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Interpretation of the results of the experiment on generation of parametric X-radiation by relativistic electrons in a single-crystal target

S.V. Blazhevich[a,*], A.V. Noskov[b]

[a] Belgorod State University, Pobedy Str., 85, Belgorod 308015, Russia
[b] Belgorod State Technological University named after V.G. Shukhov, Kostyukova Str., 46, Belgorod 308012, Russia

A B S T R A C T

The coherent X-ray radiation generated by a relativistic electron beam in a monocrystalline target of a limited thickness is investigated in the framework of two-wave approximation of the dynamical theory of diffraction in the conditions of asymmetric reflection of the radiation waves in relation to the target surface. The calculations of the radiation angular density for the conditions of the recent experiment on generation of parametric X-ray radiation (PXR) have been carried out. It has been shown, that adequate description of such experiments is possible only with use of the dynamical theory which takes into account the effects of asymmetric reflection of the radiation.

1. Introduction

When a fast charged particle crosses a monocrystalline plate, its Coulomb field is scattered in the target on a system of parallel atomic planes, generating the parametric X-ray radiation (PXR) [1–3]. The intersection of the surface of the plate by a charged particle is accompanied with the transition radiation (TR) [4], which then diffracts on a system of parallel atomic planes of the crystal, generating diffracted transition radiation (DTR) in the Bragg scattering direction [5–7]. PXR of a relativistic electron in a single crystal within the framework of the dynamical theory of X-ray diffraction, in the general case of asymmetric reflection was considered in Refs [8,9], but in these works the DTR contribution and the effects associated with the asymmetry of reflection were not considered. Transition radiation (TR) and DTR in the general case of asymmetric reflection of the electron field relative to the target surface was considered in [10]. The dynamical theory of coherent X-ray radiation of relativistic electron in a single crystal, as well as in an artificial periodic layered medium, was developed in [11–14] for the general case of the asymmetric reflection of the Coulomb field of the electron relative to the target surface, when the system of parallel reflecting layers in the target can be at arbitrary angle to the target surface. In [11–14] the coherent X-ray radiation was considered as the total effect of two radiation mechanisms, namely, PXR and DTR, and of the interference of these mechanisms. The effect of the asymmetry of the reflection on the angular density of the PXR in the conditions of multiple scattering of a beam of relativistic electrons on atoms of the medium was considered in the Bragg scattering geometry in [15]. In [16,17], the expressions describing the angular density of a PXR generated by a beam of relativistic electrons that cross a single crystal or a periodic layered medium were obtained in the Laue scattering geometry, taking into account multiple scattering of beam electrons by target atoms and the asymmetry of the radiation process.

Although it is customary to talk about the dynamical theory of PXR as the theory most correctly describing this mechanism of coherent radiation, it is traditionally believed that in the overwhelming majority of cases it is quite sufficient to use the formulas of a simpler and more understandable kinematical theory (see, for example, [18]), and dynamical theory can give only small corrections to the kinematical one. It is also believed at present that asymmetry in the radiation process requires only a slight correction of the expressions for the angular density.

The effects of the reflection asymmetry, in parametric X-ray radiation (PXR) generated by a single relativistic electron on a system of parallel atomic planes in a single crystal were discussed in [13]. In that work we compared the results of dynamical approach in coherent radiation theory to the results of kinematical one.

Since the experimental results on the angular density of the coherent X-ray radiation of relativistic electrons in a single crystal are usually presented in relative units (not in absolute ones), it may seem that it does not matter which formulas to use in the interpretation of the experiment, namely, the formulas taking into account the asymmetry of the reflection or not taking it into account (obtained for the conditions of symmetrical reflection), since the asymmetry, in general, affects the amplitude of the angular density of the PXR, and a small change in the width of the PXR angular distribution that can arise under the influence of the asymmetry, can always be associated with the errors of the

* Corresponding author.
E-mail address: blazh@bsu.edu.ru (S.V. Blazhevich).

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experiment.

The results of the experiment on the study of the angular distribution of PXR excited by electrons with an energy of 255 MeV on two different atomic planes of a single crystal of the diamond [19] have recently been published, the theoretical interpretation of which was done out using the dynamical formula for PXR taken by the authors in [20]. This formula was derived for PXR in the symmetric Laue scattering geometry; however, it was used to describe the angular distribution of the PXR both for the case of a symmetric reflection (in the conditions of the considered experiment on the system of atomic planes (2 2 0)) and for asymmetric reflection (on the system of the planes (1 1 1). The obtained in the experiment [19] angular distributions of PXR density both in the case of symmetric (on the plane 2 2 0) and asymmetric (on the (1 1 1) plane) reflections of the radiation are presented in the relative units, however, the total yield of coherent X-ray radiation is presented in absolute ones (number of photons/sec). This allowed us to investigate the adequacy of use in [19] of the dynamical formula from [20] to describe the angular distribution of PXR on the (1 1 1) planes of a single crystal of diamond.

The asymmetry of reflection affects the angular density of the PXR (see [11]). In particular, it was shown in [11] that for a fixed Bragg angle, the decrease in the angle of relativistic electron incidence to the target surface associated to a decrease in the angle between the target surface and the diffracting atomic planes leads to the increase in the spectrum width and a correspondent increase in the angular density of PXR. It means that disregard of the reflection asymmetry in the experiment under consideration will lead to an error in the results of calculation of the absolute value of the angular density of the PXR generated on the atomic planes (1 1 1).

In this connection, it seemed important to us to carry out the detailed study of the experimental data [19] and to interpret the results of this experiment based on a consistent dynamical approach.

2. Radiation process geometry

Let us consider the geometry and the main physical parameters of the coherent radiation accompanying the passage of a relativistic electron beam through the single crystal target (Fig. 1) in the condition of the experiment [19]. In the considered experiment the Laue scattering geometry was realized.

Since the interaction of each electron in the beam with the crystal under these conditions can be regarded as independent, the angular density of the radiation by the whole particle beam can be represented as the angular density of radiation by one electron, averaged over the angular distribution of the electrons in the beam and multiplied by the number of electrons in the beam.

To do such averaging, it is necessary to determine the place of the electron in the beam when describing the angular density of the coherent radiation generated by it, introducing for this purpose additional angular variables connecting the electron motion direction with the selected direction, which we call “the axis of the beam”.

