A STUDY OF MULTIPLE ATTRIBUTES DECISION MAKING METHODS FACING UNCERTAIN ATTRIBUTES

by

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Abstract

Many decision-making methods have been developed to help decision makers (DMs) make efficient decisions. One decision making method involves selecting the best choice among alternatives based on a set of criteria. Multiple Attribute Decision-Making (MADM) methods allow opportunities to determine the optimal alternative based on multiple attributes. This research aims to overcome two concerns in current MADM methods: uncertainty of attributes and sensitivity of ranking results.

Based on availability of information for attributes, a DM maybe certain or uncertain on his judgment on alternatives. Researchers have introduced the use of linguistic terms or uncertain intervals to tackle the uncertainty problems. This study provides an integrated approach to model uncertainty in one of the most popular MADM methods: TOPSIS (Technique for Order Preference by Similarity to Ideal Solution).

Current MADM methods also provide a final ranking of alternatives under consideration and, the final solution is based on a calculated number assigned to each alternative. Results have shown that the final value of alternatives may be close to each other uncertain attributes, but current methods rank alternatives according to the final scores. It exhibits a sensitivity issue related to formation of the ranking list. The proposed method solves this problem by simulating random numbers within uncertain intervals in the decision matrix. The proposed outcome is a ranking distribution for alternatives.

The proposed method is based on TOPSIS, which defines the best and the worst solution for each attribute and defines the best alternative as closest to best and farthest from the worst

solution. Random number distributions were studied under the proposed simulation solution approach. Result showed that triangular random number distribution provides better ranking results than uniform distribution.

A case study of building design selection considering resiliency and sustainability attributes was presented to demonstrate use of the proposed method. The study demonstrated that proposed method can provide better decision option for designers due to the ability to consider uncertain attributes. In addition using the proposed method, a DM can observe the final ranking distribution resulted from uncertain attribute values.

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Dedication

The thesis is dedicated to my mother who played both parents role in my life and devoted herself on me to be successful.

Chapter 1. Introduction

1.1. Background

In the field of operation research, Multi-Criteria Decision Making (MCDM) is one of the most researched fields in terms of annual publication. MCDM studies include various mathematical and hierarchical models that incorporate methods from mathematics, behavioral decision theory, economics, computer technology, software engineering and information systems (Behzadian, et al., 2012). Figure 1.1 depicts the yearly publication trend of MCDM studies from 1955 to 2007 (Bragge, et al., 2010) showing that the number of MCDM publications has increased dramatically during the last decade.

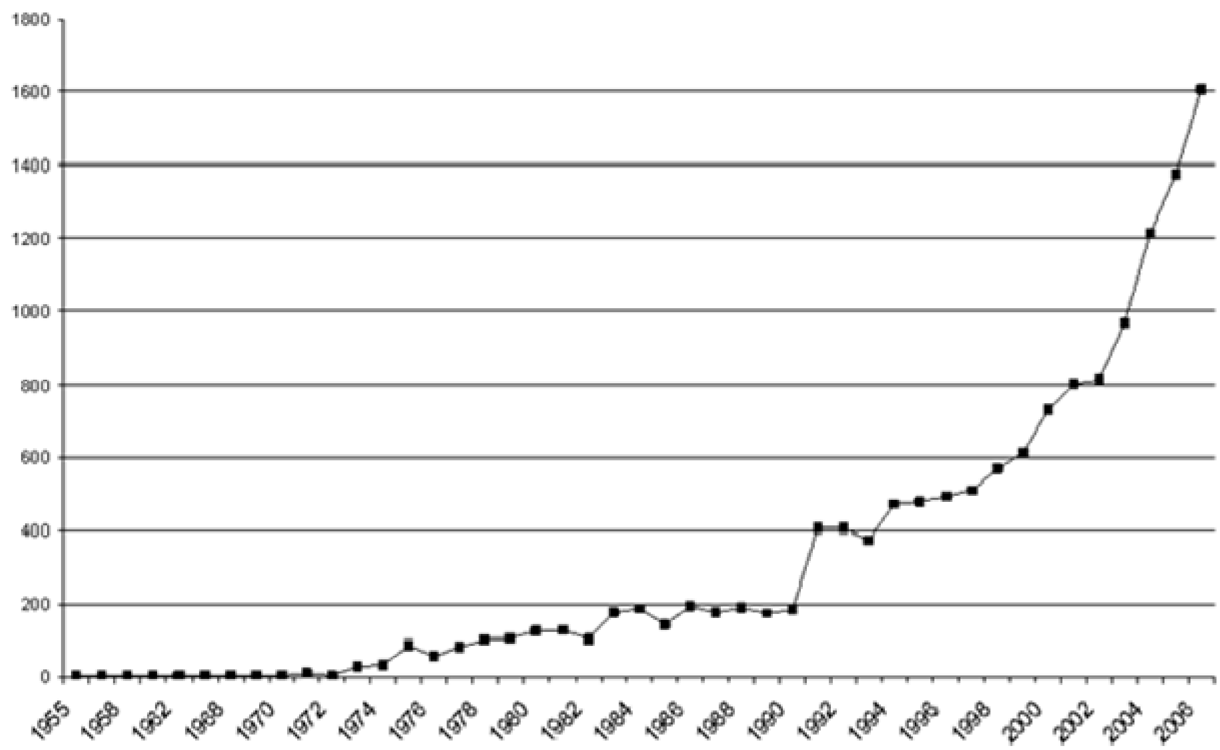


Figure 1.1 MCDM yearly publication trend (Bragge, et al., 2010)

MCDM techniques aim to give decision maker (DM) tools to make decisions when several contradictory points of view often must be taken into account (Tam, et al., 2007). MCDM methods have been designed to designate a preferred alternative, classify alternatives in a small number of categories, and/or rank alternatives in a subjective preference order (Behzadian, et al., 2012). A study by Charles, Cooper, and Ferguson (1955) was considered to be the first study on decision making (Ruiz, 2012). Modern forms of MCDM methods emerge during the 1970s, were strongly developed in the 1980s, and grew rapidly during the 1990'-s (Bragge, et al., 2010). Hwang and Yoon (1981) introduced TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), one of the most popular MADM (Multiple Attribute Decision Making) methods. Saaty (1986) introduced AHP (Analytic Hierarchy Process) to compare alternatives or attributes in a pairwise manner (Forman, et al., 2001). After Zadeh (1965) introduced fuzzy sets, researchers extended MADM methods to solve problems with uncertain attributes. Chen and Hwang (1992) developed fuzzy TOPSIS to solve MADM problems with uncertain attributes. Opricovic (1998) developed a method called VIKOR to solve MADM problems. The name VIKOR stands for VIseKriterijumska Optimizacija I Kompromisno Resenje which, in Serbian means Multi Criteria Optimization and Compromise Solution.

MCDM problems can be categorized into two groups: MADM and MODM (Multi-Objective Decision Making). MADM methods make a selection or prioritization from alternatives that are evaluated by multiple, usually conflicting, attributes (Hwang, et al., 1981). On the other hand, MODM problems aim to find the best alternative given a set of conflicting objectives (Yoon, et al., 1995). This study focused on MADM methods.

The most popular MADM methods include AHP, TOPSIS, and VIKOR. In the study of decision making, a decision is made from alternatives based on a set of criteria called attributes. For example, selection of a daily commute method may include alternatives such as use of public transportation, a bicycle, or a personal car. Attributes may include travel time, cost, convenience, health factors, and traffic. Various attributes may conflict with each other (Yoon, et al., 1995). Attributes may be quantitative, such as travel time, or qualitative, such as convenient. In some cases the attributes are not well defined. In the daily commute example, the attribute convenience may not be easily quantified.

1.2. Problem Statements

The environment in which DM makes a decision can affect a decision. For example, the decision-making environment for selecting a car differs from the environment when deciding a daily commute. External factors, such as traffic in the daily commute example, comprise the decision-making environment, potentially affecting the DM's evaluation of alternatives. Most MADM techniques assume that attributes are certain, but that assumption is not always true. In many real-world applications, a DM may be uncertain about some attributes (Bonissone, 2008). Chen (2000) and Jahanshahloo, *et al.* (2006) attempted to solve MADM problems with uncertain attributes. They defined uncertainty by fuzzy membership functions (Chen, 2000), intervals (Sayadi, et al., 2009; Jahanshahloo, et al., 2006) or some other forms.

Two problems are inherent in existing MADM methods. First, most existing MADM methods assume a deterministic environment for attributes. Chen (2000) used triangular fuzzy numbers to model linguistic terms to model uncertainty. Sometimes, however, expressing uncertainty as linguistic terms is not possible. Some uncertainty may diminish after time passes, but a decision must be made at the present time not later. For example, at the beginning of a construction project, designers may not know the energy consumption of a finished building. They can estimate based on experience and historical data, but the estimate may not be perfect. Therefore, they can express their estimations by intervals to evaluate alternatives. Jahanshahloo, *et al.* (2006) introduced intervals to solve MADM problems with uncertain attributes. They suggested that DM evaluate alternatives by considering the inherited uncertainty and express evaluations by intervals. However, they did not provide a guideline for how to generate intervals that reflect uncertainty in attributes.

Second, most MADM methods use a single scale or score to rank alternatives. The problem arises when the final scores of different alternatives are close to each other. The current methods do not consider the similarity of final scores; therefore, by changing a small amount of uncertainty, the ranking list generated by current methods may change, thereby increasing the sensitivity inherited in the existing MADM method. In addition, Yeh (2002) suggested that various MADM methods may generate different ranking results. When results obtained by various methods differ significantly, DMs may have a difficult time selecting the best choice (Jahan, et al., 2013). This study examined methods proposed by Chen (2000) and Jahanshahloo, *et al.* (2006) considering uncertain environments and improving their shortcomings. The proposed method intended to enable a DM to make decisions based on distributions of ranking using simulations to account for uncertain environment.

1.3. Proposed method

The proposed method attempted to overcome the problems discussed in the previous section for existing MADM methods. To consider uncertainty, the proposed method applied percentages of uncertainty that a DM assigns to each attribute. In the construction project example, a project manager may assign some percentage of uncertainty based on experience or past data. These percentages formed intervals of uncertainty, providing a tool to generate intervals. The larger the uncertainty, the wider the interval. Then, this study proposed to adopt the extended TOPSIS method for interval numbers developed by G. R. Jahanshahloo, *et al.* (2006) in order to obtain weighted normalized intervals.

In order to solve sensitivity issues in current MADM methods, simulations were applied to the uncertain intervals. Simulated random numbers were generated based on uniform distribution and triangular distribution. Uniform distribution considers equal probability for all values within the interval; triangular distribution assigns more probability to the most likely value within the interval. Finally the regular TOPSIS method developed by Hwang and Yoon (1981) was applied to find the result of ranking for each set of simulated runs. Ranking distribution was then formed from all simulation runs. The proposed method addressed sensitivity issues in the model via ranking distribution.

1.4. Thesis Outline

This thesis contains the following chapters. Chapter 1 defines, the problem and outlines the proposed method. Chapter 2 provides, a brief literature review of MADM methods and studies. Chapter 3 contains the proposed method and user guidelines and provides a numerical example. Chapter 4 describes a case study containing a mock construction project in which two sets of attributes related to resiliency and sustainability are considered. Chapter 5 contains conclusions and future research.

Chapter 2. Literature Review

2.1. Types of Decision Making Methods

MCDM consists of constructing a global preference relation for a set of alternatives evaluated by several criteria (Vansnick, 1986). The aim of any MCDM technique is to provide guidance to the DM in order to determine the most desired solution to a problem (Stewart, 1992). In general, MCDM can be divided into two different groups: Multi Attribute Decision Making (MADM) and Multi Objective Decision Making (MODM). In MODM problems, the decision space is often continuous, whereas the decision space in MADM problems is primarily discrete. This thesis focuses on MADM methods with uncertain attributes. Most MADM problems contain predefined number of attributes. For example, selecting a car based on price, performance and quality can be modeled as an MADM problem. The alternatives are cars under consideration, and the attributes are price, performance and, quality.

2.2. Environment of Decision Making

When solving an MADM problem, one important factor that may affect the decision outcome is the decision environment in which knowledge of attributes may be known or uncertain. The decision-making environment can be divided into three types: certainty, uncertainty, and risk. Each type is examined in details in the following paragraphs.

Certainty: In a certainty environment, a DM has full knowledge of an attribute quantified by a number. In many real-world cases, however a DM may not know a decision environment completely and it is very difficult to be 100% certain. However, a DM can assume that the knowledge of an attribute is certain to a certain degree.

Uncertainty: In an uncertain environment, a DM has less knowledge of an attribute at the time of decision. In most cases, uncertainty arises due to external factors (Chand, 2015). For example, when introducing a new product to a market, selecting between several designs is an extremely important decision. The result of a decision may be related to external factors such as competitors, consumer taste, or economy. One way to counter uncertainty is to postpone decision making until more information is available. In some cases, however, a decision cannot be postponed.

Risk: Many researchers consider risk as a special type of uncertainty (Chand, 2015); in which a DM can assign probabilities to events from past experience.

Zimmermann (2001) proposed that fuzzy sets can be used to model uncertainty. This study examined fuzzy sets since many MCDM methods use fuzzy sets to solve problems in uncertain environments. Details of fuzzy sets are briefly summarized in the following sections.

2.3. Fuzzy Set Theory

Zadeh (1965) wrote the first notes about fuzzy sets, and Salii (1965) introduced more general L-relations. Fuzzy sets, which are currently used in areas such as linguistics, decision-making, and clustering, are special cases of L-relations in which L is the unit interval [0, 1] (Latha, et al., 2015).

Zadeh (1965) defined a fuzzy set as a convenient point of departure for construction of a conceptual framework that parallels the framework used for ordinary sets. However, a fuzzy set is more general than an ordinary set and may potentially prove to have a much wider scope of applicability, particularly in fields of pattern classification and information processing.

Essentially, Zadeh's (1965) framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria. Zimmermann (2001) clarified that "imprecision" in Zadeh's statement means vagueness rather than lack of knowledge about parameters. Fuzzy set theory provides a mathematical base in which vague concepts can be defined and studied precisely (Zimmermann, 2001). Fuzzy set theory permits gradual assessment of the membership of elements in a set as described with the aid of a membership function valued in the real interval $[0, 1]$. Zimmermann (2001) classified the development of fuzzy sets into two categories:

a) "*As a formal theory which, when maturing, became more complex and specified and was enlarged by original ideas and concepts*."

b) "*As an application oriented 'fuzzy technology' that means as a tool for modeling problem solving and data mining that has proven superior to existing methods in many cases and as an attractive 'add-on' to classical approaches in other cases*."

2.3.1. Fuzzy Set

A fuzzy set $\tilde{A}$ in a set X is defined as

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$ where membership space $M = [0,1]$ the set $\tilde{A}$ is non fuzzy and $\mu_{\tilde{A}}(x)$ becomes membership grade of x in $\tilde{A}$.

2.3.2. Membership Function

A membership function is assigned to each number x in X. A fuzzy number is a fuzzy subset of the universe of discourse X that is both convex and normal (Haghighi, et al., 2011). Figure 2.1 shows a fuzzy number $\tilde{A}$ in the set of X.

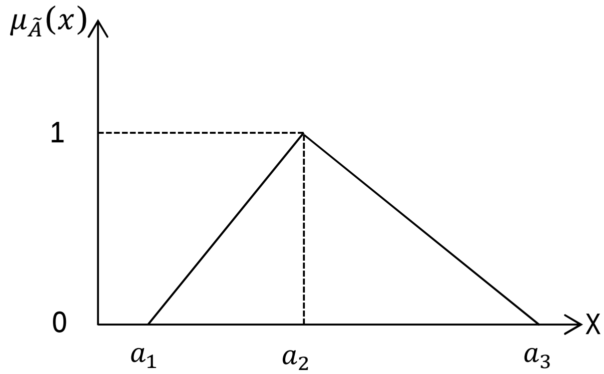


Figure 2.1 Triangular fuzzy number $\tilde{A}$

Among various shapes of fuzzy numbers, triangular fuzzy number (TFN) shown in Figure 2.1 is the most popular fuzzy membership function (Bahri, et al., 2014). A triangular fuzzy number can be defined as a triplet (a_1, a_2, a_3). The mathematic schema of this number is defined by:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq a_1; \\ \frac{x-a_1}{a_2-a_1} & a_1 < x \leq a_2; \\ \frac{a_3-x}{a_3-a_2} & a_2 < x \leq a_3; \\ 0 & x > a_3; \end{cases} \tag{2.1}$$

Mathematical operations for fuzzy numbers with positive real numbers for fuzzy numbers are defined as

$$\tilde{A} = (a_1, a_2, a_3) \text{ and } \tilde{B} = (b_1, b_2, b_3)$$

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \tag{2.2}$$

$$c \times \tilde{A} = (c \times a_1, c \times a_2, c \times a_3) \tag{2.3}$$

$$\tilde{A} \times \tilde{B} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \tag{2.4}$$

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \tag{2.5}$$

where $d(\tilde{A}, \tilde{B})$ is the distance between two fuzzy numbers of $\tilde{A}$ and $\tilde{B}$ (Haghighi, et al., 2011). Section 2.4.2 describes a MADM method to solve problems with uncertain attributes using fuzzy sets to cover uncertainty.

