SYSTEMATIC NETWORK CODING FOR LOSSY LINE NETWORKS

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Abstract

This dissertation focuses on packet-level systematic network coding (SNC) schemes to provide resilience to packet losses in lossy line networks. In theory, network coding is known to improve throughput and reliability of lossy networks. However, the translation of the network coding theory into efficient practical network coding solutions involves some critical challenges. This dissertation addresses those challenges and investigates on network coding solutions that can be utilized in practice for different instances of the lossy line networks.

The main objectives of this dissertation are: 1) to develop a matricial model that allows analytical treatment of network coding for lossy networks, 2) semi-analytical investigation of achievable throughput and reliability for line networks, a simple yet useful conceptual network model, 3) to develop practical network coding schemes for line networks that significantly outperform state-of-the-art purely forward erasure correction (FEC)-based schemes and 4) to be in line with Internet Research Task Force (IRTF) efforts and eventually contribute. To address these objectives, this dissertation provides an in-depth investigation of systematic network coding based schemes for different instances of line networks, by starting from simple one-hop networks, moving on to two-hop networks and finally generalizing the analysis to general line networks. The contributions of this thesis, such that the objectives are met are as follows.

First, we investigate the application of SNC in one-hop lossy networks. We develop a matricial model for the case without re-encoding in the network. This allows us to compare maximum distance separable (MDS) codes with SNC when used as FEC only. We derive the minimum distance of SNC and show that SNC can provide as closed as wished to MDS reliability as the field sizes is allowed to grow. We simulate practical applications at application layer of the protocol stack with two concrete results. First, it is shown that by using progressive decoding SNC achieves smaller delay than the MDS code and second, an optimal bandwidth distribution for network coding rate is obtained while applying SNC in band-limited networks.

Second, we investigate the application of SNC in two-hop lossy networks. We extend the matricial model for the networks with one intermediate node. Using the semi-analytical approach, we study and characterize the reliability and achievable rate as a function of network coding rate and capacity of the network. We simulate practical applications at

link layer of Digital Video Broadcasting via Satellite-Second Generation (DVB-S2). We propose an architectural and encapsulation framework so that network coding can be used over the state-of-the-art protocols at link layer of DVB-S2. The application of network coding for satellite communication is relevant in this case as one intermediate node (which can be a gateway or other) fits in the satellite scenario.

Third, we extend the matricial model for the network with several intermediate nodes. This allows us to understand the mathematical framework of mapping communication entities to mathematical entities at different intermediate nodes of the network. We analyze semi-analytically reliability, achievable rates, delay and complexity of network coding schemes and prove that our results are inline with information theoretical results. Finally, we develop a smart re-encoding network coding scheme which includes packet scheduling at the intermediate nodes. Our proposal is shown to provide smaller delay and smaller complexity than state-of-the-art network coding schemes.

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Dedicated in memory of my beloved grandmother Smt. Ravikanta Ravat

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Motivation and Objectives

1.1.1 Motivation

In general, performance of wireless networks is often limited by packet losses due to fading at the physical channel, shadowing, interference, noise etc. Therefore, a major challenge is to achieve efficient and reliable data transmission over wireless networks with unreliable physical links. Traditionally, schemes based on the feedback and retransmission mechanisms (for example Automatic Repeat Request (ARQ)) are used. These mechanisms rely on the philosophy of retransmission of packets in the event of loss that is conveyed through feedback. In general, these mechanisms get complicated and less efficient for various scenarios like: (i) communication networks with long round-trip times like Satellite networks, (ii) multicasting and broadcasting of delay-sensitive applications like audio/video streaming, (iii) unicasting in the network with several nodes, etc. In such scenarios where the feedback based mechanisms are not efficient, FEC codes are used to provide the reliable data delivery. The main philosophy of these codes is to send redundant packets such that the original data packets can be recovered in spite of erasures in the networks with the help of the redundant packets. FEC schemes such as MDS codes like Reed-Solomon (RS) codes [1], Fountain codes like Luby Transform (LT) codes [2] and Raptor codes [3], etc are used as packet-level coding schemes to combat packet losses.

