Study of the ^{16}O(p,γ) Reaction at Astrophysical Energies

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The Feshbach theory of the optical potential naturally leads to a microscopic description of scattering in terms of the many-body self-energy. We consider a recent calculation of this quantity for ^{16}O and study the possibility of applying it at astrophysical energies. The results obtained for the phase shifts and the ^{16}O(p,γ) capture suggest that such studies are feasible but the calculations require some improvement geared to this specific task.

1. Itroduction

The Feshbach theory of the optical potential provides a formal tool for developing a description of the scattering of nucleons from nuclei in terms of the microscopic interaction. In its most standard application, this is done by reducing the Hilbert space to a subspace that contains only the core nucleus and the additional particle in a scattering or bound single particle state. Since the scattered particle is not allowed to occupy the orbitals filled by the core nucleons, this subspace can only exhaust the part of the one-body Hilbert space the lies above the Fermi energy). Several models of nucleon-nucleus scattering (such as cluster model or folding potentials) implicitly use this approach and have been applied with significant success. However, the above limitation can be avoided by considering a Fock space (without a definite number of particles) and applying the Feshbach formalism to a subspace that contains both the scattering of a particle on top of the nuclear core and the possibility of propagating a hole excitation [2]. The resulting optical potential simply reduces to the usual many-body self-energy. The properties of this particular choice of the optical model have been reviewed by Mahaux and Sartor [3].

The above considerations suggest that the technique of using many-body Green's function can make reasonable predictions for nucleon-nucleus scattering. However, most of its applications in nuclear physics have focused on the the study of nuclear correlations [4]. In this contribution, we consider the self-energy resulting from a recent application of the self-consistent Green's function method the nucleus of ^{16}O [5, 6] and explore its predictions for proton-nucleus scattering.

2. The model

In Refs. [5, 6], the nuclear self-energy was computed within a model space $\mathcal{P}$ corresponding to the harmonic oscillator states of all orbitals up to the pf shell plus the $1g_{9/2}$

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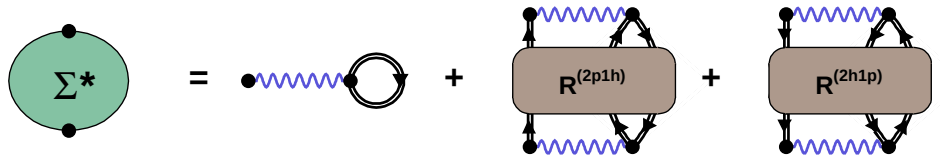

Figure 1. Feynman diagram representation of the self energy. The first diagram represent the Hartree-Fock like contribution to the mean field. The remaining ones describe core polarization effects in the particle ($R^{(2p1h)}$) and hole ($R^{(2h1p)}$) part of the spectrum.

orbital. A parameter b=1.76 fm was employed and a G-matrix interaction based on the Bonn-C potential [7] was used in the calculation. When expressed in coordinate space, this self-energy takes the form

$$\Sigma^{\star}(\mathbf{r},\mathbf{r}',\omega) = \sum_{\alpha,\beta\in\mathcal{P}} \phi_\alpha(\mathbf{r}) \left[\Sigma^{MF}_{\alpha\beta}(\omega) + \sum_p \frac{(m^{p+}_\alpha)^*\ m^{p+}_\beta}{\omega - \varepsilon^{p+} + i\eta} + \sum_h \frac{m^{h-}_\alpha\ \left(m^{h-}_\beta\right)^*}{\omega - \varepsilon^{h-} - i\eta} \right] \phi^*_\beta(\mathbf{r}') \qquad (1)$$

where $\phi_\alpha(\mathbf{r})$ are the harmonic oscillator wave functions, coupled to the nucleon spin with quantum numbers, $\alpha = \{n_\alpha, l_\alpha, j_\alpha, m_\alpha\}$ and the sum runs over all the orbits belonging to the model space [3]. We note that this choice for the space $\mathcal{P}$ is adequate to study properties of the nuclear interior when considering low energy (long range) excitations and accounting for the spectral fragmentation [6]. However the gaussian like functions $\phi_\alpha(\mathbf{r})$ are not optimal when one is concerned with properties sensitive to the nuclear surface.

