

Universal anti-Kibble-Zurek scaling in fully-connected systems

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We investigate the quench dynamics of an open quantum system involving a quantum phase transition. In the isolated case, the quench dynamics involving the phase transition exhibits a number of scaling relations with the quench rate as predicted by the celebrated Kibble-Zurek mechanism. In contact with an environment however, these scaling laws breakdown and one may observe an anti-Kibble-Zurek behavior: slower ramps lead to less adiabatic dynamics, increasing thus non-adiabatic effects with the quench time. In contrast to previous works, we show here that such anti-Kibble-Zurek scaling can acquire a universal form in the sense that it is determined by the equilibrium critical exponents of the phase transition, provided the excited states of the system exhibit singular behavior, as observed in fully-connected models. This demonstrates novel universal scaling laws granted by a system-environment interaction in a critical system. We illustrate these findings in two fully-connected models, namely, the quantum Rabi and the Lipkin-Meshkov-Glick models. In addition, we discuss the impact of non-linear ramps and finite-size systems.

Introduction.— The scrutiny of quantum matter driven out of equilibrium has led to the discovery of novel and striking phenomena [1, 2]. A comprehensive understanding of out-of-equilibrium properties of quantum systems is of crucial relevance for the further development of quantum technologies, as for example the exploitation of adiabatic evolution for quantum state preparation and computation [3, 4] or for the design and benchmark of quantum simulators [5, 6]. In this regard, a remarkable aspect of many-body quantum systems consists in the existence of quantum phase transitions (QPT) [7], which entails a sudden change in their ground state at a critical value of a control parameter. The critical point where a QPT takes place is typically accompanied by a vanishing energy gap [7], thus challenging the success of an adiabatic driving across it.

The Kibble-Zurek (KZ) mechanism, originally proposed to account for defect formation across a symmetry-breaking phase transition in the early universe, has become a cornerstone in non-equilibrium critical dynamics [8–11] and on the universal behavior of quench dynamics [12–16]. Its key prediction consists in a scaling relation between the number of defects formed upon traversing a phase transition, the equilibrium critical exponents and the quench rate. Such scaling relation is *universal* as it is solely determined by the equilibrium critical exponents of the phase transition, and hence, KZ predictions hold in different systems as confirmed in [17–22]. Indeed, the KZ mechanism also applies to the quantum realm [23–26], where scaling relations are found also for quantum excitations produced during the quench towards or across a QPT [16, 27–34]. Remarkably, although in these settings an adiabatic evolution is hindered by a vanishing energy gap, the scaling laws dictated by the KZ mechanism still imply a smaller number of excitations for slower quenches. By reducing the quench rate, the adiabatic condition is eventually achieved, although with a distinctive and smaller scaling exponent than in non-critical systems [35].

The significant experimental progress in the last decades has enabled an unprecedented degree of control, manipulation and preparation in quantum many-body systems [36–45], opening the door for the realization of quantum simulators and computers. However, the isolation from any environmental disturbance and/or experimental imperfection remains a formidable challenge. In this regard, it is worth mentioning that an interaction between the system of interest and its surroundings can have a dramatic impact in the properties of the system even when they interact weakly [46, 47], as demonstrated by the novel phenomena taking place in different dissipative critical systems [48–55]. It is therefore important to study the properties of the KZ mechanism in the presence of an environment. As observed in recent studies [56–63], the open nature of the dynamics leads to a departure from the KZ scaling prediction for the isolated case. These observations are encompassed under the term anti-Kibble-Zurek (AKZ) behavior, which refers to a linear increase of the number of excitations with the quench time. These results suggested that the AKZ behavior loses its universal fingerprints, i.e., the scaling laws as a function of the quench time no longer depend on the equilibrium critical exponents.

