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KOPP A. M., ORLOVSKYI D. L. NTU "KhPI" (Ukraine)

## BUSINESS PROCESS MODEL OPTIMIZATION USING THE CONJUGATE GRADIENT METHOD

This paper describes a model and a procedure of business process model optimization using the nonlinear conjugate gradient method. The research is based on business process modeling rules for BPMN (Business Process Model and Notation) as the de-facto standard for business process diagrams. Essential errors of business process modeling are outlined, as well as the metric used to define these shortcomings is denoted. Obtained results of business process model optimization are demonstrated.

# Introduction

Today Business Process Management (BPM) is the most popular management concept. It is based on methods and tools for modeling, analysis, improvement, and automation of business processes. A business process is considered as the set of activities that takes resources as input and produces products or services valuable for customers. Business process modeling is the fundamental technique of BPM. The purpose of business process modeling is to understand boundaries of business processes, document business processes for instructing employees, analyze business processes to find errors (so called "bottlenecks", deadlocks, etc.) and measure performance, and improve business processes to eliminate found errors [1].

## **Problem statement**

Thus, the problem of analysis and improvement of business process models becomes relevant, since early detection and correction of design errors saves costs of business process implementation and execution. The object of research is the procedure of business process design using BPMN notation, which nowadays is the standard of business process modeling. The subject of research is a model and a procedure of business process model optimization. The aim of research is to detect and correct errors of business process modeling.

## **Related work**

According to the business process modeling guidelines [2], degrees of business process model elements should be as minimal as possible. Practical implementation of this requirement means using explicit split or join connectors (gateways in terms of BPMN notation) instead of modeling multiple outgoing sequence flows or multiple incoming sequence flows for tasks respectively (Fig. 1, c, d) [3]. At the same time, the absence of incoming or outgoing sequence flows might be related to missing start or end events (Fig. 1, a, b) or, even worse, to the gaps in a business process structure [3].



Figure 1 – Essential errors of the control-flow of the business process model: a – missing start event; b – missing end event; c – missing split connector; d – missing join connector

An activity should have only one incoming and only one outgoing sequence flow. By modeling more than one incoming or outgoing sequence flow to or from an element, the diagram may become harder to read. This is why gateways should be used. It is important to split sequence flows only by using gateways, as well as joins should be modeled explicitly using gateways. Failure to do so can result in errors such as multi-merges or deadlocks [4].

Therefore, earlier we have proposed a method for business process model analysis and improvement, which is based on the modified balance coefficient used to detect the redundant sequence flows connected to activities or, vice versa, the missing sequence flows [1]. This metric is shown below:

$$K_b^F = \frac{1}{|F|} \sum_{q=1}^{|F|} \sum_{t \in \{in,out\}} |d^t(f_q) - \delta_F^t(f_q)|.$$

Where:

 $-f_q$  is the q-th task of the business process model  $f_q \in F$ , while F is the set of models' tasks;

- in means the incoming sequence flow, while out means the outgoing sequence flow;

 $-d^t(f_q)$  is the number of sequence flows of t-th type,  $t \in \{in, out\}$ , connected to the q-th task;

 $-\delta_F^t(f_q)$  is the recommended number of sequence flows of t-th type,  $t \in \{in, out\}$ , that should be connected to the *q*-th task (since BPMN diagrams are considered,  $\forall t \in \{in, out\}: \delta_F^t(f_q) = 1$ ).

The smaller this coefficient is, the less error-prone the analyzed business process model is [4]. The perfect situation is when the  $K_b^F = 0$ , which means that business process model does not have any of the mistakes shown in Fig. 1.

## **Research materials**

Hence, to define the required changes in order to eliminate possible errors within the business process model structure, the following optimization problem was formulated. The mathematical model, described by the quadratic loss function, demonstrates the minimization of deviations of the actual numbers of sequence flows connected to the tasks of the business process model  $(d^t(f_q) + r_q^t)$  from the desired numbers of sequence flows connected to the tasks of the business process model  $\delta_F^t(f_q)$ :

$$\Phi(r_q^t) = \sum_{q=1}^{|F|} \sum_{t \in \{in,out\}} \left[ \left( d^t(f_q) + r_q^t \right) - \delta_F^t(f_q) \right]^2 \to \min_{\{r_q^t\}}$$

Where  $r_q^t$  is the integer change of the number of sequence flows of t-th type,  $t \in \{in, out\}$ , connected to the q-th task, required to obtain the desired value for the  $K_b^F$  coefficient.

We came up with the idea to use the nonlinear conjugate gradient method in order to minimize the proposed loss function, since with a pure quadratic function on n variables the minimum is reached within at most *n* iterations [5]. Since the number of activities in the business process model might not exceed 50 [1], the total number of iterations required to find the best values of  $r_q^t$  might not exceed 100 iterations because there will be a goal function of  $(\max\{|F|\} \cdot |\{in, out\}|) = 50 \cdot 2 = 100$  variables to minimize.

