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Andrea Cristofoli

Niels Bohr International Academy and Discovery Center, Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, 2100 Copenhagen, Denmark

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A B S T R A C T

The aim of this note is to describe the computation of post-Minkowskian Hamiltonians in modified theories of gravity. Exploiting a recent relation between scattering amplitudes of massive scalars and potentials for relativistic point-particles we derive a contribution to post-Minkowskian Hamiltonians at second order in the Newton's constant coming from $\mathcal{R}^3$ modifications in General Relativity. Using this result we calculate the associated contribution to the scattering angle for binary black holes at second post-Minkowskian order, showing agreement in the non-relativistic limit with previous results for the bending angle of a massless particle around a static massive source in $\mathcal{R}^3$ theories.

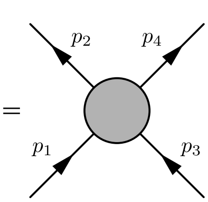
0. Introduction

The detections of gravitational waves by the LIGO and Virgo collaboration, has opened up the possibility to test Einstein's theory of General Relativity at an unprecedented level, heralding a new era in fundamental physics [1]. A central framework is the Effective One Body approach [2,3], where information from Numerical Relativity and analytical approaches are combined in order to lead to improved gravitational wave templates. Among these several inputs, it has been recently suggested [4,5] that also post-Minkowskian (PM) results, valid for weak gravitational fields and unbound velocities, can independently lead to improved modeling of bound binary dynamics. Given the growing results in post-Minkowskian physics [6–13], we would like to explore how contributions to post-Minkowskian Hamiltonians can be defined in modified theories of gravity. With no loss of generality, we here restrict ourselves on $\mathcal{R}^3$ modifications[1] to General Relativity [14–18]. Recently, these have been studied in the context of scattering amplitudes [19,20] leading to a post-Newtonian definition of the potential [21,22]. However, scattering amplitudes contain relativistic information that is lost in the passage to post-Newtonian point-particles potentials. We show how this can be restored defining a post-Minkowskian potential in cubic theories of gravity, without restricting to the case of non-relativistic point-particles. Using this result we derive the associated contribution to the fully relativistic scattering angle for binary black holes at second order in the Newton's constant. By then taking the non-relativistic limit of one particle and the massless of the other, we are able to reproduce the bending angle recently calculated in [19] for a massless particle around a static massive source.

1. Higher derivative corrections in General Relativity

A non-trivial modification of the one-loop scattering of massive scalars in cubic theories of gravity has been recently studied with amplitudes techniques in [19,20]. In what follows we focus on the contribution given by $I_1 \equiv R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu}$. As can be seen from [23], this arises as a non-trivial modification to the usual Einstein-Hilbert action which for simplicity of discussion we will parametrize by an unknown coefficient α with the dimension of length squared, following [19]. The associated classical information in the scattering of two massive scalars of masses m_1, m_2 has been calculated here [19,20]. This is given by

$$\mathcal{M}^{\alpha}(p,q) = \qquad (1)$$



$$= \mathcal{D}\Big[\mathcal{I}(m_1)\, c(m_1,m_2) + \mathcal{I}(m_2)\, c(m_2,m_1)\Big] + \ldots \qquad (2)$$

E-mail address: a.cristofoli@nbi.ku.dk.

[1] These arise as further contributions to the Ricci scalar in the Einstein-Hilbert action, where the only non-trivial modifications are given by $R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu}$ and $R^{\mu\nu\alpha}{}_{\beta}R^{\beta\gamma}{}_{\nu\sigma}R^{\sigma}{}_{\mu\gamma\alpha}$.

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where, using $s=(p_1+p_3)^2$ and $t=(p_1-p_2)^2$, we have defined

$$\mathcal{D}=\frac{i\pi^2 G_N^2\alpha^2}{\sqrt{E_1E_2E_3E_4}} \tag{3}$$

$$\mathcal{I}(m_j)=\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p_1-k)^2(p_3-k)^2(k^2-m_j^2)} \tag{4}$$

$$c(m_i,m_j)=\frac{4t^2m_i^4}{(4m_i^2-t)^2}\left[\sum_{k=1}^{3}\beta_k(m_i,m_j)t^{(k-1)}\right] \tag{5}$$

$$\beta_1(m_i,m_j)=2m_i^2\left[(m_i^2+m_j^2-s)^2-4m_i^2m_j^2\right] \tag{6}$$

$$\beta_2(m_i,m_j)=-3m_i^4+2m_i^2m_j^2+(m_j^2-s)^2 \tag{7}$$

$$\beta_3(m_i,m_j)=m_i^2-m_j^2+s \tag{8}$$

We choose the center-of-mass frame and parametrize the momenta of the scattering particles as