Here we show that in certain systems the AKZ behavior itself can acquire a universal form, and is thus in general different from a linear scaling. A driven open quantum system undergoing a QPT can show power-law relations as a function of the quench time, whose scaling is determined solely by its equilibrium critical exponents as for isolated KZ scaling laws. Such universal AKZ relation crucially depends on the critical behavior of the excited states. We illustrate our findings in two fully-connected critical systems, namely, the quantum Rabi model (QRM) [64] and the Lipkin-Meshkov-Glick (LMG) model [65], taking into account the interaction with an environment. Further, we investigate the impact of non-linear ramps and finite-size effects. Our results indicate

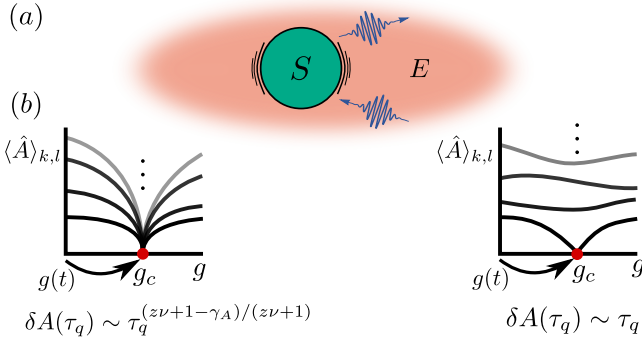


FIG. 1. (a) Schematic representation of the system S interacting with an environment E . In (b) the behavior of the excited states: at the left the excited states behave as the ground state close to g_c , thus featuring critical behavior $\langle \hat{A} \rangle_{k,l} \sim |g - g_c|^{\gamma_A}$ for a certain observable \hat{A} whose critical exponent is γ_A . Then, the excess in $\langle \hat{A}(\tau_q) \rangle_{\text{op}}$ with respect to the isolated case, i.e. $\delta A(\tau_q)$, acquires a universal AKZ scaling. For non-critical excited states, the excess $\delta A(\tau_q)$ obeys a linear scaling (see main text for details).

novel universal scaling laws emerging in the quench dynamics of an open quantum system involving a QPT.

Kibble-Zurek mechanism.— Let us denote by $\hat{H}_S(t)$ the time-dependent Hamiltonian of an isolated system which drives an initially-prepared ground state across or to the critical point g_c of a QPT by tuning a control parameter $g(t)$ in a total quench time τ_q . Due to the QPT, the energy difference between the first-excited and ground state vanishes at g_c as $\Delta_1(g) \sim |g - g_c|^{z\nu}$ [7], where z and ν denote the dynamic and correlation-length critical exponents of the QPT. This sets a time scale, $\tau_r(g) = \Delta_1^{-1}(g)$, which diverges at g_c thus impeding adiabatic dynamics for a finite quench time and eventually leading to excitations depending on τ_q , as dictated by the KZ mechanism.

The KZ mechanism is built upon the adiabatic-impulse approximation, which relies on the competition between two timescales, namely $\tau_r(g) = \Delta_1^{-1}(g)$ and $t_r(g) = \Delta_1(g)/\dot{\Delta}_1(g)$, where the latter determines the timescale on which the external parameter changes [11, 23, 35]. For a linear quench $g(t) \propto t/\tau_q$ with $g(0) < g_c$, one finds $t_r(g) \propto \tau_q |g - g_c|$. Within this simplified picture the evolution is split in two regimes as $g(t)$ approaches g_c : when $t_r(g) > \tau_r(g)$ the dynamics is fully adiabatic, while the impulse regime is found for $t_r(g) \lesssim \tau_r(g)$. In the latter regime the state *freezes* due to the lack of time to adjust to the externally-imposed $g(t)$. Since $\tau_r(g_c) \rightarrow \infty$, the state will eventually cease to follow the ground state of $\hat{H}_S(t)$ close to g_c . Hence, the population of excited states and relevant quantities after the quench will depend on τ_q . This heuristic argument is extremely useful to derive the scaling relations in the quench dynamics [66]. In particular, the boundary between the adiabatic and impulse regime takes place at \tilde{g} such that $t_r(\tilde{g}) = \tau_r(\tilde{g})$, which leads to $|\tilde{g} - g_c| \sim \tau_q^{-1/(z\nu+1)}$. Provided τ_q is sufficiently long such that diabatic excitations occur due to the QPT, the number of excitations, defined as $n_{\text{ex}} = \sum_{k>0} |c_k|^2$, will scale as

