ISSN 0025-6544, Mechanics of Solids, 2020, Vol. 55, No. 1, pp. 6–15. © Allerton Press, Inc., 2020.
Russian Text © The Author(s), 2020, published in Izvestiya Rossiiskoi Akademii Nauk. Mekhanika Tverdogo Tela, 2020, No. 1, pp. 9–20.

The Degenerate Structure of Transformation Twins and the Monocrystallinity of Part of the Thin-Plate Martensite Initiated by a Strong Magnetic Field

M. P. Kashchenko[a,b,*], **N. M. Kashchenko**[a], **and V. G. Chashchina**[a,b]

[a] *Ural Federal University, Ekaterinburg, 620002 Russia*

[b] *Ural State Forestry University, Ekaterinburg, 620100 Russia*

**e-mail: mpk46@mail.ru*

Received September 9, 2019; revised September 16, 2019; accepted September 18, 2019

Abstract—In the dynamic theory, the formation of twin martensite crystals is the result of a coordinated propagation of relatively long-wave (ℓ-waves) and short-wave (*s*-waves) displacements. The matching condition is analyzed for the γ–α martensitic transformation in iron-based alloys, taking into account the quasi-longitudinalness of the ℓ-wave carrying compression deformation. It has been shown for the first time that the previously established single-crystal effect of part of the crystals of thin-plate martensite, which arises upon cooling under the action of a strong magnetic field, can naturally be interpreted as a consequence of the formation of a degenerate structure of transformation twins.

Keywords: martensitic transformations, transformation twins, controlling wave process, degenerate twin structure, phase and group velocities, thin plate crystals, strong magnetic field

DOI: 10.3103/S0025654420010112

The object of study is the wave hierarchy, which is part of the structure of the wave process that controls the rapid growth of thin-plate martensitic crystals observed during cooling at γ–α martensitic transformation in carbon steels and a number of iron-based alloys. As a rule, these crystals have a fine structure of transformation twins, characterized by alternating orientations of the main compression axes. However, monocrystallinity variants are also observed when the second component of the twin structure is practically absent, which was paid attention to when the transformation was initiated by strong pulsed magnetic fields.

Based on the analysis of the fulfillment of the matching conditions for the actions with respect to the long-wavelength ℓ-displacements responsible for the formation of habitus planes of crystals and relatively short-wavelength s-displacements responsible for the formation of the main component of the twin structure, it is shown for the first time that coordination is achieved with strict account of the quasilongitude of ℓ-wave carrying a compressive strain, the group velocity of which substantially deviates from the direction of the phase velocity.

As a result, the observed monocrystal effect of some thin-plate crystals receives a natural explanation in the framework of dynamic theory as a consequence of the formation of a degenerate twin structure.

1. INTRODUCTION

The martensitic transformation (MT) is realized in many crystalline materials, substantially modifying their properties, and therefore is of interest both for physics and for solid mechanics. A distinctive feature of the MT is its cooperative nature. In most cases, MT occurs with signs of a first-order phase transition. However, attempts to interpret heterogeneous nucleation on the basis of traditional ideas about the existence of quasiequilibrium nuclei of a new phase, on the one hand, are not supported by reliable observations of such nuclei (the problem of unobservability of nuclei), and, on the other hand, do not adequately interpret the rich set of observed MP features. These problems are especially pronounced when trying to describe the FCC-BCC (BCT) transformation in iron alloys. It was the study of this transformation (here-

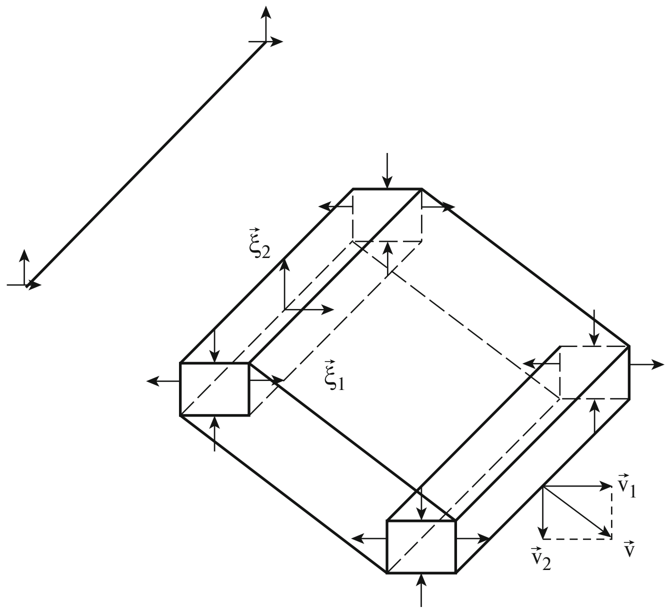

Fig. 1.

inafter, for brevity, γ–α MT) that contributed to the accumulation of the most information underlying the definition of martensitic transformation as a specific (diffusion-free) cooperative transformation [1].