2.4. MADM Methods

As shown in Figure 1.1, many MCDM methods have been developed to solve decision making problems in certain and uncertain environments. Hwang and Yoon (1995) framed a MADM problem into the following components:

Alternatives: Each decision making problem has number of alternatives that a DM tries to screen, prioritize, select, or rank in the process of decision making. For example, in deciding a daily commute, public transportation, bike, and car are alternatives of choice. The number of alternatives may differ in each case. In the decision of graduate admission for a well-known university, alternatives may include thousands of applicants, where as in the daily commute example, alternatives are limited to less than five. Various names for alternatives include choice, option, and candidate.

Attribute: Attributes are criteria used to evaluate alternatives. Each problem may have multiple attributes which may conflict with each other. The number of attributes also depends on a problem. For the problem of selecting a building design, thousands of attributes may be considered whereas in the problem of daily commute, the number of attributes is much smaller. Attributes are grouped into cost attributes and benefit attributes. An Attribute such as convenience is a benefit attribute and an attribute such as traffic is a cost attribute.

Incommensurable Units: Each attribute in the problem may have its own unit. In the car selection problem, attributes of price and gas usage have unique units of dollar and gallons per mile, respectively. Normalization solves the problem of incommensurable units.

Normalization: Because attributes may have different units, normalization is necessary to eliminate the effect of different units and bring the numbers to a common scale. Normalization allows values of alternatives x_{ij} to be transformed into a scale $[0,1]$ or into one of its sub-segments (PAVLICIC, 2001). Most MADM methods adopt very similar normalization schemes.

Attribute Weights: Some attributes are more important than others. Almost all MADM methods use weights for attributes to reflect attribute importance. A DM typically defines weights, but when the problem is complex, weights may occasionally be calculated by mathematical methods such as the AHP method (Saaty, 1986).

Decision Matrix: MADM problems are typically defined by a matrix in which columns represent attributes and rows indicate alternatives. In this decision matrix, each element of x_{ij} indicates the performance rating of the i^{th} alternative with respect to the j^{th} attribute. In the group of cost attributes alternatives with smaller scores (x_{ij}) are preferred; in the group of benefit attributes, alternatives with larger scores (x_{ij}) are preferred.

Based on availability of problem information, MADM methods can be categorized as decision making with no information, decision making with information of attributes, and decision making with information about the environment. Hwang and Yoon (1995) presented a taxonomy of MADM methods based on these categories, as shown in Figure 2.2. If no information is given to a DM, the dominance method is applicable. The dominance method is one of the simplest methods in which, an alternative is dominated if another alternative has outcomes at least as good for each attribute. Conversely, an alternative is non-dominated if no alternative excels it in all attributes considered. One or more non-dominated choice is defined as an optimal choice at the end of process.

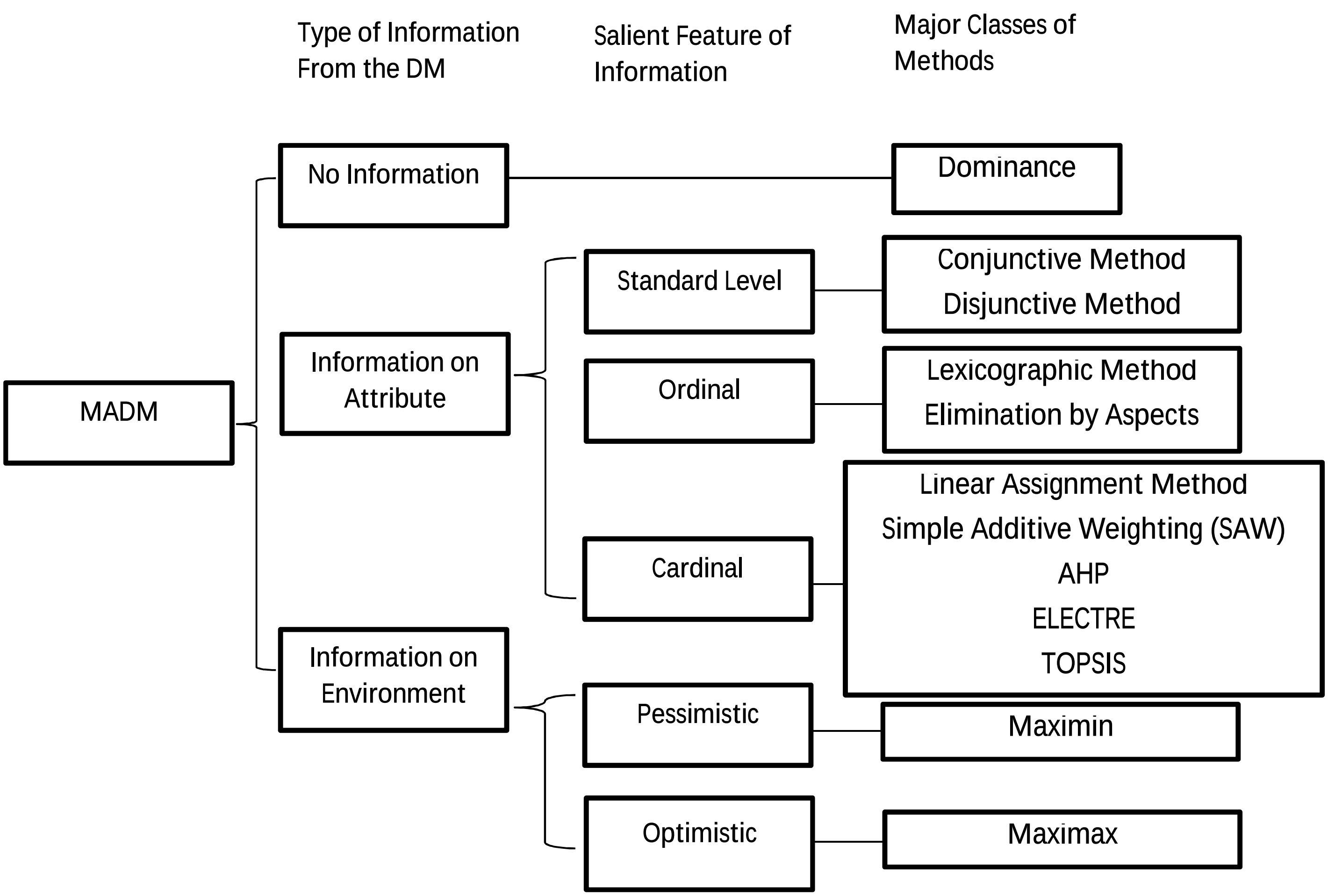


Figure 2.2 Taxonomy of methods for classical MADM methods (Yoon, et al., 1995)

If a DM has pessimistic or optimistic information on environment, Maximin or Maximax methods are applicable. For the Maximin method, the poorest attribute value for each alternative is first selected, followed by the alternative with the best value on the poorest values. For the Maximax method, however, the best attribute value for each alternative is initially selected, and then the alternative with the best values is selected as the best choice from those selected values.

If attribute information is available to the DM, then a subcategory (i.e., -the salient feature of information received by DM-) is used to further classify MADM methods. Information can be a standard level (minimum acceptable level) of each attribute or attribute weights assessed by ordinal or cardinal scales (Yoon, et al., 1995). Methods classified in the standard level group require an acceptable level of attributes defined by the DM in order to select the alternative. Attribute weights are introduced to reflect the DM thoughts on the importance of attributes. Weights can be assessed by ordinal or cardinal scales; most MADM methods use cardinal scales, as adopted in this study as well.

Uncertainty in decision making has been studied in the field of fuzzy MCDM (FMCDM). Scientists use fuzzy numbers to apply linguistic terms in order to deal with uncertainty in a problem. Fuzzy MCDM models can be used to assess alternatives based on attributes and weights (Kahraman, et al., 2015). The first notations of fuzzy MCDM were cited by Bellman and Zadeh in 1970. They applied fuzzy data to MCDM methods by defining goals and constraints as fuzzy sets (Mokhtari, 2013).

Chen and Hwang (1992) later introduced fuzzy TOPSIS based on original TOPSIS introduced by Hwang and Yoon (1981). Chen (2000) defined a table of linguistic terms in order to overcome uncertainty on attributes. In Chen's (2000) method, a DM can use linguistic terms represented by fuzzy numbers to overcome inherited uncertainty in a problem. Table 2.1 shows the equal fuzzy number of each linguistic variable defined by a DM.

Table 2.1 -Linguistic variables to reflect the uncertainty of attributes (Chen, 2000)

Linguistic variable	Triangular fuzzy number
Very Low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0)

The following sections provide brief information about the original TOPSIS method introduced by Hwang and Yoon (1981), fuzzy TOPSIS introduced by Chen (2000), an extended TOPSIS to solve interval-based numbers by Jahanshahloo, *et.al.* (2006), and VIKOR, which has similar background to TOPSIS but was introduced by different methodology.

2.4.1. TOPSIS

Among numerous MADM methods developed to solve real-world decision problems, TOPSIS continues to be applied satisfactorily in diverse application areas (Behzadian, et al., 2012). Yoon (1980; 1981) initially proposed TOPSIS, which has subsequently been used in many studies and many researches (Behzadian, et al., 2012).

The basic concept of TOPSIS is that the best alternative should have the shortest distance to the ideal solution and the farthest distance from the negative ideal solution. Therefore, before using TOPSIS, one must select the best and worst solutions for the problem of interest in an attempt to reach a solution that has the farthest distance from the worst solution and closest distance to the best solution.

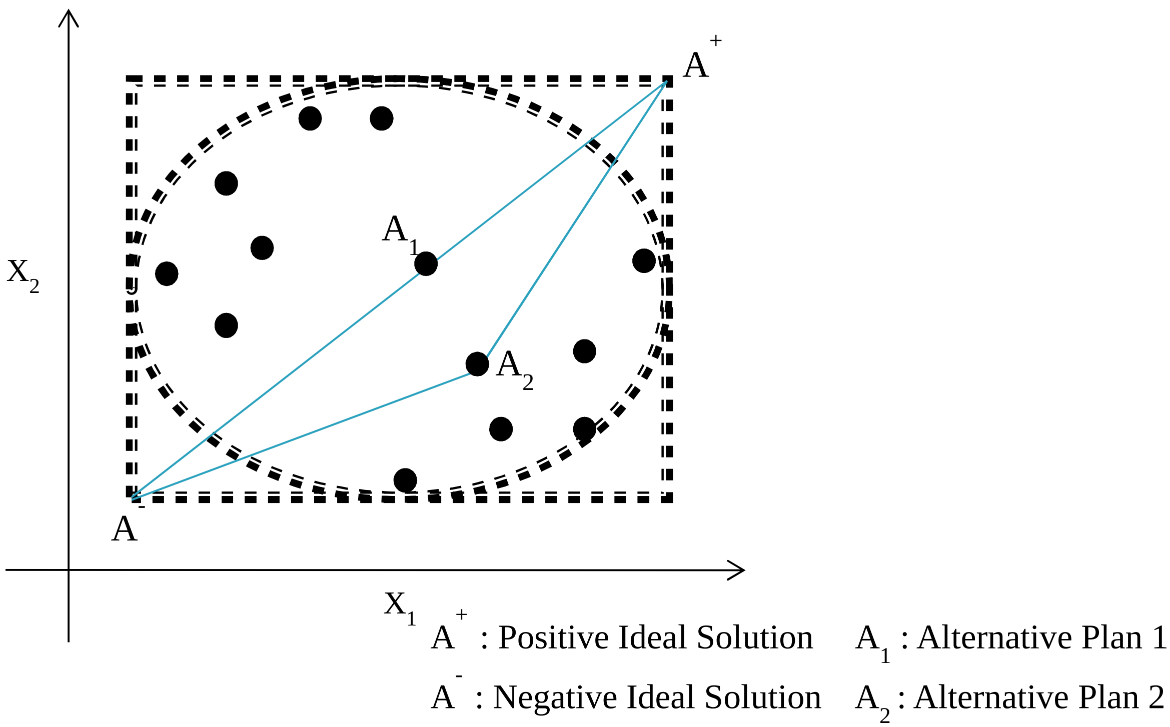


Figure 2.3 NIS and PIS distances in TOPSIS

As shown in Figure 2.3 A^{+} Positive Ideal Solution (PIS) and A^{-} Negative Ideal Solution (NIS) are defined as the best and the worst solutions, respectively. Specifically, PIS contains the best scores among all considered criteria or attributes. NIS contains the worst scores among all criteria considered. For example, if a problem has cost and benefit attributes, alternatives with lower scores are preferred for the cost attribute. For benefit criteria, however, alternatives with higher scores are preferred. One TOPSIS assumption is that the decision matrix is given. A decision matrix includes scores of each alternative regarding to all criteria. x_{ij} is the score of alternative i with respect to attribute j. A decision matrix with $\boldsymbol{n}$ alternatives and $\boldsymbol{m}$ attributes is shown in Table 2.2.

Table 2.2 Decision matrix

	Attribute			
Alternatives	C_1	C_2	...	C_m
A_1	x_{11}	x_{12}	...	x_{1m}
A_2	x_{21}	x_{22}	...	x_{2m}
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
A_n	x_{n1}	x_{n2}	...	x_{nm}

The final solution using TOPSIS can be obtained via the following steps:

Step 1: Normalize the decision matrix: Normalization is an important step to mitigate various units of all attributes into a comparable scale. Normalization allows alternatives to be combined using the comparable scale. A decision matrix can be normalized in several ways. One of the normalization methods (Behzadian, et al., 2012) is to divide the score by the square root of sum of score squared.

$$\bar{n}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n}(x_{ij})^2}} \; , i = 1, \dots , n \, , j = 1, \dots , m \qquad (2.6)$$

where i is the index for alternatives and j is the index for attributes.

Step 2: Calculate the weighted normalized matrix: Some criteria are more important than others. In this case, a DM may introduce weights to consider the preference. If one criterion is more important than the others, that criterion should have a higher weight. However, according to Equation (2.7), all weights should be added to one.

$$\sum_{j=1}^{m} w_j = 1 \tag{2.7}$$

where w_j is the weight assigned for scores under j^{th} column (attribute). The weighted normalized value is then calculated as

$$\bar{v}_{ij} = w_j . \bar{n}_{ij} \tag{2.8}$$

Step 3: Identify NIS and PIS: As discussed, this identification is an important step in the TOPSIS method. The PIS and NIS can be defined as:

$$A^{+} = PIS = \{\bar{v}_1{}^{+}, \bar{v}_2{}^{+}, \ldots, \bar{v}_m{}^{+}\} = \{\left(\max_j \bar{v}_{ij} \,\middle|\, j \in I\right), \left(\min_j \bar{v}_{ij} \,\middle|\, j \in J\right)\} \tag{2.9}$$

$$A^{-} = NIS = \{\bar{v}_1{}^{-}, \bar{v}_2{}^{-}, \ldots, \bar{v}_m{}^{-}\} = \{\left(\min_j \bar{v}_{ij} \,\middle|\, j \in I\right), \left(\max_j \bar{v}_{ij} \,\middle|\, j \in J\right)\} \tag{2.10}$$

where I is the benefit criteria group and J is the group of cost criteria.

Step 4: Calculate distances from NIS and PIS. Distances can be calculated by the Euclidean distance as in the following equations:

$$d_i^{+} = \sqrt{\sum_{j=1}^{m} (\bar{v}_{ij} - \bar{v}_j{}^{+})^2}, i = 1, \ldots, n \tag{2.11}$$

$$d_i^{-} = \sqrt{\sum_{j=1}^{m} (\bar{v}_{ij} - \bar{v}_j{}^{-})^2}, i = 1, \ldots, n \tag{2.12}$$

For the i^{th} alternative, the distance to the PIS is defined by d_i^+ using Equation (2.11) while its distance to the NIS defined by d_i^- is calculated by Equation (2.12).

Step 5: Calculate the closeness coefficient. The closeness coefficient can be calculated by:

$$\bar{R}_i = \frac{\bar{d}_i^-}{\bar{d}_i^- + \bar{d}_i^+} \quad , i = 1, \ldots, n \tag{2.13}$$

Note that $0 \le \bar{R}_i \le 1$ where $\bar{R}_i = 0$ when $A_i = NIS$, and $\bar{R}_i = 1$ when $A_i = PIS$.

Step 6: Rank the alternatives: Alternatives can be ranked in descending order of their closeness coefficient. The alternative with the largest $\bar{R}_i$ value is the best solution because it has the farthest distance from the worst solution.

