Fixed-point action for fermions in QCD

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We report our progress constructing a fixed-point action for fermions interacting with SU(3) gauge fields.

1. INTRODUCTION

The ultimate goal of any improvement program for QCD is to find a perfect action with no lattice artifacts: the renormalized trajectory (RT) of some renormalization group transformation (RGT), parameterized by g^2 the gauge coupling and m the quark mass. That is too difficult a task at present, and so we attempt instead to find the one-loop perfect fixed point (FP) action (at $\beta = \infty$) and to follow the trajectory in m. Fixed point (FP) actions[1] for non-Abelian gauge theories have proven their ability to improve scaling for physical observables[2] and provide the starting point for the construction of actions for full QCD. The outline of our work with fermions is similar to that of the MIT group[3] but differs from it in specifics. There is not enough space for us to describe it in detail, and so we present here only an impressionistic overview of our progress to date.

We begin with a set of fermionic (ψ_n, $\bar{\psi}_n$) and gauge field ($U_\mu(n)$) variables and integrate them out to construct an action involving coarse-grained variables Ψ_{n_b}, $\bar{\Psi}_{n_b}$ and $V_\mu(n_b)$ using a renormalization group kernel

$$T = \beta T_g(U,V) + \kappa \sum_{n_b} (\bar{\Psi}_{n_b} - \Omega_{n_b,n}\bar{\psi}_n)(\Psi_{n_b} - \Omega_{n_b,n}\psi_n) \qquad (1)$$

We assume a fine action

$$S = \beta S_g(U) + \bar{\psi}_i \Delta(U)_{ij}\psi_j. \qquad (2)$$

S_g and T_g are the action and blocking kernel for the gauge fields. The renormalization group equation

$$e^{-S'} = \int d\psi d\bar{\psi} dU e^{-(T+S)} \qquad (3)$$

has a pure gauge FP at $g^2 = 0$ ($\beta \to \infty$). In that limit the gauge action dominates the integral; its RG equation is given by the same steepest-descent equation as for a pure gauge model

$$S^{FP}(V) = \min_{\{U\}} \left(S^{FP}(U) + T(U,V) \right), \qquad (4)$$

while the fermions sit in the gauge-field background. Their action remains quadratic in the field variables, and the transformation of the fermion action is given most easily in terms of the propagator

$$(\Delta'(V))^{-1}_{n_b,n'_b} = \frac{1}{\kappa}\delta_{n_b,n'_b} + \Omega(U)_{n_b,n}(\Delta(U))^{-1}_{n,n'}\Omega(U)_{n',n'_b} \qquad (5)$$

where U and V are related through Eqn. 4. In all our work we have focussed on a scale factor 2 RG transformation, for gauge fields the so-called "Type-1" transformation of Ref. [2].

2. FREE MASSLESS FERMIONS

The formalism for free fermions has been given by Wiese[4]. We begin with a continuum action for fermions which has no doublers and is chirally symmetric. We select a blocking kernel Ω, iterate it to find a fixed point action, and then tune parameters in Ω to make the action maximally local. We have used one in which Ω is restricted to a hypercube: Ω_{ij} is nonzero only if $j = i \pm \mu$, $i \pm \mu \pm \nu, \ldots\ i \pm \mu \pm \nu \pm \lambda \pm \sigma$. Each site communicates to $3^4 - 1 = 80$ neighbors. This RGT explicitly breaks chiral symmetry, and so the resulting FP action will not be chirally invariant. However, the spectrum should be chiral.

There are many good parameterizations, resulting in fairly local FP actions. However, one ultimately wants to use these actions in simulations, and the action must be somehow truncated. There are a number of (subjective) criteria to select a good RGT, based on the properties of the truncated action (which the RGT does not know about): a good dispersion relation, $E(p) = |\vec{p}|$ out to large $|\vec{p}|$ with no complex roots, good free-field thermodynamics, $P = 1/3\sigma T^4$ even at large discretization, etc. The"most natural" truncation of our FP actions is to a hypercube. The free field action then can be parameterized as

$$\Delta_0(x) = \lambda(x) + \sum_\mu \gamma_\mu \rho_\mu(x) \qquad (6)$$

with five nonzero λ's and four nonzero ρ's, corresponding to each of the nonzero offsets. An example of a dispersion relation for a "typical" hypercubic action is compared to the Wilson action in Figs. 1 and 2. We show both branches of the hypercubic action's dispersion relation; all roots are real. The non-truncated FP action has a perfect dispersion relation $E = |\vec{p}|$ for all $\vec{p}$.

3. FREE MASSIVE FERMIONS

For massive fermions, we want an action which is on an RT for some RGT (with $m \to 2m$ at each step). To reach the RT, one can begin with an action which has a very small mass but is otherwise close to a FP action, perform a series of blockings, and follow it out.

One complication with this procedure is that an action which is local for small mass can block into an action for large mass which is very nonlocal. To avoid this, we take an RG transformation whose parameters are functions of the mass (for example, the parameter κ in Eqn. 5 is allowed to vary from step to step) and tune the parameter(s) to insure a local action at each blocking step. The resulting λ's and ρ's are smooth functions of the mass. Again, the dispersion relation for hypercubic approximations to RT actions are well behaved out to large $|\vec{p}|$.