Currently most of the communication networks use FEC schemes in end-to-end fashion where only the source and the sink are involved in the coding process. Intermediate nodes in the network are used only for routing the packets. However, routing at the intermediate nodes is not the optimal solution. It has been established recently that by employing coding at the intermediate nodes, higher transmission rates and higher reliability are achievable than by simply routing. This approach is referred to as network coding [4]. Network coding extends traditional network operations from routing and store-and-forward to more powerful operations that allow for coding information at intermediate nodes. The coding coefficients, which are used for encoding, are sent along with the packets as side information [5] and are used at the sink to decode the original data packets. Network coding provides the opportunity to enhance both the reliability and transmission rates in the existing wireless networks.

The translation of the theory of network coding into efficient practical network coding solutions involves some vital challenges. This thesis focuses on real-time or near real-time transmission (as opposite to large file transfers). Mainly two primary challenges in practical network coding solutions can be identified:

- Practical network coding solutions should provide overall small delay, small complexity and small overhead of sending coding coefficients. These are the three main factors influencing near real-time transmission in the communication systems. Although the state-of-the-art network coding solutions can provide higher reliability and higher transmission rates than routing but in general they involved high delay, high complexity and high overhead. Network coding strategies should be designed by taking into account these factors in order to provide reliable, robust and resilient solution.

- Practical network coding solutions should take into consideration the underlying network protocol stack that forms the backbone of the communication system. The current state-of-the-art network coding solutions often do not take into consideration the constraints imposed by different protocols at different layers. Wireless network protocol architecture primarily consists of upper layers (layers independent of air interface radio access technology) and lower layers (layers dependent of air interface radio access technology). Network coding strategies should be designed in the upper layers by taking into account the considerations of the application's developer who has an access to the data flowing in these layers and in the lower layers by taking into account the considerations of network operators who have the access to data flowing in the lower layers.

Inspired by the need for the practical network coding solutions, in this thesis we make a series of contributions towards current state-of-the-art network coding techniques in wireless networks. In particular, our focus is on the wireless line network which is a simple topology model, yet commonly found as logical abstractions of realistic network. Our goal is to propose network coding solutions that can tackle the aforementioned challenges and are applicable in practical communication systems. Our work proposes systematic network coding solutions and characterizes the achievable rates, reliability, delay, complexity and overhead in multimedia transmission over the lossy line networks.

1.1.2 Objectives

In this dissertation, we have the following objectives based on the challenges described above.

- **Objective 1: Develop a matricial model that allows analytical treatment of network coding for lossy networks. The model should be applicable at any layer of the protocol stack.**
  - Our first objective is to understand and develop the mathematical structure behind the network coding schemes for lossy networks. To this aim, we set out ourselves to develop a generic matricial model that will allow us to study different network coding schemes on a common mathematical framework by mapping communication entities to mathematical entities. Moreover, this model should provide the flexibility of application of different network coding schemes across different layers of the protocol stack.
- **Objective 2: Semi-analytical investigation of achievable throughput and reliability for line networks, a simple yet useful conceptual network model.**
  - Our next objective is to provide an in-depth analysis of achievable throughput and reliability of network coding schemes for line networks. This should be achieved by using the matricial model developed in the previous objective and by utilizing semi-analytical methods thereby conducting assessment and analysis based on theoretical as well as simulation approach. The results we obtain with our developed mathematical framework and semi-analytical methods will be properly compared with information-theoretical bounds available in the literature. The focus of study is line network topology, a simple yet useful conceptual network model.
- **Objective 3: Develop practical network coding schemes for line networks that significantly outperform state-of-the-art purely FEC-based schemes.**
  - After the study of network coding matricial model, semi-analytical investigation of its achievable throughput and benchmarking against information-theoretical results, our next objective is to develop practical network coding schemes that are able to provide performance improvements over state-of-the-art FEC based schemes. These schemes should take into account the specific constraints of the practical scenarios of interest (which we also identify (see 1.1.3)) in order to provide concrete solutions for the efficient use of network coding in current network instantiations.

- **Objective 4: To be in line with Internet Research Task Force (IRTF) efforts.**
  - The research conducted in this thesis is not undertaken in isolation but coherence with IRTF efforts in network coding. As a consequence, some of the contributions of this thesis will be presented at IRTF Network Coding reserach group.

1.1.3 Requirements

Our thesis pursues a theoretically grounded work on network coding that ultimately should lead to concrete and efficient network coding schemes. For this reason, technical requirements have been identified to narrow down the search of algorithmic feasibility while matching them to realistic applications. To this aim, a number of requirements have been set as follows:

- **To exploit systematic random coding:** The systematic random linear coding uses two phases namely systematic phase and non-systematic phase. The source first sends original (systematic) packets during the systematic phase followed by randomly coded packets during the non-systematic phase. The use of systematic coding and random coding eventually provides us algorithmic advantages for encoding and decoding such that complexity and delay of the network coding schemes is minimized.