To describe the radiation generated by a relativistic electron traveling at a velocity **V** through a crystal plate (Fig. 1), we introduce the angular variables ψ, θ and θ_0 related correspondently to the velocity of a relativistic electron and to the unit vectors **n** (in the direction of the photon emitted close to the electron velocity) and $\mathbf{n_g}$ (in the direction of the photon emitted near the Bragg scattering direction):

$$\begin{aligned}
&\mathbf{V} = \left(1 - \tfrac{1}{2}\gamma^{-2} - \tfrac{1}{2}\psi^2\right)\mathbf{e}_1 + \psi, \ \ \mathbf{e}_1\psi = 0\\
&\mathbf{n} = \left(1 - \tfrac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \theta_0, \ \ \mathbf{e}_1\theta_0 = 0, \ \ \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B\\
&\mathbf{n_g} = \left(1 - \tfrac{1}{2}\theta^2\right)\mathbf{e}_2 + \theta, \ \ \mathbf{e}_2\theta = 0
\end{aligned} \tag{1}$$

where Θ is the radiation angle counted from the axis of the radiation detector $\mathbf{e}_2$, $\boldsymbol{\psi}$ is the electron deflection angle in the beam, counted from the electron beam axis $\mathbf{e}_1$, Θ_0 is the angle between the direction of propagation of the incident photon and the $\mathbf{e}_1$ axis, $\gamma = 1/\sqrt{1 - V^2}$ is Lorentz -factor of a particle. The angular variables are decomposed into components parallel and perpendicular to the plane of the figure $\Theta = \Theta_{||} + \Theta_{\perp}$, $\Theta_0 = \Theta_{0||} + \Theta_{0\perp}$, $\psi = \psi_{||} + \psi_{\perp}$.

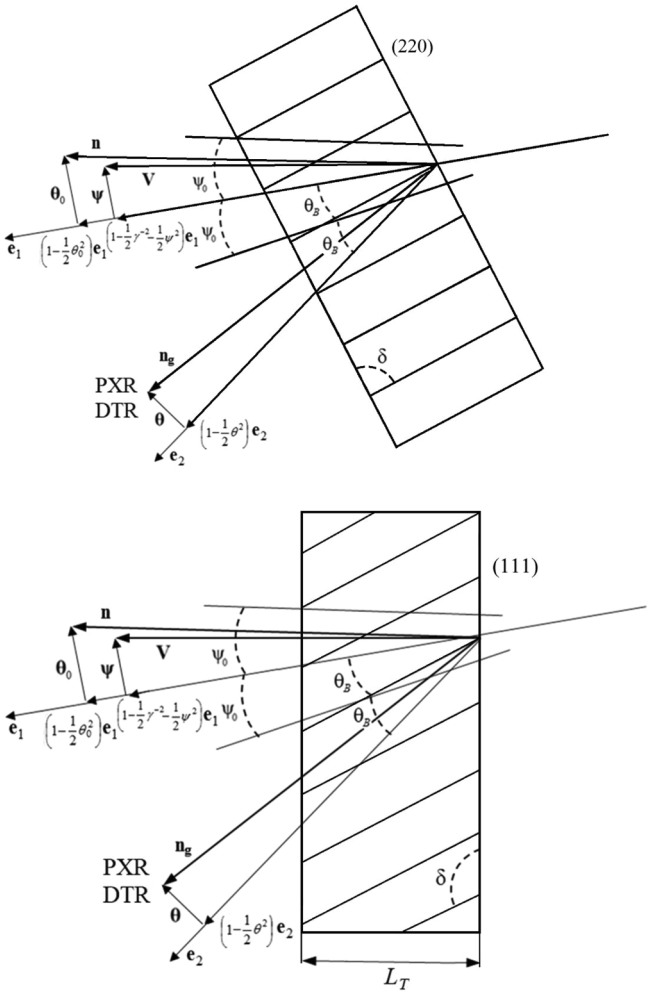

Fig. 1. The radiation process geometry for symmetric (atomic planes (2 2 0)) and asymmetric (atomic planes (1 1 1)) reflection of the radiation waves.

3. The comparison of the photon absolute yields obtained in the experiment and in theoretical calculations

In the work [19], the results of the measurements of the angular distribution of the coherent X-ray radiation excited by the beam of 255 MeV electrons in the 50 μm thick diamond crystal on the systems of atomic planes (1 1 1) and (2 2 0) near the direction of Bragg scattering are presented.

It is assumed in [19] that the main contribution to coherent X-ray radiation excited by electrons with an energy of 255 MeV is provided by parametric X-ray radiation. The calculations of the averaged angular density of the PXR were made using the formula $I^{conv}(\theta_x, \theta_y)$ of [20], which has the form

$$\begin{aligned}
\left\langle \frac{d^2N_{PXR}}{d\theta_x d\theta_y} \right\rangle &\equiv I^{conv}(\theta_x, \theta_y)\\
&= \frac{1}{2\pi\sigma'_{\exp,x}\sigma'_{\exp,y}} \int\int I(\theta_x - q_x, \theta_y - q_y) \exp\Bigg(\\
&\quad -\frac{q_x^2}{2(\sigma'_{\exp,x})^2}\Bigg) \exp\left(-\frac{q_y^2}{2(\sigma'_{\exp,y})^2}\right) dq_x dq_y,
\end{aligned} \tag{2}$$

where

$$I(\theta_x, \theta_y) = \frac{\alpha\omega_B}{4\pi c \sin^2(\theta_B)}\left[\frac{\theta_x^2}{4(1+W_{||}^2)} + \frac{\theta_y^2}{4(1+W_{\perp}^2)}\right]$$

$$W_{||,\perp} = \frac{1}{2|\chi_g|P_{||,\perp}}\left(\theta_x^2 + \theta_y^2 + \theta_{PXR}^2 - \frac{|\chi_g|^2 P_{||,\perp}^2}{\theta_x^2 + \theta_y^2 + \theta_{PXR}^2}\right)$$

$$P_{||} = \cos(2\theta_B),\ P_{\perp} = 1,\ \theta_{PXR} = \sqrt{\gamma^{-2} + \frac{\omega_p^2}{\omega^2}} \tag{3}$$