$$\begin{aligned} &p_1^\mu=(E_1,\vec{p}\,), \quad p_2^\mu=(E_1,\vec{p}\,') \\ &p_3^\mu=(E_2,-\vec{p}\,), \; p_4^\mu=(E_2,-\vec{p}\,') \end{aligned} \tag{9}$$

$$\vec{q}\equiv\vec{p}\,'-\vec{p} \tag{10}$$

$$|\vec{p}|=|\vec{p'}|\equiv p \quad , \quad |\vec{q}|\equiv q \tag{11}$$

We now proceed to define a post-Minkowskian potential in the context of this modified theory of gravity using a recent relation between post-Minkowskian amplitudes and Hamiltonians [13]. The simplicity of this computation here lies in the lack of the Born subtraction, as there is no tree level amplitude to iterate that scales in the same way as (2). We can thus define a post-Minkowskian potential to second order in G_N and in the coupling α as

$$V_{2PM}^{I_1}(p,r)=\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\mathcal{M}^\alpha(p,q) \tag{12}$$

By performing a proper k^0 integration on (4), the scalar triangle integral becomes [6,24]

$$\mathcal{I}(m_j)=-\frac{i}{32m_jq}+\ldots \tag{13}$$

where the ellipsis denotes quantum contributions.

To leading order in q the associated post-Minkowskian potential is[2]

$$V_{2PM}^{I_1}(p,r)=\frac{\pi^2G_N^2\alpha^2}{32E_1E_2}\int\frac{d^3q}{(2\pi)^3}\left[\frac{c(m_1,m_2)}{m_1}+\frac{c(m_2,m_1)}{m_2}\right]\frac{e^{i\vec{q}\cdot\vec{r}}}{q} \tag{14}$$

$$=\frac{\pi^2G_N^2\alpha^2}{128E_1E_2}\left(\frac{\beta_1(m_1,m_2)}{m_1}+\frac{\beta_1(m_2,m_1)}{m_2}\right)\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}q^3 \tag{15}$$

$$V_{2PM}^{I_1}=\frac{3\alpha^2}{32E_1E_2}\frac{G_N^2}{r^6}\left(\frac{\beta_1(m_1,m_2)}{m_1}+\frac{\beta_1(m_2,m_1)}{m_2}\right) \tag{16}$$

In the non-relativistic limit, our post-Minkowskian potential reduces to

$$V_{2PM}^{I_1}(p,r)=\frac{3\alpha^2}{4}\frac{G_N^2p^2}{r^6}\frac{(m_1+m_2)^3}{m_1m_2}+\ldots \tag{17}$$

[2] The reason we only keep the leading term in q is due by $\hbar$ counting. For a detailed analysis on how to restore the proper classical limit from an amplitude calculation see [25].

in agreement with the post-Newtonian calculation in [19]. For the sake of completeness we also report the post-Minkowskian contribution to the potential given by the remaining cubic term $R^{\mu\nu\alpha}{}_{\beta}R^{\beta\gamma}{}_{\nu\sigma}R^{\sigma}{}_{\mu\gamma\alpha}$. This has been recently calculated in [19] as coming from the topological invariant $G_3=R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu}-2R^{\mu\nu\alpha}{}_{\beta}R^{\beta\gamma}{}_{\nu\sigma}R^{\sigma}{}_{\mu\gamma\alpha}$. The result has been found equal to

$$V_{2PM}^{G_3}(p,r)=\frac{12\alpha^2G_N^2}{E_1E_2}\frac{m_1^2m_2^2(m_1+m_2)}{r^6} \tag{18}$$

In a natural way, the same procedure for defining a post-Minkowskian potential can be applied for more general modified theories of gravity.

2. The scattering angle

At second post-Minkowskian order in G_N, the Hamiltonian for a binary system of spinless binary black holes, including contributions from cubic gravity, is given by

$$H_{2PM}^\alpha(p,r)=\sqrt{p^2+m_a^2}+\sqrt{p^2+m_b^2}+V_{2PM}(p,r)+V_{2PM}^\alpha(p,r) \tag{19}$$

where $V_{2PM}(p,r)$ has been calculated here [6,13], being V_{2PM}^α the sum of (16) and (18). Since the motion lies on a plane, we can introduce the following coordinates on the phase space (r,ϕ,p_r,p_ϕ) so as to express the momentum in the center of mass frame as

$$p^2=p_r^2+\frac{p_\phi^2}{r^2} \quad , \quad p_\phi=L \tag{20}$$

being L the angular momentum of the system, which is a conserved quantity.

The associated Hamilton-Jacobi equation is given by

$$\sqrt{p^2+m_a^2}+\sqrt{p^2+m_b^2}+V_{2PM}(p,r)+V_{2PM}^\alpha(p,r)=E \tag{21}$$

with E being the energy, another constant of motion.