- **Coherent transmission but no feedback**: We do not consider any feedback in the network to avoid the complexity and inefficiency of feedback-based mechanisms. However, we do allow the receiver to have channel side information between the receiver and transmitter. The knowledge of channel side information is utilized by the receiver to recover the lost packets and to increase the reliability in the lossy networks.

- **Focus on single-transmitter and single-receiver**: Our overall focus in this dissertation is on the line networks with single-transmitter and single-receiver. We believe that once this simplest case is completely characterized, other more complex ones can be tackled.

- **Focus on real-time (not in file transfer)**: We focus on real-time or near real-time applications (which is not the case of file transfers). This impacts the design of network coding solutions which should not only provide higher throughput and reliability but also have a small delay, small complexity and small overhead. These three are the essential factors influencing near real-time packet transmission.

These requirements generate interesting tradeoffs and additional considerations. These tradeoffs will be systematically tackled and properly discussed in the thesis.

1.2 Contributions of the dissertation

We have made an in-depth investigation of systematic network coding schemes in line networks, by starting from a simple one-hop networks, moving on to two-hop networks and finally generalizing the analysis to lossy line networks. This allowed us to develop our proposals and draw conclusions applicable to complex networks while ensuring that our analysis is validated in all types of scenarios. The logic undertaken towards the above three objectives (while respecting our self-imposed requirements) is reflected in the four chapters that form the main contributions of the thesis.

- Chapter 3 focusses on systematic network coding for one-hop lossy network. The main contributions from this chapter are as follows.
  - Towards the first objective, we develop a matricial model for the case without re-encoding in the network. This allows us to compare MDS codes with SNC when used as FEC only.
  - Towards the second objective, we derive the minimum distance of SNC and show that SNC can provide reliability very close to the MDS code.
  - Towards the third objective, we simulate practical applications at application layer of the protocol stack. First, it is shown that by using progressive decoding SNC achieves smaller delay than the MDS codes and second, an optimal bandwidth distribution for network coding rate is obtained while applying SNC at application layer in band-limited networks.
- Chapter 4 focusses on systematic network coding for two-hop lossy network. The main contributions from this chapter are as follows.
  - Towards the first objective, we extend the matricial model for the networks with one intermediate node. This allows us to understand the mathematical structure behind the systematic network coding when used at the intermediate node as well.
  - Towards the second objective, we study the reliability and achievable rate in the line networks with one intermediate node. Using the semi-analytical approach, we study and characterize the reliability and achievable rate as a function of network coding rate and capacity of the network.
  - Towards the third objective, we simulate a practical application at link layer of Digital Video Broadcasting via Satellite-Second Generation (DVB-S2). We propose an architectural and encapsulation framework so that network coding can be used over the state-of-the- art protocols at link layer of DVB-S2. The application of network coding for the satellite communication is relevant in this

case as one intermediate node (which can be a gateway or other) fits in the satellite scenario.

- Chapter 5 focusses on systematic network coding for general line networks. The main contributions from this chapter are as follows.

  - Towards the first objective, we extend the matricial model for the network with several intermediate nodes. This allows us to understand the mathematical framework of mapping communication entities to mathematical entities at different intermediate nodes of the network.
  - Towards the second objective, we analyze reliability, achievable rates, delay and complexity of network coding schemes. We prove that our results are inline with information theoretical results.
  - Towards the third objective, we develop a smart re-encoding network coding scheme which includes packet scheduling at the intermediate nodes. Our proposal is shown to provide smaller delay and smaller complexity than state-of-the-art network coding schemes.

- Finally, towards the objective 4, chapter 6 describes the recent network coding contributions to IRTF. We discuss a network coding architecture and several use cases for future deployment of practical network coding solutions for better internet and its evolution.

The work leading to this thesis has been presented in different scientific publications. Following are the list of contributions.

Journals

1. **P. Saxena** and M. A. Vázquez-Castro, “DARE: DoF-Aided Random Encoding for Network Coding over Lossy Line Networks” under review in IEEE wireless communications letters, 2014.
2. **P. Saxena** and M. A. Vázquez-Castro, “Link layer random network coding for DVB-S2X/RCS” under review in IEEE wireless communications letters, 2014.
3. M. A. Pimentel-Niño, **P. Saxena** and M. A. Vázquez-Castro, “Multimedia delivery for situation awareness provision over satellite” submitted in the special issue of Hindawi on recent advances in streaming multimedia content delivery, October 2014.