θ_x and θ_y are the angles determining the distribution of radiation at the IP detector (Imaging Plate), $\sigma'_{exp,x}$ and $\sigma'_{exp,y}$, are the experimental angular resolution in the horizontal and vertical directions, ω_B is the Bragg frequency, θ_B is the Bragg reflection angle, χ_g is the Fourier component of the dielectric susceptibility for the reciprocal-lattice vector g. The formula (2) determines the angular density of PXR excited by an electron per unit length of its path in the target. In [16], within the framework of the dynamical theory of diffraction, for the general case of asymmetric reflection of the electron field relative to the target surface the expression is obtained that describes the angular density of the PXR generated by the beam of relativistic electrons in a single crystal. In order to take into account the effect of the initial angular divergence of the beam and the multiple scattering of electrons by medium atoms, this expression is averaged over the expanding beam of rectilinear trajectories of radiating electron along the path of the electron in the target $L_e = L_T/\sin(\delta - \theta_B)$, where L_T is the target thickness, using the angular distribution function of the electrons in the beam

$$f(\psi, t) = \frac{1}{2\pi}\frac{e^{-\frac{\psi_{||}^2}{2\cdot(\psi_{0||}^2 + \psi_{ms||}^2 t)}}}{\sqrt{\psi_{0||}^2 + \psi_{ms||}^2 t}}\cdot\frac{e^{-\frac{\psi_{\perp}^2}{2\cdot(\psi_{0\perp}^2 + \psi_{ms\perp}^2 t)}}}{\sqrt{\psi_{0\perp}^2 + \psi_{ms\perp}^2 t}} \tag{4}$$

Where $\psi_{0||}$ and $\psi_{0\perp}$ are the initial angular divergences of the electron beam in horizontal and vertical planes correspondently, $\psi_0 = \sqrt{\psi_{0||}^2 + \psi_{0\perp}^2}$ is the initial divergence of the electron beam in volume. $\psi_{ms||}^2$ and $\psi_{ms\perp}^2$ are the mean squared projections of multiple scattering angle per units of the electron path in the target ψ_{ms}^2: $\psi_{ms||}^2 = \psi_{ms\perp}^2 = \frac{\psi_{ms}^2}{2}$, $\psi_{ms}^2 = \frac{E_{ms}^2}{m^2\gamma^2}\frac{1}{L_R}\left(1 + 0.038\cdot\ln\left(\frac{t}{L_R}\right)\right)^2$ is the mean square of the multiple scattering angle of the electron per unit length of the trajectory [21], $E_{ms} \approx \frac{4\pi m^2}{e^2} \approx 21\,\text{MeV}$, L_R is the radiation length, $\psi_0 = \sqrt{\psi_{0||}^2 + \psi_{0\perp}^2}$ is the initial divergence of the electron beam.

For asymmetric angular distribution the averaging was carried out by the formula

$$\left\langle \frac{d^2N_{PXR}^{(s)}}{d\theta_\perp d\theta_{||}} \right\rangle = \frac{1}{2\pi L_e}\int_0^{L_e} dt \int_0^{L_e}\int d\psi_\perp d\psi_{//}\frac{e^{-\frac{\psi_{||}^2}{2\cdot(\psi_{0||}^2 + \psi_{ms||}^2 t)}}}{\sqrt{\psi_{0||}^2 + \psi_{ms||}^2 t}}\cdot \frac{e^{-\frac{\psi_{\perp}^2}{2\cdot(\psi_{0\perp}^2 + \psi_{ms\perp}^2 t)}}}{\sqrt{\psi_{0\perp}^2 + \psi_{ms\perp}^2 t}}\int \frac{d^2N_{PXR}^{(s)}}{d\theta_\perp d\theta_{||}} \tag{5}$$

where $\frac{d^2N_{PXR}^{(s)}}{d\theta_\perp d\theta_{||}}$ is the angular density of PXR generated by a single electron incident on the crystal at an angle $\psi(\psi_\perp, \psi_{//})$ to the axis of the electron beam. Substituting the expression for $\frac{d^2N_{PXR}^{(s)}}{d\theta_\perp d\theta_{||}}$ derived in [16] into (5), we obtain the expression describing the angular density of the PXR with allowance for the multiple scattering of the beam electrons by the atoms of the medium:

$$\left\langle \frac{d^2N_{PXR}^{(s)}}{d\theta_\perp d\theta_{||}} \right\rangle \equiv I(\theta_\perp, \theta_{||}) = \frac{e^2\varepsilon^2}{8\pi^2\sin^2\theta_B\chi_0'' L_e}\sum_{s=1}^{2}\nu^{(s)2}\int\int d\psi_\perp d\psi_{//}\Omega^{(s)2} \times\left(\frac{\left(1 - \exp\left\{-2b^{(s)}\rho^{(s)}\frac{(\Sigma+1)^2 - 2\kappa^{(s)}\nu^{(s)}(\Sigma+1) + \nu^{(s)2}}{\nu^{(s)2} + (\Sigma+1)^2\varepsilon}\right\}\right)}{(\Sigma+1)^2 - 2\kappa^{(s)}\nu^{(s)}(\Sigma+1) + \nu^{(s)2}} \int_0^{L_e}\frac{e^{-\frac{\psi_{||}^2}{2(\psi_{0||}^2 + \psi_{ms||}^2 t)}}}{\sqrt{\psi_{0||}^2 + \psi_{ms||}^2 t}}\cdot\frac{e^{-\frac{\psi_{\perp}^2}{2(\psi_{0\perp}^2 + \psi_{ms\perp}^2 t)}}}{\sqrt{\psi_{0\perp}^2 + \psi_{ms\perp}^2 t}}dt\right) \tag{6}$$

where the following notation is introduced $\Omega^{(1)} = \theta_\perp - \psi_\perp$, $\Omega^{(2)} = \theta_{//} + \psi_{//}$, $\nu^{(s)} = \frac{\chi_g' C^{(s)}}{\chi_0'}$, $\kappa^{(s)} = \frac{\chi_g'' C^{(s)}}{\chi_0''}$, $\varepsilon = \frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)}$, $\rho^{(s)} = \frac{\chi_0''}{|\chi_g'|C^{(s)}}$, $b^{(s)} = \frac{1}{2\sin(\delta-\theta_B)}\frac{L}{L_{ext}^{(s)}}$, where $L_{ext}^{(s)} = 1/\omega_B\,|\chi_g'|\,C^{(s)}$ is the extinction length,

$$\sum \equiv \sum(\theta_\perp, \theta_{//}, \psi_\perp, \psi_{//}, \gamma) = \frac{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2}{|\chi_0'|},\ C^{(1)} = 1,\ C^{(2)} = \cos 2\theta_B \tag{7}$$

$\chi_g = \chi_g' + i\chi_g''$ is the Fourier coefficients of the expansion of the dielectric susceptibility of a single crystal over the reciprocal-lattice vectors **g**: $\chi(\omega, \mathbf{r}) = \chi_0(\omega) + \sum_{g\neq 0}\chi_g(\omega)\exp(i\mathbf{gr})$, $\chi_0 = \chi_0' + i\chi_0''$ is the average dielectric susceptibility of the single crystal. Expressions (6) describe the angular density of the PXR summed over the polarizations (for $s = 1$ the expression (6) describes the σ- polarized fields and for $s = 2$ the π-polarized fields).