According to the nonlinear conjugate gradient method, we need to define the initial point  $r_q^{t0}$  and the tolerance value  $\varepsilon$ :

$$r_q^{t0} = 0, q = \overline{1, |F|}, t \in \{in, out\}; \ \varepsilon = 10^{-6}.$$

The steepest direction is the following:

 $\nabla \Phi(r_q^{t0}) = 2 \cdot \left( d^t(f_q) - \delta_F^t(f_q) + r_q^{t0} \right), q = \overline{1, |F|}, t \in \{in, out\}.$ 

So it is required to perform a line search in this direction until it reaches the minimum of  $\Phi$  in order to find the step length  $\lambda_a^t$ :

$$\lambda_q^t = \arg\min_{\lambda_q^t} \Phi(r_q^{t0} - \lambda_q^t \cdot \left[2 \cdot \left(d^t(f_q) - \delta_F^t(f_q) + r_q^{t0}\right)\right]), q = \overline{1, |F|}, t \in \{in, out\}.$$

The minimum of  $\Phi$  is obtained when the gradient is 0:

$$\nabla \Phi(\lambda_q^t) = 2 \cdot \left( d^t(f_q) - \delta_F^t(f_q) - \lambda_q^t \cdot \left[ 2 \cdot \left( d^t(f_q) - \delta_F^t(f_q) \right) \right] \right) = 0, q = \overline{1, |F|}, t \in \{in, out\},$$

$$d^t(f_q) - \delta_F^t(f_q) = \lambda_q^t \cdot \left[ 2 \cdot \left( d^t(f_q) - \delta_F^t(f_q) \right) \right], q = \overline{1, |F|}, t \in \{in, out\},$$

$$\lambda_q^t = \frac{d^t(f_q) - \delta_F^t(f_q)}{2 \cdot \left( d^t(f_q) - \delta_F^t(f_q) \right)} = \frac{1}{2}, q = \overline{1, |F|}, t \in \{in, out\}.$$

Then the position should be updated:  $t = t^{t} + t^{0} + t^{0}$ 

$$r_q^t = r_q^{t0} - \lambda_q^t \cdot \nabla \Phi(r_q^{t0}), q = 1, |F|, t \in \{in, out\},$$
$$r_q^t = r_q^{t0} - \frac{1}{2} \cdot \left[2 \cdot \left(d^t(f_q) - \delta_F^t(f_q) + r_q^{t0}\right)\right] = \delta_F^t(f_q) - d^t(f_q), q = \overline{1, |F|}, t \in \{in, out\}.$$
he equation below demonstrates that the tolerance criterion  $\varepsilon$  is reached:

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$$\|\nabla \Phi(r_q^t)\| = \sqrt{\sum_{q=1}^{|F|} \sum_{t \in \{in,out\}} \left(2 \cdot \left(d^t(f_q) - \delta_F^t(f_q) + \delta_F^t(f_q) - d^t(f_q)\right)\right)^2} = \sqrt{\sum_{q=1}^{|F|} \sum_{t \in \{in,out\}} 0^2} = 0 < \varepsilon.$$

Thus, it is required to perform  $|F| \cdot |\{in, out\}|$  iterations to find values  $r_q^t$  that minimize the introduced loss function  $\Phi$ :

$$r_q^t = \delta_F^t(f_q) - d^t(f_q), q = \overline{1, |F|}, t \in \{in, out\}.$$

Proposed procedure of business process model optimization based on the nonlinear conjugate gradient method is shown in Fig. 2.



Outlined procedure was applied to analyze sample business process models provided at [3]. Obtained results that describe structure of input models and the required changes are shown in Tab. 1. Generated recommendations were used to improve quality of the considered business process models.

Table 1 – Obtained results of	of business process m	odel analysis and optimization
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Model name	$K_b^F$	Input model's tasks $\left\{ \left( f_q, d^{in}(f_q), d^{out}(f_q) \right)   f_q \in F \right\}$	Required changes $\{(f_q, r_q^{in}, r_q^{out})   f_q \in F\}$
Multi-merges negative 2	1	$\{(f_1, 1, 2), (f_2, 2, 1)\}$	$\{(f_1, 0, -1), (f_2, -1, 0)\}$
Multi-merges positive 2	0.5	$\{(f_1, 1, 1), (f_2, 2, 1)\}$	$\{(f_1, 0, 0), (f_2, -1, 0)\}$
Explicit joins negative	0.5	$\{(f_1, 1, 1), (f_2, 1, 1), (f_3, 2, 1)\}$	$\{(f_1, 0, 0), (f_2, 0, 0), (f_3, -1, 0)\}$
Conclusion			

In this paper we have proposed the model and the procedure of business process model optimization based on the nonlinear conjugate gradient method. Future research includes extension of the proposed solution, so it will be possible to detect and correct another types of business process modeling errors.

#### References

1. A. Kopp and D. Orlovskyi, "A Method for Business Process Model Analysis and Improvement," In ICTERI 2019 PhD Symposium, Kherson, Ukraine, 2019, pp. 1-10.

2. J. Krogstie, "Quality of business process models," In IFIP Working Conference on The Practice of Enterprise Modeling, Springer, Berlin, Heidelberg, 2012, pp. 76-90.

3. А. Копп и Д. Орловский, "Анализ и оптимизация моделей бизнес-процессов в нотациях ЕРС и ВРММ," Технічні науки та технології, Т. 14, № 4, С. 145-152, 2018.

4. Process structure – BPMN modelling guidelines. [Online]. Available: https://www.modelingguidelines.org/categories/process-structure/. [Accessed Sept. 28, 2019].

5. R. W. Cottle, M. N. Thapa, Linear and Nonlinear Optimization. Springer, 2017.