The most striking feature of the spontaneous (upon cooling) γ–α MT is the supersonic (with respect to longitudinal waves) growth rate of martensite crystals. This circumstance immediately allows us to exclude from consideration the dislocation models of crystal growth, non-alternatively indicating the wave nature of crystal growth control. Thus, it was necessary to clarify the specifics of the heterogeneous start of crystal growth, leading to the emergence of a wave process that controls the formation of the crystal and interpretation of the observed set of morphological characters. This program was carried out during the development of the dynamic MT theory [2–4]. The central role in this is played by the concept of the initial excited (vibrational) state (IES). An IES arises in the elastic fields of individual dislocations and has the shape of an elongated parallelepiped with edge orientations close to the orientations of the eigenvectors ξ_1, ξ_2 and ξ_3 of the dislocation nucleation center (DNC) elastic field tensor. Moreover, in the region favorable for the occurrence of an IES, the two eigenvalues of the strain tensor have different signs, and the third strain is small ($\varepsilon_1 > 0$, $\varepsilon_2 < 0$, $\varepsilon_3 \ll |\varepsilon_{1,2}|$). The absolute values of $|\varepsilon_{1,2}|$ are considered comparable with threshold values $\varepsilon_{th} \sim 10^{-4}$–$10^{-3}$ of interphase barriers. Then, upon cooling to a temperature M_s (below the phase equilibrium temperature T_0) in the region with a reduced threshold strain value, a jump occurs in the case of an agreed fast jump of atoms to new equilibrium positions with excitation of vibrations in orthogonal directions close to ξ_1, ξ_2. Such oscillations generate wave beams with orthogonal wave vectors and velocities $\mathbf{v}_1$, $\mathbf{v}_2$. Moreover, the overlapping region of wave beams (in the form of an elongated rectangular parallelepiped) is significant, in which a threshold deformation of the tensile-compression type is realized. The described two-wave pattern for the formation of the inverse image of the plate martensitic crystal is shown in Fig. 1, where the straight-line segment bounded by a pair of ⊥ symbols corresponds to a dislocation in which IES appears in the elastic field.

As follows from Fig. 1, flat crystal boundaries (phase boundaries — habitus planes or, in short, habitus) in such a model correspond to the movement of the lines of intersection of the wavefront fronts at a speed $\mathbf{v} = \mathbf{v_1} + \mathbf{v_2}$. Thus, the occurrence of IES gives rise to the simplest version of the control wave process (CWP). Further, this pair of quasi-longitudinal waves with relatively large wavelengths will be called ℓ-waves. It is easy to show that the orientation of the normal N to the habit plane is given by the equation

$$\mathbf{N}_{1,2\ell} \parallel \mathbf{n}_{2\ell} \pm æ_{\ell\ell}\mathbf{n}_{1\ell}, \quad æ_{\ell\ell} = \frac{\mathbf{v}_{2\ell}}{v_{1\ell}}, \quad |\mathbf{n}_{1,2\ell}| = 1, \quad \mathbf{n}_{1\ell} = \frac{\mathbf{v}_{1\ell}}{v_{1\ell}}, \quad \mathbf{n}_{2\ell} = \frac{\mathbf{v}_{2\ell}}{v_{2\ell}}. \tag{1.1}$$

However, martensitic crystals can also have a fine structure of transformation twins. Transformation twins arise directly during the growth of martensitic crystals in the form of alternating lamellar components. They are observed both in the case of pronounced signs of phase transitions of the first kind (for example, in iron alloys [1]), and at MTs close to phase transitions of the second kind (for example, in a number of non-ferrous metal alloys [5]). A typical difference between the components of the twin structure (TS) is the difference in the orientations of the principal axes of deformation, as a rule, orthogonal to each other in the initial phase. At relatively low (subsonic) growth rates of martensitic crystals, it is permissible to use dislocation representations in interpreting DS as a growth mechanism (see, for example, [6]). At high growth rates of crystals with TS, it is natural to rely on dynamic (wave) interpretations using the concept of CWP. In previous works, it was shown that the concept of CWP, after the inclusion of shorter s-waves in the CWP structure, allows us to describe not only ideal (strictly regular) TS [3, 7, 8], but, based on the model of formation of regular TS, put and solve the problem of interpreting real heterogeneous TS variants [9–13]. One of the results of this analysis is the conclusion about the fragmentation of the domain structure, with each fragment being generated by a single spontaneously arising excited cell initiating s waves. Dynamic theory allows us to consider the limiting case of a degenerate twin structure (DTS), in which the main component dominates [14].

It is well known [1] that, when classifying morphotypes observed with γ–α MT in iron alloys according to habitus orientations, three crystal variants with habituses close to $\{557\}_\gamma$, $\{225\}_\gamma$ and $\{259\}_\gamma$ –$\{31015\}_\gamma$ (crystallographic notations are given in the basis of the initial γ-phase). Moreover, the latter group can be realized both in the form of thin-plate and lenticular crystals. Recall that in the case of lenticular crystals, habitus is characterized by a thin plate central part (for brevity, “midrib”). As an analysis of experimental data [2] shows, the growth rate of midrib (as well as thin-plate crystals) is supersonic, while some effective (average) speed, comparable to the formation of the lenticular shell of midrib, can be smaller in order of magnitude. That is, according to the kinetics of the formation of the lenticular crystal, as early as in the pioneering work [15], 2 stages were distinguished. This conclusion was unconditionally confirmed in studies on the influence of a strong pulsed magnetic field on the formation of martensite [16]. It was shown that the initially formed thin-plate crystal, with subsequent isothermal exposure due to lateral growth, turns into a lenticular crystal, playing the role of midrib in it. The conclusion about the actual correspondence between the midrib and the thin-plate crystal was also confirmed in [17]. As a rule, thin-plate crystals (and midribs) are completely twin. However, as noted in [16], in some cases it is not possible to fix twins in the midrib, which, regardless of the orientation of the electron beam, looks monocrystal. This fact, for compatibility with the conclusions of the phenomenological crystallographic approach [18, 19], requires a formal extension of this approach. Indeed, according to [18, 19], the formation of crystals with the $\{259\}_\gamma$ -$\{31015\}_\gamma$ habit is impossible without a well-defined ratio between the volumes of the main and twin components (we denote it as β) close to β = 1.5. From the standpoint of the dynamic theory, the habit orientation is associated exclusively with ℓ-waves, and the transition to degenerate TS corresponds to the monocrystallinity of the midrib of some crystals.