We give the identical notation for the formulas (2), (3) [19] and (5), (6):

$$\theta_\perp \equiv \theta_y,\ \theta_{//} \equiv \theta_x,\ \psi_{//} \equiv q_x,\ \psi_\perp \equiv q_y,\ C^{(2)} \equiv P_{||},\ C^{(1)} \equiv P_\perp$$

It should be noted that in the formula (2) the parameters $\sigma'_{exp,y}$ and, $\sigma'_{exp,x}$, describing the angular resolution of the experiment for measurements along the coordinate y and ×, respectively, take into account the initial divergence of the electron beam and its transverse dimensions and the angles of multiple scattering of electrons by the medium material, averaged over thickness of the target $\sigma'_{exp,s} = \sqrt{\left(\frac{\sigma_s}{L}\right)^2 + (\sigma'_s)^2 + (\sigma'_{MS})^2}$, where σ_s is the transverse dimension parameter (one standard deviation) of the electron beam on the target, $\sigma'_s$is the angular divergence of the electron beam along the transverse coordinate, σ'_{MS} is the multiple scattering angle averaged over the thickness of the target, $s = x$ or y and L is the distance from the target to the detector (Image plate). The estimates of these quantities, obtained in [19] on the basis of experimental data and used in (2), give the following values of the angular resolution:

$$\sigma'_{exp,y} = 1\,\text{mrad},\ \sigma'_{exp,x} = 0.6\,\text{mrad}$$

In the formulas (5) and (6), the multiple scattering angle is used as a function of the path t traversed by the electron in the target $\psi_{ms}^2\cdot t$. It defines the formation of the angular distribution of the radiation along the path of the electron beam in the target together with the projection of the mean squared angles of the initial divergence of the electron beam incident on the target $\psi_{0\perp}^2$ and $\psi_{0||}^2$. The initial divergence of the incident electron beam was estimated from the experimental data by formulas $\psi_{0||} \equiv \sigma 1_{exp,x}$and $\psi_{0||\perp} \equiv \sigma 1_{exp,y}$, where $\sigma 1_{exp,s} = \sqrt{\left(\frac{\sigma_s}{L}\right)^2 + (\sigma'_s)^2}$, as $\psi_{0\perp} = 0.9\,\text{mrad}$, $\psi_{0||} = 0.32$ mrad and were used in theoretical calculations carried on according to formula (6).

The important parameter $\varepsilon = \frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)}$ in expressions (6) and (7) determines the degree of asymmetry of the reflection of the electron field in the crystal plate relative to the target surface. The parameter δ is the angle between the target surface and the reflecting atomic planes.

Note that the expressions (2) and (3) do not contain the asymmetry parameter. However, when calculating the angular density of the PXR, it is necessary to multiply expression (2) by the path of the electron in

the target $L_e = L_T/\sin(\delta - \theta_B)$, that is, the parameter δ entering the definition of the asymmetry parameter ε determines only the path of the electron in the target, that changes as the asymmetry does.

The parameter $\rho^{(s)}$, characterizing the degree of X-ray absorption in the crystal, is equal to the ratio of the extinction length $L_{ext}^{(s)} = 1/\omega\ |\chi_{\mathbf{g}}'|\ C^{(s)}$ to the absorption length $L_{abs} = 1/\omega\chi_0''$ of the X-ray: $\rho^{(s)} = L_{ext}^{(s)}/L_{abs} = \chi_0''/(|\chi_{\mathbf{g}}'|\ C^{(s)})$. The parameter $b^{(s)}$ characterizing the thickness of the crystalline plate, is equal to half the ratio of the electron path in the target $L_e = L_T/\sin(\delta - \theta_B)$ to the extinction length $L_{ext}^{(s)}$. The parameter $\nu^{(s)}$, taking values in the $0 \le \nu^{(s)} \le 1$ interval, determines the degree of reflection of waves from the crystal, which is caused by the nature of the interference of waves reflected from different planes (constructive ($\nu^{(s)} \approx 1$) or destructive ($\nu^{(s)} \approx 0$)). The parameter $\varkappa^{(s)}$ determines the degree of manifestation of the effect of anomalous weak photoabsorption (the Bormann effect) in the passage of X-ray photons of PXR and DTR through a crystal target [6].

The total yield of the radiation measured in solid angle of 30×30 mrad^2 at the average electron beam current of 70nA is presented in [19] in absolute units and equals the value of 1.2×10^5 photons per second for the atomic plane (1 1 1) and 3.2×10^4 photons per second for the plane (2 2 0). Since the formulas (2) and (6) allow us to calculate the absolute values of the angular radiation density (the number of photons per electron in the solid angle of 1 sr), then substituting them correspondently into expressions

$$N'_{PXR} = L_e \cdot \int_{-0.03}^{0.03}\int_{-0.03}^{0.03} I^{conv}(\theta_x, \theta_y)d\theta_x d\theta_y \tag{8a}$$

and

$$N_{\mathrm{PXR}} = \int_{-0.03}^{0.03}\int_{-0.03}^{0.03} I(\theta_\perp, \theta_{||})d\theta_\perp d\theta_{||} \tag{8b}$$

we can calculate the total yield of the PXR and compare it with the experimentally measured yield. In this case, in (6), the asymmetry parameter for radiation on atomic planes (2 2 0) takes on a value $\varepsilon = 1$ (symmetric case) ($\delta = 90^0$, $\theta_B = 16.\ 1^o$), and for radiation on atomic planes (1 1 1) $\varepsilon = 0.66$ (asymmetric case, $\delta = 125.\ 3^0$, $\theta_B = 16.\ 1^o$). Numerical calculations yielded the following results:

$$\begin{aligned} N'_{PXR(220)} &= 8.325\cdot10^{-7}\tfrac{photons}{electron}\\ N_{PXR(220)} &= 8.312\cdot10^{-7}\tfrac{Photons}{electron}\\ N'_{PXR(111)} &= 4.581\cdot10^{-6}\tfrac{photons}{electron}\\ N_{PXR(111)} &= 2.968\cdot10^{-6}\tfrac{Photons}{electron} \end{aligned} \tag{9}$$

For the experimental conditions [19] (electron current $I = 7\cdot10^{-9}A$), the calculated numbers of photons emitted per second ($n_{PXR} = N_e\cdot N_{PXR} = \frac{I}{e}\cdot N_{PXR}$) are presented for comparison with the experiment in Table 1.