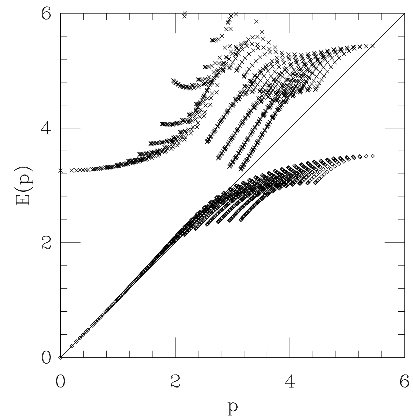

Figure 1. Dispersion relation $E(p)$ vs $|p|$ for a typical hypercubic approximate FP action.

4. THE FP VERTEX

As a first stage in the construction of a FP action for full QCD, we find the FP vertex: That is, we assume smooth gauge fields, expand $U_\mu(x) = 1 + iA_\mu(x) + \ldots$, and solve the RG equation Eq. 5 to lowest nontrivial order in A_μ. The relevant formula has been given by Bietenholz and Wiese[5]. The fermionic action is

$$\begin{aligned} S = & \sum_{n,n'} \bar{\psi}(n)\Delta_0(n-n')\psi(n') \\ + & \ i\sum_{n,n',m} \bar{\psi}(n)\Delta_1^\mu(n-n', m-n') \\ & \times\psi(n')A_\mu(m). \end{aligned} \qquad (7)$$

We solve Eq. 5 by iteration, beginning with a simple free action and its associated gauge invariant vertex. Two independent (but approximate) programs provide cross checks, and we have determined the FP vertex to within a few per cent level for several RGT's. The vertex $\Delta_1^\mu(n-n', m-n')$ is quite complicated and contains in addition to scalar and vector terms, many kinds of "clover"

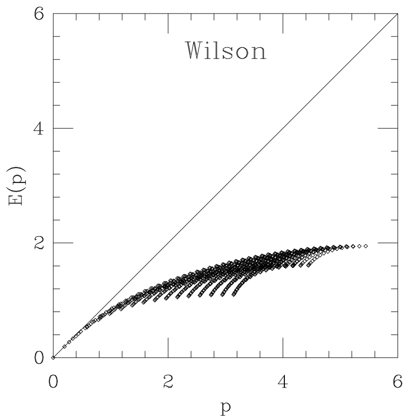

Figure 2. Dispersion relation $E(p)$ vs $|p|$ for massless Wilson fermions.

terms and other operators. However, most of these coefficients are very small. The contributions can be represented as sums of paths of U's. For all but the nearest neighbor term, an (apparently) good approximation to the action is a simple superposition of all minimum length paths of links connecting fermions on sites n and n', while for the nearest neighbor terms, the action involves roughly equal weights of the simple link $U_\mu(x)$ and a sum of length-3 paths $U_\nu(x)U_\mu(x+\nu)U_\nu^\dagger(x+\mu)$. We can also construct the vertex for nonzero mass by iterating the RGT out along the trajectory as described in the previous section. The relative weight of different paths contributing to the vertex shows little variation with quark mass while the overall normalization is set by the parameters of the free action.

5. NONPERTURBATIVE FP ACTION

With the perturbative vertex in hand we will construct the full nonperturbative FP action. This must be done by solving Eqn. 5 numerically, then fitting the action to a set of operators. The perturbative vertex fixes the operators for smooth gauge fields. We have not yet done this part of the project, and we do not know yet whether it is sufficient just to exponentiate the FP vertex, or if other operators are needed.

We have written various pilot matrix inverters for FP fermion actions. The cost of matrix inversion, compared to the Wilson action, is about a factor of twenty per site, which will have to be overcome by the gain going to a bigger lattice spacing.

We believe the major unsolved problem is to find a parameterization of the action which is appropriate for coarse configurations, where we will do simulations.

The final test of an action involves numerical simulation, of course. We plan to compute spectroscopy at fixed physical volume and quark mass (fixed π/ρ mass ratio) at varying lattice spacing, without performing any chiral extrapolations. Our fiducial will be the excellent staggered spectroscopy of the MILC collaboration[6].

This work was supported by the U.S. Department of Energy, the U. S. National Science Foundation, and the Swiss National Science Foundation.

REFERENCES

1. P. Hasenfratz and F. Niedermayer, Nucl. Phys. B414 (1994) 785.
2. T. DeGrand,A. Hasenfratz, P. Hasenfratz, F. Niedermayer, Nucl. Phys. B454 (1995) 587; Nucl. Phys. B454 (1995) 615; A. Hasenfratz and T. DeGrand, these proceedings.
3. See the contributions of Bietenholz, Brower, Chandrasekharan, and Wiese to this volume.
4. U. J. Wiese, Phys. Lett. B315 (1993) 417.
5. W. Bietenholz and U. J. Wiese, Nucl. Phys. B464 (1996) 319.
6. The MILC collaboration, in these proceedings.