This basic TOPSIS method is based on the assumption that all attributes are certain at the time of decision making. Several methods have been proposed to solve problems in uncertain environments in which some or all of the attributes are uncertain. Many proposed TOPSIS methods have adopted fuzzy sets for uncertain environments.

2.4.2. Chen's (2000) Fuzzy TOPSIS

Chen and Hwang (1992) proposed one of the most important improvements to the TOPSIS method that addresses uncertain environment issues using fuzzy sets. Triangular fuzzy membership functions were considered for uncertain attributes. The x_{ij} scores in a decision matrix were modeled in fuzzy values rather than crisp numbers, as shown in Table 2.3. Chen (2000) used linguistic variables shown in Table 2.1 for the importance weight of attributes and linguistic variables for ratings in the decision matrix.

This fuzzy TOPSIS method defines three numbers for each cell of a decision matrix. The left number is the lowest support of fuzzy number and the right number is the highest value of support of fuzzy number. The middle number is the most likely value for an alternative of an attribute. For example in (6.3, 7, 7.7), 6.3 is the lowest value of support, 7 is the most likely value and 7.7 is the highest value of a cell. The solution for Chen's (2000) fuzzy TOPSIS can be obtained using the following steps:

Step 1: Normalize the table. The equations to normalize the scales are

$$\bar{n}_{ij}^{L} = \frac{x_{ij}^{L}}{\sqrt{\sum_{i=1}^{n}((x_{ij}^{L})^2 + (x_{ij}^{M})^2 + (x_{ij}^{U})^2)}}, j = 1, \dots, m, \quad i = 1, \dots, n \tag{2.14}$$

$$\bar{n}_{ij}^{M} = \frac{x_{ij}^{M}}{\sqrt{\sum_{i=1}^{n}((x_{ij}^{L})^2 + (x_{ij}^{M})^2 + (x_{ij}^{U})^2)}}, j = 1, \dots, m, \quad i = 1, \dots, n \tag{2.15}$$

$$\bar{n}_{ij}^{U} = \frac{x_{ij}^{U}}{\sqrt{\sum_{i=1}^{n}((x_{ij}^{L})^2 + (x_{ij}^{M})^2 + (x_{ij}^{U})^2)}}, j = 1, \dots, m, \quad i = 1, \dots, n \tag{2.16}$$

where $\bar{n}_{ij}^{L}, \bar{n}_{ij}^{M}, \bar{n}_{ij}^{U} \in [0,1]$

Table 2.3 Fuzzy decision matrix

	Attribute			
Alternatives	C_1	C_2	…	C_m
A_1	$[x_{11}^L, x_{11}^M, x_{11}^U]$	$[x_{12}^L, x_{12}^M, x_{12}^U]$	…	$[x_{1m}^L, x_{1m}^M, x_{1m}^U]$
A_2	$[x_{21}^L, x_{21}^M, x_{21}^U]$	$[x_{22}^L, x_{22}^M, x_{22}^U]$	…	$[x_{2m}^L, x_{2m}^M, x_{2m}^U]$
⋮	⋮	⋮	…	⋮
A_n	$[x_{n1}^L, x_{n1}^M, x_{n1}^U]$	$[x_{n2}^L, x_{n2}^M, x_{n2}^U]$	…	$[x_{nm}^L, x_{nm}^M, x_{nm}^U]$

Step 2: *Calculate weighted normalized matrix*. If a set of weights is assumed to be $W = [w_1, w_2, \dots, w_m]$ for attributes where $\sum_{j=1}^{m} w_j = 1$, then the weighted values after normalizing are calculated by the following equations:

$$\bar{v}_{ij}^L = w_j . \bar{n}_{ij}^L \tag{2.17}$$

$$\bar{v}_{ij}^M = w_j . \bar{n}_{ij}^M \tag{2.18}$$

$$\bar{v}_{ij}^U = w_j . \bar{n}_{ij}^U \tag{2.19}$$

Step 3: Identify PIS and NIS. The PIS and NIS can be defined by the following equations, respectively:

$$\bar{A}^+ = \{v_1^+, v_2^+, \dots, v_m^+\} = \{(\max_i \bar{v}_{ij}^U | j \in I), (\min_i \bar{v}_{ij}^L | j \in J)\} \tag{2.20}$$

$$\bar{A}^- = \{v_1^-, v_2^-, \dots, v_m^-\} = \{(\max_i \bar{v}_{ij}^U | j \in J), (\min_i \bar{v}_{ij}^L | j \in I)\} \tag{2.21}$$

where I is associated with benefit attributes, J is associated with cost attributes, v_1^+ is the PIS, and v_1^- is NIS associated with the first attribute.

As $\bar{n}_{ij}^L, \bar{n}_{ij}^M, \bar{n}_{ij}^U \in [0,1]$, Chen (2000) defined PIS and NIS by Equation (2.22) and (2.23) respectively.

$$\bar{A}^+ = \{((1,1,1)|\, j \in I)\,, ((0,0,0)|j \in J)\} \tag{2.22}$$

$$\bar{A}^- = \{((0,0,0)|\, j \in I)\,, ((1,1,1)|j \in J)\} \tag{2.23}$$

New definitions of PIS and NIS based on Equations (2.22) and (2.23) are FPIS and FNIS, respectively. In the proposed method, PIS and NIS generated from Equations (2.20) and (2.21) were used to define the PIS and NIS. In the original PIS and NIS, the numbers came from original data from the decision matrix. In FPIS and FNIS, however, the numbers were chosen regardless of the decision matrix.

Step 4: Calculate the distance of alternatives from NIS and PIS. The distances of alternatives from PIS and NIS can be calculated by Equation (2.24) and (2.25), respectively.

$$d_i^+ = \sqrt{\frac{1}{3}[(\bar{v}_{i1}^L - v_1^+)^2 + (\bar{v}_{i1}^M - v_1^+)^2 + (\bar{v}_{i1}^U - v_1^+)^2]} + \cdots + \sqrt{\frac{1}{3}[(\bar{v}_{im}^L - v_m^+)^2 + (\bar{v}_{im}^M - v_m^+)^2 + (\bar{v}_{im}^U - v_m^+)^2]} \tag{2.24}$$

$$d_i^- = \sqrt{\frac{1}{3}[(\bar{v}_{i1}^L - v_1^-)^2 + (\bar{v}_{i1}^M - v_1^-)^2 + (\bar{v}_{i1}^U - v_1^-)^2]} + \cdots + \sqrt{\frac{1}{3}[(\bar{v}_{im}^L - v_m^-)^2 + (\bar{v}_{im}^M - v_m^-)^2 + (\bar{v}_{im}^U - v_m^-)^2]} \tag{2.25}$$

Step 5: Calculate the closeness coefficient and rank alternatives. Calculation of the closeness coefficient is similar to Equation (2.13) where $\bar{d}_i^-$ denotes the distance of alternative from NIS and $\bar{d}_i^+$ is representative of the distance of the alternative from PIS. Ranking the alternatives in this method is similar to the original TOPSIS based on the closeness coefficient shown in Equation (2.13). The largest value of the closeness coefficient is the best solution.

A numerical example of Chen's (2000) fuzzy MADM with four alternatives, three criteria, and decision matrix is shown in Table 2.4. The assumptions were made that the first and third attributes were profit attributes and the second attribute was a cost attribute. Also, weights considered by the DM for attributes were 0.3, 0.4, and 0.3 for the first, second, and third attribute, respectively.

Table 2.4 Decision matrix for fuzzy TOPSIS

	Attribute/Weights		
Alternatives	C1/0.3	C2/0.4	C3/0.3
A_1	(6.3 7 7.7)	(1.5 3 4.5)	(0.8 8 15.2)
A_2	(0.9 1 1.1)	(3.5 7 10.5)	(0.5 5 9.5)
A_3	(8.1 9 9.9)	(1 2 3)	(0.3 3 5.7)
A_4	(4.5 5 5.5)	(3 6 9)	(0.5 5 9.5)

The solution to the problem is obtained using the following steps:

Step 1: Normalize the table. The normalized table according to equations provided is shown in Table 2.5.

Table 2.5 Normalized decision matrix for fuzzy TOPSIS

	Attribute/Weights		
Alternatives	C1/0.3	C1/0.3	C1/0.3
A_1	(0.29 0.32 0.35)	(0.08 0.16 0.24)	(0.03 0.33 0.63)
A_2	(0.05 0.04 0.05)	(0.18 0.37 0.56)	(0.02 0.20 0.39)
A_3	(0.45 0.41 0.45)	(0.05 0.10 0.16)	(0.01 0.12 0.23)
A_4	(0.51 0.23 0.25)	(0.16 0.32 0.48)	(0.02 0.20 0.39)

Values in the first cell for A1 and C1 can be calculated as

$$\bar{n}_{11}^{L} = \frac{6.3}{\sqrt{6.3^2 + 7^2 + 7.7^2 + 0.9^2 + 1^2 + 1.1^2 + 8.1^2 + 9^2 + 9.9^2 + 4.5^2 + 5^2 + 5.5^2}} = 0.2902$$

$$\bar{n}_{11}^{M} = \frac{7}{\sqrt{6.3^2 + 7^2 + 7.7^2 + 0.9^2 + 1^2 + 1.1^2 + 8.1^2 + 9^2 + 9.9^2 + 4.5^2 + 5^2 + 5.5^2}} = 0.3225$$

$$\bar{n}_{11}^{U} = \frac{7.7}{\sqrt{6.3^2 + 7^2 + 7.7^2 + 0.9^2 + 1^2 + 1.1^2 + 8.1^2 + 9^2 + 9.9^2 + 4.5^2 + 5^2 + 5.5^2}} = 0.3547$$

Step 2- Calculate the weighted normalized scales. After applying the weights to the decision matrix, the updated decision matrix is shown in Table 2.6.

Table 2.6 Weighted normalized decision matrix for fuzzy TOPSIS

	Attribute/Weights		
Alternatives	C1/0.3	C1/0.3	C1/0.3
A_1	(0.087 0.097 0.106)	(0.032 0.065 0.097)	(0.010 0.101 0.191)
A_2	(0.015 0.014 0.015)	(0.076 0.151 0.227)	(0.006 0.063 0.120)
A_3	(0.136 0.124 0.137)	(0.022 0.043 0.065)	(0.004 0.038 0.072)
A_4	(0.155 0.069 0.076)	(0.065 0.130 0.194)	(0.006 0.063 0.120)

Values in the first cell for A1 and C1 can be calculated as

0.3* (0.29, 0.32, 0.35) = (0.087, 0.097, 0.106)

Step 3: Define the NIS and PIS. Based on Equations 2.20 and 2.21, NIS and PIS can be calculated as

$$\bar{A}^{+} = \{v_1^{+}, v_2^{+}, v_3^{+}\} = \{0.155, 0.022, 0.191\}$$

$$\bar{A}^{-} = \{v_1^{-}, v_2^{-}, v_3^{-}\} = \{0.014, 0.227, 0.004\}$$

Values in $\bar{A}^{+}$can be calculated as

$$v_1^{+} = \max\{0.087, 0.097, \ldots, 0.076\} = 0.155$$

$$v_2^{+} = \min\{0.032, 0.065, \ldots, 0.194\} = 0.022$$

$$v_3^{+} = \max\{0.010, 0.101, \ldots, 0.120\} = 0.191$$

Step 4: Calculate the distances of alternatives from NIS and PIS. The distances calculated by Equations (2.24) and (2.25) are shown in Table 2.7.

Table 2.7 Distances from NIS and PIS in fuzzy TOPSIS

Alternatives	$\bar{d}_j^-$	$\bar{d}_j^+$
A_1	0.369	0.226
A_2	0.173	0.420
A_3	0.346	0.207
A_4	0.280	0.324

Values of $\bar{d}_1^-$ and $\bar{d}_1^+$ of A1 are calculated as:

$$d_1^- = \sqrt{\frac{1}{3}[(0.087-0.014)^2+(0.097-0.014)^2+(0.106-0.014)^2]}+\sqrt{\frac{1}{3}[(0.032-0.227)^2+(0.065-0.227)^2+(0.097-0.227)^2]}$$

$$+\sqrt{\frac{1}{3}[(0.010-0.004)^2+(0.101-0.004)^2+(0.191-0.004)^2]} = 0.369$$

$$d_1^+ = \sqrt{\frac{1}{3}[(0.087-0.155)^2+(0.097-0.155)^2+(0.106-0.155)^2]}+\sqrt{\frac{1}{3}[(0.032-0.022)^2+(0.065-0.022)^2+(0.097-0.022)^2]}$$

$$+\sqrt{\frac{1}{3}[(0.010-0.191)^2+(0.101-0.191)^2+(0.191-0.191)^2]} = 0.226$$

Step 5: Calculate the closeness coefficient and rank alternatives. Closeness coefficients and ranking of alternatives are shown in Table 2.8 Using Equation 2.13,

$$\bar{R}_1 = \frac{0.369}{0.369+0.226} = \frac{0.369}{0.595} = 0.619$$

Table 2.8 Closeness coefficient and final ranking in fuzzy TOPSIS

Alternatives	$\bar{R}_j$	Rank
A_1	0.619	2
A_2	0.292	4
A_3	0.625	1
A_4	0.463	3

The DM chooses A3 as the best alternative although the $\bar{R}$ value for A1 is very similar to that of A3.

2.4.3. Extended TOPSIS Method by Jahanshahloo *et al.* (2006) with Interval Numbers

Chen (2000) used fuzzy membership functions to define uncertainty in the problem. Chen (2000) assumed that fuzzy parameters have known membership functions. However, a DM may not be able to specify membership functions or probability distribution in an inexact environment (Sayadi, et al., 2009). Uncertainty can also be defined using intervals. Interval numbers are more suitable for decision-making problems in an imprecise and uncertain environment because they are the simplest form to represent uncertainty in the decision matrix (Sayadi, et al., 2009). Jahanshahloo, *et al* (2006) proposed use of intervals for TOPSIS. They assumed that the score of alternative i with regard to criterion j was not known exactly but belonged to an interval between$[x_{ij}^L, x_{ij}^U]$ in which x_{ij}^L is the lower limit and x_{ij}^U is the upper limit of interval.

Table 2.9 shows a decision matrix for an interval-based problem with $\boldsymbol{n}$ alternatives and $\boldsymbol{m}$ attributes.

Table 2.9 Interval-based decision matrix

	Attribute			
Alternatives	C_1	C_2	…	C_m
A_1	$[x_{11}^L, x_{11}^U]$	$[x_{12}^L, x_{12}^U]$	…	$[x_{1m}^L, x_{1m}^U]$
A_2	$[x_{21}^L, x_{21}^U]$	$[x_{22}^L, x_{22}^U]$	…	$[x_{2m}^L, x_{2m}^U]$
$\vdots$	$\vdots$	$\vdots$	…	$\vdots$
A_n	$[x_{n1}^L, x_{n1}^U]$	$[x_{n2}^L, x_{n2}^U]$	…	$[x_{nm}^L, x_{nm}^U]$

Similarly, weights $W = [w_1, w_2, \dots, w_m]$ where $\sum_{j=1}^{m} w_j = 1$ are introduced for attributes. The solution for interval-based TOPSIS is similar to the steps for the original TOPSIS with minor modifications. The following steps provide a summary of this method.

Step 1: Normalize the table. Similar to the original TOPSIS method, normalization is necessary here, but the equation is different. For each interval, the upper limit and lower limit must be normalized. $\bar{n}_{ij}^L$ is the normalized value of the lower limit and $\bar{n}_{ij}^U$ represents the normalized value of the upper limit, calculated by the following equations:

$$\bar{n}_{ij}^L = \frac{x_{ij}^L}{\sqrt{\sum_{i=1}^{n}\left(\left(x_{ij}^L\right)^2 + \left(x_{ij}^U\right)^2\right)}}, j = 1, \dots, m, \quad i = 1, \dots, n \tag{2.26}$$

$$\bar{n}_{ij}^U = \frac{x_{ij}^U}{\sqrt{\sum_{i=1}^{n}\left(\left(x_{ij}^L\right)^2 + \left(x_{ij}^U\right)^2\right)}}, j = 1, \dots, m, \quad i = 1, \dots, n \tag{2.27}$$

All normalized values fall in the interval of $[0,1]$.

Step 2: Find the weighted normalized values. Weighted normalized values are calculated by the following equations:

$$\bar{v}_{ij}^{L} = w_j . x_{ij}^{L} \quad (2.28)$$

$$\bar{v}_{ij}^{U} = w_j . x_{ij}^{U} \quad (2.29)$$

where the importance (weight) of attribute j is introduced by w_j and $\sum_{j=1}^{m} w_j = 1$.