$n_{\text{ex}} \sim \tau_q^{-d\nu/(z\nu+1)}$, where $|\psi(\tau_q)\rangle = \sum_{k=0} c_k |\phi_k(g(\tau_q))\rangle$ is the final state and $\hat{H}_S(t) = \sum_{k=0} \epsilon_k |\phi_k(g(t))\rangle \langle \phi_k(g(t))|$, for a system with d spatial dimensions. In general, an observable \hat{A} whose ground-state expectation value follows $\langle \hat{A} \rangle_0 \sim |g - g_c|^{\gamma_A}$ close to g_c being γ_A its associated critical exponent, will display KZ scaling according to $\langle \psi(\tau_q) | \hat{A} | \psi(\tau_q) \rangle \sim \tau_q^{-(d\nu+\gamma_A)/(z\nu+1)}$ if $g(\tau_q) = g_c$ or $\tau_q^{-d\nu/(z\nu+1)}$ for $g(\tau_q) > g_c$ [35, 67–69]. These universal scaling relations are the key KZ predictions for isolated systems. In the following we consider fully-connected models, i.e. $d = 0$, so that KZ scaling appears only when $g(\tau_q) = g_c$ [70, 71].

Universal anti-Kibble-Zurek scaling.— Let us consider now the open quantum system dynamics and assume in particular a weak interaction with a Markovian environment, such that $\dot{\hat{\rho}} = -i[\hat{H}_S(t), \hat{\rho}] + \mathcal{D}[\hat{\rho}]$, where $\mathcal{D}[\bullet]$ accounts for the dissipative dynamics via a proper (Lindblad) structure, with an overall rate κ which expresses the strength of the system-environment interaction [46, 47] (see Fig 1(a)) [72]. In the weak coupling limit and for a finite quench time, the expectation values of the resulting open-system observables (denoted with subscript op), can be split in two contributions,

$$\langle \hat{A}(\tau_q) \rangle_{\text{op}} \equiv \text{Tr}[\hat{\rho}(\tau_q)\hat{A}] = \langle \hat{A}(\tau_q) \rangle + \delta A(\tau_q), \quad (1)$$

where $\langle \hat{A}(\tau_q) \rangle \equiv \langle \psi(\tau_q) | \hat{A} | \psi(\tau_q) \rangle$ is the isolated (fully coherent) contribution, while $\delta A(\tau_q)$ accounts for the excess introduced by the dissipative dynamics [58, 59, 62, 63]. As aforementioned, resorting to KZ arguments one obtains scaling predictions for $\langle \hat{A}(\tau_q) \rangle$ in terms of γ_A , d , z and ν . On the other hand, $\delta A(\tau_q)$ stems from the contact with the environment and it produces excitations at a constant rate κ per unit of time,

$$\delta A(\tau_q) \approx \kappa \tau_q \sum_{k,l=0} h_{k,l} \langle \hat{A} \rangle_{k,l}, \quad (2)$$

for sufficiently small $\kappa \tau_q$, and where $h_{k,l}$ takes account of how the dissipative dynamics populate different excited states and their coherences, while $\langle \hat{A} \rangle_{k,l} \equiv \langle \phi_k(g_f) | \hat{A} | \phi_l(g_f) \rangle$ with $g_f \equiv g(\tau_q)$; see [73] for details. If the excited states show critical behavior, that is, if $\langle \hat{A} \rangle_{k,l} \sim |g - g_c|^{\gamma_A}$, assuming the same critical exponent γ_A for any k, l , then $\delta A(\tau_q) \sim \kappa \tau_q |g_f - g_c|^{\gamma_A}$. Note that this is the case for fully-connected systems [70, 74–78]. Finally, relying on the adiabatic-impulse approximation and introducing $|\tilde{g} - g_c| \sim \tau_q^{-1/(z\nu+1)}$ one finds

$$\delta A(\tau_q) \sim \tau_q^{(z\nu+1-\gamma_A)/(z\nu+1)}. \quad (3)$$

This is the key result of the paper. The contribution due to the dissipative dynamics introduces a universal AKZ scaling, as its value is given by the equilibrium critical exponents of the QPT. In contrast, if the excited states do not show critical features (cf. Fig 1(b)), Eq. (2) yields a linear scaling $\delta A(\tau_q) \sim \tau_q$, as for a one-dimensional transverse-field Ising model, which agrees with previous observations [58, 59, 61–63]. In [73] one can find further details about the derivation of Eq. (3), the proof that it can hold even when only few excited states are critical, and the analysis of the scaling of the optimal quench