Book Chapter

1. **P. Saxena** and M. A. Vázquez-Castro, "Network coding advantage over MDS codes for multimedia transmission via erasure satellite channels", Lecture notes of the institute for computer sciences, social informatics and telecommunications engineering, (Springer 2013), Volume 123, 2013, pp 199-210, ISBN: 978-3-319-02761-6.

Conferences

1. **P. Saxena** and M. A. Vázquez-Castro, "Network coding advantage over MDS codes for multimedia transmission via erasure satellite channels", The 5th International conference on personal satellite services (PSATS 2013), Tolouse (France), June 2013.
2. **P. Saxena** and M. A. Vázquez-Castro, "RNC advantage over MDS codes for adaptive multimedia communications", in International conference on random network codes and design over GF(q), Ghent (Belgium), September 2013.
3. M. A. Pimentel-Niño, **P. Saxena** and M. A. Vázquez-Castro, "QoE driven adaptive video with overlapping network coding for best effort erasure satellite links" accepted in 31st AIAA international communications satellite systems conference, Florence (Italy), October 2013.
4. **P. Saxena** and M. A. Vázquez-Castro, "Random Linear Network Coding over Satellite" in Conference on algebraic approaches to storage and network coding, Barcelona, Feb 2014.

Contributions to IRTF

1. **P. Saxena** and M. A. Vázquez-Castro, "Network coding contributions to IRTF", March 2015.

1.3 Outline of the dissertation

The outline of this dissertation is as follows. In chapter 1, we introduce the overall motivation and objectives of this doctoral thesis. Chapter 2 covers the state-of-the-art on network coding for the understanding of the contributions made in the rest of the work. In chapter 3, we investigate network coding schemes for one-hop lossy networks. Chapter 4 focusses on networks with one intermediate node and chapter 5 focusses on networks with several intermediate nodes. In chapter 6, the recent network coding contributions to IRTF is presented and chapter 7 concludes this thesis.

Chapter 2

Preliminaries on network coding

In this chapter, we will introduce the preliminaries on network coding to understand the contributions made in the rest of the thesis. Section 2.1 discusses the background and seminal work on network coding. Section 2.2 presents the existing network coding related work in line networks. Section 2.3 discusses the state-of-the-art work on practical applications of network coding and section 2.4 concludes this chapter.

2.1 Background on network coding

Network coding extends traditional network operations from routing and store-and-forward to more powerful operations that allow for coding information at intermediate nodes. Network coding was introduced in [4] in the seminal work, for lossless networks, which shows that the min-cut capacity of the network can be achieved by allowing coding at the intermediate nodes. Later in [6], it is shown that the linear network coding, in which encoding and decoding are based on linear operations on the data packets, is sufficient to achieve the capacity of the network. Further, it is also shown that random network coding (RNC) [7], where information packets transmitted in the network are random linear combinations of the original data packets, is asymptotically capacity achieving if the finite field from which the coding coefficients of the linear combinations are chosen is sufficiently large. In order to achieve the min-cut capacity, the choice of finite field size is critical. It is bounded by the number of receivers [8], [9], [10]. The use of higher finite field size affects the computational complexity of network coding and makes it computationally expensive in the network with several receivers. The initial work on network coding was mainly focussed on multicast transmission in lossless networks. Later, there have been several efforts to understand the performance of network coding in lossy networks.

2.2 Network coding for lossy line networks

In the case of lossy networks, [11] explored the theoretical benefits of random linear coding based schemes in lossy line networks. The authors show that if we allow intermediate nodes to transmit random linear combinations of the incoming packets over a finite field $GF(q)$, then the transmission rate approaches to the min-cut capacity as q goes to infinity and block length goes to infinity. This analyis is shown to be valid for both unicat and multicast transmissions. The asymptotic anaysis is done in [11] by considering q goes to infinity and block length goes to infinity. [12] and [13] then used non-asymptotic approach of studying coding schemes in line networks. [12] proposed coding schemes that can achieve min-cut capacity when using a constant field size and [13] explored coding schemes in line networks when intermediate nodes can process blocks of finite size. In order to acheive the constant end-to-end rate, [13] provides the relationship between the block length and the size of the network to achieve constant end-to-end rate.