Step 3: Define the PIS and NIS. As mentioned PIS and NIS can be calculated by the following equations:

$$\bar{A}^{+} = \{v_1^{+}, v_2^{+}, \dots, v_m^{+}\} = \{(\max_i \bar{v}_{ij}^{U} | j \in I), (\min_i \bar{v}_{ij}^{L} | j \in J)\} \quad (2.30)$$

$$\bar{A}^{-} = \{v_1^{-}, v_2^{-}, \dots, v_m^{-}\} = \{(\max_i \bar{v}_{ij}^{U} | j \in J), (\min_i \bar{v}_{ij}^{L} | j \in I)\} \quad (2.31)$$

where I is associated with the benefit attribute and J is associated with cost attribute and v_1^{+} is the PIS and v_1^{-} is NIS associated with first attribute. FPIS and FNIS defined in Equations (2.22) and (2.23) can also be used in this step as discussed.

Step 4: Calculate the distances of alternatives from PIS and NIS. Distances of alternatives from NIS and PIS are calculated by n-dimensional Euclidean distance as described in the following equations:

$$\bar{d}_i^{+} = \sqrt{\sum_j (\bar{v}_{ij}^{L} - v_j^{+})^2 + \sum_j (\bar{v}_{ij}^{U} - v_j^{+})^2}, j = 1, \dots, m, \quad i = 1, \dots, n \quad (2.32)$$

$$\bar{d}_i^{-} = \sqrt{\sum_j \left(\bar{v}_{ij}^{U} - v_j^{-}\right)^2 + \sum_j \left(\bar{v}_{ij}^{L} - v_j^{-}\right)^2}, j = 1, \dots, m, \quad i = 1, \dots, n \quad (2.33)$$

where $\bar{d}_i^{+}$ is the distance of i^{th} alternative of PIS and $\bar{d}_i^{-}$ is the distance of i^{th} of NIS.

Step 5: Calculate the closeness coefficients. Final ranking of alternatives depends on the closeness coefficient, a unique number associated with each alternative. It is calculated as

$$\bar{R}_i = \frac{\bar{d}_i^-}{\bar{d}_i^- + \bar{d}_i^+} \quad , i = 1, \ldots, n \tag{2.34}$$

where $0 \leq \bar{R}_i \leq 1$. The special cases are $\bar{R}_i = 0$ when $A_i = NIS$, and $\bar{R}_i = 1$ when $A_i = PIS$. A preferred alternative is closest to its PIS and farthest from its NIS. The closer that $\bar{R}_i$ is to 1, the better.

Chen’s fuzzy TOPSIS and Jahanshaahloo’s interval based TOPSIS both used the TOPSIS method developed by Hwang and Yoon (1981). Both methods attempted to solve MADM problems with uncertain attributes. The main difference between methods, though, is that Chen (2000) defined uncertainty by fuzzy membership functions, where Jahanshahloo, *et al.* (2006) introduced uncertain attribute values by intervals. Beside this difference, both modified methods solved MADM problems similarly.

2.4.4. VIKOR

Opricovic (1998) proposed another MADM method, called VIKOR based on measure of closeness to the ideal solution. VIKOR finds a compromised solution among conflicting attributes. The basic function of VIKOR adopts the $L_P - metric$ calculation as an aggregation function proposed by Yu (1973). In a MADM problem with n alternatives and m criteria, the score of each alternative regard with each criteria is $x_{ij} (i = 1,2, \ldots, n; j = 1,2, \ldots, m)$. The equation of discrete form of $L_P - metric$ distance is defined by:

$$L_{P,i} = (\sum_{j=1}^{m} (w_j \frac{x_j^* - x_{ij}}{x_j^* - x_j^-})^P)^{\frac{1}{P}}, \qquad 1 \leq P \leq \infty; \quad i = 1,2, \ldots, n \tag{2.35}$$

$$x_j^* = \{x_1^*, x_2^*, \ldots, x_m^*\} = \{(\max(x_{ij})_i | j \in I), (\min_i(x_{ij}) | j \in J)\}, \; j = 1,2, \ldots, m \tag{2.36}$$

$$x_j^- = \{x_1^*, x_2^*, \ldots, x_m^*\} = \{(\min(x_{ij})_i | j \in I), (\max_i(x_{ij}) | j \in J)\}, j = 1,2, \ldots, m \tag{2.37}$$

where I is associated with benefit attribute, J is associated with cost attribute, w_j is the corresponded weight of each attribute, and x_j^* and x_j^- are identical to PIS and NIS in the TOPSIS method. In this method, x_{ij} is an evaluation of corresponded attribute for alternatives which plays the same role as x_{ij} plays in the TOPSIS method.

The following steps summarize the VIKOR method:

Step 1: Under the assumption of availability of the decision matrix and scores of each alternative regarding to attributes, calculate S_i and R_i. Two important factors in VIKOR are group utility (S_i) and individual regret (R_i), calculated using the following equations:

$$S_i = L_{1,i} = \sum_{j=1}^{M}\left(w_j \frac{x_j^* - x_{ij}}{x_j^* - x_j^-}\right), i = 1,2,\dots,N;\ j = 1,2,\dots,M \tag{2.38}$$

$$R_i = L_{\infty,i} = \max_j \left(w_j \frac{x_j^* - x_{ij}}{x_j^* - x_j^-}\right), i = 1,2,\dots,N;\ j = 1,2,\dots,M \tag{2.39}$$

Step 2: Calculate Q_i

$$Q_i = v\frac{S_i - S^*}{S^- - S_i} + (1-v)\frac{R_i - R^*}{R^- - R_i} \tag{2.40}$$

where the definition of S^*, S^-, R^*, R^- and v are

$$S^- = \max_i S_i \tag{2.41}$$

$$S^* = \min_i S_i \tag{2.42}$$

$$R^- = \max_i R_i \tag{2.43}$$

$$R^* = \min_i R_i \tag{2.43}$$

Variable v in Equation (2.40) is used as the weighting strategy for a majority of attributes or the maximum group utility. Variable v is usually set at 0.5 without loss of generality (Liao, et al., 2013). Q_i in Equation (2.40) considers individual regret and group utility.

Step 3: Sort the alternatives based on decreasing order in three categories: S_i, R_i, and Q_i. The results are three ranking lists of alternatives.

Step 4: Obtain the best choice or a compromise solution. The choice with smaller Q_i is the better solution. However, the best solution requires two qualifications to obtain a unique solution:

Qualification 1: $(Q_{A_1} - Q_{A_2}) \geq \frac{1}{m-1}$ where A_1 and A_2 are the first and second positions at ranking list.

Qualification 2: The best solution based on Q_i should be the best ranked in S_i and R_i as well.

When these qualifications are not met simultaneously, VIKOR generates a set of compromised solutions as follows:

If the first qualification is not obtained, then all alternatives in $\{A_1, A_2, \dots, A_I\}$ are compromised solutions where the value of I is obtained by Equation (2.45).

$$(Q_I - Q_{A_1}) < \frac{1}{M-1} \tag{2.45}$$

If the second qualification is not obtained, then both A_1 and A_2 are compromised solutions.

One of the most important differences between VIKOR and TOPSIS is that TOPSIS does not consider individual regret between alternatives while VIKOR considers it as an important factor in calculations. Individual regret defines how far an alternative is from the ideal solution. Parameter P in Equation (2.35) plays the role of individual regret. VIKOR relies on this parameter because it uses $L_{1,i}$ (as S_i in Equation 2.38) and $L_{\infty,i}$ (as $R_i$in Equation 2.39) to formulate ranking measure.

Another significant difference between TOPSIS and VIKOR is that the solution proposed by VIKOR is closest to the ideal solution, whereas the solution provided by TOPSIS is not necessarily closest to the ideal solution.

Chen's fuzzy TOPSIS and Jahanshaahloo's interval based TOPSIS both extended the original TOPSIS to solve problems with uncertain attributes. Chen (2000) used fuzzy membership functions to address uncertainty in the problem that is dependent on complex mathematical concepts. Jahanshahloo *et. al.* (2006) used intervals to address the uncertainty; however, they did not provide a guideline for how to generate intervals. Both methods selected the best alternative based on a final value derived from the equations provided. The final values of alternatives obtained to find the best solution may be close to each other, but any existing MADM method selects the alternative with the highest value. This decision-making process reveals an inherent sensitivity issue in these methods discussed in Chapter Three. The proposed method overcomes this concern by repeating the ranking processes via simulations. The proposed method also addresses the issue of needing a guideline to generate uncertain intervals, as discussed in the following chapter.

Chapter 3. Proposed TOPSIS Method with Uncertain Attributes

3.1. Introduction

Chapter Two described several MADM methods for certain and uncertain environments. However, Chen and Hwang's (1992) fuzzy TOPSIS, required that fuzzy numbers be defined for uncertain attributes, but fuzzy numbers are not easily defined. Jahanshahloo, *et al.* (2006) extended TOPSIS to solve MADM problems with uncertain attributes via intervals. However, both methods ranked alternatives based on a single number. This chapter proposes a new approach to model uncertainty and a simulation procedure to generate a ranking distribution. The proposed method defines uncertainty by intervals, maintaining the three-number fuzzy number syntax with the middle number as the most likely value. In addition simulation of numbers in each interval of uncertain attributes according to a triangular or uniform distribution is proposed, resulting in each iteration containing one decision matrix with all attribute values simulated.

The proposed method is also based on TOPSIS, one of the most adopted methods in decision making (Rahimi, et al., 2007). TOPSIS was chosen for this study to tackle uncertainty in attributes for the following reasons:

- TOPSIS is commonly used, resulting in access to numerous published papers (Behzadian, et al., 2012).
- Many software packages, such as R, include the TOPSIS algorithm.
- TOPSIS was developed to deal with uncertain attributes as fuzzy membership functions or uncertain intervals (Chen. 2000, Jahanshahloo, et al. 2006).

- Among all MCDM methods, TOPSIS and VIKOR have been developed most recently. TOPSIS offers a ranking at the end whereas VIKOR offers three lists of rankings. DMs may be confused about final decision due to the complexity of rules in VIKOR (Liao, et al., 2013). Therefore, TOPSIS was selected as the base method in this study to tackle uncertainty in attributes.

3.2. Proposed Method on Uncertain Attributes

Solving MADM problems with uncertain attributes was the main subject of this study. Uncertainty can be modeled by fuzzy numbers, vague sets, or intervals. Uncertainty can also be defined based on percentages defined by DMs, who can subjectively define the uncertainty percentage for each attribute. For a construction project, a DM must select one design (alternative) based on criteria such as maintenance expenses, waste output, and CO_2 emission. Estimates of attribute values may not be clear at the beginning of a project, but evaluation of uncertain attributes can be obtained based on experience or historical data. In this case, DMs may assign a percentage of uncertainty to scores. For example, DMs may assign 70% uncertainty to an attribute, showing that very limited information exists for this attribute. Therefore, the range generated from this uncertainty value will be larger than those with small uncertainty values.

This study proposed the following method for handling uncertain attributes. First, a percentage of uncertainty was used to model uncertain attributes, a DM was asked to define the most likely attribute value. Similar to the method described by Jahanshahloo, *et al*(2006), the uncertainty percentage was used to build an interval around the most likely value. Simulations were then used to generate random values within intervals, and the original TOPSIS was applied to obtain one ranking of alternative based on the simulated decision matrix. Finally, the final ranking of alternatives was determined by examining ranking distributions of multiple simulation runs. The proposed method consisted of four parts: defining uncertainty, finding weighted normalized values by interval-TOPSIS method, applying simulation runs, and applying original TOPSIS. Details of the proposed method are discussed in the following sections. Steps of the proposed method are shown in Figure 3.1.

3.2.1. Defining Uncertainty

Step 1: Fill the decision matrix with the most likely scores. As discussed in Section 2.4.2, one assumption in the original TOPSIS method is that the decision matrix is provided. In general, scores of $\boldsymbol{n}$ alternatives and $\boldsymbol{m}$ criteria must be determined in the decision matrix, as shown in Table 3.1.

Table 3.1 Decision matrix

	Attribute			
Alternatives	C_1	C_2	…	C_m
A_1	x_{11}	x_{12}	…	x_{1m}
A_2	x_{21}	x_{22}	…	x_{2m}
$\vdots$	$\vdots$	$\vdots$	…	$\vdots$
A_n	x_{n1}	x_{n2}	…	x_{nm}

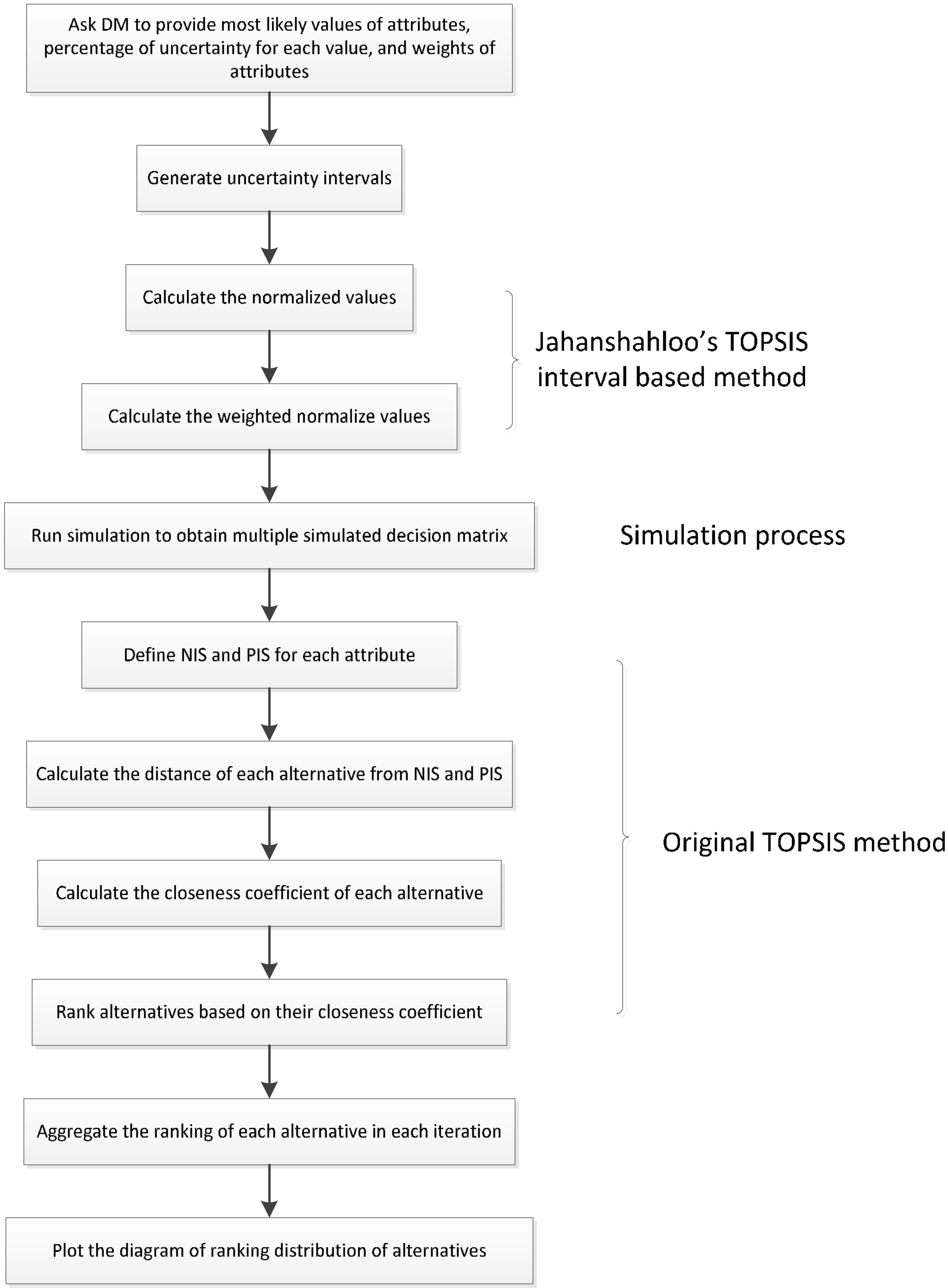


Figure 3.1 Flowchart of the proposed method

Step 2: Define uncertainty. Assuming $\boldsymbol{m}$ criteria and $\boldsymbol{n}$ alternatives allows DMs to assign some percentage for uncertainty on x_{ij}. Let $\propto_1$ %, $\propto_2$ % and $\propto_3$ % be the percentage of uncertainty of the first, second and third attribute, respectively. $\propto_1$ % is the uncertainty percentage about C_1. In this case, the interval for x_{11} is $[x_{11} - \frac{\propto_1\%}{2} \times x_{11}, x_{11} + \frac{\propto_1\%}{2} \times x_{11}]$. The difference between defining intervals and fuzzy membership is that defining intervals is much easier than defining certain fuzzy membership functions (Sayadi, et al., 2009). The final score for the first alternative and the first criterion is modeled by an uncertainty range. Based on this definition, the new matrix is defined in Table 3.2.