It can be seen that in the symmetric case the formulas (8a) and (8b) give identical results close to the experimental one $n'_{PXR(220)} = n_{PXR(220)} \approx n_{220}^{\exp}$. In the asymmetric case, the absolute yields of photons calculated by formulas (8a) and (8b)$n'_{PXR(111)}$ $n_{PXR(111)}$ differ significantly, while the result obtained by formula (8b) practically coincides with the experimental one.

Table 1
The total yield of PXR excited by an electron beam (I_e = 7nA) to a solid angle of 30×30 mrad^2, (number of photons per second). The target is a single-crystal diamond of $5*10^{-5}$m thickness.

	(2 2 0) plane	(1 1 1) plane
Experiment [19], $n^{\exp}$	3.2×10^4	1.2×10^5
Calculation by the formula (8a), n'_{PXR}	3.630×10^4	2.004×10^5
Accuracy of coincidence with experiment, %	13.4	67
Calculation by the formula (8b), n_{PXR}	3.636×10^4	1.325×10^5
Accuracy of coincidence with experiment, %	13.6	10.4

Thus, the calculations show that formula (2) cannot be used to calculate the absolute value of the angular density of PXR in the general case of asymmetric reflection of the electron field in the crystal with respect to the target surface, and formula (6) correctly describes the angular density of PXR in both the symmetric and asymmetric cases.

Since the difference between the results of the calculations of the output of PXR photons by means of formulas (8a) and (8b) is associated with the asymmetry of reflection of radiation waves on a system of atomic planes in a crystal, we should pay attention to the fact that in formula (8b) there is a common factor in the form of the asymmetry parameter epsilon in the second degree, which distinguishes it from the formula of kinematic [18,22] and traditional dynamic theory of PXR [20,23] describing the reflection asymmetry by the factor in the form of the asymmetry parameter of epsilon in the first degree. This difference was explained earlier by the authors in their work [11,13], where it was shown that the additional degree of the epsilon appears as a result of the broadening effect of the spectral peak of the PXR with a change in the reflection asymmetry (the parameter epsilon), which, when integrating the spectral function of the PXR in frequency, leads to the correspondent growth in the PXR angular density. As can be seen from the comparison with the experiment, this coefficient completely compensates for the discrepancy between the experiment and theory, which is observed when formulas (2) and (8a) are used. A similar asymmetry effect also occurs in diffracted transition radiation [11].

4. Comparison of the angular densities of PXR calculated by different formulas and measured in the experiment

As shown above, for the total yield of PXR generated on the atomic plane (1 1 1), the discrepancy between the experiment and the calculation by formula (8a) [20] exceeds 50%, while for the plane (2 2 0) it does not exceed 14%. Hence the question arises by what the way the good agreement between the theory and the experiment was obtained in [19] for distributions of the angular density of PXR generated by relativistic electrons in considered single-crystal target not only on the system of diamond atomic planes (2 2 0), but also on (1 1 1). The result in [19] can be explained by the fact that the “cross-linking” of the experimental data and the calculation results in [19] was carried out for the (2 2 0) and (1 1 1) planes independently under the assumption that the calculated and experimental curves should coincide in both cases. For this purpose, a linear regression was used, linking the theoretical and experimental dependencies according to the condition of the minimum for the sum of root-mean-square deviations at all points of the angular distribution (see [19]). One of the regression parameters (free term in the linear dependence) actually determines the background contribution in the experimental angular distribution of radiation, and the second (the coefficient in front of the calculated intensity value) coordinates the scales of intensity units in the experimental and in calculated dependences.

Since the units of measurement used are the same for both the (2 2 0) plane and the (1 1 1) plane, the same regression parameters should be used in both cases. Obviously, the considerable discrepancy between the theory and the experiment in the absolute values of the total yield of PXR generated on the system of atomic planes (1 1 1) means that formula (2) in this case incorrectly describes the experiment.

The cross-linking of the angular dependences of the PXR obtained in the experiment and calculated theoretically seems to be natural for the case of reflecting on the atomic planes (2 2 0), and the values of the regression parameters obtained in this case must be applied also for the comparison of theory and experiment in the case of PXR on atomic planes (1 1 1). Such cross-linking can be used because of a rather small distinction of the results of theoretical calculations and the results of the experiment allowing such actions in the limits of the errors of the experiment and the calculations.

Note that all theoretical calculations of the angular distribution of

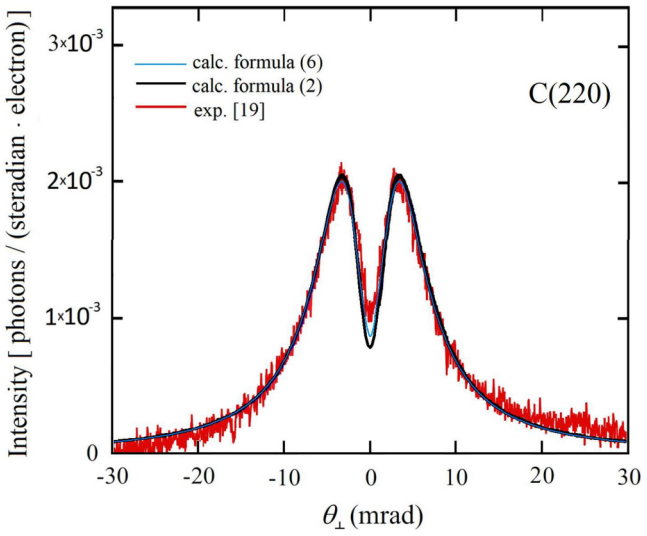

Fig.2. Comparison of the experimental data [19] and the results of calculation of the PXR by formula (2) and formula (6): the angular dependence of the intensity (angular density) of the coherent radiation on the atomic planes of the crystal (2 2 0) (symmetrical reflection), ($\theta_{||} = 0$). The point ($\theta_{\perp} = 0, \theta_{||}=0$) corresponds to the direction of Bragg reflection.