Several other interesting work has been done to investigate network coding theory information theoretically. The main conclusion of these works is the frequent use of random linear coding based schemes. RNC provides several benefits theoretically. The main philosophy of using RNC as a capacity-achieving network coding scheme in the wireless networks is that it allows the practical application of network coding in the distributed manner and for the networks whose topologies are not known.

Although RNC is a capacity achieving code and provides several benefits over lossy line networks, it does not utilize efficiently the computational resources. It has three main limitations: high delay, high complexity and high overhead. The high decoding complexity is due to the use of Gaussian elimination (GE) algorithm to solve a system of linear equations using densely filled decoding matrix with non-zero elements from GF(q). The high delay is due to the time which receiver waits for the arrival of the complete block in order to start the decoding process. The high overhead is due to the coding coefficients which are attached as a side information with the coded packets. These three limitations impose constraints on RNC to be used as a practical network coding solution for multimedia transmission over lossy line networks. In order to recover from these limitations, there are mainly two directions of work in the literature. The first direction focuses on network coding schemes for transmission of large files in the lossy networks and the second direction focuses on the network coding schemes for real or non-real time streaming over the lossy networks. In the next section, we will discuss the related work in both these directions.

2.3 Practical application of network coding

2.3.1 Network coding for file transmission

The first direction focuses on the efficient transmission of large files using chunks (generations or classes) in order to reduce the encoding and decoding complexity. A chunk is a subset of the original packets. By dividing a large file into smaller disjoint chunks [5], and performing coding and decoding operations only on the packets from the same chunk, the encoding and decoding complexity can be substantially reduced. When the feedback from the sink is allowed, these chunks can be scheduled sequentially from the source. However, feedback from the sink is required to acknowledge the receiving of each chunk. In order to reduce heavy feedbacks, random scheduling of chunks is proposed in which the node chooses a chunk at random and transmits a coded packets for that chunk [14]. While the random scheduling of disjoint chunks is shown to have a good performance asymptotically, the performance quickly deteriorates for practical chunk sizes, as some chunks may take too long to decode. The use of overlapping chunks is shown to improve the throughput for practical chunk sizes [15], [16] where already decoded chunks can be used in the decoding of other chunks. Further, low complexity batched sparse (BATS) codes are proposed in [17], [18] which extends the idea of fountain codes to the realm of networks and utilizes both network coding and the properties of overlapping chunks by using belief propagation decoding where packets from the already decoded batches can help to decode the packets from the other batches. As described in this section, several work have been proposed in order to have practical network coding solution for transmitting large files over the lossy wireless networks.

2.3.2 Network coding for near real-time streaming

The second direction focuses on the efficient transmission of streaming media using the network coding. Specially, SNC has been investigated recently as a powerful practical network coding solution for the efficient multimedia streaming over the lossy line networks. If the systematic coding is used, the sink can receive both the uncoded and coded packets. There are three main benefits of using the systematic coding. Firstly, by receiving the systematic packets, the sink does not have to wait for the complete block to start recovering the packets. The packets, which are received in their original form, are recovered instantly which decreases the overall per-packet delay. Secondly, the sink has to decode only the packets which has not arrived in their original form. Hence, some rows of the decoding matrix is singleton and contain only one non-zero element. In this case, the decoding is done over a sparse decoding matrix which contains several zero elements, which reduces the decoding complexity significantly. Finally, the systematic packets do not have overhead of coding coefficients, as these are not the encoded packets. This reduces the overall overhead significantly. Therefore, SNC can overcome all the limitations imposed by RNC for multimedia delivery in the lossy line networks. In this section, we will discuss SNC related work in one-hop, two-hop and multiple-hop line networks.

In the case of one-hop lossy networks, [19] and [20] have explored the benefits of SNC on mobile and laptop devices. It is shown in their work that SNC can achieve higher transmission rates than RNC for practical block sizes and packet sizes. Further, in [21] it is also shown that SNC, which uses binary field for encoding, can achieve transmission rates similar to RNC which uses a higher field size for encoding. By using the binary field size, the encoding and decoding complexity in SNC can be further reduced as compared to that in RNC. However, in all these papers, the comprehensive analysis of SNC in multimedia delivery and characterization of various performance metrics (reliability, achievable rates, complexity, delay and overhead) is missing. Moreover, the use of SNC as a practical network coding scheme requires the analysis of its application in the protocol stack of the communication system. This has not been addressed in the state-of-the-art work in one-hop lossy networks. Therefore, it is required to address the objectives which are described in the Chapter 1 to fulfill the need of practical network coding solutions for the one-hop lossy networks.