The purpose of this work is to show that the dynamic theory allows one to select s-waves active in the formation of TS and to compare the effect of the formation of a single crystal midrib with the transition to a degenerate twin structure, clarifying how a strong magnetic field contributes to the manifestation of the effect.

2. BASIC IDEAS ABOUT THE OCCURRENCE OF TS AND DTS

Recall that, during γ–α MT, the transformation twins contact along the $\{110\}_\gamma$ planes. Such planes define pairs of s-waves propagating along the orthogonal axes $\langle 100\rangle_\gamma$ and $\langle 010\rangle_\gamma$ of the initial phase (direction Δ in the first Brillouin zone) with velocities $v_{s\Delta}$. Indeed, the orientations of the normals

$$\mathbf{N}_{1,2s} \parallel [110\,],\ [1\,\bar{1}0\,] \tag{2.1}$$

to the twin planes are trivially found from (1.1) with substitutions

$$æ_{\ell\ell} \to æ_{ss} = 1, \quad \mathbf{n}_{1\ell} \to \mathbf{n}_{1s} = [010], \quad \mathbf{n}_{2l} \to \mathbf{n}_{2s} = [100]. \tag{2.2}$$

The mechanism for removing degeneracy with respect to the orientations of twinning planes is discussed in detail in [3]. If the occurrence of an IES initiating ℓ waves is accompanied by a more or less synchronized occurrence within the localization region of an IES and an excited region in the form of a thin oscillating parallelepiped initiating the excitation of s-wave pairs, then the resulting CWP can correctly describe the formation of thin plate twin crystals (or twin central regions lenticular crystals). Moreover, the main components of the TS are physically isolated, since it is they that are initiated by the action of waves, while twin layers arise due to the coherent coupling of the contacting regions of the lattice.

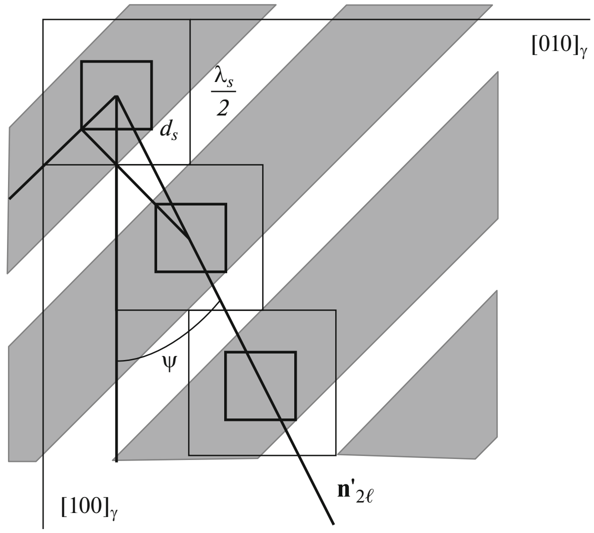

Fig. 2.

The process of induced reproduction of an excited s-cell occurs after two legs of the triangle travel in the $[1\bar{1}0]_\gamma$ and $[110]_\gamma$ directions at the same time by a superposition of s-waves, and the hypotenuse of the triangle with a velocity equal to the $v'_{2\ell}$-projection velocity ℓ- wave $\mathbf{v}_{2\ell}$, bearing compression deformation, onto the plane (001). In Fig. 2, this projection corresponds to the position of the unit vector $\mathbf{n}'_{2\ell}$, which constitutes the acute angle ψ with the direction $[100]_\gamma$.

When the leg speed is $\sqrt{2}v_{s\Delta}$, we obtain the condition

$$v_{s\Delta} = v'_{2\ell}\cos\psi. \tag{2.3}$$

In the harmonic description of the threshold strain, it is assumed that the loss of stability of the lattice of the initial phase occurs in the region with a transverse dimension $d_s < \lambda_s/2$. Then for the ratio β_{tw} of the shares of the components of the TS we get

$$\beta_{tw} = 4\tilde{d}_s/(1+\tan\psi - 4\tilde{d}_s), \quad \tilde{d}_s = d_s/\lambda_s < 1/2. \tag{2.4}$$

Note that relation (2.4) corresponds, in the general case, to the formation of a regular layered structure (including the twin structure). The example shown in Fig. 2 relates to the ratio of component fractions $\beta_{tw} = 2$.

According to (2.4), the case of DTS meets the requirement

$$1 + \tan\psi - 4\tilde{d}_s = 0. \tag{2.5}$$

Such an option, leading to the singularity $\beta_{tw} \to \infty$, is associated with a continuous description that does not take into account the discreteness of the crystalline medium. In Fig. 3, taken from [3], a system of alternating shifts is presented in the main (wider) and additional components of the regular layered structure with a component ratio of 2/1. The coherence of the conjugation of the components, as is evident from Fig. 1, indicates the possibility of removing the singularity by creating a dislocation in place of the twin layer.

Indeed, the minimum layer thickness between the main components of the TS is equal to the distance between the nearest atomic planes $(110)_\gamma$, i.e., $a/\sqrt{2}$, where a is the FCC lattice parameter. If the relative displacement of neighboring planes reaches the same value corresponding to the Burgers vector of the complete dislocation, it is natural to expect the birth of a dislocation loop, the main segments of which are screw oriented. It is clear that the shear strain in the twin component in this case is $\tan\varphi_{tw} = 1$. Since the twin component is surrounded by a pair of the main components of the TS, each of the main components makes an equal contribution $a/2\sqrt{2}$ to the relative displacement of neighboring planes. It was shown earlier [3] that, during twinning, the first fast stage of deformatio n is associated with the action of s-waves; moreover, if the velocities are equal, the compression and tensile strains should be equal: $\varepsilon_{1s} = |\varepsilon_{2s}| = \varepsilon_s$.