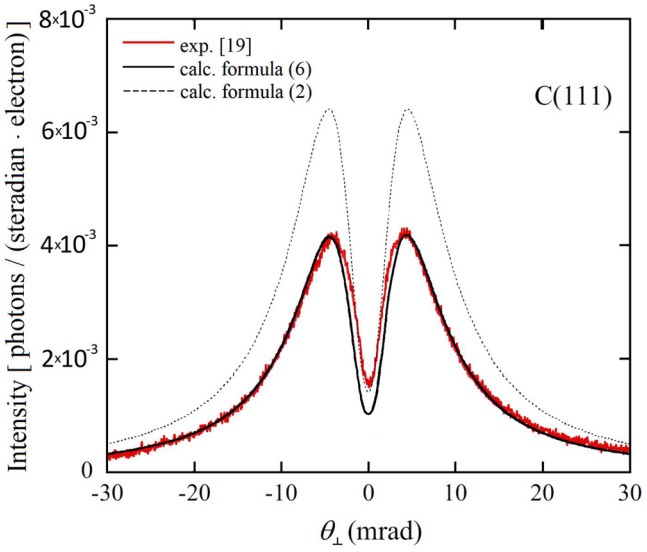

Fig. 3. The angular dependence of the intensity (angular density) of the coherent radiation on the atomic planes of the crystal (1 1 1) (asymmetric reflection) in the plane perpendicular to the figure Fig. 1, ($\theta_{||} = 0$). The point ($\theta_{\perp} = 0$, $\theta_{||} = 0$) corresponds to the direction of Bragg reflection. The comparison of the experimental data [19] with the results of calculation of the PXR using formula (2) and formula (6).

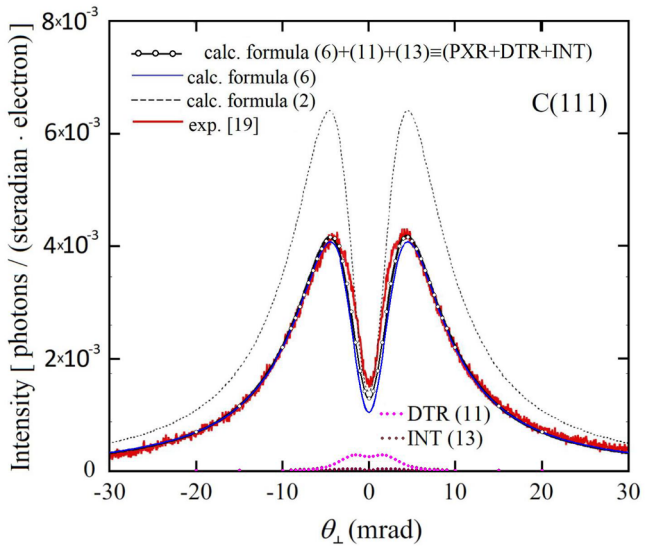

Fig. 4. The contributions of the DTR (formula (11)) and the interference term (PXR + DTR) (formula (13)) into the angular dependence of the intensity (angular density) of the coherent radiation by the relativistic electron on the atomic planes (1 1 1) of the crystal (symmetrical reflection) on the angle $\theta_{\perp}$ in the plane perpendicular to the plane of the Fig. 1, ($\theta_{||} = 0$). The point ($\theta_{\perp} = 0$, $\theta_{||} = 0$) corresponds to the direction of Bragg reflection.

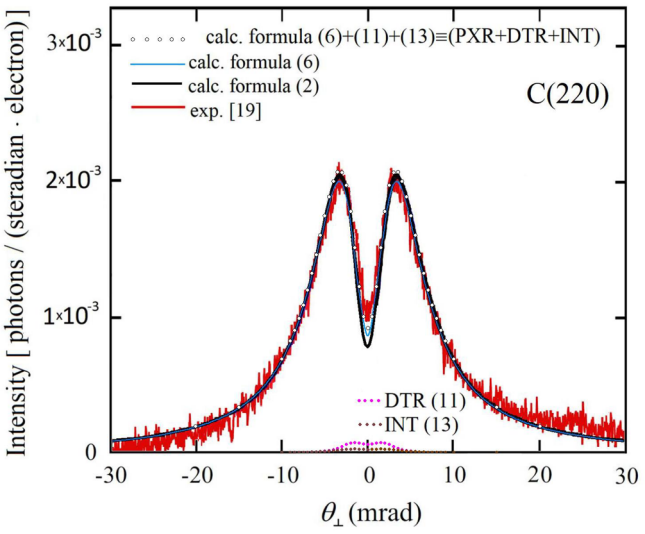

Fig. 5. The contributions of the DTR (formula (11)) and the interference term (PXR + DTR) (formula (13)) into the angular dependence of the intensity (angular density) of the coherent radiation by relativistic electrons on the atomic planes (2 2 0) of the crystal (symmetrical reflection) on the angle $\theta_{\perp}$ in the plane perpendicular to plane of the Fig. 1, ($\theta_{||} = 0$). The point ($\theta_{\perp} = 0$, $\theta_{||} = 0$) corresponds to the direction of Bragg reflection.

the intensity of PXR were carried out in absolute units. After cross-linking the results of the calculation and the results of the experiment, these absolute units have determined the intensity scale on the graphs of the angular dependence in Figs. 2–5.

Comparison of the total radiation yields calculated from formulas (8a) and (8b) and measured in the experiment gives an estimate of the error in the values of the radiation intensity. The values of the total radiation yield obtained after cross-linking the calculation results and experimental data are presented in Table 2.

The calculations of the angular dependence of the intensity for the PXR according to the formulas (2) and (6) on the (2 2 0) plane and according to the formula (6) on the (1 1 1) plane after the cross-linking with the dependence measured experimentally show almost complete match of the theory and the experiment (see Figs. 2 and 3) with the exception of a narrow region of radiation angles close to the Bragg angle, in which, in addition to PXR, other radiation mechanisms can contribute to the angular density of the coherent radiation. The radiation in this region of the angular distribution of the PXR is treated separately in the next section of this article. Table 2 and Fig. 3, demonstrate that the discrepancies between the results of the experiment and the calculations for the PXR on the (1 1 1) plane remain significant both for the angular dependence of the intensity calculated by means of the formula (2) and for the total yield of the radiation calculated by the formula (8a) even after this cross-linking, as one could expect. On the other hand, the formulas (6) and (8b) correctly describe the experiment

Table 2
The total yield of PXR excited by an electron beam (I_e = 7 nA) into a solid angle of 30 × 30 mrad^2, (number of photons/s). The coefficient of cross-linking of the theory and experiment a = 0.88.