In Eq. (1), the mean field part of the optical potential, Σ^{MF}, corresponds to the Hartree-Fock diagram of Fig. 1. The remaining diagrams are usually referred as core polarization contributions [3]. They were computed in Ref. [6] by including the coupling of single particle motion (i.e. quasiparticles and quasiholes) to different types of collective motion, namely excitations of the A-particle core, two-particle and two-hole states. At low enough energies they can be expressed as a discrete sum of poles, see Eq. (1). There $\varepsilon^{i\pm}$ determine the excitation energies of the resonances that do not have a mean field character.

Thus, Eq. (1) gives us a model for the optical potential that acts in the full single particle Hilbert space. However, before using it in practical applications one has to correct for the fact that the calculations of Ref. [6] did not include the electromagnetic interaction and were based on a two-body realistic interaction, which is not sufficient to account for spin-orbit splitting. Hence, in this work we augment the self-energy (1) with the Coulomb potential for a uniformly charged sphere of radius $R_c = 3.2$ fm and add a correction $U(\mathbf{r},\mathbf{r}')$. The scattering equation takes a Schrödinger-like form [$\hbar$=c=1 and μ is the reduced mass]

$$\left\{ \frac{-\nabla^2}{2\mu} + V_{Coul.}(\mathbf{r}) \right\} \psi(\mathbf{r}) \ + \ \int d\mathbf{r}' \ \{U(\mathbf{r},\mathbf{r}') \ + \ \Sigma^{\star}(\mathbf{r},\mathbf{r}',E_{cm})\} \ \psi(\mathbf{r}') \ = \ E_{cm}\ \psi(\mathbf{r})\ , \quad (2)$$

which, for $E_{cm} < 0$, also describes the bound states of ^{17}F. The potential

$$U(\mathbf{r},\mathbf{r}') = \sum_\alpha \delta\varepsilon_\alpha\ \phi_\alpha(\mathbf{r}')\phi^*_\alpha(\mathbf{r}') \qquad (3)$$

[3]The isospin degrees of freedom are not shown explicitly here.

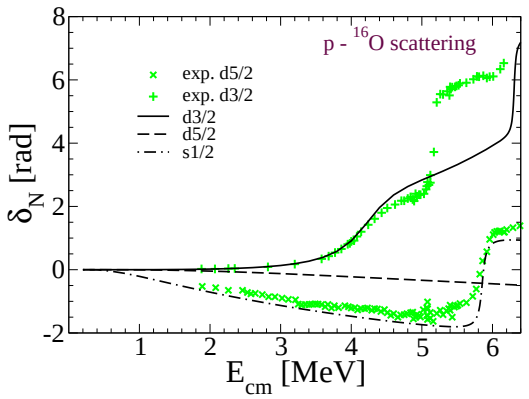

Figure 2. Phase shifts for proton - ^{16}O scattering in the $s_{1/2}$, $d_{5/2}$ and $d_{3/2}$ partial waves, as a function of the center of mass energy. The experimental results are from Ref. [8].

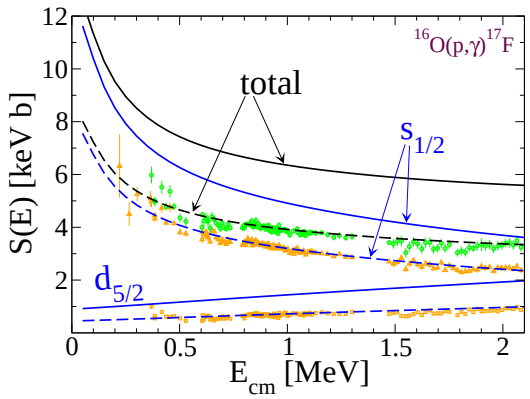

Figure 3. Astrophysical factor for the ^{16}O(p,γ) capture. The solid curves give the theoretical results for capture to the $d_{5/2}$ and $s_{1/2}$ states and the total capture. The dashed curves represent the same results after rescaling by a constant as described in the text. The experimental results are from Ref. [9].