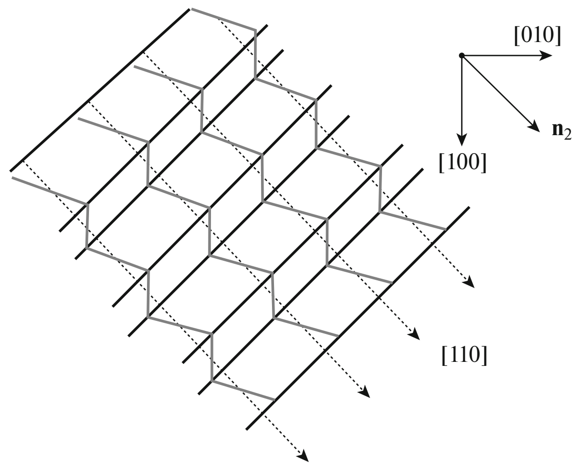

Fig. 3.

Equality applies not only to the threshold ε_{sth}, but also to the final strain values of the first stage ε_{sf}. This means that the strain value of the resulting shear in the main component of the TS with a thickness $d_s\sqrt{2}$ is $\tan\varphi_{\text{bas}} = \varepsilon_{sf}$ at the first stage of deformation already. It follows that the birth of dislocations is possible if the following condition is satisfied:

$$d_s\sqrt{2}\varepsilon_{sf} = a/2\sqrt{2}, \quad d_s\varepsilon_{sf} = a/4. \tag{2.6}$$

Recall [3], in the case when the twin component actually forms, it is natural to assume that the final Bane compressive deformation ($|\varepsilon_{2B}| \le 0.2$) in the twin region is achieved already at the first stage of deformation, while only a smaller deformation can be achieved in the main component stretching. In the case of the DTS, instead of the twin martensitic interlayer, dislocations are generated in the intended CWP layered structure, which requires somewhat less strain ε_{sf} than $|\varepsilon_{2B}|$ (but of the same order). The refinement of ε_{sf} is possible if we consider that, according to (2.4), the maximum value is $d_{s\max} = \lambda_s/2$. Therefore, the determination of the value of λ_s from the additional condition would allow using (2.6) to find the value of ε_{sf}. Such an additional condition is relation (2.3) for the velocities of s- and ℓ-waves. The implementation of the indicated algorithm allows us to explain [20] the appearance of a dislocation structure in martensite crystals with habits close to $\{557\}_\gamma$. Moreover, it is not difficult to show, analyzing the law of phonon dispersion for the Δ-direction, that condition (2.3) can be exactly satisfied when the orientation of the vector $\mathbf{n}'_{2\ell}$ is close to $[110]_\gamma$, that is, when $\psi \le \pi/4$. It is clear that maintaining the consistency of the action of s- and ℓ waves facilitates overcoming the interphase barrier, which has a finite value at a temperature M_s.

3. MATCHING THE VELOCITIES OF s- AND ℓ-WAVES WHILE TAKING INTO ACCOUNT THE QUASILONGAGE OF ℓ-WAVES

Note that, for twinning in crystals with the habit $(3\ 10\ 15)_\gamma - (259)_\gamma$, condition (2.3), which is optimal for the formation of a regular TS, does not hold in the longitudinal wave approximation for the observed elastic moduli [21]. However, as analysis [3] shows, the real wave normals $\mathbf{n}_{1,2\ell}$ relate to quasi-longitudinal waves, and taking into account quasi-longitudinals significantly affects the refined (quantitative) description of morphological characters. Along with the orientation of the normals, knowledge of the polarization vectors $\mathbf{u}_{1,2\ell}$ is also required. In addition, it is clear that the threshold process is associated with the propagation of energy, and hence with group wave velocities $\mathbf{v}_g$. It is known [22] that the projection of the group velocity onto the direction of the wave normal coincides with the phase velocity. The values of the phase velocities of elastic waves from known elastic modules are found from the Christoffel equation, and the eigenvectors of the Christoffel tensor are collinear to the polarization vectors of the waves, and the eigenvalues specify the squares of the velocities. In principle, this information is sufficient to verify the possibility of satisfying condition (2.3), in which $v'_{2\ell}$ is assumed to be the projection of the group velocity of the wave carrying the compression strain onto the $(001)_\gamma$ plane.

The simplest and most obvious way to demonstrate the consequences of taking into account the quasilongage of ℓ-waves is the case where the unit wave normal vector $\mathbf{n}_{2\ell}$ of the quasilongitudinal wave lies in the symmetry plane of the cubic lattice $(001)_\gamma$, and the vector $\mathbf{n}_{1\ell}$ of the longitudinal wave is collinear $[001]_\gamma$. Then for $\mathbf{n}_{1\ell} = [001]_\gamma$, $\mathbf{n}_{2\ell} = [\cos\psi, \sin\psi, 0]_\gamma$, according to (1.1), we obtain

$$\mathbf{N}_{1,2\ell} \parallel [\cos\psi \sin\psi \pm æ_{\ell\ell}]_\gamma. \tag{3.1}$$