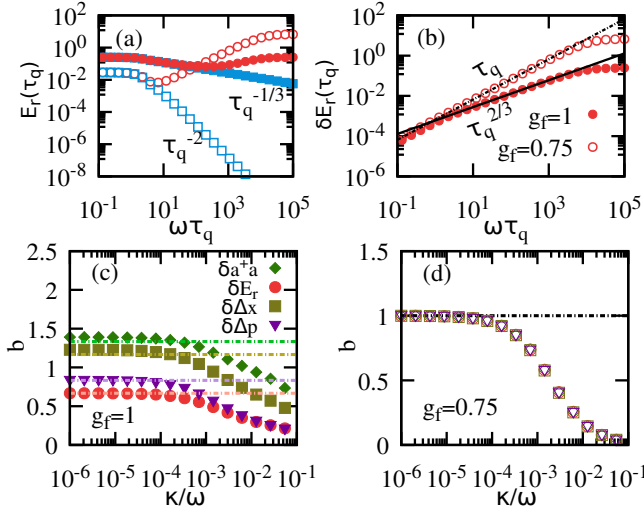


FIG. 2. (a) Residual energy (in units of ω) for the isolated case $\kappa = 0$, $\langle E_r(\tau_q) \rangle$ (blue squares), and for $\kappa = 10^{-4}\omega$ and $T = 10\omega$, $\langle E_r(\tau_q) \rangle_{\text{op}}$ (red circles) for $g_f = 1$ (full points) and $g_f = 0.75$ (open points) as a function of τ_q . In (b) we show the excess $\delta E_r(\tau_q)$ for the two cases, unveiling the linear AKZ scaling for $g_f = 0.75$ (dashed line) and its universal form when $g_f = 1$, $\delta E_r(\tau_q) \sim \tau_q^{2/3}$ (solid line). In panel (c) we show the fitted exponent b as a function of κ , $\delta A(\tau_q) \propto \tau_q^b$ for different quantities \hat{A} with $g_f = 1$, $\omega\tau_q \in [10^3, 10^4]$ and a $T = 0$ bath. The universal AKZ predictions are indicated by dashed lines (see main text). Panel (d) shows the fitted exponents b when $g_f = 0.75$, which follow $\delta A(\tau_q) \sim \tau_q$.

time that minimizes Eq. (1) with the rate κ [62]. In the following, we show the validity of Eq. (3) in some case study.

Example.— We illustrate the universal AKZ scaling laws in two $d = 0$ systems which exhibit a mean-field QPT, namely, the QRM [64] and the LMG model [65]. Models involving a mean-field QPT are of significance in diverse experimental platforms [29, 45, 79–82]. Although the QRM comprises two degrees of freedom, a spin and a single bosonic mode, it is possible to find a QPT in a suitable parameter limit [15, 70, 73, 77]. In contrast, the LMG comprises N two-level systems with a long-range interaction [65, 75, 76, 83, 84]. In the thermodynamic limit, denoted here by $\eta \rightarrow \infty$, the Hamiltonian of both models in one of the phases, $0 \leq g \leq g_c = 1$, can be written as [73]

$$\hat{H}^{(0)}(g) = \omega \hat{a}^\dagger \hat{a} - \frac{g^2 \omega}{4} (\hat{a} + \hat{a}^\dagger)^2, \quad (4)$$

