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Soheil R. Nadimi and Dennis L. Weisman

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Non-Exclusionary Input Prices

By

Soheil R. Nadimi
Ph.D. Candidate

and

Dennis L. Weisman
Professor

Department of Economics

Kansas State University

Waters Hall

Manhattan, KS 66506-4001

Correspondence:

weisman@ksu.edu

(785) 532-4588 (V)

(785) 539-7225 (F)

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Abstract

This paper models a vertically-integrated provider that is a monopoly supplier of an input that is essential for downstream production. An input price that is "too high" can lead to inefficient foreclosure and one that is "too low" creates incentives for non-price discrimination. The range of non-exclusionary input prices is circumscribed by the input prices generated on the basis of upper-bound and lower-bound displacement ratios. The admissible range of the ratio of downstream to upstream price-cost margins is increasing in the degree of product differentiation and reduces to a single ratio in the limit as the products become perfectly homogeneous.

1. Introduction

In traditional infrastructure industries, including telecommunications, electric power and natural gas, it is common for an upstream monopolist to supply an input that is essential for downstream production. An input price that is "too high" can give rise to inefficient foreclosure,[1] whereas an input price that is "too low" can induce the vertically-integrated provider (VIP) to engage in sabotage or non-price discrimination.[2] The primary objective of this paper is to examine the role of product differentiation in circumscribing the range of non-exclusionary input prices.

2. Notation and Definitions

There is a single VIP that serves as a monopolist in the upstream input market and a single independent downstream provider. The downstream demand functions for the VIP and the independent provider are given by $Q^V(P^V, P^I)$ and $Q^I(P^I, P^V)$, where P^i, $i = V$ and I denotes the respective downstream prices for the VIP and the independent rival. The downstream outputs of the VIP and the independent downstream provider are imperfect substitutes so that $Q^i_{P^j} > 0$ for $i, j = V$ and I, $i \neq j$, where the subscripts denote partial derivatives. There are no income effects.

[1] Weisman (2002) and Hausman and Tardiff (1995).
[2] Mandy and Sappington (2007) and Sibley and Weisman (1998).

The price and constant marginal cost of the input are denoted by w and c, respectively. The production technology is fixed-coefficient: each unit of downstream output requires one unit of the VIP-supplied input and one unit of a complementary input. The cost of each unit of the complementary input is denoted by s^i, $i = V$ and I. Let $d > 0$ denote the increment by which the VIP raises the per-unit cost of its rival through non-price discrimination. Finally, let $C(d)$ denote the cost of non-price discrimination for the VIP, with $C(0) = 0$, $C'(0) = 0$, $C'(d) > 0$, and $C''(d) > 0 \; \forall d > 0$.

The profit functions for the VIP and the independent rival, which are assumed to satisfy standard regularity conditions that ensure a unique optimum, are given, respectively, by:

$$\Pi^V = Q^I(P^I, P^V)[w - c] + Q^V(P^V, P^I)[P^V - c - s^V] - C(d), \text{ and} \tag{1}$$

$$\Pi^I = Q^I(P^I, P^V)[P^I - w - s^I - d]. \tag{2}$$

Assumption 1. $\left|\frac{\partial Q^i}{\partial P^i}\right| > \frac{\partial Q^i}{\partial P^j}, i, j = V, I, i \neq j.$

Assumption 1 imposes the standard regularity condition that own-price effects dominate cross-price effects (Vives, 1999, p. 157).

Definition 1. (Displacement Ratio)

The displacement ratio is the absolute value of the change in the output of the independent rival associated with a one-unit increase in the output of the VIP (Armstrong et. al., 1996).

In the differentiated products setting under examination, there are two downstream prices and therefore two displacement ratios.

Lemma 1. The upper-bound displacement ratio: $\sigma_u = \left| \frac{\partial Q^I}{\partial P^I} \Big/ \frac{\partial Q^V}{\partial P^I} \right|$.

Lemma 2. The lower-bound displacement ratio: $\sigma_l = \left| \left(\frac{\partial Q^I}{\partial P^V} + \frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial P^V} \right) \Big/ \frac{\partial Q^V}{\partial P^V} \right|$.