Taking into account that the position of the average index in the N normal record corresponds to the unit position when recording the main compression axis (in our case, $[100]_\gamma$), for definiteness we will consider crystals with a habit $(10\ 3\ 15)_\gamma$. Then $\tan\psi = 3/10$, and from (2.5) we determine the values $\tilde{d}_s = d_s/\lambda$ corresponding to the formation of the DTS:

$$\tilde{d}_s = d_s/\lambda_s = (1 + \tan\psi)/4 = 0.325. \tag{3.2}$$

From the Christoffel equation [22] with the direction $\mathbf{n}_{2\ell} = [\cos\psi, \sin\psi, 0]_\gamma$, which makes the angle ψ with the axis $[100]_\gamma$, the expression for the phase velocity $v_\ell(\psi)$ follows:

$$\frac{v_\ell(\psi)}{v_\Delta} = \sqrt{1 + \left(\frac{C_L}{C_{11}} - 1\right)(\sin 2\psi)^2}, \tag{3.3}$$

where C_L and C_{11} are elastic modules defining the velocities of longitudinal waves in the directions of the symmetry axes of the second (v_Σ) and fourth (v_Δ) orders, respectively, for $\psi = \pi/4$ and $\psi = 0$. It is easy to verify that when substituting in (3.3) the values of the measured elastic moduli [21] in the region of relatively small angles

$$\frac{v_\ell(\psi)}{v_\Delta} > \frac{1}{\cos(\psi)}. \tag{3.4}$$

For crystals with habits close to $\{31015\}_\gamma - \{2\ 5\ 9\}_\gamma$, the angle ψ varies from ≈16.7° to ≈21.8°, and inequality (3.4) is satisfied only when it grows stronger when passing from v_Δ to $v_s < v_\Delta$. But this conclusion refers to the approximation of plane longitudinal waves, for which the phase and group velocities coincide. However, for the normal $\mathbf{n}_{2\ell}$ belonging to the plane of symmetry, the wave of interest to us is quasi-longitudinal with the polarization vector $\mathbf{n}_{2\ell}$ lying in the plane of symmetry. The vector $\mathbf{n}_{2\ell}(\psi)$ is the proper Christoffel tensor vector, corresponds to the eigenvalue $[v_\ell(\psi)]^2$ and makes the angle ψ_u with the axis $[100]_\gamma$. The matrix elements of the Christoffel tensor have the form:

$$\begin{gathered}\rho\Lambda_{11} = C_{44} + (C_{11} - C_{44})\cos^2\psi, \quad \rho\Lambda_{22} = C_{44} + (C_{11} - C_{44})\sin^2\psi, \\ \rho\Lambda_{12} = \rho\Lambda_{21} = [(C_{12} + C_{44})\sin 2\psi]/2,\end{gathered} \tag{3.5}$$

where ρ is the density of the material, and C_{11}, C_{12}, C_{44} are the independent elastic moduli of the cubic crystal. Using (3.3) and (3.5) we obtain

$$\tan\psi_u = \frac{C_{11} + (C_L - C_{11})(\sin 2\psi)^2 - C_{44} + (C_{44} - C_{11})(\sin\psi)^2}{(C_{12} + C_{44})\sin 2\psi/2}, \tag{3.6}$$

$$u_1 = \frac{1}{\sqrt{1 + (\tan\psi_u)^2}}, \quad u_2 = \frac{\tan\psi_u}{\sqrt{1 + (\tan\psi_u)^2}}. \tag{3.7}$$

For known projections n_k and u_j of unit vectors of the wave normal and polarization of the ℓ-wave, the components $v^g_{\ell i}$ of the group velocity vector $\mathbf{v}^g_\ell$ are found, according to [22], using the equation

$$(\rho v_i) v^g_{\ell i} = C_{ijkm} u_j u_m n_k, \tag{3.8}$$

where v_ℓ is the magnitude of the phase velocity, C_{ijkm} is the tensor of the elastic moduli, and summation is performed over repeated indices. Using (3.6)–(3.8) for the angle ψ_g between the group velocity $\mathbf{v}^g_\ell$ and the direction $[100]_\gamma$, we obtain

$$\tan\psi_g = \frac{n_2(C_{11}u_2^2 + C_{44}u_1^2) + u_1 u_2 n_1 (C_{44} + C_{12})}{n_1(C_{11}u_1^2 + C_{44}u_2^2) + u_1 u_2 n_2 (C_{12} + C_{44})}. \tag{3.9}$$

Table 1

$\tan\psi = 0.3$	$\tan\psi_u \approx 0.85643$	$\tan\psi_g \approx 0.88206$	$\psi_g - \psi$	$v_\ell(\psi)/v_\Delta$	$v_\ell^g(\psi)/v_\Delta$
$\Psi \approx 16.699°$	$\Psi_u \approx 40.577°$	$\Psi_g \approx 41.414°$	24.715°	1.0925	1.2027

The orientations of the polarization vectors and the group velocity of the quasilongitudinal wave were calculated using the data [21] on the elastic moduli of the Fe-31.5Ni alloy (in TPa): $C_L = 0.218$, $C_{44} = 0.112$, $C' = 0.027$ at the temperature $M_s = 239$ K. Then

$$C_{11} = C_L + C' - C_{44} = 0.133, \quad C_{12} = C_L - C' - C_{44} = 0.079, \quad C_{44} = 0.112. \tag{3.10}$$

The calculation results based on data (3.10) are shown in Table 1.