By solving now in p^2 we can express the momentum in the center of mass frame as

$$p^2=p^2(E,L,\alpha,r) \quad , \quad p^2=p_0^2+\frac{G_Nf_1}{r}+\frac{G_N^2f_2}{r^2}+\frac{G_N^2\alpha^2f_\alpha}{r^6}+\ldots \tag{22}$$

where the ellipsis denotes higher contributions in G_N and

$$\begin{aligned} &p_0^2=\frac{(p_1\cdot p_2)^2-m_1^2m_2^2}{s} \\ &f_1=-\frac{2c_1}{\sqrt{s}} \quad , \quad f_2=-\frac{1}{2\sqrt{s}}\left(\frac{c_{\triangle_a}}{m_a}+\frac{c_{\triangle_b}}{m_b}\right) \end{aligned} \tag{23}$$

$$f_\alpha=-\frac{3}{16E}\left(\frac{\beta_1(m_1,m_2)}{m_1}+\frac{\beta_1(m_2,m_1)}{m_2}\right)-\frac{24m_1^2m_2^2(m_1+m_2)}{E} \tag{24}$$

At this point, by considering the angular variable ϕ, it is straightforward to derive the following expression for its total change during a scattering

$$\Delta\phi=\pi+\chi \quad , \quad \frac{\chi(E,L)}{2}=-\int_{r_{min}}^{+\infty}dr\frac{\partial p_r}{\partial L}-\frac{\pi}{2} \tag{25}$$

where r_{min} is the positive root for $p_r=0$.

In order to evaluate (25) we proceed perturbatively by expanding both the integrand and the extreme of integration in G_N, where

$$r_{min} = \frac{L}{p_0} + \ldots \quad , \quad p_r = \sqrt{p_0^2 - \frac{L^2}{r^2}} + \ldots \tag{26}$$

being the leading term of r_{min} equivalent to the impact parameter b.

This expansion gives rise to divergent integrals which can be handled only by means of the *Hadamard Partie finie*[3] (Pf) of the latter as shown by Damour in [5,26]. Restricting to the contribution to (25) due to $\mathcal{R}^3$ one finds

$$\frac{\chi^{\alpha}_{2PM}}{2} = -\frac{L G_N^2 \alpha^2 f_\alpha}{2} \,\mathbf{Pf} \int_{r_0}^{+\infty} \frac{dr}{r^8} \left(p_0^2 - \frac{L^2}{r^2}\right)^{-\frac{3}{2}} \tag{27}$$

Changing variables to $u = \frac{1}{r}$ the integral becomes

$$\frac{\chi^{\alpha}_{2PM}}{2} = -\frac{G_N^2 \alpha^2 f_\alpha}{2L^2} \,\mathbf{Pf} \int_0^{u_0} du \frac{u^6}{(u_0^2 - u^2)^{\frac{3}{2}}} \quad , \quad u_0 \equiv \frac{1}{b} \tag{28}$$

The remaining integration is straightforward, leading to

$$\frac{\chi^{\alpha}_{2PM}}{2} = \frac{15\pi G_N^2 \alpha^2 f_\alpha}{32 L^2 b^4} \tag{29}$$

$$\frac{\chi^{\alpha}_{2PM}}{2} = -\frac{45\pi G_N^2 \alpha^2}{512 L^2 b^4 E}\left(\frac{\beta_1(m_1,m_2)}{m_1} + \frac{\beta_1(m_2,m_1)}{m_2} + 128 m_1^2 m_2^2 (m_1+m_2)\right) \tag{30}$$

Equation (30) has to be considered as an additional contribution to the fully relativistic scattering angle at second order in G_N coming from a cubic theory of gravity. In particular, by taking the non-relativistic limit of our result with the additional condition $m_1 = m$ and $m_2 = 0$, we have

$$\chi^{\alpha}_{2PM} = -\frac{45 G_N^2 \alpha^2 \pi m^2}{32 b^6} + \ldots \tag{31}$$

which agrees with the non-relativistic contribution derived in [19] for the bending angle of a massless particle around a static massive source.[4] In this case, the G_3 contribution to the potential is found to be absent for the bending angle of a massless particle, but not in the fully relativistic scattering angle of two massive particles as it can be seen from (30).

3. Conclusion

We have derived the post-Minkowskian contribution to relativistic point-particles Hamiltonians in modified theories of gravity. We have restricted ourselves to the case of $\mathcal{R}^3$ modifications, although similar changes are expected to appear also for $\mathcal{R}^2$ terms [27–29]. The derived post-Minkowskian contribution, once expanded for small velocities, is in agreement with the recent post-Newtonian computation [19]. The simplicity of the calculation has taken advantage of a recent relation between post-Minkowskian amplitudes and Hamiltonians for relativistic point-particles [13]. Indeed, the computation has required no effective field theory matching as well as no need to known the operator reproducing the $\mathcal{R}^3$ modifications in an effective field theory of scalar fields. We have also derived an additional contribution to the fully relativistic scattering angle of black holes at second order in G_N arising from $\mathcal{R}^3$, showing agreement in the non-relativistic limit with a result derived in [19] for the bending angle of a massless particle around a static massive source. It would be interesting to systematically explore similar results in other alternative formulations of General Relativity.