Definition 2. (Product Homogeneity)

The degree of product homogeneity is given by $\theta = \frac{\sigma_l}{\sigma_u} \in (0,1)$.

Assumption 2. The displacement ratios, $\sigma_i, i = l, u,$ are constants.

The VIP is generally required by the antitrust or regulatory authority to satisfy a price floor (P-F) constraint. This constraint requires that the downstream price for the VIP be no lower than the incremental cost of providing downstream output plus the net contribution foregone (opportunity cost) in not providing the upstream input. The opportunity cost in this setting is computed on the basis of σ_l because it is the change in the VIP's price (P^V) rather than the rival's price (P^I) that induces the change in the VIP's output. This constraint requires that the lower-bound displacement ratio be no greater than the ratio of downstream to upstream price-cost margins (*r*).

Definition 3. (P-F Constraint)

$$P^V \geq c + s^V + \sigma_l[w - c] \Leftrightarrow w \leq c + \sigma_l^{-1}[P^V - c - s^V] \Leftrightarrow r = \frac{P^V - c - s^V}{w - c} \geq \sigma_l.$$ [3]

An input price that is too low relative to the output price can give rise to non-price discrimination and underscores the need for a complementary, price-ceiling (P-C) constraint. This constraint requires that the upper-bound displacement ratio be no less than the ratio of downstream to upstream price-cost margins.

[3] The input price that results when this last relation holds with equality is a form of the efficient component pricing rule or ECPR.

Definition 4. (P-C Constraint)

$$P^V \le c + s^V + \sigma_u[w-c] \Leftrightarrow w \ge c + \sigma_u^{-1}[P^V - c - s^V] \Leftrightarrow r = \frac{P^V - c - s^V}{w-c} \le \sigma_u.$$

The upper-bound displacement ratio (σ_u) enters the analysis here because it is the change in the rival's price (P^I), triggered by the non-price discrimination and the resultant increase in its costs, that diverts demand from the rival to the VIP.

A binding P-C constraint defines the lower bound input price, $\underline{w}$, and a binding P-F constraint defines the upper bound input price, $\overline{w}$.

Definition 5. (Lower/Upper Bound Input Prices and Margin Ratios)

a) The lower-bound input price (upper-bound margin ratio) is given by

$$\underline{w}(\sigma_u^{-1}) = c + \sigma_u^{-1}[P^V - c - s^V] \Leftrightarrow \overline{r} = \frac{P^V - c - s^V}{w-c} = \sigma_u .$$

b) The upper-bound input price (lower-bound margin ratio) is given by

$$\overline{w}(\sigma_l^{-1}) = c + \sigma_l^{-1}[P^V - c - s^V] \Leftrightarrow \underline{r} = \frac{P^V - c - s^V}{w-c} = \sigma_l .$$

3. Formal Model

The VIP and the independent rival compete in a three-stage, Bertrand-Nash game.[4] In the first stage, the regulator chooses the input pricing rule, $w = c + k[P^V - c - s^V]$, where k is the inverse displacement ratio. In the second stage, the VIP and the independent rival simultaneously choose profit-maximizing prices. In the third stage, the VIP chooses the profit-maximizing level of non-price discrimination (d).

The necessary first-order conditions for the second stage of the game are given by:

[4] Weisman (2013) examines a similar problem in which the VIP is the leader and the rival is the follower.

$$\frac{\partial \Pi^V}{\partial P^V} = \frac{\partial Q^I}{\partial P^V}[w-c] + \frac{\partial w}{\partial P^V} Q^I(P^I, P^V) + \frac{\partial Q^V}{\partial P^V}[P^V - c - s^V] + Q^V(P^V, P^I) = 0; \qquad (3)$$

$$\frac{\partial \Pi^I}{\partial P^I} = \frac{\partial Q^I}{\partial P^I}[P^I - (w+d) - s^I] + Q^I(P^I, P^V) = 0. \qquad (4)$$

Lemma 3. At the Nash equilibrium defined by (3) and (4), $\frac{\partial P^{I*}}{\partial d} > 0$ and $\frac{\partial P^{V*}}{\partial d} > 0$.