is chosen in analogy to the work of Ref. [6] and acts by simply shifting the energy of the principal mean field orbitals. In practice, this corresponds to modifying the depth of the optical potential independently for each different partial wave. In the present work we found that one can fit the energy levels of the orbitals in the p and sd shells by choosing $\delta\varepsilon_{1p_{3/2}}$=-2.62 MeV, $\delta\varepsilon_{1p_{1/2}}$=-2.25 MeV, $\delta\varepsilon_{2s_{1/2}}$=+8.52 MeV, $\delta\varepsilon_{1d_{5/2}}$=-3.75 MeV and $\delta\varepsilon_{1d_{3/2}}$=+8.4 MeV.

3. Results

Figure 2 shows the phase shifts for the scattering of positive parity waves resulting from Eq. (2). The position of the bound states in ^{17}F and of the resonances shown in the plot has been forced to agree with the experimental data employing the above choice of $U(\mathbf{r},\mathbf{r}')$. This is necessary for obtaining the correct asymptotic behaviour of the bound wave functions. The position of the first $s_{1/2}$ resonance is determined by the first ε^{p+} pole in Eq. (1). This was originally predicted at 6.3 MeV, ∼0.5 MeV above the experimental value. The background contribution to the phase shifts is instead a prediction of the theory. As it can be seen, the mean field tends to be slightly too repulsive for the $s_{1/2}$ partial wave and slightly too attractive in the $d_{3/2}$ case. Considering the simple expansion of Eq. (1) in terms of a few harmonic oscillator wave functions, the model gives a reasonable description of the phase shifts. Eq. (2) was also solved for the valence orbitals of the last proton bound states of ^{17}F. The radial wave functions behave asymptotically as

$$f_{lj}(r) \longrightarrow_{r\to\infty} C_{lj}\frac{W_{-\eta,l+1/2}(r)}{r}\ , \qquad (4)$$

where $W_{-\eta,l+1/2}$ is a Whittaker function, η the Sommerfield parameter and the asymptotic normalization constants (ANCs) are predicted to be $C_{s_{1/2}}$= 98.2 fm$^{-1/2}$ and $C_{d_{5/2}}$= 1.59 fm$^{-1/2}$.

The astrophysical factor for ^{16}O(p,γ) is computed from these wave functions and the solutions of Eq. (2) for the relevant scattering waves. The results for the transitions to the bound states of ^{17}F are reported in Fig. 3 with solid lines. The curves overestimate the experimental results, due to the large values obtained for the ANCs. In contrast, the relative spectroscopic factors are found to be 0.921 for the $s_{1/2}$ state and 0.878 for the $d_{5/2}$ case. This agrees with what is expected from the halo nature of these orbitals. Moreover, we find that the shape of the theoretical curves agree well with the experimental data if we rescale the result for capture to the $s_{1/2}$ state by 0.63 and the one for $d_{5/2}$ by 0.49. This is depicted by the dashed lines in Fig. 3.

At zero energy, the astrophysical factor is determined by ANCs. The analytical study of the asymptotic wave functions done in Ref. [10] yields the following approximation,

$$S(0) \;=\; 0.37 C^2_{s_{1/2}} \;+\; 1.58\times 10^{-3} C^2_{d_{5/2}} \; . \tag{5}$$

After rescaling $C_{s_{1/2}}$ and $C_{d_{5/2}}$ as described above Eq. (5) gives $S(0)$=10.37 keV b, in agreement with their estimate [10].

The discrepancies found between theory and experiment are not unexpected. Part of these are due to the fact that the expansion of Eq. (1), originally chosen to study properties sensitive to the interior of the nucleus, gives a poor description of the nuclear surface to which halo states are particularly sensitive. A better choice of the basis would involve repeating completely the calculations of Ref. [6]. Nevertheless, it is plausible that this approach would lead to useful predictions of the scattering process.

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