where \hat{a} and \hat{a}^\dagger stand for the bosonic mode in the QRM and for the Holstein-Primakoff transformed pseudo-angular momenta in the LMG, and with ω its energy scale ($\hbar = 1$). For the purposes of this Letter it is enough to consider one phase since KZ scaling laws appear when $g(\tau_q) = g_c$ [70, 71], and so we consider $g(t) = g_f t / \tau_q$ with $g_f \leq g_c$. The Eq. (4) describes the low-energy subspace, where we find that the eigenstates are $|\phi_k(g)\rangle = \hat{S}[s(g)]|k\rangle$ with $|k\rangle$ the k th eigenstate of $\hat{a}^\dagger \hat{a}$, $\hat{S}[s] = e^{\frac{1}{2}(s^* \hat{a}^2 - s \hat{a}^{\dagger 2})}$ and $s(g) = \frac{1}{4} \ln(1 - g^2)$, so that $\hat{H}^{(0)}(g) = \sum_k \epsilon_k(g) |\phi_k(g)\rangle \langle \phi_k(g)|$ and $\epsilon_k(g) = k\omega \sqrt{1 - g^2}$.

The low-energy excited states inherit thus the critical properties of the QPT. In particular, the number of bosons diverges $\langle \hat{a}^\dagger \hat{a} \rangle_k \sim |g - g_c|^{-1/2}$ for $|\phi_k(g)\rangle$, while the position and momentum quadrature become $\Delta x_k \sim |g - g_c|^{-1/4}$ and $\Delta p_k \sim |g - g_c|^{1/4}$, respectively, such that $\Delta x_k \Delta p_k = (2k + 1)$. Hence, $\gamma_{\Delta p} = -\gamma_{\Delta x} = \gamma_{a^\dagger a} / 2 = 1/4$. In addition, we compute the residual energy $E_r(\tau_q) \equiv \text{Tr}[\hat{\rho}(\tau_q) \hat{H}^{(0)}(g(\tau_q))] - E_{\text{gs}}(g(\tau_q))$ whose critical exponent is given by $\gamma_{E_r} = z\nu$ as it is related to the energy gap $\Delta_1 \sim |g - g_c|^{z\nu}$, with $z\nu = 1/2$. Note that the energy gap for the k th eigenstate is $\Delta_k(g) = \epsilon_k(g) - \epsilon_0(g) = k\omega \sqrt{1 - g^2}$ and thus universal AKZ scaling for δE_r is expected too. Substituting γ_A for these quantities in Eq. (3), we obtain the predicted AKZ scaling, namely, $\delta \hat{a}^\dagger \hat{a}(\tau_q) \sim \tau_q^{4/3}$, $\delta \Delta x \sim \tau_q^{7/6}$, $\delta \Delta p \sim \tau_q^{5/6}$ and $\delta E_r \sim \tau_q^{2/3}$ provided $g(\tau_q) = g_c$. The universal AKZ scaling breaks down if $g_f < g_c$, recovering the linear relation $\delta A(\tau_q) \sim \tau_q$.

In order to verify the AKZ scaling relations, we consider the system interacting weakly with a *Markovian* bath at temperature T , although similar results are found considering a more realistic scenario [73], including possible memory effects via the non-perturbative approach developed in [85, 86]. Thus, consider for now the dynamics fixed by the master equation [46, 47]

$$\dot{\hat{\rho}}(t) = -i[\hat{H}^{(0)}(g(t)), \hat{\rho}(t)] + \mathcal{D}_{\hat{a}}[\hat{\rho}(t)] + \mathcal{D}_{\hat{a}^\dagger}[\hat{\rho}(t)], \quad (5)$$