Table 3.2 Interval-based decision matrix

	Attribute			
Alternatives	C_1	C_2	...	C_m
A_1	$[x_{11}^L, x_{11}^U]$	$[x_{12}^L, x_{12}^U]$	...	$[x_{1m}^L, x_{1m}^U]$
A_2	$[x_{21}^L, x_{21}^U]$	$[x_{22}^L, x_{22}^U]$	...	$[x_{2m}^L, x_{2m}^U]$
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
A_n	$[x_{n1}^L, x_{n1}^U]$	$[x_{n2}^L, x_{n2}^U]$	...	$[x_{nm}^L, x_{nm}^U]$

In the interval-based decision matrix, x_{ij}^L means the lower limit of interval and x_{ij}^U means the upper limit of the uncertainty interval.

3.2.2. Finding Weighted Normalized Values by Interval-TOPSIS Method

Step : Normalize values. The table can be normalized using Equations (3.1) and (3.2). $\bar{n}_{ij}^{L}$ is the normalized value of the lower limit and $\bar{n}_{ij}^{U}$ is the normalized value of the upper limit, calculated by the following equations:

$$\bar{n}_{ij}^{L} = \frac{x_{ij}^{L}}{\sqrt{\sum_{i=1}^{n}(\left(x_{ij}^{L}\right)^{2}+\left(x_{ij}^{U}\right)^{2})}}, j = 1, \dots, m, \quad i = 1, \dots, n \qquad (3.1)$$

$$\bar{n}_{ij}^{U} = \frac{x_{ij}^{U}}{\sqrt{\sum_{i=1}^{n}(\left(x_{ij}^{L}\right)^{2}+\left(x_{ij}^{U}\right)^{2})}}, j = 1, \dots, m, \quad i = 1, \dots, n \qquad (3.2)$$

Normalized values fall in the interval of $[0,1]$.

Step 4: Define the weights of criteria and find weighted normalized values. Because DMs may consider certain attributes to be more important than others, a larger weight should be assigned to reflect their importance. Let $w_j, j = 1, \dots, m$ be the weight for each criteria and $\sum_{j=1}^{m} w_j = 1$. A weight adjusted decision matrix is obtained by multiplying each interval by the corresponding weight according to Equations (3.3) and (3.4).

$$\bar{v}_{ij}^{L} = w_j . x_{ij}^{L} \qquad (3.3)$$

$$\bar{v}_{ij}^{U} = w_j . x_{ij}^{U} \qquad (3.4)$$

3.2.3. Application of Simulation Runs

Step 5: Applying simulation. According to the method described by Jahanshahloo, *et al.*(2006), NIS and PIS were initially defined in thid study and then the distances of each alternative from NIS and PIS and the closeness coefficient of each alternative were calculated. Then the alternatives were ranked based on their closeness coefficients. As shown in Section 2.4.3, the final rankings of alternatives were obtained according to their closeness coefficients. Figure 3.2 shows a numerical example of two alternatives ($n = 2$) when the same uncertainty values were assigned to each alternative. The X-axis is various uncertainty values and the Y-axis is the computed $\bar{R}_i$ (Equation 2.34). As shown in the figure, when the uncertainty increased, the rank of the first and the second alternative changed.

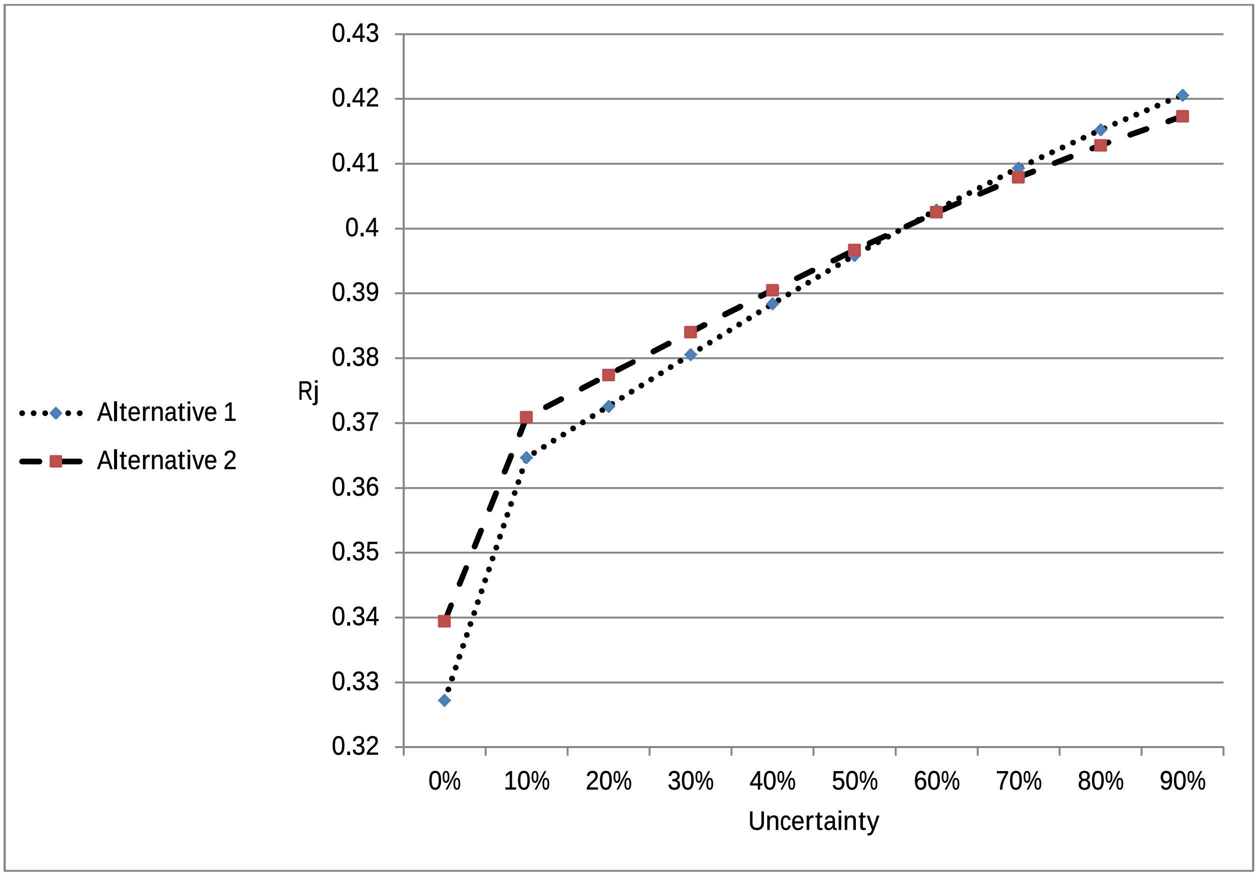


Figure 3.2 Changing ranks by increasing the percentage of uncertainty

In order to overcome this concern, this study proposed use of simulation to generate many solutions in the form of ranking distribution. One hundred numbers in each interval were randomly generated, and the original TOPSIS method was performed 100 times to generate 100 ranking results. Final rankings were then summarized in a ranking distribution represented in a bar chart. As a numerical example with four alternatives, the final result showed 5 times first rank, 20 times second rank, 45 times third rank and 30 times fourth rank in 100 iterations for the first alternative as shown in Figure 3.3.

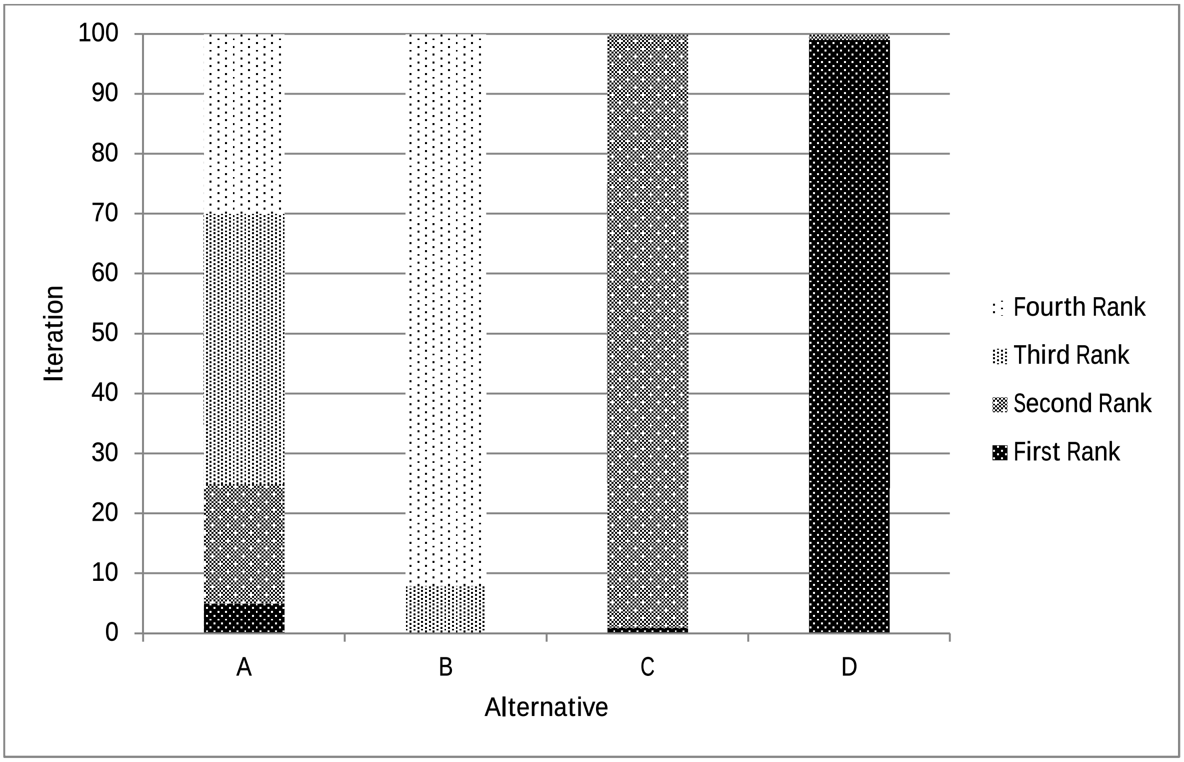


Figure 3.3 Final ranking for alternatives by 100 iterations

The next step was to obtain a rank based on a matrix resulting from the random numbers simulated. In order to obtain the random numbers and simulate the problem, two strategies of generating numbers were studied. The first method used a uniform distribution that randomly selected a number within the interval. However, this method did not consider the most likely score in the decision matrix. The second method used a triangular distribution that selected a random number within the interval that gave higher probability to the middle number according to triangular fuzzy membership function. The middle number was the most likely attribute number assigned by DM. The second was chosen for this study because the triangular method selected a number close to the middle number of the interval due to its most likely occurrence. However, the uniform method randomly selected numbers in an interval with equal probability. Using the numerical example in Section 3.3, Figures 3.4 and 3.5 show results from both random numbers selection.

Implementation of the proposed simulation studies was done using the language R. In order to generate a random number in R from a uniform distribution in the interval of $[a, b]$, $\mathrm{runif}(1) * (\mathrm{a} - \mathrm{b}) + \mathrm{a}$ was used. The R function $runif(1)$ generated a random number based on a uniform distribution between zero and one. In the case of creating random numbers based on a triangular distribution, the R function $qtriangle(runif(1), a, b, \frac{a+b}{2})$ was used.

3.2.4. Application of Original TOPSIS on Simulated Numbers

After obtaining random values, the rest of the steps were identical to the regular TOPSIS method discussed in Section 2.4.1, including defining NIS and PIS, calculating distances of each alternative from NIS and PIS, calculating the closeness coefficients of each alternative, and ranking the alternatives. Values obtained by simulation, were normalized and weighted by corresponding associated weights of attributes.

3.3. Numerical Example Solution Provided by Multiple Methods

In this section, a numerical problem is solved using the proposed method and then compared to Chen and Hwang's (1992) fuzzy TOPSIS. The numerical problem under consideration had four alternatives (A_1, A_2, A_3 and A_4) and three attributes (C_1, C_2 and C_3) in addition to various assigned weights. Various confidence values were assigned to each attribute. Table 3.3 shows information needed for MADM analysis. The attributes came with their weights and uncertainty as attribute, weight, and uncertainty, respectively.

Table 3.3 Decision matrix

	Attribute/Weight/Uncertainty		
Alternatives	C_1/0.35/60% (Benefit attribute)	C_2/0.25/70% (Cost attribute)	C_3/0.40/30% (Benefit attribute)
A_1	3	5	4
A_2	6	5	1
A_3	7	3	3
A_4	2	3	8

Table 3.3 shows that C_3 had 30% uncertainty on his attribute and that C_3 is more important than the other attributes due to its weight (0.4). Less information was available for attribute C_2 compared to the other attributes. Finally, the assumption was made that C_2 is a cost attribute and C_1 and C_3 are benefit attributes. MADM analysis was performed for two methods:

the proposed method and Chen's (2000) fuzzy TOPSIS method discussed in Chapter Two.

3.3.1. The Proposed Method

Step 1: The decision matrix is provided in Table 3.3.

Step 2: Calculate the intervals. The first step to obtaining the intervals is identical for all methods. Based on the definition provided in Chapter Three, the decision matrix containing intervals is shown in Table 3.4.

Table 3.4 Interval based decision matrix

	Attribute/Weight/Uncertainty		
Alternatives	C_1/0.35/60%	C_2/0.25/70%	C_3/0.40/30%
A_1	[2.1 3.9]	[3.25 6.75]	[3.4 4.6]
A_2	[4.2 7.8]	[3.25 6.75]	[0.85 1.15]
A_3	[4.9 9.1]	[1.95 4.05]	[2.55 3.45]
A_4	[1.4 2.6]	[1.95 4.05]	[6.8 9.2]

For example, the interval for the first alternative in regards to the first attribute is calculated as

$3 \pm (3 \times 0.6)/2 = [2.1,3.9]$.

Step 3: Normalize the scales. The normalized table is shown in Table 3.5.

Table 3.5 Normalized interval-based decision matrix

	Attribute/Weight/Uncertainty		
Alternatives	C_1/0.35/60%	C_2/0.25/70%	C_3/0.40/30%
A_1	[0.144, 0.267]	[0.263, 0.546]	[0.251, 0.339]
A_2	[0.287, 0.534]	[0.263, 0.546]	[0.063, 0.085]
A_3	[0.335, 0.623]	[0.158, 0.328]	[0.188, 0.254]
A_4	[0.096, 0.178]	[0.158, 0.328]	[0.501, 0.678]

For example, the lower limit in the first cell between alternative A_1 and attribute C_1 is calculated as

$$\bar{n}_{11}^{L} = \frac{2.1}{\sqrt{2.1^2 + 3.9^2 + 4.2^2 + 7.8^2 + 4.9^2 + 9.1^2 + 1.4^2 + 2.6^2}} = 0.144$$

Step 4: Calculate weighted normalized values. Table 3.6 contains weighted normalized values. For example, the weighted normalized for the first cell is calculated as

0.35*[0.144, 0.267] = [0.050, 0.093]

Step 5: Generate random values. The next step was to simulate different numbers and perform simple TOPSIS for each iteration (defined as 100). Then the regular TOPSIS with the simulated numbers was used. In this study, the processes that defined the PIS and NIS, calculated the distances, calculated the closeness coefficient, and ranked alternatives were coded in R language.

Table 3.6 Weighted normalized interval-based decision matrix

	Attribute/Weight/Uncertainty		
Alternatives	C_1/0.35/60%	C_2/0.25/70%	C_3/0.40/30%
A_1	[0.050, 0.093]	[0.066, 0.137]	[0.100, 0.136]
A_2	[0.101, 0.187]	[0.066, 0.137]	[0.025, 0.034]
A_3	[0.117, 0.218]	[0.039, 0.082]	[0.075, 0.102]
A_4	[0.034, 0.062]	[0.039, 0.082]	[0.200, 0.271]

After obtaining the ranking result of each iteration, the total ranking of each alternative was summarized to form Figures 3.4 and 3.5, which show alternative ranking distributions based on 100 times iterations by uniform and triangular membership functions. According to Figures 3.4 and 3.5, alternative A_4 obtained the most first ranks by either the triangular random numbers or the uniform random numbers. Therefore, A_4 was deemed the best alternative, followed by A_3 as has the most second-place outcome. The triangular distribution assigns more weight on most-likely value within the uncertain interval rather than the uniform distribution. Therefore, triangular distribution was proposed for use in order to provide better ranking distribution.