	(2 2 0) plane	(1 1 1) plane
Experiment [19]	3.2×10^4	1.2×10^5
Calculation by the formula (8a)	$0.88 \times (3.630 \times 10^4) = 3.194 \times 10^4$	$0.88 \times (2.004 \times 10^5) = 1.764 \times 10^5$
Accuracy of coincidence with experiment, %	0.17	47
Calculation by the formula (8b)	$0.88 \times (3.636 \times 10^4) = 3.1997 \times 10^4$	$0.88 \times (1.325 \times 10^5) = 1.166 \times 10^5$
Accuracy of coincidence with experiment, %	0.01	2.8

in the case of PXR on the atomic plane (1 1 1) same as on plane (2 2 0). It means that disregard of the reflection asymmetry in the experiment under consideration leads to the significant error in the results of calculation of the absolute value of the angular density of the PXR generated on the atomic planes (1 1 1). It should be noted that in figures Figs. 2–5 only experimentally measured dependences are taken from [19]. All other curves were obtained as the result of our calculations.

5. Calculation of the contribution of DTR to coherent radiation

As noted in [19], the discrepancies between the theory and the experiment in the angular density of PXR near the Bragg direction can mean the presence in the experimental angular dependence of the radiation intensity of additional contribution from other radiation mechanisms, namely diffracted transition radiation (DTR) and diffracted bremsstrahlung (DBR) generated by relativistic electrons in the target. In this connection, we have calculated the contributions of the DTR and the interference term PXR + DTR to the coherent radiation under consideration.

We used the formulas for the angular distribution of the intensity of the coherent radiation of a relativistic electron excited in a single crystal by a beam of relativistic electrons, which were obtained in Ref. [14]. These formulas represent the angular density of coherent radiation in the form of the sum of the contributions of the PXR, DTR, and the term describing the interference result of the PXR and DTR. Averaging of the angular radiation density from the distribution of electrons in the beam and taking into account the multiple scattering of electrons in the target were carried out for the angular density of the DTR according to the formula similar to (5):

$$\left\langle \frac{d^2N^{(s)}_{\mathrm{DTR}}}{d\theta_\perp d\theta_{||}} \right\rangle = \frac{1}{2\pi L_e^*} \int_0^{L_e^*} dt \int\int d\psi_\perp d\psi_{//} \frac{e^{-\frac{\psi_{||}^2}{2(\psi_{0||}^2+\psi_{s||}^2 t)}}}{\sqrt{\psi_{0||}^2+\psi_{s||}^2 t}} \cdot \frac{e^{-\frac{\psi_\perp^2}{2(\psi_{0\perp}^2+\psi_{s\perp}^2 t)}}}{\sqrt{\psi_{0\perp}^2+\psi_{s\perp}^2 t}} \frac{d^2N^{(s)}_{\mathrm{DTR}}}{d\theta_\perp d\theta_{||}} \tag{10}$$

where $L_e^* = \min(L_e, l_c)$, $l_c \approx \frac{2}{\omega(\gamma^{-2}-\chi_0')}$ is the coherence length (the radiation forming length).

Substituting the expression for $\frac{d^2N^{(s)}_{\mathrm{DTR}}}{d\theta_\perp d\theta_{||}}$ [16] to (10) we obtain full formula for DTR angular density

$$\left\langle \frac{d^2N^{(s)}_{\mathrm{DTR}}}{d\theta_\perp d\theta_{||}} \right\rangle = \frac{e^2\,|\chi_{\mathbf{g}}'|}{16\pi^3 \sin^2\theta_B L_e^*} \sum_{s=1}^{2} C^{(s)} \int_{-\infty}^{\infty} R^{(s)}_{DTR} d\xi \times \int\int d\psi_\perp d\psi_{//} \Omega^{(s)2} \left(\frac{1}{\Omega^{(1)2}+\Omega^{(2)2}+\gamma^{-2}} - \frac{1}{\Omega^{(1)2}+\Omega^{(2)2}+\gamma^{-2}-\chi_0'} \right)^2 \int_0^{L_e^*} \frac{e^{-\frac{\psi_{||}^2}{2(\psi_{0||}^2+\psi_{s||}^2 t)}}}{\sqrt{\psi_{0||}^2+\psi_{s||}^2 t}} \cdot \frac{e^{-\frac{\psi_\perp^2}{2(\psi_{0\perp}^2+\psi_{s\perp}^2 t)}}}{\sqrt{\psi_{0\perp}^2+\psi_{s\perp}^2 t}} dt \tag{11}$$

where

$$R^{(s)}_{DTR} = \frac{4\varepsilon^2}{\xi^2+\varepsilon} \exp\left(-b^{(s)}\rho^{(s)}\frac{1+\varepsilon}{\varepsilon}\right) \times \left[\sin^2\left(b^{(s)} \frac{(\sqrt{\xi^2+\varepsilon})}{\varepsilon} \right) + sh^2\left(b^{(s)}\rho^{(s)} \frac{(1-\varepsilon)\xi^{(s)} + 2\varepsilon\kappa^{(s)}}{2\varepsilon\sqrt{\xi^2+\varepsilon}} \right) \right], \tag{12}$$

$$\xi^{(s)}(\omega) = \frac{2\sin^2\theta_B}{V^2\,|\chi_{\mathbf{g}}'|\,C^{(s)}} \left(\frac{\omega(1-\theta_{//}\cot\theta_B)}{\omega_B} - 1 \right) + \frac{1-\varepsilon}{2\nu^{(s)}}$$

$R^{(s)}_{DTR}$ is the DTR spectral function.

The expression describing the interference of PXR and DTR get such a form

$$\left\langle \frac{d^2N_{INT}}{d\theta_\perp d\theta_{||}} \right\rangle = \frac{e^2\,|\chi_{\mathbf{g}}'|}{16\pi^3 \sin^2\theta_B L_e} \sum_{s=1}^{2} C^{(s)} \int_{-\infty}^{\infty} R^{(s)}_{INT} d\xi \times \int\int d\psi_\perp d\psi_{//} \Omega^{(s)2} \left(\frac{1}{\Omega^{(1)2}+\Omega^{(2)2}+\gamma^{-2}} - \frac{1}{\Omega^{(1)2}+\Omega^{(2)2}+\gamma^{-2}-\chi_0'} \right) \times \int_0^{L_e} \frac{e^{-\frac{\psi_{||}^2}{2(\psi_{0||}^2+\psi_{s||}^2 t)}}}{\sqrt{\psi_{0||}^2+\psi_{s||}^2 t}} \cdot \frac{e^{-\frac{\psi_\perp^2}{2(\psi_{0\perp}^2+\psi_{s\perp}^2 t)}}}{\sqrt{\psi_{0\perp}^2+\psi_{s\perp}^2 t}} dt \tag{13}$$

