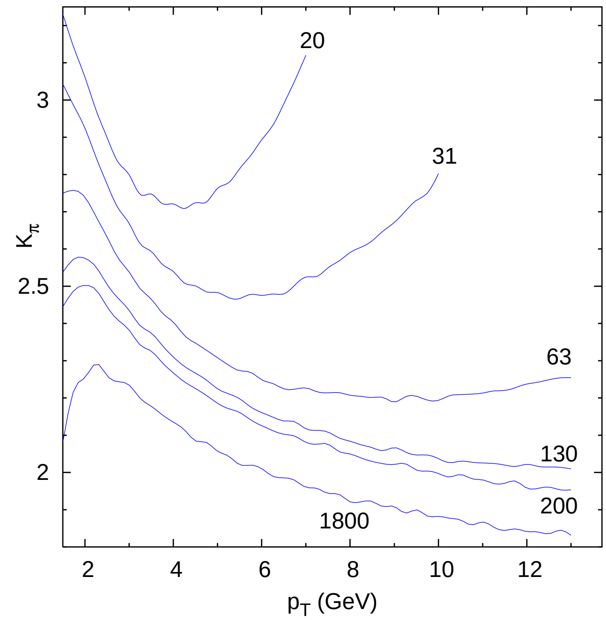

FIG. 3: K factor at $\langle k_\perp^2 \rangle = 0$ and energies from $\sqrt{s} = 20$ GeV to 1800 GeV.

the scale parameters $Q = Q_r = Q_f = p_T/2$ and neglecting intrinsic transverse momentum (as in [3]). Decreasing the scales (but still keeping them hard for the applicability of pQCD) the agreement can be improved, however, without intrinsic transverse momentum, even if lower scales are chosen, the data are still underestimated.

The lower line in Fig. 1. presents the NLO calculation *with* a partonic intrinsic transverse momentum distribution of width $\langle k_\perp^2 \rangle$ =1 GeV^2, and shows a nice agreement with the data. We note, however, that there is a delicate interplay between the choice of the scales and the intrinsic transverse momentum $\langle k_\perp^2 \rangle$, needed to reproduce the data [18]. Typically, increasing the scales increases the value of $\langle k_\perp^2 \rangle$.

B. Intrinsic transverse momentum

In this section we summarize the results on the intrinsic transverse momentum width of the partons in the nucleon fitting the NLO pQCD calculations to the data, generalizing the LO scale choice of [11] to $Q = Q_r = \kappa p_T/z_c$, $Q_f = \kappa p_T$, where κ is a $\mathcal{O}(1)$ number. In this work, we fix $\kappa = 2/3$ at NLO level. Our previous study [11] also showed, that the reproduction of the Cronin peak in pA collision requires the width of the intrinsic transverse momentum distribution to be on the order of $\langle k_\perp^2 \rangle \approx 2$ GeV^2 at energies $\sqrt{s} \sim 30$ GeV, which is achieved by the above choice of scales. A similar value of $\langle k_\perp^2 \rangle$ was extracted from the experimental analysis of jet-angle distribution [19].

Analyzing the $pp \to \pi + X$ experimental data [17, 19, 20], we deduced the best fit value $\langle k_\perp^2 \rangle$ for each experiment, similarly to what is shown in Fig. 1, minimizing the $\chi^2(D/T - 1)$ (data over theory) ratio in the range $3 - 6$ GeV. The result is presented in Fig. 2, separately indicating the runs from different experiments and shows a need for a considerable amount of partonic transverse momentum. We also checked, that similarly to the LO results [1], the extracted width does not depend on the charge of the pion.

Usually, changing the order of a calculation requires changing of the underlying scales. Comparing NLO result to the previous LO ones [1] one notices that in order to keep the average transverse momentum width, we had to increase the scales, dictated by the experimentally obtained Cronin peak [11]. Keeping LO scales ($\kappa = 1/2$) would lead to a substantial reduction of the width, originating in the mechanism of NLO graphs to automatically generate transverse momenta. It is remarkable, that fitting to the nuclear reaction data requires the same width independent of the order of the calculation used!

Recent dAu data at RHIC [21] also indicate that at $\sqrt{s} = 200$ GeV more transverse momentum is necessary, than $\langle k_\perp^2 \rangle \approx 0 - 0.5$ GeV^2, indicated in Fig. 2. This shows, that the scale parameter κ possibly may also depend on the energy, and a higher, $\kappa = 4/3$ scale with $\langle k_\perp^2 \rangle = 2.5$ GeV^2 reproduces well both the pp and the dAu data without jet quenching effects [22]. The dependence of the scale parameter on the c.m. energy and its consequences will be studied in a separate paper.