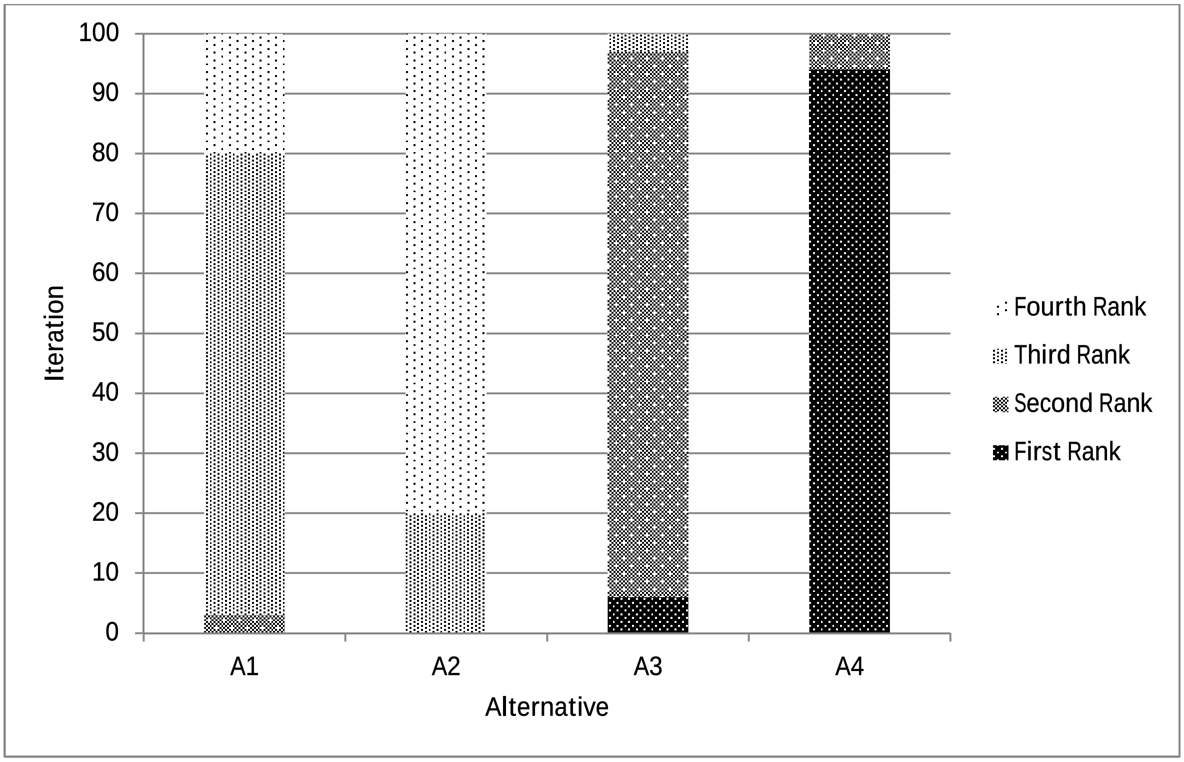


Figure 3.4 Final ranking for 100 simulated runs by triangular random numbers

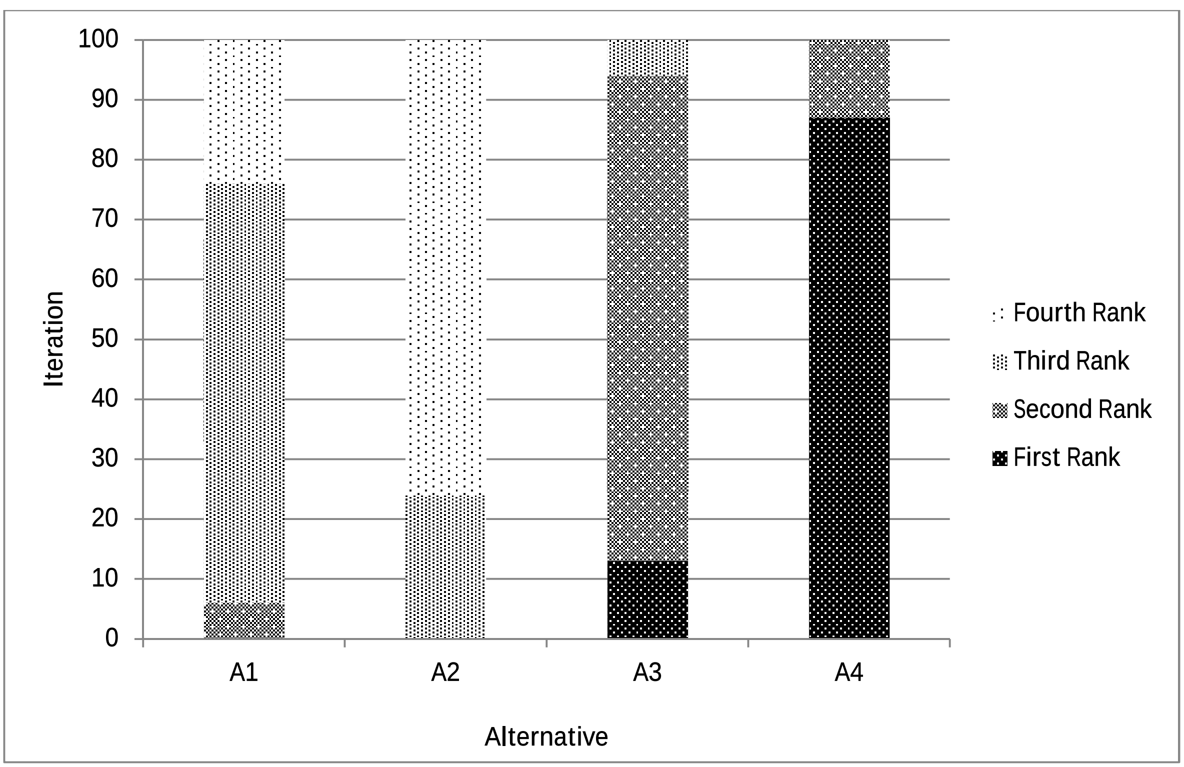


Figure 3.5 Final ranking for 100 simulated runs by uniform random numbers

3.3.2. Chen's (2000) Fuzzy TOPSIS

Step 1: Integrate uncertainty with decision matrix. This step, which calculates the intervals, was identical to the proposed method. The values are shown in Table 3.4.

Step 2: Calculate fuzzy scales. In order to solve the problem using fuzzy TOPSIS, the intervals were converted into fuzzy memberships. Triangular fuzzy membership was chosen in this study to represent fuzzy values. Upper and lower numbers in intervals were same as before, and the middle number was calculated as the average of the upper limit and lower limit of the interval. Fuzzy scales are shown in Table 3.7.

Table 3.7 Fuzzy decision matrix in fuzzy TOPSIS

	Attribute/Weight/Uncertainty		
Alternatives	C_1/0.35/60%	C_2/0.25/70%	C_3/0.40/30%
A_1	(2.1, 3 ,3.9)	(3.25, 5, 6.75)	(3.4, 4, 4.6)
A_2	(4.2, 6, 7.8)	(3.25, 5, 6.75)	(0.85, 1, 1.15)
A_3	(4.9, 7, 9.1)	(1.95, 3, 4.05)	(2.55, 3, 3.45)
A_4	(1.4, 2, 2.6)	(1.95, 3, 4.05)	(6.8, 8, 9.2)

For example, the fuzzy number for the first cell was calculated as (2.1, 3, 3.9), with 2.1 and 3.9 as the upper limit and lower limit of the first cell in Table 3.7. The middle number was calculated as (2.1+3.9)/2=3.

Step 3: Normalize the scales. The normalized table is provided in Table 3.8.

Table 3.8 Normalized decision matrix in fuzzy TOPSIS

	Attribute/Weight/Uncertainty		
Alternatives	C_1/0.35/60%	C_2/0.25/70%	C_3/0.40/30%
A_1	(0.119, 0.170, 0.221)	(0.219, 0.337, 0.454)	(0.205, 0.242, 0.278)
A_2	(0.250, 0.340, 0.442)	(0.219, 0.337, 0.454)	(0.051, 0.060, 0.069)
A_3	(0.337, 0.397, 0.515)	(0.131, 0.202, 0.273)	(0.154, 0.181, 0.208)
A_4	(0.393, 0.113, 0.147)	(0.131, 0.202, 0.273)	(0.411, 0.483, 0.556)

For example, the first element in the first cell originated from the following calculation:

$$\bar{n}_{11}^{L} = \frac{x_{11}^{L}}{\sqrt{\sum_{i=1}^{n}((x_{i1}^{L})^2 + (x_{i1}^{M})^2 + (x_{i1}^{U})^2)}} \rightarrow$$

$$\bar{n}_{11}^{L} = \frac{2.1}{\sqrt{2.1^2 + 3^2 + 3.9^2 + 4.2^2 + 6^2 + 7.8^2 + 4.9^2 + 7^2 + 9.1^2 + 1.4^2 + 2^2 + 2.6^2}} = 0.119$$

Step 4: Calculate weighted normalized values. Weighted normalized values are shown in Table 3.9.

Table 3.9 Weighted normalized decision matrix in fuzzy TOPSIS

	Attribute/Weight/Uncertainty		
Alternatives	C_1/0.35/60%	C_2/0.25/70%	C_3/0.40/30%
A_1	(0.042, 0.059, 0.077)	(0.055 0.084, 0.114)	(0.082,0.097,0.111)
A_2	(0.087, 0.119, 0.155)	(0.055 0.084, 0.114)	(0.021, 0.024, 0.028)
A_3	(0.132, 0.139, 0.180)	(0.033, 0.050, 0.068)	(0.062, 0.072, 0.083)
A_4	(0.137, 0.040, 0.052)	(0.033, 0.050, 0.068)	(0.164, 0.193, 0.222)

For example, the first number in the first cell was calculated as 0.35*0.119=0.042

Step 5: Identify NIS and PIS. Definitions of NIS and PIS are provided based on the definitions of FNIS and FPIS discussed in Section 2.4.3. The following terms show the PIS and NIS for each attribute:

$$\bar{A}^+ = \{v_1^+, v_2^+, v_3^+\} = \{1, 0, 1\}$$

$$\bar{A}^- = \{v_1^-, v_2^-, v_3^-\} = \{0, 1,0\}$$

Step 6: Calculate the distances of NIS and PIS. Distances of each alternative of NIS and PIS are calculated in Table 3.10.

Table 3.10 Distances from PIS and NIS in fuzzy TOPSIS

Alternatives	$\bar{d}_i^-$	$\bar{d}_i^+$
A_1	1.074	1.931
A_2	1.063	1.943
A_3	1.174	1.829
A_4	1.232	1.784

For example, the values of $\bar{d}_1^-$ and $\bar{d}_1^+$ were calculated as

$$d_1^- = \sqrt{\frac{1}{3}[(0.042-0)^2+(0.059-0)^2+(0.077-0)^2]} + \sqrt{\frac{1}{3}[(0.055-1)^2+(0.084-1)^2+(0.114-1)^2]}$$

$$+ \sqrt{\frac{1}{3}[(0.082-0)^2+(0.097-0)^2+(0.111-0)^2]} = 1.074$$

$$d_1^+ = \sqrt{\frac{1}{3}[(0.042-1)^2+(0.059-1)^2+(0.077-1)^2]} + \sqrt{\frac{1}{3}[(0.055-0)^2+(0.084-0)^2+(0.114-0)^2]}$$

$$+ \sqrt{\frac{1}{3}[(0.082-1)^2+(0.097-1)^2+(0.111-1)^2]} = 1.931$$

Step 7: Calculate the closeness coefficient and rank alternatives. The last step was to calculate the closeness coefficient and rank the alternatives. Final results are shown in Table 3.11.

Using Equation (2.13), $\bar{R}_1 = \frac{1.074}{1.074+1.931} = 0.357$. Table 3.11 shows that alternative A_4 is the best choice.

Table 3.11 Closeness coefficient and final ranking of alternatives in fuzzy TOPSIS

Alternatives	$\bar{R}_i$	Rank
A_1	0.357	3
A_2	0.353	4
A_3	0.390	2
A_4	0.408	1

Using the proposed method, a DM can observe which alternative has the most first-rank outcome. This process is an important factor in decision making because a DM has more information. In the original TOPSIS, fuzzy TOPSIS, and VIKOR methods, a DM only had a ranking list. When alternatives are close to each other, only one ranking outcome does not provide adequately describe sensitivity of the result. Using the proposed method, however, a DM obtains ranking results, as shown in Figures 3.4 and 3.5.

Chapter 4. A Case Study

4.1. Introduction

This chapter describes a case study that selected a building design with consideration of resiliency and sustainability. In the context of MADM, candidates were various building designs and attributes were related to multi-hazard resilience and sustainability factors impacting building structures. Multi-hazard resilience and sustainability have recently become two of the most discussed topics among building design and construction researchers as well as the general public (Saunders, et al., 2015). The concepts of multi-hazard resilience and sustainability in building design have originated from multiple societal disciplines (architecture, science, engineering, economics, and sociology) and project scales such as individual buildings and infrastructure, institutions, communities, and regions. However, resilience and sustainability have seldom been holistically considered together for building design. These two categories interact with multiple attributes to evaluate alternatives considered in a study. However, quantifying those assessments does not provide absolute certainty because some concepts (alternatives) may evolve over time. For example, this study considered attributes such as the LEED (Leadership in Energy & Environmental Design) score and life cycle analysis to evaluate sustainability of alternatives. One tool to evaluate the LEED score of alternatives is energy consumption of a building as related to many different items, such as HVAC systems. When a DM decides that the HVAC systems are not yet established and therefore he does not has 100% accurate estimation, leading to uncertainty about the estimations. This uncertainty applies to all attributes, thereby complicating the decision-making process. Based on the nature of building design, a DM can assign percentages of uncertainty to each attribute related to resiliency or sustainability. In this chapter, a case study demonstrates application of the proposed method to solve the decision-making problem of building design under uncertainty. The following sections describe the case study and a solution generated from the proposed method. The terms resiliency and sustainability are described in Sections 4.2 and 4.3, respectively.

4.2. Multi-Hazard Resilience

According to the United States Geological Survey (USGS) (2013), "*Every year in the United States, natural hazard events threaten lives and livelihoods, resulting in deaths and billions of dollars in damage*." Natural hazards include earthquakes, extreme winds (hurricanes and tornadoes), landslides (mudslides), floods, volcanos, and wild fires.

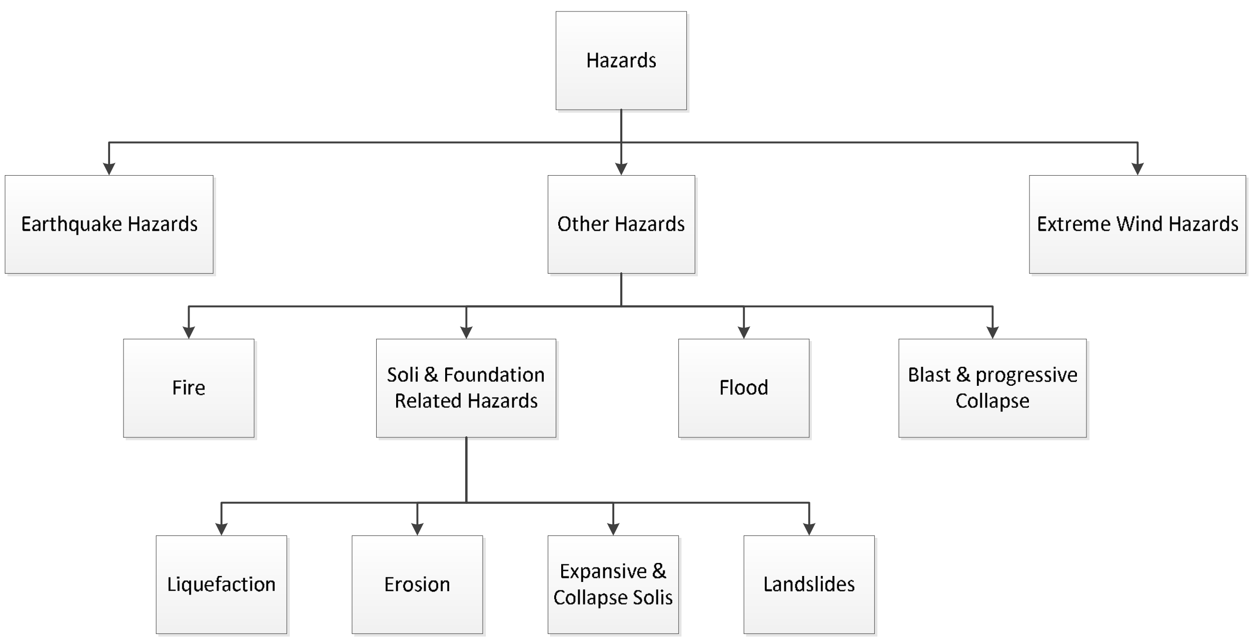


Figure 4.1 Taxonomy of multi-hazards

Among these hazards, earthquakes extreme winds, and floods can cause extensive and severe building damages and collapse, and loss of life. The following sections briefly describe each natural hazard. Figure 4.1 provides a taxonomy of hazards studied in this section.