where $\mathcal{D}_{\hat{a}}[\bullet] = \Gamma_{\hat{a}}(2\hat{a} \bullet \hat{a}^\dagger - \{\hat{a}^\dagger \hat{a}, \bullet\})$ is the Lindblad operator associated to the jump operator \hat{a} and with rates $\Gamma_{\hat{a}} = \kappa(N_{\text{th}} + 1)/2$ and $\Gamma_{\hat{a}^\dagger} = \kappa N_{\text{th}}/2$, with $N_{\text{th}} = (e^{\omega/k_B T} - 1)^{-1}$ the number of thermal excitations at temperature T . As the initial state is considered to be the ground state of $\hat{H}^{(0)}(0)$, we exploit the Gaussian-preserving nature of Eq. (5). For that, we employ the Wigner characteristic function $\chi(\beta, \beta^*, t) = \text{Tr}[e^{\beta \hat{a}^\dagger - \beta^* \hat{a}} \hat{\rho}(t)]$ with $\beta, \beta^* \in \mathbb{C}$ to calculate the evolution. The Fokker-Planck equation $\dot{\chi}(\beta, \beta^*, t) = \mathcal{X}[\chi(\beta, \beta^*, t)]$ allows for a Gaussian Ansatz, $\chi(\beta, \beta^*, t) = e^{i\mathbf{u}^t \mu(t) - \frac{1}{2} \mathbf{u}^t \Sigma(t) \mathbf{u}}$ with $\mathbf{u} = (\beta, \beta^*)^t$, and first and second moments $\mu(t) = (q_0(t), q_1(t))^t$ and $\Sigma(t) = (\sigma_{00}(t), \sigma_{01}(t); \sigma_{10}(t), \sigma_{11}(t))$, respectively. Defining $2\sigma(t) = \sigma_{00}(t) + \sigma_{11}(t)$, and since $q_{0,1}(0) = 0$, we obtain [73]

$$\dot{\sigma}(t) = 2\Gamma_- \sigma(t) + \Gamma_+ + iG(t)(\sigma_{01}(t) - \sigma_{10}(t)) \quad (6)$$

$$\dot{\sigma}_{10}(t) = 2(i\omega - iG(t) + \Gamma_-) \sigma_{10}(t) + i2G(t)\omega \sigma(t) \quad (7)$$

with $\sigma_{01}(t) = \sigma_{10}^*(t)$, $G(t) = g^2(t)\omega/2$, $\Gamma_\pm = \Gamma_{\hat{a}^\dagger} \pm \Gamma_{\hat{a}}$, $\sigma(0) = 1/2$ and $\sigma_{01}(0) = 0$. From $\chi(\beta, \beta^*, t)$ we calculate the quantities of interest, e.g. $\text{Tr}[\hat{a}^\dagger \hat{a} \hat{\rho}(t)] = \sigma(t) - \frac{1}{2}$ and $\text{Tr}[(\hat{a} + \hat{a}^\dagger)^2 \hat{\rho}(t)] = 2\sigma(t) - \sigma_{01}(t) - \sigma_{10}(t)$ [73]. Solving Eqs. (6)-(7) under $g(t)$ with different quench times τ_q allows us to obtain $\langle \hat{A}(\tau_q) \rangle$ for the isolated case ($\kappa = 0$) and $\langle \hat{A}(\tau_q) \rangle_{\text{op}}$ for a chosen $\kappa \neq 0$ and T , with $\hat{A} \in \{\hat{a}^\dagger \hat{a}, \Delta x, \Delta p, E_r\}$. Then, we calculate the excess due to the dissipation as $\delta \hat{A}(\tau_q) = \langle \hat{A}(\tau_q) \rangle_{\text{op}} - \langle \hat{A}(\tau_q) \rangle$ to corroborate the AKZ scaling prediction (cf. Eq. (3)).

In Fig. 2(a) we show the results for $\langle E_r(\tau_q) \rangle_{\text{op}}$ together with $\langle E_r(\tau_q) \rangle$ for $\kappa = 10^{-4}\omega$ and $T = 10\omega$ when the quench,