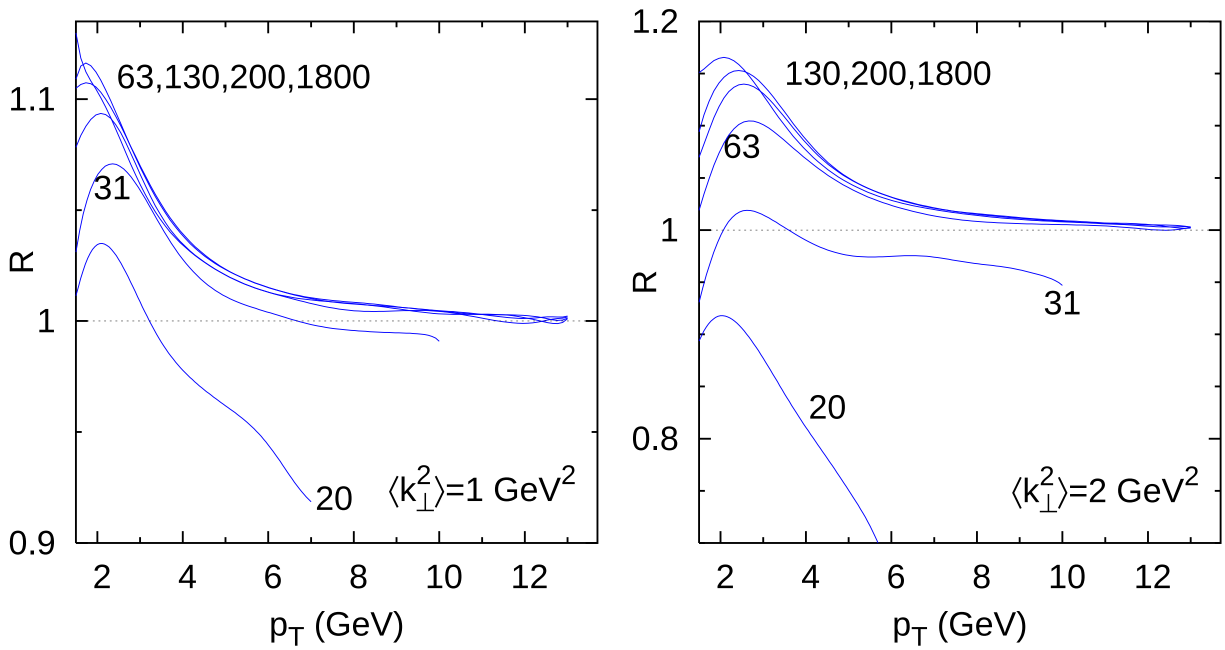

FIG. 4: Ratio of the pionic K factor at $\langle k_\perp^2\rangle = 1$ GeV^2 (left) and 2 GeV^2 (right) to K factor at $\langle k_\perp^2\rangle = 0$ at energies from $\sqrt{s} = 20$ GeV to 1800 GeV.

C. K factor

Since most calculations (especially, for nucleus-nucleus collisions) are still based on LO, it is useful to provide a well-founded K factor for these faster calculations. Fig. 3 shows the ratio of the full NLO calculation to the Born term with no intrinsic transverse momentum. Indeed, at high energy and transverse momenta K_π approaches the well known value of 2, however, in the low transverse momentum region it has a strong p_T dependence. As a first approximation, for $\sqrt{s} \gtrsim 60$ GeV, the pionic K factor can be taken energy independent [23].

Since the intrinsic transverse momentum may have a different effect on the Born term than on the higher order (HO) processes, the latter contributing the dominant part, it is worth to study the dependence of the K factor on the width $\langle k_\perp^2\rangle$. We present this behavior in Fig. 4, indicating the ratio $R = K(\langle k_\perp^2\rangle)/K(0)$ at $\langle k_\perp^2\rangle = 1$ and 2 GeV^2 (with $\kappa = 2/3$ scale parameter) and at different energies from $\sqrt{s} = 20$ to 1800 GeV. While at low energies the ratio R shows a large decrease with increasing $\langle k_\perp^2\rangle$ (mainly due to the efficiency of the intrinsic transverse momentum at low energies in enhancing the Born term faster than the HO corrections), from $\sqrt{s} \gtrsim 60$ GeV its value is always above 1, indicating, that HO corrections are gaining more from transverse momentum, than the Born term. The maximal enhancement is peaked at $p_\perp \approx 2-3$ GeV, with 10% correction at $\langle k_\perp^2\rangle = 1$ GeV^2 and 20% correction at $\langle k_\perp^2\rangle = 2$ GeV^2, while for $p_T \gtrsim$ 4 GeV, the transverse momentum driven correction vanishes, validating LO calculations in this range with $\langle k_\perp^2\rangle$ independent K factors [7, 11].

IV. CONCLUSIONS

In this paper we introduced initial transverse momentum distributions into the parton-based description of hadron production in NLO level pQCD calculations and demonstrated, that such an extension is necessary even in NLO to reproduce the pp experimental data. From the analysis of most experiments in the 3 GeV $\lesssim p_T \lesssim 6$ window a width of intrinsic transverse momentum distribution on the order of $\langle k_\perp^2\rangle \approx$ 2 GeV^2 was fitted. The obtained precision of the description of pion production in pp collisions is high enough to find possible collective effects in nuclear collisions.

We presented the pionic K-factor (full NLO cross section to the Born term) for several energies and transverse momentum values, and found a pronounced dependence on p_T in the 3 GeV $\lesssim p_T \lesssim 6$ window, where nuclear effects show up most prominently. Furthermore, we investigated in detail the modifications of the Born and higher order terms due to the presence of an intrinsic transverse momentum and concluded, that above certain energy in the above mentioned p_T window the higher order contribution raises faster, than the Born term and hence the K-factor increases compared to its value without intrinsic transverse momentum. For $p_T \gtrsim 4$ GeV this enhancement may be neglected, justifying $\langle k_\perp^2\rangle$ independent K-factor calculations, however, at smaller transverse momenta the correction goes up to 20%, just in the order of the measured nuclear enhancement. The detailed study of the nuclear enhancement, and its role in fixing the scales of an NLO calculation will be carried out separately.

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