4.2.1. Earthquake Hazards

Earthquake is one of the most studied natural hazards in research. A large amount of research has been conducted on earthquake hazard mitigation since the 1989 Loma Prieta earthquake and the 1994 Northridge earthquake, both in California. Among them most notably are the SAC (SEAOC ATC CUREe) Steel Project funded by the Federal Emergency Management Agency (FEMA), and the NEES (Network for Earthquake Engineering Simulation) Program funded through the National Science Foundation (NSF). SAC is a joint venture of Structural Engineers Association of California (SEAOC), the Applied Technology Council (ATC), and the Consortium of Universities for Research in Earthquake Engineering (CUREE). The joint venture, was formed in 1994 after the Northridge earthquake, received funding from FEMA and the California Office of Emergency Services (OES). SAC focused on steel moment frames and largely updated AISC (American Institute of Steel Construction) seismic design provisions. A few new connections were prequalified as the result of the SAC project, including the reduced beam section (RBS), which has become very popular in seismic applications. More recently, NSF created the George E. Brown, Jr. Network for Earthquake Engineering Simulation (NEES) "*to give researchers the tools to learn how earthquakes and tsunami impact the buildings, bridges, utility systems and other critical components of today's society*." (Chamot, 2004). The NEES network infrastructure includes 14 earthquake engineering and tsunami research facility sites at universities throughout the United States available for testing on-site, in the field, or through telepresence and cyberinfrastructure operations that connect the work of experimental facilities, researchers, educators and students. The Division of Civil, Mechanical and Manufacturing Innovation (CMMI) of NSF has supported more than 160 NEES research awards through annual research program solicitations (Chamot, 2004).

Seismic design for buildings and other structures is performed using the Seismic Design Hazard maps developed and updated by USGS. Current seismic design is based on collapse prevention (life safety) philosophy, aiming a uniform low collapse probability for structures built throughout the United States. The design hazard maps give spectral acceleration values according to geographic locations. USGS seismic design hazard maps were adopted by the American Society of Civil Engineers (ASCE) in its publication ASCE 7 (ASCE, 2006) – Minimum Design Loads for Buildings and Other Structures. ASCE 7 was then adopted by the International Building Code (IBC) published by the International Code Council (ICC). Most state and local jurisdictions have adopted IBC as their model building code. ASCE 7 (ASCE, 2006), contains dozens of preapproved types of lateral load-resisting structural systems that can be readily used in seismic design with corresponding design parameters and detailing requirements. These systems are commonly referred to as Prequalified Seismic Design Structural Systems. Structural systems not listed can be used per the approval of building officials based on experimental and/or analytical evidence. Specific material trade organizations, such as the American Concrete Institute (ACI), the American Institute of Steel Construction (AISC), and the American Wood Council (AWC), published their standards (e.g., AISC 358 – Prequalified Connections for Special and Intermediate Steel Moment Frames for Seismic Applications) and specifications (e.g., AISC 341 – Seismic Provisions for Structural Steel Buildings and ACI 318 –

Building Code Requirements for Structural Concrete) that further prescribed seismic design requirements for structural systems using these materials.

The focus of seismic design traditionally has been on ductility and energy dissipation. Ductile systems are preferred in seismic design because they can withstand large inelastic deformation under strong ground excitation without major damage or collapse. Ductility is correlated to a structure's energy dissipating capacity. In general, materials and structures with increased energy dissipating capacity can sustain large inelastic deformation without failure. Instead of using prescribed design provisions in building codes and related references, the performance based design approach sets performance targets and designs the structure to meet performance targets. Performance targets equal or exceed minimum performance requirements of the code. This method provides great flexibility in design and performance targets (e.g., life safety and property damage control) and is able to be set at different levels of hazard, often yielding optimal designs. Recent and current research on seismic design has explored new analysis methods, such as push-over analysis and, incremental dynamic analysis, and new concepts, such as fragility, for assessing building performance. FEMA's recent publication, FEMA P695 – Quantification of Building Seismic Performance Factors- introduced a quantified methodology for assessing a building structure's seismic performance, that can be used to quantify seismic design parameters for new systems. Self-centering structures have also been studied recently for their advantages of reduced repair cost and decreased downtime of buildings after major earthquakes.

4.2.2. Extreme Wind Hazards

Wind hazard is another extensively studied natural hazard. Extreme winds can cause extensive casualty and property loss. Two main types of damaging winds are, hurricanes and tornadoes. Hurricanes often affect the East Coast and Mexican gulf states, while tornadoes primarily impact the southern and mid-western states (tornado alley). Hurricanes cause damages to buildings and structures in a large region or regions while tornadoes affect relatively smaller areas with much stronger winds and devastating damages. Both types of extreme winds exert high pressure (inward and outward) to building structural and nonstructural systems, causing potential failures of components (roofs, facades, windows, and doors) and/or main structural frames. Both types of extreme winds generate wind-borne debris that is hazardous to humans and other structures. Hurricanes are often accompanied by extensive rainfalls that cause flooding in large regions

Current building design codes address wind resistance design by assigning design basic wind speed according to geographic locations (wind speed maps) and calculating pressures associated with the wind based on topographic configuration of the site and height and geometric configuration of the buildings. In hurricane-prone areas, wind-borne debris resistance is taken into consideration. However, current building codes lack tornado resistance design provisions (Kuligowski, et al., 2014). Some existing tornado resistance design guidelines and documents exist but without general acceptance and adoption. Therefore, tornadoes often cause devastating, and catastrophic damages and fatalities, such as in May 2011 in Joplin, Missouri, and May 2013 in Moore, Oklahoma, both tornadoes which hit local areas with dozens of fatalities and significant amounts of property losses.

4.2.3. Other Hazards

4.2.3.1. Fire

Fire can be an individual event or an accompanying event from other hazards, such as aftermath of earthquakes. Fire resistance has been incorporated into building design for decades. Fire ratings are in accordance with building occupancies. Structures and components are fireproofed based on fire ratings. In recent years, performance of structures (especially steel structures) under fire has been studied by researchers.

4.2.3.2. Blast and Progressive Collapse

The effects of blasts on buildings have been studied extensively in recent decades, mostly as anti-terrorism measures. The studies have focused on damage control of buildings and prevention of progressive collapse of structures. Stand-off distance and alternative load path are the two main considerations in blast and progressive collapse design.

4.2.3.3. Flooding

Floods often accompany extreme wind events, especially hurricanes. Floods typically do not cause direct damage to building structures upon the first hit. However, damage of non-structural components (exterior, interior of buildings, and mechanical and electrical systems) can cause great inconvenience to building occupants and communities and cause massive property loss and fatalities. Hurricane Katrina and Hurricane Sandy are examples of such a catastrophic event.

4.2.3.3. Soil and Foundation Related Hazards

Gonzales de Vallejo and Ferrer (2011) identified geological or meteorological processes and their associated risks. Geo-hazard risks include liquefaction, erosion, expansive and collapsible soils, landslides, collapse, and subsidence. The following section briefly overviews each of these risks and how in situ soils may be identified as susceptible.

4.2.3.3.1. Liquefaction

Soil liquefaction due to earthquakes occurs in loose to moderately dense saturated cohesion-less soils and sensitive clays. Soil loses shear strength and will fail when in a high stress state, such as under a foundation. Detrimental effects to soils and foundations due to liquefaction can be grouped into three categories: ground subsidence, lateral spread, and damage induced by buoyancy (Huang, et al., 2013). Settlement is particularly troublesome to structures where the settlement is non-uniform, leading to titling or cracking of the structure. Lateral spread typically occurs in floodplains. Damage induced by buoyancy causes underground utilities to be uplifted and damaged. Many advanced tests are available to identify liquefaction potential of a site.

4.2.3.3.2. Erosion

Soil erosion due to flooding undermines building foundations, Extent of soil erosion depends on soil properties, site geometry, hydraulic conditions, vegetation, and many other factors. Hydrodynamic aspects of soil erosion and transport are well understood, as are erosion mechanisms of coarse grain soils. Erodibility of fine-grained soils, however, is much more difficult to predict, and current methods are either conservative or require specialized testing. Soil erosion can be deterred by covering susceptible soils with aggregate or rip-rap, grass, or other native plants with shallow root systems, or engineered products such as a synthetic erosion blanket or geotextile.

4.2.3.3.3. Expansive and collapsible soils

Expansive and collapsible soils similarly undergo volume change with the addition of water. Most shallow foundation problems are due to expansive soils. Expansive soils typically are highly plastic clays that shrink and swell due to seasonal moisture changes or water infiltration. Four common options are used to design foundations in expansive soils: stiffened slab on grade, elevated structural slab on piers, a raft foundation, or a post tensioned slab. Collapsible soils, or metastable soils, are unsaturated soils that undergo large volume change when saturated. Most collapsible soils are wind deposited sands and/or silts. Design of foundations for collapsible soils is similar to expansive soils in that foundation depth can simply be below the depth of anticipated wetting.

4.2.3.3.4. Landslides

Landslides are fundamentally gravity driven soil or rock failures along the weakest plane. Landslides can be caused by earthquakes, excessive rainfall or rapid snow melt, undercutting of a slope, or excessive loading on a slope. Although landslides occur in all 50 states, areas such as the West Coast are more susceptible to landslides. The USGS has a landslide hazard program that reports and monitors landsides across the United States. Developers utilize landslide hazard maps to determine if their structures will be built in an area susceptible to landslides.

4.3. Sustainability

Sustainability has attracted much attention since the late 1970s in consideration of energy and resource uses, ecological system preservation, and societal development. The World Commission on Environment and Development of the United Nations says that sustainable development "… *meets the needs of the present without compromising the ability of future* *generations to meet their own needs*" (WCED, 1987). Sustainability relates not only to the environment, but it also encompasses economic, social, and political development of human societies. In building design, construction and operation, sustainability means minimization or effective, efficient use of materials, water, and energy to minimize the impact of buildings on the environment while providing a healthy environment and satisfactory service to building occupants. The building sector has incorporated sustainable design practice since the 1990s. Currently, some countries utilize several sustainable building design guidelines and rating systems (Fowler, et al., 2006), most of which address multiple categories, including site selection, energy use, water use, and indoor environment quality. In the United States, the US Green Building Council (USGBC) publishes and implements the LEED rating system (USGBC 2009, USGBC 2014), and trains and certifies design professionals for LEED sustainable building design and construction practice.

The LEED 2009 Rating System (USGBC 2009) gives credits in six categories of sustainable design (five basic categories plus one for innovation), including: Sustainable Sites, Water Efficiency, Energy and Atmosphere, Materials and Resources, Indoor Environmental Quality, and Innovation in Design. Each category has prerequisites (except Innovation in Design) that must be complied with in order to be LEED-certified and various credits with scores ranging from 1 to 19 (most credits can just score one point). Sustainable Sites encourages minimizing building footprints, reducing impact on the surrounding environment, and improving stormwater management practices. Water Efficiency encourages reduction in water use, and Energy and Atmosphere encourages minimizing energy use, reducing ozone depleting gas releases, and increasing use of renewable energy. Materials and Resources encourages building and material reuse, increased recycling, use of local materials and rapidly renewable materials, and good waste management practice. Indoor Environmental Quality encourages a healthier and more pleasant indoor environment for building occupants, including increased air quality, daylight, and views. Innovation in Design encourages novel design approaches in sustainability in any of the above categories or beyond.

The goal of sustainability is to minimize the impact of buildings on the environment. A primary tool for assessing building impact is the Life Cycle Assessment (LCA). LCA, which has been used in the manufacturing industry for quite long time, conducts a cradle to grave analysis to assess the impact of material use for the product. LCA for buildings analyzes energy and water use and environmental impact of construction materials production, construction of the building, operation of the building, and demolition of the building (Junnila et al., 2006; Bribian et al., 2009; Ortiz et al., 2009). Energy use throughout a building's life includes energy used in building operation as well as embodied energy used during material production. Same is true for water use. Li *et al*. (2010) proposed a quantitative LCA-based environmental impact assessment for construction processes. Environmental impact includes ozone depleting gas and greenhouse gas releases. Other environmental impacts, such as light pollution and heat island effect, are also included in the assessment, depending on building location and type.

4.4. The Case Study

The building considered in this case study was an office building in Los Angeles, California. The building had a square floor plan, with five 30 ft. bays in each direction; plan dimensions were 150 ft. by 150 ft. The building was six-stories tall, with the story height of the bottom level at 18 ft. and the story height of the other levels at 13 ft. Six designs were considered in order to find an optimal building considering all multi-hazard resilient and sustainable criteria. Several attributes or design factors were essential for selecting the best design. The attributes were grouped into two categories: sustainability and resiliency. Attributes in the sustainability group were related to the building designs impact on the environment, while attributes in the resiliency group were related to the amount of building resiliency in the midst of natural disasters.

Attributes in the resiliency group were further divided into groups based on information available over time. Uncertainty for the DM in this case study was tied to time, meaning that the DM felt more uncertain about attributes that were scheduled later in project rather than attributes that were evaluated at the beginning of the project. For example, as shown in Table 4.1, the DM had less uncertainty on Attribute 1 (ductility) compared to Attribute 9 (Story Drift (Wind)) because, the DM has more information about ductility of the designs. However, the amount of certainty the DM had on scores of Story Drift (Wind) was less because he can only roughly estimate it without precise computation, which will take place after a design is chosen.

Table 4.1 Decision matrix of the case study

		Resiliency		Sustainability				Resiliency		
		Att1/Weight	Att2/Weight	Att3/Weight	Att4/Weight	Att5/Weight	Att6/Weight	Att7/Weight	Att8/Weight	Att9/Weight
Row	Alternatives	Ductility/0.2	Wind-borne/0.1	LEED scores/0.1	Life Cycle Analysis/0.1	Embodied Energy/0.05	Material Use/0.05	Fragility FEA/0.2	Story Drift FEA/0.1	Story Drift(Wind)/0.1
1	Steel-Braced Frame No.1	0.9/0.1	0.8/0.1	0.7/0.1	0.7/0.2	0.5/0.3	0.6/0.3	0.4/0.5	0.7/0.5	0.6/0.5
2	Steel-Braced Frame No.2	0.8/0.1	0.8/0.1	0.7/0.1	0.6/0.2	0.5/0.3	0.5/0.3	0.6/0.3	0.6/0.3	0.5/0.3
3	Steel-Braced Frame No.25	0.9/0.2	0.8/0.1	0.7/0.1	0.8/0.2	0.5/0.3	0.6/0.3	0.5/0.5	0.8/0.5	0.6/0.5
4	Steel-Braced Frame No.26	0.9/0.2	0.9/0.2	0.7/0.1	0.8/0.2	0.6/0.3	0.7/0.3	0.4/0.5	0.7/0.3	0.5/0.3
5	Steel-Moment No.1	0.9/0.1	0.8/0.1	0.7/0.1	0.7/0.2	0.7/0.3	0.8/0.3	0.5/0.4	0.8/0.4	0.7/0.4
6	Steel-Moment No.2	0.8/0.1	0.8/0.1	0.7/0.1	0.6/0.2	0.8/0.3	0.8/0.3	0.6/0.4	0.8/0.5	0.7/0.5

Table 4.1 represents the decision matrix with attributes and alternatives. Each cell contains two numbers: the first number represents the most likely alternative score, and the second number represents the uncertainty score. For example, the first cell containing evaluation of alternative one (Steel-Braced Frame No.1) regarding the first attribute (Ductility) has values of 0.9/0.1 of which 0.9 is the most likely alternative score and 0.1 is the uncertainty score.

In this case study, the design engineer provided evaluation of all cells in the decision matrix. Embodied Energy, Material Use, Fragility FEA, Story Drift FEA, and Story Drift (Wind) were in the cost group of attributes and the others were in the benefit group of attributes. Each score ranged between 0 to 1 with 0 being the least desirable score in the benefit attributes (most desirable score in cost attributes) and 1 being the most desirable score in the benefit attributes (least desirable score in cost attributes).

Attributes were selected based on final design goals of the resilience against multi-hazard and sustainability for environmental friendliness. The alternatives were the building's structural systems, which were in the category of Lateral Force Resisting System (LFRS) or, if focusing on seismic loads of the building, were in the category of Seismic Force Resisting System (SFRS). ASCE 7 - Minimum Design Loads for Buildings and Other Structures- contains dozens of different SFRS systems are preapproved for use in seismic applications (ASCE, 2006) .

In this case study, material selection was limited to steel; therefore only a few systems were used for the building in the high seismic design category. Corresponding systems based on the standard table in ASCE 7 (ASCE, 2006) formed the following design candidates:

- B. Building Frame Systems, No. 1 Steel eccentrically braced frames.

- B. Building Frame Systems, No. 2 Steel special concentrically braced frames.
- B. Building Frame Systems, No. 25 Steel buckling-restrained braced frames.
- B. Building Frame Systems, No. 26 Steel special plate shear walls.
- C. Moment-Resisting Frame Systems, No. 1 Steel special moment frames.
- C. Moment-Resisting Frame Systems, No. 2 Steel special truss moment frames.

These established and approved systems can be chosen for various design and analysis methods and detailing requirements.