The first proposition establishes that the VIP does not engage in non-price discrimination for any input price that is greater than or equal to the lower-bound input price.

Proposition 1. At the Nash equilibrium, $d^* = 0 \ \forall k \geq \sigma_u^{-1} \Rightarrow w \geq \underline{w}$.

The second proposition establishes that the VIP engages in non-price discrimination for any input price that is strictly less than the lower-bound input price.

Proposition 2. At the Nash equilibrium, $d^* > 0 \ \forall k < \sigma_u^{-1} \Rightarrow w < \underline{w}$.

The third proposition establishes that the VIP engages in neither type of market exclusion for input prices that satisfy both the P-F and P-C constraints.

Proposition 3. The VIP does not engage in market exclusion $\forall k \in [\sigma_u^{-1}, \sigma_l^{-1}] \Rightarrow w \in [\underline{w}, \overline{w}]$.

The fourth proposition reveals that the range of admissible margin ratios reduces to a single ratio in the limit as the degree of product differentiation vanishes.

Proposition 4. In the limit as $\theta \to 1, \underline{r} \to \overline{r}$.

Corollary 1. In the limit as $\theta \to 1$, the non-exclusionary margin ratio is unique and satisfies the "equal-margin rule."

The "equal-margin rule" requires that the input price be set so as maintain equality between the VIP's retail and wholesale margins, or $P^V - c - s^V = w - c$.

To facilitate a closed-form solution, we specify a linear demand system of the form

$$Q^V(P^V, P^I) = a^V - b^V P^V + g^V P^I. \tag{5}$$

$$Q^I(P^I, P^V) = a^I - b^I P^I + g^I P^V, \tag{6}$$

where $a^V, a^I, b^V, b^I, g^V, g^I > 0$.

Proposition 5. The Nash-Equilibrium prices are given by

$$P^V(k) = -\frac{2a^V + \left((c+d+s^I - k(s^V + c)\right)(g^V - b^I k) + 2(c+s^V)(b^V - g^I k) + a^I(k + \frac{g^V}{b^I})}{(3g^I + g^V - b^I k)k - 4b^V + \frac{g^I g^V}{b^I}}. \tag{7}$$

and

$$P^I(k) = -\frac{(b^V - kg^I)\left[g^I(c+s^V) - b^I(ck + ks^V - 2s^I) + 2b^I(c+d) + a^I\right] + b^I k(a^I k + a^V) + a^V g^I + a^I b^V}{g^I g^V - b^I\left[4b^V + b^I k^2 - k(3g^I + g^V)\right]}. \tag{8}$$

Proposition 6. The VIP's Nash-Equilibrium profit function is given by

$$\Pi^V(k) = \frac{\begin{pmatrix} a^I(g^V + b^I k) + b^I(c+d+s^I)(g^V - b^I k) + 2a^V b^I \\ + \ (c+s^V)\left[g^I g^V + b^I(g^I k - 2b^V)\right] \end{pmatrix}^2 \times (b^V - g^I k)}{\left(g^I g^V - b^I\left[4b^V + b^I k^2 - k(3g^I + g^V)\right]\right)^2} - C(d). \tag{9}$$

The following example illustrates the manner in which the range of non-exclusionary input prices varies with the degree of product differentiation.

Example 1. The demand functions for the VIP and the independent rival are symmetric in (5) and (6) with $a^V = a^I = 20$, $b^V = b^I = 2$ and $g^V = g^I = g \in (0,2)$. Also, $s^V = s^I = c = 1$. The non-exclusionary input prices and margin ratios are shown below.[5]

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[5] *Lemmas1* and *2* along with (5), (6) and (A7) imply that $\sigma_u = \frac{b^I}{g^V}$ and $\sigma_l = 2b^I \times \left([(g^I)^2 + 8b^I b^V]^{\frac{1}{2}} - g^I \right)^{-1}$.