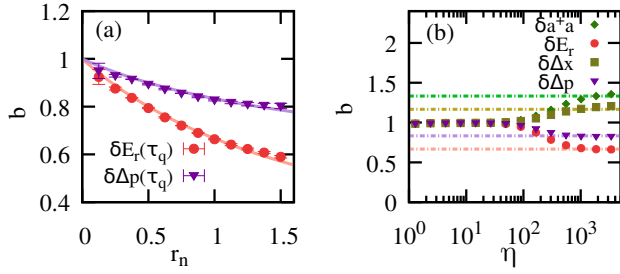


FIG. 3. (a) Numerically determined AKZ scaling exponent b under a non-linear ramp $g(t) \propto (t/\tau_q)^{r_n}$ as a function of r_n , for $\delta E_r(\tau_q)$ (red circles) and $\delta \Delta p(\tau_q)$ (magenta triangles) together with the universal AKZ prediction (solid lines), $2/(r_n + 2)$ and $(r_n + 4)/(2r_n + 4)$, respectively. The fits performed in $\omega\tau_q \in [10^4, 10^5]$ with $\kappa = 10^{-6}\omega$ and $T = 0$. In panel (b) we show the dependence of b with the QRM system size η for $\kappa = 10^{-5}\omega$ and $\omega\tau_q \in [10^3, 10^4]$.

$g(t) = g_f t/\tau_q$, ends either at $g_f = 0.75$ or at the critical point $g_f = 1$. As expected, $\langle E_r(\tau_q) \rangle$ follows the KZ prediction $\tau_q^{-z\nu/(z\nu+1)} = \tau_q^{-1/3}$ as $z\nu = 1/2$ when $g_f = 1$, and the adiabatic scaling τ_q^{-2} for $g_f < 1$ [70]. For $\kappa \neq 0$, $\langle E_r(\tau_q) \rangle_{\text{op}}$ deviates from its isolated value. In Fig. 2(b) we show the excess $\delta E_r(\tau_q)$: for $g_f < 1$ it follows a linear scaling $\delta E_r(\tau_q) \sim \tau_q$, while for $g_f = 1$ it shows the universal AKZ scaling given in Eq. (3), $\delta E_r(\tau_q) \sim \tau_q^{(z\nu+1-\gamma_{E_r})/(z\nu+1)} = \tau_q^{2/3}$. Note that for $\tau_q\kappa \gtrsim 1$, $\langle E_r(\tau_q) \rangle_{\text{op}}$ saturates to a constant value and so does $\delta E_r(\tau_q)$, since τ_q is long enough to reach a steady state. Similar behavior is observed for other quantities. We compare the numerically determined exponent b from a fit to $\delta A(\tau_q) = a\tau_q^b$ with the AKZ scaling prediction $(z\nu + 1 - \gamma_A)/(z\nu + 1)$ as a function of κ for $\hat{A} \in \{\hat{a}^\dagger \hat{a}, \Delta x, \Delta p, E_r\}$ in the interval $\omega\tau_q \in [10^3, 10^4]$ and $g_f = 1$. This is plotted in Fig. 2(c) choosing a $T = 0$ bath, while Fig. 2(d) shows the results when $g_f = 0.75$. The latter illustrates how the universal AKZ is lost in favor of the linear scaling when the excited states are not critical. As for the standard KZ scaling, some quantities may exhibit stronger corrections to its leading scaling, as found here for $\delta \hat{a}^\dagger \hat{a}(\tau_q)$ and $\delta \Delta x(\tau_q)$ [73].

Non-linear ramps.— The scaling presented in Eq. (3) is valid for linear ramps. However, as for the standard KZ predictions [87, 88], non-linear ramps $g(t) \propto (t/\tau_q)^{r_n}$ with an exponent $r_n > 0$ non-trivially modify the universal AKZ scaling laws. Here we choose $g(t) = g_f(1 - (1 - t/\tau_q)^{r_n})$. The adiabatic-impulse boundary becomes $|\tilde{g} - g_c| \sim \tau_q^{-r_n/(z\nu r_n + 1)}$ as it can be shown relying on adiabatic perturbation theory [73], or by scaling arguments [87, 88]. Then, provided $g(\tau_q) = 1$ this non-linear ramp leads to [73]

$$\delta A(\tau_q) \sim \tau_q^{(z\nu r_n + 1 - \gamma_A r_n)/(z\nu r_n + 1)}. \quad (8)$$

As an example we consider the scaling for $\delta E_r(\tau_q)$ and $\delta \Delta p$, which are expected to follow $\delta E_r(\tau_q) \sim \tau_q^{2/(r_n+2)}$ and $\delta \Delta p(\tau_q) \sim \tau_q^{(r_n+4)/(2r_n+4)}$. In Fig. 3(a) we show the fitted exponent b as a function of r_n (see caption for parameters), which agree very well with the universal AKZ prediction. In particular, for $r_n = 1/2$, we find $b = 0.794(3)$ and $0.893(3)$ for

$\delta E_r(\tau_q)$ and $\delta \Delta p$, close to the predicted values $4/5$ and $9/10$. Similar results are obtained for other quantities [73].