Commenting on the data in the table, we note, firstly, that the values of the group and phase velocities are related by the equation

$$v_\ell^g = v_\ell(\psi)/\cos(\psi_g - \psi). \tag{3.11}$$

Secondly, in contrast to inequality (3.4) for phase velocity, in the case of group velocity we obtain the opposite inequality

$$v_\ell^g/v_\Delta \approx 11.2027 < 1/\cos\Psi_g \approx 1.333. \tag{3.12}$$

This means that taking into account the phonon dispersion, leading to a monotonic decrease in the group velocity of s-waves v_s^g with an increase in the quasimomentum q_s in the Δ direction, will single out a specific value q_s for which (after replacing $v_\Delta \to v_s^g$) inequality (3.12) turns into equality. Therefore, if the group velocity $\mathbf{v}^g$ is considered as the velocity of the ℓ-wave, condition (2.3) for matching the wave velocities can be satisfied for well-defined s-waves.

Let us find the quasimomentum of the s-wave for which requirement (2.3) is satisfied. For this, we use the previously obtained analytical interpolation of the phonon dispersion law $\varepsilon_\mathbf{k}$ in the Δ direction [10], which is in good agreement with both sound velocity measurements [21] and neutron research data [23]. The dispersion law $\varepsilon_\mathbf{k}$ along $\langle 001\rangle_\gamma$ for $0 \le k \le k_{\max} = 2\pi/a$ (a is the lattice parameter) is approximated in the dimensionless variables y and x by the function

$$1 - y = (1 - x)^p, \quad y = \varepsilon_k/(\varepsilon_k)_{\max}, \quad x = k/k_{\max}. \tag{3.13}$$

For example, for an Fe30Ni alloy with an FCC lattice, agreement with experimental data is achieved at $p \approx 1.733$. For group $v_s^g(x) = dy/dx$ velocities of s-waves, we get:

$$v_s^g(x) = dy/dx = p(1 - x)^{p-1}, \quad v_s^g(0) = v_\Delta = p. \tag{3.14}$$

Assuming $v_\ell^g/v_s^g = 1.3334273 = (v_\ell^g/v_\Delta)(v_\Delta/v_s^g) = 1.202712(v_\Delta/v_s^g)$, we find $(v_s^g/v_\Delta) \approx 0.901971$. Then, for $p = 1.733$, from (3.14) we obtain $x \approx 0.1323140$, that is, $q_s = x{\cdot}q_{\max} \approx 0.265\pi/a$. Therefore, $\lambda_s = 2\pi/q_s \approx 7.5a$. According to (2.5), at $\tan\psi = \tan\psi_g = 0.88205925$, the formation of the DTS corresponds to the value of $\tilde{d}_s = d_s/\lambda_s \approx 0.4705$, which significantly exceeds $\tilde{d} = 0.325$ in (3.2) found in the longitudinal wave approximation. Obviously, when passing to the DTS and $d_s = d_{s\max} \approx 0.941\lambda_s/2$ is a larger fraction of $\lambda_s/2$. And finally, from (2.6) we obtain $\varepsilon_{sf} = 0.066$. Thus, during the formation of a DTS already at the first stage of the formation of a thin-plate crystal (or midrib), the deformation initiated by the contribution of s-waves reaches $\varepsilon_s = 0.066$ at the boundaries of the main component of the TS.

4. DISCUSSION OF THE RESULTS

Taking into account the dispersion of s-waves and the quasi-longitudinalness of ℓ-waves allows us to satisfy condition (2.3) for matching the waves. Since the induced initiation of a new s-cell is a threshold process, it is natural to expect that a nonequilibrium system, striving for a new metastable stable state, will use all available energy resources with maximum efficiency, this is achieved by taking into account the direction of energy propagation of the quasi-longitudinal ℓ-wave.

In [10], the quasilongitude of the ℓ-wave was not taken into account, and therefore, full agreement between the wave velocities reflected by condition (2.3) was not obtained. Nevertheless, it is clear that the whole spectrum of harmonics is mapped to the pulsed nature of the initial "bursts" [24], and not just a single harmonic. Therefore, the mismatch of the velocities of s- and ℓ-waves remains important as one of the reasons for the modulation of the TS and its fragmentation.

The effect of monocrystality of midrib implies the absence of a twin component, and not the ideal perfection of the crystal lattice, that is, the term "monocrystality" in this context has a conditional character. Upon transition to the DTS taking into account the discreteness of the medium, instead of the ideal closure of the main components of the TS, a series of interlayers appears (for example, in the form of dislocation loops), and, therefore, there should be a violation of monocrystality with a repetition rate of interlayers between the main components of the TS. Note that in [25], the observation of small dislocation loops in the martensite structure of high-carbon steel was reported.

In the considered example, for more clarity, the wave normal of a quasilongitudinal ℓ-wave (bearing compression deformation) is located in the symmetry plane $(001)_\gamma$, and the normal of the longitudinal ℓ-wave (bearing tensile strain) is directed along the $[001]_\gamma$ axis. This made it possible to obtain an easily visible analytical description. With allowance for deviations from the indicated ℓ-wave orientations, the length of real s-waves satisfying (2.3) can increase, then the repetition rate of the interlayers in the DTS will decrease, and the degree of approximation to the midrib single crystal will increase. It is also useful to keep in mind that the indication of crystallographic habit indexes, for example, the $\{3\ 10\ 15\}_\gamma$ family, refers to some center of the actual habit distribution. Such a "scattering" of orientations reflects the heterogeneity of the elastic fields of the dislocation loops, which leads to variations in the IES and the CWP variants that inherit information about elastic fields in the localization region of the starting state. This means that the variation of the quantity $\tan\psi$ in (2.5) will also be real. In particular, it is clear that an increase in the values of $\tan\psi$ (at $0 \le \psi \le \pi/4$) contributes to an increase in the value of $\tilde{d}_{\max}$, which is comparable with the transition to the DTS. Recall that the calculation of the group velocity direction (see Sec. 3) leads to an angle ψ_g close to $\pi/4$. The limiting value $\tilde{d}_{\max} = 1/2$ in the description of the threshold strain ε_{th} in the harmonic approximation is associated with the case $\varepsilon_{th} = 0$ at the boundaries of the s-cell with the width $\lambda_s/2$. Thus, an increase in $\tilde{d}_{\max}$, in combination with the formation of a DTS, may indirectly indicate a decrease in $\varepsilon_{\rm th}$, or, in a more general formulation, the occurrence of conditions for over-barrier movement.