g	σ_u^{-1}	σ_l^{-1}	θ	$\underline{w}$	$\overline{w}$	$P^V(\underline{w})$	$P^V(\overline{w})$	$\overline{r}$	$\underline{r}$	$\overline{r}-\underline{r}$
0.5	0.25	1.29	0.19	2.42	10.03	7.67	8.98	4.00	0.77	3.23
1.0	0.5	1.19	0.42	5.50	15.13	11.00	13.91	2.00	0.84	1.16
1.2	0.6	1.15	0.52	7.90	18.92	13.50	17.64	1.67	0.87	0.80
1.4	0.7	1.11	0.63	11.97	25.21	17.67	23.87	1.43	0.90	0.53
1.6	0.8	1.07	0.75	20.20	37.75	26.00	36.36	1.25	0.93	0.32
1.8	0.9	1.03	0.87	45.10	75.29	51.00	73.84	1.11	0.97	0.14
1.9	0.95	1.02	0.93	95.05	150.31	101.00	148.84	1.05	0.98	0.07

4. Conclusion

Market exclusion is a concern when input prices are “too high” because it can result in inefficient foreclosure as well as when input prices are “too low” because it can create incentives for sabotage. Upper/lower-bound displacement ratios are used to generate a range of non-exclusionary input prices. The admissible range of the ratio of downstream to upstream margins is increasing in the degree of product differentiation and reduces to a single ratio in the limit as the products become perfectly homogeneous. An important implication for competition policy is that both price-floor and price-ceiling constraints may be necessary to protect against market exclusion in certain settings.

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Appendix

Proof of Lemma 1.

$dQ^V(P^I,P^V)=\dfrac{\partial Q^V}{\partial P^V}dP^V+\dfrac{\partial Q^V}{\partial P^I}dP^I$. Set $dQ^V=1$ and $dP^V=0$, then $dP^I=\left[\dfrac{\partial Q^V}{\partial P^I}\right]^{-1}$.

$dQ^I(P^I,P^V)=\dfrac{\partial Q^I}{\partial P^V}dP^V+\dfrac{\partial Q^I}{\partial P^I}dP^I$. Set $dP^V=0$, and substituting for dP^I yields

$\left|dQ^I\right|=\left|\dfrac{\partial Q^I}{\partial P^I}dP^I\right|=\left|\dfrac{\partial Q^I}{\partial P^I}\Big/\dfrac{\partial Q^V}{\partial P^I}\right|$. □

Proof of Lemma 2.

$dQ^V(P^I,P^V)=\dfrac{\partial Q^V}{\partial P^V}dP^V+\dfrac{\partial Q^V}{\partial P^I}dP^I$. Set $dQ^V=1$ and $dP^I=0$, then $dP^V=\left[\dfrac{\partial Q^V}{\partial P^V}\right]^{-1}$.

$dQ^I(P^I,P^V)=\dfrac{\partial Q^I}{\partial P^V}dP^V+\dfrac{\partial Q^I}{\partial P^I}dP^I$.

Recognizing that $dP^I=\dfrac{\partial P^I}{\partial P^V}dP^V$ and substituting for dP^V yields

$\left|dQ^I\right|=\left|\dfrac{\partial Q^I}{\partial P^V}dP^V+\dfrac{\partial Q^I}{\partial P^I}\dfrac{\partial P^I}{\partial P^V}dP^V\right|=\left|\left(\dfrac{\partial Q^I}{\partial P^V}+\dfrac{\partial Q^I}{\partial P^I}\dfrac{\partial P^I}{\partial P^V}\right)\Big/\dfrac{\partial Q^V}{\partial P^V}\right|$. □

Proof of Lemma 3.