Finite-size effects.— The previous results have been obtained in the thermodynamic limit, $\eta \rightarrow \infty$. Finite-size systems however do not feature true singularities, e.g., they possess finite energy gap at the critical point. This leads to deviations from KZ scaling laws and to a maximum quench time in order to apply KZ arguments. It is therefore advisable to investigate how these AKZ scaling laws emerge as the system size increases. For that, we consider $\hat{H}(g) = \hat{H}^{(0)}(g) + \eta^{-1}\hat{H}^{(1)}(g) + O(\eta^{-2})$ where $\hat{H}^{(0)}(g)$ is again the Hamiltonian for $\eta \rightarrow \infty$, Eq. (4). The first-order correction $\hat{H}^{(1)}(g)$ comprises up to quartic terms in \hat{a} and \hat{a}^\dagger , although its specific form depends on the model, namely $\hat{H}_{\text{QRM}}^{(1)}(g) = \frac{g^4\omega}{16}f_{\text{QRM}}(\hat{a}, \hat{a}^\dagger)$, and $\hat{H}_{\text{LMG}}^{(1)}(g) = \frac{g^2\omega}{8}f_{\text{LMG}}(\hat{a}, \hat{a}^\dagger)$. In order to capture the main finite-size effects we keep only quadratic terms upon normal ordering in $\hat{H}_{\text{QRM,LMG}}^{(1)}(g)$ that preserves the Gaussian form of $\chi(\beta, \beta^*, t)$ but evolving now under modified equations of motion. In particular, $f_{\text{QRM}}(\hat{a}, \hat{a}^\dagger) \approx 12\hat{a}^\dagger \hat{a} + 6(\hat{a}^2 + \hat{a}^{\dagger 2}) + 3$, while $f_{\text{LMG}}(\hat{a}, \hat{a}^\dagger) \approx 4\hat{a}^\dagger \hat{a} + \hat{a}^2 + \hat{a}^{\dagger 2}$. For the QRM (the analogous results for the LMG are shown in [73]), the equations of motion follow from Eqs. (6)-(7) replacing $G(t)$ by $G_{\text{QRM}}(t) = g^2(t)\omega/2 - 12g^4(t)\omega/\eta$. Hence, in the $\eta \rightarrow \infty$ limit, Eqs. (6)-(7) are recovered. The results plotted in Fig. 3(b) reveal a smooth crossover of the exponent b from linear, $b = 1$, to its universal AKZ scaling value as the system size increases ($\eta \gtrsim 10^3$). The system size at which the crossover takes place in this fully connected model depends on the chosen quench time interval [73].

Conclusions.— We have shown that the system-environment interaction can lead to universal scaling laws upon a ramp towards the critical point of a QPT, similar to KZ scaling laws for isolated systems. Even a weak system-environment interaction yields an AKZ scaling, i.e., slower ramps provoke more excitations. Remarkably, provided the excited states of the quantum system display singular behavior, the AKZ scaling acquires a universal form, i.e., relevant observables scale in a power-law fashion with the quench time, whose scaling is determined solely by the equilibrium critical exponents of the QPT. If the excited states do not display critical behavior, a linear AKZ scaling is recovered [62]. We illustrated these findings in fully-connected, i.e. zero-dimensional, interacting systems such as the QRM and LMG undergoing dissipation, which are examples of superradiant [70, 78, 89] and ferromagnetic QPT, respectively. The results are also extended to non-linear ramps, and we further showed how the universal AKZ scaling laws emerge increasing the system size. The reported results may stimulate further research to underpin the role of open quantum dynamics in critical phenomena, and how its universal traits are modified when including a non-negligible interaction with a bosonic or fermionic environment of local or non-local nature, with distinct coupling directions [90], and systems with different spatial dimensions.

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