Turning to the discussion of the influence of a strong magnetic field on the martensitic transformation, the thermodynamic and dynamic reasons for the possible effect should be separated. First of all, we note that the influence of the field is most pronounced for alloys having low temperatures M_s and having a significant magnetostrictive effect. In such alloys, a strong magnetic field leads to a sharp increase in M_s, which is explained from thermodynamic positions [16] as a result of a shift in the temperature of phase equilibrium due to differences in the phase magnetization (the magnetization of martensite is noticeably greater than that of austenite). This explanation is substantially supplemented in dynamic theory. Firstly, there is reason to believe that the alloy compositions correspond to a state with a large value of the critical grain diameter D_c (with grain diameters $D \le D_c$, the transformation does not occur up to absolute zero temperature). Therefore, for values of D exceeding D_c (but of the same order), the MT proceeds at low temperatures. It was shown in [26–28] that the value of D_c is determined by a combination of physically significant parameters, in particular, by the modulus of the difference between the average energy $\bar{\varepsilon}_d$ of nonequilibrium d-electrons (active in phonon generation) and the chemical potential μ. An increase of $|\bar{\varepsilon}_d - \mu|$ in a strong magnetic field (μ decreases due to volume magnetostriction) is accompanied by a sharp decrease in D_c and a corresponding increase in M_s. Secondly, a strong magnetic field leads to a significant contribution to the inverse population of d-electron states, which ensures an increase in the oscillation amplitudes (and hence the deformations) of the generated elastic waves. This aspect of the influence of the field is most clearly expressed in the orientation effect, when the field is oriented along the 4-order symmetry axis ($[001]_\gamma$), only martensite crystals are formed, whose habituses are the smallest angles with the field direction [2, 16, 29, 30]. In our opinion, it is this aspect that is most essential for ensuring the over-barrier movement and, in particular, the implementation of the DTS, if the value of $\tilde{d}$ is close to $\tilde{d}_{\max}$.

Concerning the prospects of using dynamic models of the formation of TS and DTS, note should be taken of the formation of crystals with habits close to $\{110\}$. Such (or close) habits are observed in nonferrous alloys [5]. Since the description of the habitus is given by a pair of waves traveling in the orthogonal directions $\langle 100 \rangle$, their velocities are equal, and therefore, the strains are equal when moving to the finish values. This circumstance allows advancement in the analytical description of the tetragonality of martensitic crystals, a change in their specific volume [14] and leads to good agreement with the results for martensitic transformation in the equiatomic $Ni_{50}Mn_{50}$ alloy [31]. Note that with respect to iron alloys, this habitus option is interesting in connection with the formation of a zone of thin crystals adjacent to the midbread of lenticular crystals.

5. CONCLUSION

The dynamic theory of the formation of regular layered structures during MT, including transformation twins, makes it possible to carry out the passage to the degenerate structure of transformation twins taking into account the discreteness of the crystalline medium. It has been shown for the first time that in the case of the formation of thin-plate crystals with habites $\{3\ 10\ 15\}_\gamma$ (or midribs of lenticular crystals), it is possible to satisfy the matching condition for the speeds of short-wave (s) and long-wave (ℓ) displacements, taking into account the quasi-length of ℓ-waves, leading to significant deviations polarization vectors and group velocity from the wave normal. The absence of a twin component in some crystals (the midrib monocrystal effect), which was previously discovered in experiments on the influence of a strong pulsed magnetic field on the course of a MT, is interpreted as a transition to a degenerate twin structure. This transition is facilitated by the appearance of an additional contribution to the generation of elastic waves by nonequilibrium electrons associated with the influence of a strong magnetic field.

ACKNOWLEDGMENTS

The authors are grateful to the participants of the XXI Winter School on Continuum Mechanics (Perm, February 18-22, 2019) and the international conference MGCTF 2019 (St. Petersburg, July 1-5, 2019) for discussing the results.