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Appendix A. Hadamard Partie finie

In order to evaluate perturbatively the scattering angle, one needs to expand both the integrand as well as the extreme of integration in G_N. As noticed in [5,26], the procedure gives rise to spurious divergences which can be handled in a proper way only by means of the *Hadamard Partie finie* of the latter. In order to see how this procedure applies, let's consider the 1PM scattering angle

$$\frac{\chi}{2} = L \int_{r_{min}}^{+\infty} \frac{dr}{r^2 p_r(r)} - \frac{\pi}{2} \tag{32}$$

$$= L \int_0^{+\infty} \frac{dr}{r^2} \frac{\theta(r - r_{\min})}{p_r} - \frac{\pi}{2} \tag{33}$$

The integrand in Eq. (33) has to be Taylor expanded in G_N both for p_r as well as for r_{min}, giving[5]

$$\theta(r - r_{min}) = \theta(r - b) + \delta(r - b)\frac{G_N f_1}{2p_0^2} + \ldots \tag{34}$$

$$\frac{1}{p_r} = \frac{1}{(p_0^2 - \frac{L^2}{r^2})^{\frac{1}{2}}} - \frac{f_1 G_N}{2(p_0^2 - \frac{L^2}{r^2})^{\frac{3}{2}} r} + \ldots \tag{35}$$

Plugging these into Eq. (33) one obtains

$$\frac{\chi}{2} = L \int_0^{+\infty} \frac{dr}{r^2}\left[\theta(r-b) + \delta(r-b)\frac{G_N f_1}{2p_0^2}\right] \times \left[\frac{1}{(p_0^2 - \frac{L^2}{r^2})^{\frac{1}{2}}} - \frac{f_1 G_N}{2(p_0^2 - \frac{L^2}{r^2})^{\frac{3}{2}} r}\right] - \frac{\pi}{2} \tag{36}$$

$$= -\frac{L G_N f_1}{2} \int_b^{+\infty} \frac{dr}{r^3} \frac{1}{(p_0^2 - \frac{L^2}{r^2})^{\frac{3}{2}}} + \frac{L G_N f_1}{2p_0^2} \int_0^{+\infty} \frac{dr}{r^2} \frac{\delta(r-b)}{\sqrt{p_0^2 - \frac{L^2}{r^2}}} \tag{37}$$

$$= -\frac{L G_N f_1}{2b^2 p_0^3} \int_1^{+\infty} \frac{dx}{(x^2-1)^{\frac{3}{2}}} + \frac{L G_N f_1}{2b^2 p_0^3} \int_0^{+\infty} \frac{dx}{x} \frac{\delta(x-1)}{\sqrt{x^2-1}} \tag{38}$$

[3] For further details, see Appendix A.

[4] The authors in [19] have used a convention for the deflection angle which differs by a minus sign compared to ours.

[5] We have used the fact that $r_{min} = b - \frac{G_N f_1}{2p_0^2} + \ldots$ which can be derived by using the definition of r_{min}.

where in the last line we have changed variable using $r = xb$.

At this point, we can notice the presence of two divergent contributions to the scattering angle. In order to deal with them, one needs to regularize these integrals. The *Hadamard Partie finie* consists of regularize them in the following way

$$\chi = \lim_{\Lambda\to 1}\left(-\frac{LG_N f_1}{2b^2 p_0^3}\int_{\Lambda}^{+\infty}\frac{dx}{(x^2-1)^{\frac{3}{2}}} + \frac{LG_N f_1}{2b^2 p_0^3}\int_{0}^{+\infty}\frac{dx}{x}\frac{\delta(x-\Lambda)}{\sqrt{x^2-1}}\right) \tag{39}$$

In doing so, one obtains

$$\chi = \lim_{\Lambda\to 1}\left(-\frac{LG_N f_1}{2b^2 p_0^3}\left[\frac{\Lambda}{\sqrt{\Lambda^2-1}}-1\right] + \frac{LG_N f_1}{2b^2 p_0^3}\left[\frac{1}{\Lambda\sqrt{\Lambda^2-1}}\right]\right) \tag{40}$$

$$= \frac{G_N f_1}{2Lp_0} \tag{41}$$

which give us the desired finite contribution to the scattering angle. As shown in [5,26], this technique can be generalized to any PM order providing a powerful tool for the perturbative evaluation of the scattering angle.

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