Each attribute should be evaluated independently in regards to uncertainties because, the uncertainty for one attribute may not be affected by pinning down options for another attribute. For example, attributes for sustainability may not be related to attributes for resilience. Even within one category, such as resilience, those related to seismic resilience may not have any effect on hurricane resilience. Therefore, uncertainties are largely independent.

One advantage of the proposed method is that the uncertainty of attributes can differ for each alternative. The percentages of uncertainty means the amount of uncertainty the DM has on scores. For example, the uncertainty the DM assigned on scores for Attribute 2 (Wind-borne Impact) was 10% for all alternatives except Alternative 4, which was set at 20% meaning that the DM was 80% certain on this score compared to 90% on the other alternative scores.

4.5. Solution provided by Proposed Method

The solution based on the proposed method is provided in the following steps:

Step 1: Calculate the intervals. The intervals of each cell were generated based on uncertainty provided in Table 4.1. Table 4.2 shows intervals generated from uncertainty and scores provided in Table 4.1. For example, the interval for the first alternative in regards to the first attribute is calculated as

$0.9 \pm (0.9 \times 0.1)/2 = [0.855{,}0.945]$.

Step 2 Normalize the values. The normalized table used Equations (3.1) and (3.2) are as Table 4.3. For example, the lower limit in the first cell between Alternative 1 and Attribute 1 is calculated as

$$\bar{n}_{11}^{L} = \frac{0.85}{\sqrt{0.85^2 + 0.94^2 + 0.76^2 + 0.84^2 + 0.81^2 + 0.99^2 + 0.85^2 + 0.94^2 + +0.76^2 + +0.84^2}}$$

$$\bar{n}_{11}^{L} = 0.2836340$$

Step 3: Calculate weighted normalized values. Weighted normalized values are shown in Table 4.4. For example, the weighted normalized value for the first cell is calculated as

0.2*[0.28, 0.31] = [0.056, 0.062]

Table 4.2 Interval based decision matrix

		Resiliency		Sustainability				Resiliency		
		Att1/Weight	Att2/Weight	Att3/Weight	Att4/Weight	Att5/Weight	Att6/Weight	Att7/Weight	Att8/Weight	Att9/Weight
Row	Alternatives	Ductility/0.2	Wind-borne/0.1	LEED scores/0.1	Life Cycle Analysis/0.1	Embodied Energy/0.05	Material Use/0.05	Fragility FEA/0.1	Story Drift FEA/0.1	Story Drift(Wind)/0.1
1	Steel-Braced Frame No.1	[0.855,0.945]	[0.76,0.84]	[0.655,0.735]	[0.63,0.77]	[0.425,0.575]	[0.51,0.69]	[0.3,0.5]	[0.525,0.875]	[0.45,0.75]
2	Steel-Braced Frame No.2	[0.76,0.84]	[0.76,0.84]	[0.655,0.735]	[0.54,0.66]	[0.425,0.575]	[0.425,0.575]	[0.51,0.69]	[0.51,0.69]	[0.425,0.575]
3	Steel-Braced Frame No.25	[0.81,0.99]	[0.76,0.84]	[0.655,0.735]	[0.72,0.88]	[0.425,0.575]	[0.51,0.69]	[0.375,0.625]	[0.6,1]	[0.45,0.75]
4	Steel-Braced Frame No.26	[0.81,0.99]	[0.81,0.99]	[0.655,0.735]	[0.72,0.88]	[0.51,0.69]	[0.595,0.805]	[0.3,0.5]	[0.595,0.805]	[0.425,0.575]
5	Steel-Moment No.1	[0.855,0.945]	[0.76,0.84]	[0.655,0.735]	[0.63,0.77]	[0.595,0.805]	[0.68,0.92]	[0.4,0.6]	[0.64,0.96]	[0.56,0.84]
6	Steel-Moment No.2	[0.76,0.84]	[0.76,0.84]	[0.655,0.735]	[0.54,0.66]	[0.68,0.92]	[0.68,0.92]	[0.48,0.72]	[0.6,1]	[0.525,0.875]

Table 4.3 Normalized values

		Resiliency		Sustainability				Resiliency		
		Att1/Weight	Att2/Weight	Att3/Weight	Att4/Weight	Att5/Weight	Att6/Weight	Att7/Weight	Att8/Weight	Att9/Weight
Row	Alternatives	Ductility/0.2	Wind-borne/0.1	LEED scores/0.1	Life Cycle Analysis/0.1	Embodied Energy/0.05	Material Use/0.05	Fragility FEA/0.1	Story Drift FEA/0.1	Story Drift(Wind)/0.1
1	Steel-Braced Frame No.1	[0.28,0.31]	[0.26,0.29]	[0.27,0.30]	[0.25,0.31]	[0.19,0.26]	[0.21,0.29]	[0.16,0.27]	[0.20,0.33]	[0.20,0.34]
2	Steel-Braced Frame No.2	[0.25,0.27]	[0.26,0.29]	[0.27,0.30]	[0.22,0.26]	[0.19,0.26]	[0.17,0.24]	[0.28,0.34]	[0.19,0.26]	[0.19,0.26]
3	Steel-Braced Frame No.25	[0.26,0.32]	[0.26,0.29]	[0.27,0.30]	[0.29,0.35]	[0.19,0.26]	[0.21,0.29]	[0.20,0.34]	[0.22,0.38]	[0.20,0.34]
4	Steel-Braced Frame No.26	[0.26,0.32]	[0.28,0.34]	[0.27,0.30]	[0.29,0.35]	[0.23,0.32]	[0.25,0.34]	[0.16,0.27]	[0.22,0.3]	[0.19,0.26]
5	Steel-Moment No.1	[0.28,0.31]	[0.26,0.29]	[0.27,0.30]	[0.25,0.31]	[0.27,0.37]	[0.28,0.38]	[0.22,0.33]	[0.24,0.36]	[0.26,0.39]
6	Steel-Moment No.2	[0.25,0.27]	[0.26,0.29]	[0.27,0.30]	[0.22,0.26]	[0.31,0.42]	[0.28,0.38]	[0.26,0.40]	[0.22,0.38]	[0.24,0.40]

Table 4.4 Weighted normalized values

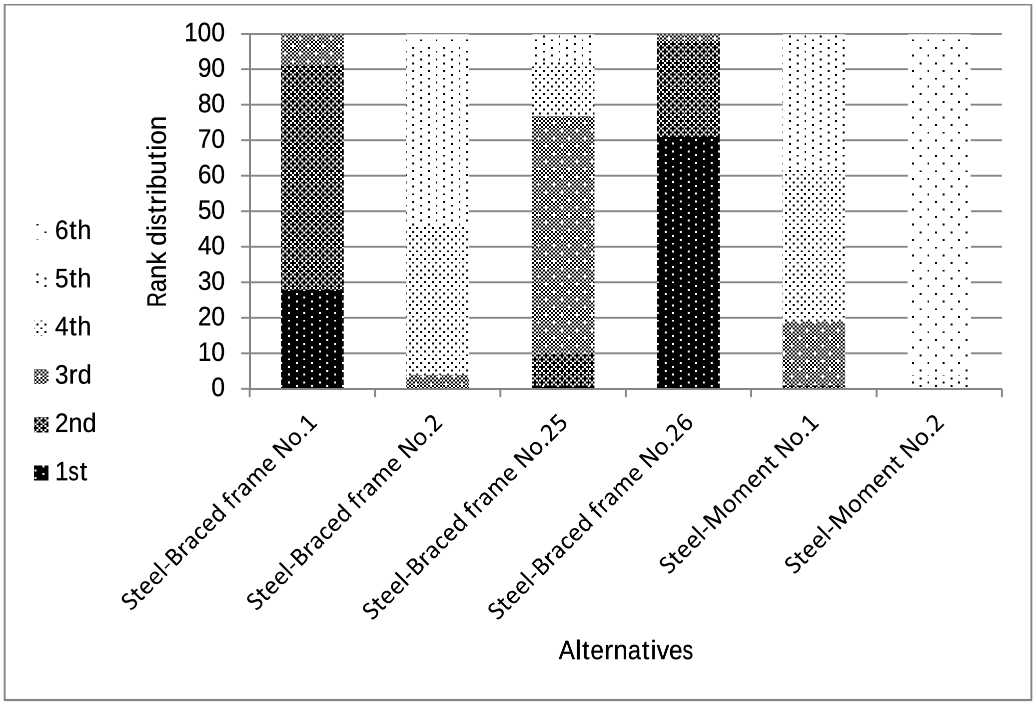
		Resiliency		Sustainability				Resiliency		
		Att1/Weight	Att2/Weight	Att3/Weight	Att4/Weight	Att5/Weight	Att6/Weight	Att7/Weight	Att8/Weight	Att9/Weight
Row	Alternatives	Ductility/0.2	Wind-borne/0.1	LEED scores/0.1	Life Cycle Analysis/0.1	Embodied Energy/0.05	Material Use/0.05	Fragility FEA/0.1	Story Drift FEA/0.1	Story Drift(Wind)/0.1
1	Steel-Braced Frame No.1	[0.056,0.062]	[0.026,0.029]	[0.027,0.03]	[0.025,0.031]	[0.009,0.013]	[0.01,0.014]	[0.033,0.055]	[0.020,0.033]	[0.020,0.034]
2	Steel-Braced Frame No.2	[0.05,0.055]	[0.026,0.029]	[0.027,0.03]	[0.022,0.026]	[0.009,0.013]	[0.008,0.012]	[0.056,0.076]	[0.019,0.026]	[0.019,0.026]
3	Steel-Braced Frame No.25	[0.053,0.065]	[0.026,0.029]	[0.027,0.03]	[0.029,0.035]	[0.009,0.013]	[0.01,0.014]	[0.041,0.069]	[0.022,0.038]	[0.020,0.034]
4	Steel-Braced Frame No.26	[0.053,0.065]	[0.028,0.034]	[0.027,0.03]	[0.029,0.035]	[0.011,0.016]	[0.012,0.017]	[0.033,0.055]	[0.022,0.030]	[0.019,0.026]
5	Steel-Moment No.1	[0.056,0.062]	[0.026,0.029]	[0.027,0.03]	[0.025,0.031]	[0.013,0.018]	[0.014,0.019]	[0.044,0.066]	[0.024,0.036]	[0.026,0.039]
6	Steel-Moment No.2	[0.05,0.055]	[0.026,0.029]	[0.027,0.03]	[0.022,0.026]	[0.015,0.021]	[0.014,0.019]	[0.053,0.080]	[0.022,0.038]	[0.024,0.040]

Step 4: Generate random numbers. As discussed in Chapter 3, triangular distribution provides a distribution of rankings for alternatives that is better than result from a uniform distribution. In this study, a total of 100 iterations were performed to generate 100 decision matrices based on the proposed triangular distribution random number generator.

Step 5: Applying original TOPSIS method to rank alternatives. The original TOPSIS method was applied to each decision matrix. NIS and PIS were found, distances of alternatives from NIS and PIS and closeness coefficients were calculated, and alternatives were ranked. The original TOPSIS was executed 100 times, one for each decision matrix generated in the previous step, as shown in Figure 4.2. Alternative Steel-Braced Frame No.26 was the best choice among all candidates, followed by Steel-Braced Frame No.1 with the second-place of first-rank among alternatives. The proposed method guarantees introduction of the best alternative despite random distribution.

In this case study, the proposed method selected Steel-Braced Frame No.26 as the best alternative because it obtained more first-rank than the other alternatives. However, the second-place design, Steel-Braced Frame No.1 obtained 28 first ranks. Other existing MADM methods can obtain only one ranking result. For example, Chen's (2000) fuzzy TOPSIS and the method developed by Jahanshahloo, *et al*(2006) both provide a ranking list. Figure 3.2 in Chapter Three demonstrated that by taking account the uncertainty into MADM models, the final result used by current MADM methods to rank alternatives may be close to each other. Therefore, ranking can change when the uncertainty number increases, demonstrating inherent sensitivity in the current methods. The proposed method provides a ranking distribution for MADM problems with uncertainty in attributes, thereby overcoming inherent sensitivity in these problems. Using the ranking distribution provided by the proposed method, DMs are presented with more ranking information that takes uncertainty into account.

Figure 4.2 Rank distribution based on triangular random numbers



Chapter 5. Conclusions

5.1 Summary of the Research

This research studied several MADM methods for problems with uncertain attributes. This study primarily focused on the TOPSIS method, one of the most popular MADM methods. Fuzzy concepts have been adopted in some MADM methods for uncertain attributes. Chen's (2000) and the interval-based TOPSIS developed by Jahanshahloo, *et al* (2006) are two important TOPSIS methods in order to develop method to tackle the uncertainty in attributes.

Chen's (2000) fuzzy TOPSIS used linguistic terms to solve the uncertainty issue. Specifically, linguistic terms were converted into fuzzy numbers and used in the original TOPSIS method. The method by Jahanshahloo, *et al* (2006) used intervals to model uncertainty into interval-based scores and then the original TOPSIS method was applied to the lower bound and upper bound of the interval.

Although both methods extended TOPSIS to solve uncertain problems, Chen's (2000) fuzzy TOPSIS method was more difficult to use in term of modeling. A study by Sayadi, *et al* (2009) confirmed that interval numbers are more suitable to deal with decision-making problems in uncertain environments. The method by Jahanshahloo, *et al.*(2006) did not provide a guideline to obtain uncertain intervals; they just extended the TOPSIS method to solve interval-based scores. The proposed method uses intervals introduced by Jahanshahloo, *et al.*(2006) and provided a guideline for how to obtain uncertain intervals. The proposed method used uncertainty percentages defined by a DM to generate uncertain intervals. This simple, logical method quantifies uncertainty in mathematical models, and when uncertainty increases in the model, the numerical interval in the decision matrix logically increases.

This research also studied VIKOR, one of the newest methods in the field of MADM. VIKOR is advantageous compared to TOPSIS because VIKOR considers individual regret between alternatives; TOPSIS does not consider individual regret. However, a disadvantage of VIKOR is the complexity of alternative rankings using three ranking lists of alternatives; TOPSIS generates only one unique ranking list.

Chapter Three proposed generation interval numbers based on uncertainty values and decision matrix provided by a DM. The method developed by Jahanshahloo, *et al* (2006) was then used to generate weighted normalized values. Simulation runs provided random numbers within uncertain intervals, and two random generator distributions based on uniform distribution and triangular distribution were studied. Triangular distribution was recommended because it assigns more weight on the most likely value within the uncertain interval.

Random numbers according to distributions were generated to form multiple different decision matrices. This study simulated and generated the decision matrix 100 times with the possibility of generating more decision matrices if needed. The method was coded in R language. The original TOPSIS method developed by Hwang and Yoon (1981) was used to generate a ranking list. Ranking distribution was obtained by summing the ranking for each alternative. An alternative with maximum first-rank outcomes was selected as the best choice. However, in cases when some alternatives may not be feasible, other alternatives should be selected based on ranking distribution. A numerical example provided in Chapter Three was solved by the proposed method and Chen's (2000) fuzzy TOPSIS method. Discussions were provided to motivate use of the proposed method.

Chapter Four contained a case study related to building design and architecture engineering in which various alternatives were considered for construction of an office building. Designs were judged against multiple attributes grouped into two categories: sustainability and resiliency. Due to the nature of the case study, all attributes contained uncertainty, with some attributes containing more uncertainty than others.

For current practice building designers compare alternatives in multiple steps over time. In each step they consider attributes that are certain, first comparing alternatives based on attributes that are fixed and do not have uncertainty. Then between designs, designers come up to a limited amount of alternatives. When more information is provided on remaining attributes, building designers compare the chosen alternatives, re limiting their choices. The process is repeated until they come up to a solution.

Because the decision should be made at the beginning of the project (not in steps), the proposed method simultaneously considers all attributes. A DM provides the most likely scores for all alternative attribute combinations based on experience, but based on availability of information about specific different attributes, the DM assigns unique uncertainty values on scores. In this case study, the uncertainty for the resiliency attributes such as fragility and story drift are larger than those resiliency attributes related to ductility and wind-borne impact.

Ranking distribution of alternatives via simulation provides a guideline for choosing the best alternative. Ranking distribution also provides much richer information since the most likely best alternative should process more first-place and second-place rankings than other alternatives.

5.2 Future Studies

The proposed method can be applied to various MADM methods. Sayadi, et al. (2009) extended the VIKOR to solve interval-based problems. Similarly, the proposed method can also be applied to VIKOR. Because this study focused only on the TOPSIS method, further study should be conducted for the VIKOR method.

The assessment used on the case study was based on the experience of the DM with high uncertainty percentages. After a while the amount of uncertainty will decrease and the scores will be more precise than previous estimates. Therefore, the proposed method can be feasibly applied at a later time when more information on attributes is available, leading to more precise solution. By comparing the final choice revealed by the proposed method and the method building engineers use to evaluate alternatives (in multiple steps), the proposed method can be validated.

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