REFERENCES

1. G. V. Kurdyumov, L. M. Utevskiy, and R. I. Entin, *Transformations in Iron and Steel* (Nauka, Moscow, 1977) [in Russian].
2. M. P. Kashchenko, *Wave Model of Martensite Growth upon the g—a Transformation in Iron-Based Alloys,* 2nd ed. (NITs Regul. i Khaotich. Dinamika, Izhevsk. Inst. Komp'yuternykh Issledovanii, Izhevsk, 2010) [in Russian].
3. M. P. Kashchebko and V. G. Chashchina, *Dynamic Model of Twinned Martensitic Crystal Formation at g—a Transformation in Iron Alloys* (UGLTU, Ekaterinburg, 2009) [in Russian].
4. M. P. Kashchenko and V. G. Chashchina, "Dynamic model of supersonic martensitic crystal growth," Phys. Usp. **54** (4), 331–349 (2011).
5. H. Warlimont and L. Delaey, *Martensitic Transformations in Copper-Silver-and Gold-Based Alloys* (Pergamon Press, Oxford, 1974; Nauka, Moscow, 1980).
6. M. A. Shtremel', *Strength of Alloys,* Part II: *Deformation* (MISIS, Moscow, 1997) [in Russian].
7. M. P. Kashchenko, V. G. Chashchina, and S. V. Vikharev, "Dynamic model of the formation of twinned martensite crystals: I. Control wave process and the removal of degeneracy in twin-boundary orientation," Phys. Met. Metallogr. **110** (3), 200–209 (2010).
8. M. P. Kashchenko, V. G. Chashchina, and S. V. Vikharev, "Dynamic model of the formation of twinned martensite crystals: II. Pretransition states and relationships between the volumes of the twin components," Phys. Met. Metallogr. **110** (4), 305–317 (2010).
9. M. P. Kashchenko and V. G. Chashchina, "Key role of transformation twins in comparison of results of crystal geometric and dynamic analysis for thin-plate martensite," Phys. Met. Metallogr. **114** (10), 821–825 (2013).
10. M. P. Kashchenko, I. F. Latypov, and V. G. Chashchina, "Correlation of velocities of the waves controlling the thin-plate a-martensite formation and the modulation of the transformation twin structure," Lett. Mat., No. 2, 146–150 (2017).
11. M. P. Kashchenko and V. G. Chashchina, "Effect of the attenuation of short-wavelength displacements on the formation of transformation twins in a-martensite crystals," Phys. Met. Metallogr. **118** (4), 311–315 (2017).
12. M. P. Kashchenko, N. M. Kashchenko, and V. G. Chashchina, "Dynamic options for forming transformation twins," Mat. Today: Proc. **4**, 4605–4610 (2017).
13. M. P. Kashchenko, N. M. Kashchenko, and V. G. Chashchina, "Effect of change in the wavelengths of short wave shifts on the formation of a twin structure fragment in thin lamellar a-martensite crystals," Phys. Met. Metallogr. **119** (1), 1–5 (2018).
14. M. P. Kashchenko, N. M. Kashchenko, and V. G. Chashchina, "The formation of martensite crystals with a degenerate structure of transformation twins," Lett. Mat., No. 4, 424–429 (2018).
15. R. P. Bunshah and R. F. Mehl, "Rate of propagation of martensite," Trans. AIME **197**, 1251–1258 (1953).

16. V. M. Schastlivtsev, Yu. V. Kaletina, and E. A. Fokina, *Martensitic Transformation in a Magnetic Field* (URO RAN, Ekaterinburg, 2007) [in Russian].
17. A. Shibata, T. Murakami, S. Morito, T. Furuhara, et al., "The origin of midrib in lenticular martensite," Mat. Trans., No. 6, 1242–1248 (2008).
18. M. S. Wechsler, D. S. Lieberman, and T. A. Read, "On the theory of the formation of martensite," J. Met., No. 11, 1503–1515 (1953).
19. J. S. Bowles and J. K. Mackenzie, "The crystallography of martensite transformations I," Acta Met., No. 1, 129–137 (1954).
20. M. P. Kashchenko, N. M. Kashchenko, and V. G. Chashchina, "Interpretation of packet martensite crystals in iron alloys as crystals with a degenerate structure of transformation twins," in *Proceedings of Scientific Conference XXI Winter School on Continuum Mechanics* (PFITS URO RAN, Perm, 2019), p. 139.
21. G. Haush and H. Warlimont, "Single crystalline elastic constants of ferromagnetic face centered cubic fe-ni invar alloys," Acta Met., No. 4, 401–414 (1973).
22. F. I. Fedorov, *Theory of Elastic Waves in Crystals* (Nauka, Moscow, 1965) [in Russian].
23. E. D. Hallman and B. N. Brockhause, "Crystal dynamics of nickel-iron and copper-zinc alloys," Canadian J. Phys. **47** (10), 11–17 (1969).
24. M. P. Kashchenko, N. M. Kashchenko, and V. G. Chashchina, "Initial excited state and divergence of wave beams as factors responsible for the realization of the wave process that controls the growth of transformation twins," Phys. Metals Metallogr. **120** (5), 422–428 (2019).
25. F. F. Satdarova, "Dislocation structure of martensitic transformation in carbon steel," Phys. Metals Metallogr. **117** (4), 355–363 (2016).
26. M. P. Kashchenko and V. G. Chashchina, "Problem of critical grain size under $\gamma \rightarrow \alpha$ martensitic transformation. thermodynamic analysis with allowance for spatial scales typical for the stage of nucleation of martensite," Fiz. Mesomekh. **13** (1), 29–35 (2010).
27. M. P. Kashchenko and V. G. Chashchina, "Dependence of the temperature of the start of $\gamma \rightarrow \alpha$ martensitic transformation on the grain size," Fiz. Mesomekh. **13** (1), 37–45 (2010).
28. M. P. Kashchenko and V. G. Chashchina, *Dynamical Model of the* $\gamma \rightarrow \alpha$ *Transformation and a Solution to the Critical Grain Size Problem* (Izhevsk. Inst. Komp'yuternykh Issledovanii, Izhevsk, 2010) [in Russian].
29. M. P. Kashchenko, "Interpretation of some characteristic morphological indicators of martensite of systems iron-nickel, iron-carbon using the phonon maser model," Fiz. Met. Metalloved. **58** (5), 862–869 (1984).
30. A. A. Leont'ev, V. M. Schastlivtsev, and L. N. Romashev, "Habitus and orientation of martensite crystals formed under action of magnetic field," Fiz. Met. Metalloved. **58** (5), 950–957 (1984).
31. M. P. Kashchenko, N. M. Kashchenko, V. G. Chashchina, et al., "Dynamic scenarios of the formation of martensite with the {110} habits in the $Ni_{50}Mn_{50}$ alloy," Phys. Metals Metallogr. **120** (8), 782–789 (2019).