$$R^{(s)}_{INT} = -\frac{2\varepsilon}{\xi^{(s)2}+\varepsilon} \mathrm{Re}\Big((\xi^{(s)} - \sqrt{\xi^{(s)2}+\varepsilon}) \frac{1-\exp[-ib^{(s)}M_2(\xi^{(s)}) - b^{(s)}\rho^{(s)}\Delta^{(1)}]}{M_2(\xi^{(s)}) - i\rho^{(s)}\Delta^{(1)}} \times (\exp[ib^{(s)}M_2(\xi^{(s)}) - b^{(s)}\rho^{(s)}\Delta^{(1)}] - \exp[ib^{(s)}M_1(\xi^{(s)}) - b^{(s)}\rho^{(s)}\Delta^{(2)}]) \Big), \tag{14}$$

where, $R^{(s)}_{INT}$ is the interference spectral function, which describes the influence of interference on the spectrum of the total radiation,

$$M_{1,2}(\xi^{(s)}) = \frac{1}{|\chi_{\mathbf{g}}'|\,C^{(s)}} (\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0') + \frac{\xi^{(s)} \pm \sqrt{\xi^{(s)2}+\varepsilon}}{\varepsilon} \tag{15}$$

It can be seen from the Figs. 4 and 5 that the contribution of the DTR to the coherent X-ray emission excited by the beam of electrons with an energy of 255 MeV in the crystal of a diamond is comparatively small, but its allowance brings to the significant approximation of the values of the coherent radiation intensity calculated theoretically to the values measured in the experiment [19] for the directions close to the Bragg reflection. The remaining discrepancy allows us to expect also the contribution of diffracted bremsstrahlung (DBR) to the intensity of the radiation in that region of the emission angles.

6. Conclusion

In this paper, we compare the published in [19] results of the experimental study of the angular distribution of coherent X-ray radiation excited by relativistic 255-MeV electrons in a single-crystal diamond target on the systems of parallel atomic planes (2 2 0) and (1 1 1) with the results of theoretical calculations carried out in the frame of dynamical theory taking into account the effect of asymmetry of the radiation waves reflection.

Our calculations of absolute yield of PXR photons in a given solid angle in Bragg scattering direction show excellent agreement with the results of the experiment presented in [19] for both cases of symmetric (2 2 0 plane) and asymmetric (1 1 1 plane) reflection of the radiation waves, while the results of calculations by traditional dynamical formula [20] demonstrate a significant discrepancy with the experiment in the case of asymmetric reflection on a system of atomic planes (1 1 1).

It is shown that the quantitative interpretation of the experimental results on the generation of coherent radiation excited by relativistic electrons in a single crystal on a system of parallel atomic planes located at an angle with respect to the target surface (for the Laue scattering geometry) or $\delta \neq 0$ (for the Bragg scattering geometry) should be carried out only within the framework of a dynamical theory that takes into account the effects of the asymmetry of X-ray waves reflection.

The carried out detailed calculation of the multiple scattering of electrons by target atoms made it possible to bring the results of the calculated angular distribution of the PXR closer to the experimental data. The remaining discrepancy in the region of small emission angles was partially compensated by taking into consideration the contribution of the diffracted transition radiation arising on the front boundary of the target. It means that the contribution of DBR to coherent radiation also is expected.

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References

[1] M.L. Ter-Mikaelian, High-Energy Electromagnetic Processes in Condensed Media, Wiley, New York, 1972.
[2] G.M. Garibian, C. Yang, J. Exp. Theor. Phys. 61 (1971) 930.
[3] V.G. Baryshevsky, I.D. Feranchuk, J. Exp. Theor. Phys. 61 (1971) 944.
[4] V.L. Ginzburg, I.M. Frank, J. Exp. Theor. Phys. 15 (1946) 15.
[5] A. Caticha, Phys. Rev. A 40 (1989) 4322.
[6] N. Nasonov, Phys. Lett. A 246 (1998) 148.
[7] X. Artru, P. Rullhusen, Nucl. Instr. Meth. B 145 (1998) 1.
[8] V.G. Baryshevsky, O.M. Lugovskaj, Phys. At. Nucl. 66 (2) (2003) 409.
[9] V.G. Baryshevsky, I.D. Feranchuk, A.P. Ulyanenkov, Parametric X-Ray Radiation in Crystals, Springer, 2005, p. 171.
[10] S.V. Blazhevich, A.V. Noskov, Nucl. Instr. Meth. B 252 (2006) 69.
[11] S.V. Blazhevich, A.V. Noskov, Nucl. Instr. Meth. B 266 (2008) 3770.
[12] S. Blazhevich, A. Noskov, J. Exp. Theor. Phys. 109 (2009) 901.
[13] S.V. Blazhevich, A.V. Noskov, J. Surf. Invest. X-ray, Synchrotron Neutron Techn. 5 (4) (2011) 718.
[14] S.V. Blazhevich, I.V. Kolosova, A.V. Noskov, J. Exp. Theor. Phys. 114 (2012) 547.
[15] S.V. Blazhevich, A.V. Noskov, S.N. Nemtsev, Russian Phys. J. 59 (7) (2016) 1098.
[16] S.V. Blazhevich, N.I. Moskalenko, T.V. Kos'kova, E.A. Tkachenko, A.V. Noskov, J. Surf. Invest.: X-ray, Synchrotron Neutron Tech. 11 (1) (2017) 49.
[17] S.V. Blazhevich, T.V. Koskova, A.V. Noskov, J. Exp. Theor. Phys. 122 (1) (2016) 5.
[18] I.D. Feranchuk, A.V. Ivashin, J. Physique. 46 (1985) 1981.
[19] Y. Takabayashi, K.B. Korotchenko, Yu.L. Pivovarov, T.A. Tukhfatullin, Nucl. Instr. and Meth. B 402 (2017) 79.
[20] R. Yabuki, H. Nitta, T. Ikeda, Y.H. Ohtsuki, Phys. Rev. B 63 (2001) 174112.
[21] N.P. Shulga, S.P. Fomin, JETP. 86 (1) (1998) 32.
[22] K.-H. Brenzinger, C. Herberg, B. Limburg, et al., Physica A 358 (1997) 107.
[23] D. Feranchuk, O. Lugovskaya, A. Ulyanenko, Nucl. Instr. Meth. B 234 (2005) 148–158.