Totally differentiating (3) and (4) with respect to *d* yields the linear system $A\cdot X=B$:

$$\overbrace{\begin{bmatrix}\overbrace{\dfrac{\partial^2\Pi^V}{\partial P^{V2}}}^{-} & \overbrace{\dfrac{\partial^2\Pi^V}{\partial P^V\partial P^I}}^{+}\\ \underbrace{\dfrac{\partial^2\Pi^I}{\partial P^I\partial P^V}}_{+} & \underbrace{\dfrac{\partial^2\Pi^I}{\partial P^{I2}}}_{-}\end{bmatrix}}^{A}\overbrace{\begin{bmatrix}\dfrac{\partial P^V}{\partial d}\\ \dfrac{\partial P^I}{\partial d}\end{bmatrix}}^{X}=\overbrace{\begin{bmatrix}0\\ \dfrac{\partial Q^I}{\partial P^I}\end{bmatrix}}^{B}. \qquad (A1)$$

Sufficient second-order conditions for a maximum require that

$$\frac{\partial^2\Pi^V}{\partial P^{V2}}<0,\ \frac{\partial^2\Pi^I}{\partial P^{I2}}<0 \text{ and } |A|=\frac{\partial^2\Pi^V}{\partial P^{V2}}\frac{\partial^2\Pi^I}{\partial P^{I2}}-\frac{\partial^2\Pi^V}{\partial P^V\partial P^I}\frac{\partial^2\Pi^I}{\partial P^I\partial P^V}>0.$$

Appealing to Cramer's rule yields

$$\frac{\partial P^I}{\partial d}=\frac{1}{|A|}\begin{vmatrix}\frac{\partial^2\Pi^V}{\partial P^{V2}} & 0\\ \frac{\partial^2\Pi^I}{\partial P^I\partial P^V} & \frac{\partial Q^I}{\partial P^I}\end{vmatrix}=\frac{\frac{\partial^2\Pi^V}{\partial P^{V2}}\frac{\partial Q^I}{\partial P^I}}{|A|}>0,\ \frac{\partial P^V}{\partial d}=\frac{1}{|A|}\begin{vmatrix}0 & \frac{\partial^2\Pi^V}{\partial P^V\partial P^I}\\ \frac{\partial Q^I}{\partial P^I} & \frac{\partial^2\Pi^I}{\partial P^{I2}}\end{vmatrix}=\frac{-\frac{\partial^2\Pi^V}{\partial P^V\partial P^I}\frac{\partial Q^I}{\partial P^I}}{|A|}>0.$$

since $\frac{\partial^2\Pi^V}{\partial P^V\partial P^I}>0$ when prices are strategic complements. □

Proof of Proposition 1.

From (1), the necessary first-order condition for *d* is given by

$$\Pi_d^V:\left[\frac{\partial Q^I}{\partial P^I}\frac{\partial P^I}{\partial d}+\frac{\partial Q^I}{\partial P^V}\frac{\partial P^V}{\partial d}\right](w-c)+Q^I\frac{\partial w}{\partial P^V}\frac{\partial P^V}{\partial d}+\left[\frac{\partial Q^V}{\partial P^V}\frac{\partial P^V}{\partial d}+\frac{\partial Q^V}{\partial P^I}\frac{\partial P^I}{\partial d}\right][P^V-c-s^V]$$
$$+Q^V\frac{\partial P^V}{\partial d}-C'(d)\le 0;\ \text{and}\ d\left(\Pi_d^V\right)=0. \tag{A2}$$

Let $k=\Delta+\sigma_u^{-1}$, where $\Delta\ge 0$. Substituting for $\sigma_u^{-1}=\left|\frac{\partial Q^V}{\partial P^I}\Big/\frac{\partial Q^I}{\partial P^I}\right|=-\left(\frac{\partial Q^V}{\partial P^I}\Big/\frac{\partial Q^I}{\partial P^I}\right)$

along with $\underline{w}=c+(\Delta+\sigma_u^{-1})[P^V-c-s^V]$ and (A2) yields

$$\left[\frac{\partial Q^I}{\partial P^I}\frac{\partial P^I}{\partial d}+\frac{\partial Q^I}{\partial P^V}\frac{\partial P^V}{\partial d}\right](\Delta+\sigma_u^{-1})[P^V-c-s^V]+Q^I(\Delta+\sigma_u^{-1})\frac{\partial P^V}{\partial d}+$$
$$\left[\frac{\partial Q^V}{\partial P^V}\frac{\partial P^V}{\partial d}+\frac{\partial Q^V}{\partial P^I}\frac{\partial P^I}{\partial d}\right][P^V-c-s^V]+Q^V\frac{\partial P^V}{\partial d}-C'(d)\le 0. \tag{A3}$$

