Heuristics for a Green Orienteering Problem

Saverio Basso[1], Marco Casazza[1], and Alberto Ceselli[1]

[1]Università degli Studi di Milano, Dipartimento di Informatica, Italy.

We address a routing problem where a vehicle with limited time, loading capacity and battery autonomy can optionally serve a set of customers, each providing a profit. Such a problem is of particular relevance both because of its practical implications in sustainable transportation and its use as a sub-problem in *Green Vehicle Routing* column generation algorithms. We propose a dynamic programming approach to obtain both primal and dual bounds to the value of the optimal solutions, a fast greedy heuristics and a very large scale neighbourhood search procedure.

1 Introduction

Pushed by the constant increase in gasoline costs, and eco-sustainability awareness of consumers, the market share of vehicles powered by alternative fuels is increasing steadily. In traditional routing problems, fuel autonomy is typically assumed sufficient to serve all customers because of both the big size of the tank and the short refuelling time. However, such conditions do not hold for vehicles running on alternative fuels such as electric batteries, whose full recharge can take up to several hours. Therefore, an explicit planning of the refuelling stops is required to satisfy time constraints, such as in [3].

Within this context we address the *Green Orienteering Problem (GOP)*, a variant of the *Orienteering Problem* [2] involving a single vehicle running on electric batteries only. Each time the vehicle visits a customer it collects a profit. However, to travel between customers the vehicle consumes both battery and time resources. Recharge stations are available on the network, which are equipped with different recharging technologies, offering different trade-off between recharge time and cost. During its route, the vehicle can stop at recharge stations, selecting a particular technology and the amount of energy to be recharged. A fixed cost is always paid at each recharge. The aim is to find a route that maximizes the difference between collected profits and recharge costs, in such a way that (a) a time limit is not exceeded, (b) the loading capacity of the vehicle is not exceeded when serving customer demands, and (c) the vehicle never runs out of battery.

Our GOP is of particular relevance since it arises as a sub-problem in *Green Vehicle Routing Problems* [1] when they are solved exploiting column generation techniques. We propose a methodology based on dynamic programming to obtain both primal and dual bounds to the value of optimal solutions, a fast greedy algorithm, and a very large scale neighbourhood search procedure.

2 Modelling

The GOP can be formulated as follows: let $G = (V \cup R, E)$ be an undirected graph, where V is the set of customer vertices, R is the set of station vertices, and $E = \{(i, j) \mid i, j \in V \cup R\}$ is a set of edges connecting them. To perform customer visits we are given a single vehicle of limited loading capacity Q and limited time availability T, equipped with a battery of maximum charge B. The vehicle starts and ends at a depot $o \in R$. Each time the vehicle visits a customer i, it serves a demand q_i and collects a profit p_i using s_i units of time. Also, when the vehicle travels along an edge $(i, j) \in E$ it consumes $d_{(i,j)}$ units of battery charge and $t_{(i,j)}$ units of time. We assume $q_i = p_i = 0$ for each $i \notin V$.

The vehicle cannot travel if the battery charge reaches zero, but it can be recharged at each station vertex $r \in R \setminus \{r_0\}$ using one out of a set K of technologies. Each technology $k \in K$ provides b_k battery charge units for each unit of time at a cost c_k for each unit of battery recharged. Also, a fixed cost f is paid at each recharge. Mixing technologies during the same recharge stop is forbidden.