Rewrite (A3) in the following form:

$$\left[\Delta\left(\frac{\partial Q^I}{\partial P^I}\frac{\partial P^I}{\partial d}\right)+\left(\frac{\partial Q^I}{\partial P^V}\frac{\partial P^V}{\partial d}\right)\left(\Delta-\left(\frac{\partial Q^V}{\partial P^I}\Big/\frac{\partial Q^I}{\partial P^I}\right)\right)\right][P^V-c-s^V]+Q^I\left(\Delta-\left(\frac{\partial Q^V}{\partial P^I}\Big/\frac{\partial Q^I}{\partial P^I}\right)\right)\frac{\partial P^V}{\partial d}$$
$$\left(\frac{\partial Q^V}{\partial P^V}\frac{\partial P^V}{\partial d}\right)[P^V-c-s^V]+Q^V\frac{\partial P^V}{\partial d}-C'(d)\leq 0. \tag{A4}$$

At an optimum for (1), $\Pi^V_{P^V}=0$ or

$$\frac{\partial Q^I}{\partial P^V}\left(\Delta-\left(\frac{\partial Q^V}{\partial P^I}\Big/\frac{\partial Q^I}{\partial P^I}\right)\right)[P^V-c-s^V]+Q^I\left(\Delta-\left(\frac{\partial Q^V}{\partial P^I}\Big/\frac{\partial Q^I}{\partial P^I}\right)\right)$$
$$+\frac{\partial Q^V}{\partial P^V}[P^V-c-s^V]+Q^V=0. \tag{A5}$$

Substituting (A5) into (A4) and appealing to *Lemma 3* yields

$$\Delta\underbrace{\left(\frac{\partial Q^I}{\partial P^I}\frac{\partial P^I}{\partial d}\right)}_{-}\underbrace{[P^V-c-s^V]}_{+}-C'(d)\leq 0. \tag{A6}$$

Since $\Delta\geq 0$, (A6) implies that $d^*=0$ by complementary slackness from (A2). □

Proof of Proposition 2.

From (A6), $\Delta<0$ implies that $-C'(d)<0\Rightarrow d^*>0$. □

Proof of Proposition 3.

By Proposition 1, $d^*=0\ \forall k\geq\sigma_u^{-1}\Rightarrow w\geq\underline{w}$. By Proposition 2, $d^*>0\ \forall k<\sigma_u^{-1}\Rightarrow w<\underline{w}$.

If $k>\sigma_l^{-1}$ then $w>\overline{w}$ and the P-F constraint is violated. The result follows. □

Proof of Proposition 4.

$\theta\to 1\Rightarrow\sigma_l\to\sigma_u\approx 1$ and $\underline{r}\to\overline{r}$. □

Proof of Corollary 1.

$\theta\to 1\Rightarrow\sigma_l\to\sigma_u\approx 1$. Satisfaction of the P-C and P-F constraints requires that

$\frac{P^V - c - s^V}{w-c} \le 1$ and $\frac{P^V - c - s^V}{w-c} \ge 1 \Rightarrow \frac{P^V - c - s^V}{w-c} = 1 \Rightarrow P^V - c - s^V = w - c$, the "equal-margin rule." □

Proof of Proposition 5.

Substituting for w in (4), appealing to (6) and solving for P^I yields

$$P^I = \frac{a^I + b^I\left(c + d + s^I + k[P^V - c - s^V]\right) + P^V g^I}{2b^I}. \tag{A7}$$

Substituting for w and P^I along with solving (3) and (4) simultaneously for the linear system in (5) and (6) yields the Nash-equilibrium prices in (7) and (8). □

Proof of Proposition 6.

Substituting (5) - (8) into (1) yields the VIP's reduced-form profit function in (9). □