A route $\rho = ((i_1, \delta_1, k_1), \dots, (i_n, \delta_n, k_n))$ is a sequence of triplets describing the customers visited, the order of visits, and charge information, where $i \in V \cup R$ is a vertex and δ is the time spent recharging the vehicle at vertex i using technology k. When i is a customer, recharge is forbidden: δ is fixed to 0 and k is set to a dummy value. A route is feasible if:

- there is an edge between vertices of two following triplets: $\exists (i_\sigma, i_{\sigma+1}) \in E, \forall \sigma = 1 \dots n-1$;
- it does not exceed the time limit: $\sum_{\sigma=1}^{n-1} t_{(i_\sigma, i_{\sigma+1})} + \sum_{\sigma=1}^{n} s_{i_\sigma} + \sum_{\sigma=1}^{n} \delta_\sigma \leq T$;
- it does not exceed the capacity: $\sum_{\sigma=1}^{n} q_{i_\sigma} \leq Q$;
- the battery level is always between 0 and B: $\sum_{\sigma=1}^{\sigma'-1} \delta_\sigma b_{k_\sigma} - d_{(i_\sigma, i_{\sigma+1})} \geq 0, \forall \sigma' = 2 \dots n$ and $\delta_{\sigma'} b_{k'_\sigma} + \sum_{\sigma=1}^{\sigma'-1} \delta_\sigma b_{k_\sigma} - d_{(i_\sigma, i_{\sigma+1})} \leq B, \forall \sigma' = 2 \dots n$.

A route is optimal if it is feasible and its value, computed as the difference between collected profits and recharge costs, is maximum: $\max \quad \sum_{\sigma=1 \mid i_\sigma \in V}^{n} p_{i_\sigma} - \sum_{\sigma=1 \mid i_\sigma \in R}^{n} f + \delta_\sigma b_{k_\sigma} c_{k_\sigma}$.

3 Algorithms

We propose a methodology to obtain both primal and dual bounds to the value of the optimal solutions of GOP. We start from a simple observation:

Observation 1. *If a utopia technology having recharge speed $\hat{b} = \max_{k \in K} b_k$ and recharge cost $\hat{c} = \min_{k \in K} c_k$ exists, it would always be profitable to select it.*

That is, all the other technologies would be dominated. We accordingly define the *Utopia Technology GOP (UT-GOP)*, where we suppose that at each station, such an additional utopia technology is available (and therefore always selected), and we prove that:

Observation 2. *any sequence of visits yielding a feasible UT-GOP route, and in particular an optimal one, yields a feasible GOP route as well.*

Proposition 1. *The value of an optimal UT-GOP solution is a dual bound to GOP.*

To solve to optimality the UT-GOP we define a label correcting algorithm having the following structure:

Label structure: partial routes starting from the depot r_0 and ending in vertex i are encoded as labels $\lambda = (i, \tilde{V}, p, x, y, z)$ where p is the profit of the partial route computed as the difference of the collected profits and the recharge costs, $\tilde{V}$ is the set of customer vertices visited in the partial route, x is the potential battery level, y is the residual time, and z is the residual capacity.

Initialization: an initial label $\lambda = (r_0, \emptyset, 0, B, T, Q)$ is created and marked as 'open'.

Extension: at each iteration an open label $\lambda = (i, \tilde{V}, p, x, y, z)$ having maximum p value is selected and for each edge $(i, j) \in E$ a new label $\lambda' = (j, \tilde{V}', p', x', y', z')$ is created. If vertex j is a customer, that is $j \in V$, we set $p' = p + p_j - d_{(i,j)}\hat{c}$, $\tilde{V}' = \tilde{V} \cup \{j\}$, $x' = x - d_{(i,j)}$, $y' = y - t_{(i,j)} - s_j - d_{(i,j)}/\hat{b}$, and $z' = z - q_j$. Otherwise, if $j \in R$, $p' = p - f - d_{(i,j)}\hat{c}$, $\tilde{V}' = \tilde{V}$, $x' = x - d_{(i,j)}$, $y' = y - t_{(i,j)} - d_{(i,j)}/\hat{b}$, and $z' = z$. Label λ is marked as 'closed' while λ' as 'open'. Extension is not performed when $x' < 0$, $y' < 0$, or $z' < 0$, which encode an infeasible route.

Recharge: when a label $\lambda = (i, \tilde{V}, p, x, y, z)$ is created at a vertex $i \in R$, the potential battery level is fully charged by setting $x = B$.

Dominance: after extension, if two labels $\lambda' = (i, \tilde{V}', p', x', y', z')$ and $\lambda'' = (i, \tilde{V}'', p'', x'', y'', z'')$ are found, having $p' \geq p''$, $\tilde{V}' \subseteq \tilde{V}''$, $x' \geq x''$, $y' \geq y''$, and $z' \geq z''$, and at least one of these inequalities is strict, then label λ'' is deleted being sub-optimal.

Stopping criteria: we stop when no 'open' label is left.

According to Observation 2 by optimizing the UT-GOP we obtain a sequence of visits allowing to build a feasible GOP route, thereby obtaining a primal bound. In particular, we need to adjust the technology selection and amount of recharge done at each station. To perform such an adjustment we formulate a new optimization problem where given an incomplete route ρ, we have to find δ and k values in such a way that the route is still feasible and it minimizes the recharge costs.

Let $\tilde{R}$ be the set of recharge stations visited in ρ. W.l.o.g. if a station r is visited more than once, $\tilde{R}$ contains its copies r', r'', and so on. The problem of adjusting recharge quantities is formulated as follows:

$$\min \quad \sum_{r\in\tilde{R}}\sum_{k\in K} c_k b_k y_{rk} \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in K} x_{rk} \leq 1 \qquad \forall r \in \tilde{R} \tag{2}$$

$$\sum_{r\in\tilde{R}}\sum_{k\in K} y_{rk} \leq T - \sum_{\sigma=1}^{n-1} t_{(i_\sigma, i_{\sigma+1})} \tag{3}$$

$$\sum_{\sigma=1|i_\sigma\in\tilde{R}}^{\sigma'-1}\sum_{k\in K} b_k y_{i_\sigma k} - \sum_{\sigma=1}^{\sigma'-1} d_{(i_\sigma, i_{\sigma+1})} \geq 0 \qquad \forall \sigma' = 2\ldots n \tag{4}$$

$$\sum_{\sigma=1|i_\sigma\in\tilde{R}}^{\sigma'}\sum_{k\in K} b_k y_{i_\sigma k} - \sum_{\sigma=1}^{\sigma'-1} d_{(i_\sigma, i_{\sigma+1})} \leq B \qquad \forall \sigma' = 2\ldots n \tag{5}$$

$$b_k y_{rk} \leq B x_{rk} \qquad \forall r \in \tilde{R}, k \in K \tag{6}$$

$$y_{rk} \geq 0, x_{rk} \in \mathbb{B} \qquad \forall r \in \tilde{R}, k \in K \tag{7}$$

where y_{rk} is the time spent at station r recharging the vehicle with technology k and x_{rk} is a binary variable that is 1 if technology k is used at station r, 0 otherwise. The objective function (1) minimizes recharge costs. Constraints (2) forbid mixing technologies in a single station. Constraint (3) enforces the time limit. Constraints (4) and (5) ensure that the vehicle does not travel with empty battery and the maximum battery level is never exceeded, respectively. Constraints (6) enforce that variables x_{rk} are set to 1 when a technology is used in a station.

Greedy heuristic and very large scale neighbourhood search. We also propose a greedy heuristic for GOP that iteratively moves between vertices until the vehicle runs out of battery or time resources. At each step, the algorithm selects one out of three possible operations: (a) move the vehicle to the neighbour customer maximizing the difference between the collected profit and the travelling cost, (b) take a detour to a recharge station if no customer can be visited with current resources, or (c) go back to depot if the previous operations invalidate route feasibility.

Once a route is built, we re-optimize the technology selection and energy recharge amounts by optimizing a MIP sub-problem similar to (1) – (7), that corresponds to exploring a neighbourhood whose size is exponential in the number of stations. Such an optimization is performed by means of general purpose MIP solvers.

References

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