In the case of two-hop lossy networks, [22] and [23] show that SNC is a capacity achieving code when block length goes to infinity. In this work, SNC is also analysed using blocks of finite lengths and it is shown that SNC can achieve higher transmission rates than RNC in the two-hop lossy networks. However, in the two-hop lossy networks as well, a comprehensive and in-depth analysis of SNC as a practical network coding solution and characterization of SNC with different performance metrics is missing. In order to investigate its practical usage, it is also needed to analyse the application of SNC in the protocol stack of the communication system. Therefore, it is required to address the objectives of the thesis as described in the Chapter 1 to fulfill the need of practical network coding solutions for the two-hop lossy networks.

In the case of multiple-hop line networks, [24] explored the benefits of SNC as compared to RNC. It is shown in their results that as the number of node increases, the advantage of SNC as compared to RNC shrinks and SNC behaves similar to RNC. When there are several lossy links, many systematic packets are lost during the systematic phase. The sink receives fewer systematic packets and therefore all the advantages of SNC over RNC diminish. In order to recover this limitation, it is required to investigate new network coding schemes which can provide the benefits of low-delay, low-complexity and low-overhead and in addition can achieve higher rates and reliability than routing. Therefore, the primary objectives of the thesis should be addressed for the multiple-hop lossy line networks in order to have the practical network coding solutions.

In chapter 5, we have proposed SS-SNC as a practical network coding solution for multimedia delivery providing several benefits over SNC. Our proposal is based on the systematic concatenation of outer and inner codes with three fold objectives: (i) channel coding with outer code to counter packet losses, (ii) network coding with inner code to achieve higher transmission rates and (iii) systematic coding in order to have a low-complexity and low-delay network coding solution. Some recent work on the concatenation of outer and inner codes includes BATS codes [17], FUN codes [25] and Fulcrum network codes [26]. BATS codes and FUN codes are based on dividing the source blocks into batches. Our work is different from [17] and [25] as we focus on low-delay and low-complexity solution for real time multimedia streaming like video streaming which usually have small block sizes. Hence dividing this small block into batches may add to unwanted complexity and delay. Fulcrum network codes are designed to provide multimedia delivery to heterogeneous receivers with different processing capabilities with the coding design based on the concatenation of two separate finite fields. SS-SNC is different from Fulcrum network codes as it does not add design complexity nor sacrifices achievable rates w.r.t routing while minimizes delay, complexity and overhead which are the key ingredients for efficient multimedia streaming.

2.4 Conclusions

In this chapter, we have introduced the context and preliminaries on network coding. The aim is to introduce to readers the state-of-the-art in network coding so that the contributions in the rest of the thesis is sufficiently understood and justified. We have discussed the seminal work on network coding and the current literature that is focussed upon network coding for file transmission and near real time streaming.

Chapter 3

Systematic network coding for one-hop lossy networks

3.1 Contributions and Outline

In this chapter, we focus on systematic network coding solutions for the one-hop lossy networks. In general, from the coding point of view any communication network is a one-hop lossy network when only the source and the sink are involved in the coding process and intermediate nodes only forward the packets. Thus, it is very common to encounter one-hop lossy networks in real-time communication systems.

RS codes are the most extensively used state-of-the-art codes for one-hop lossy networks. RS codes are the MDS codes [1] and they are optimal in terms of erasure correction performance. However, a construction of the RS code is based on a finite algebraic arithmetic, therefore the sink has to wait for all the packets to start the decoding process. Moreover, the extension of RS codes for re-encoding at several intermediate nodes requires decoding and encoding at every node. This would result into additional delay and complexity.

In order to counter these limitations of RS coding, random linear coding [7] based network coding schemes have recently attracted the attention of the research community. These schemes have two advantages over RS codes. First, their construction is based on the random structure therefore the sink does not have to wait for all the packets to start the decoding process. In this case, the sink can follow progressive decoding and can start decoding as soon as it receives the first packet. Second, the extension of random linear coding based schemes to lossy networks with several intermediate nodes do not require decoding and encoding at every node. Thanks to their random structure, re-encoding could be done at the intermediate nodes without decoding the complete block. In this chapter, we will present an in-depth investigation on systematic network coding schemes in the one-hop lossy networks.

3.1.1 Contributions of the chapter

We present the contributions of this chapter that meets the overall objectives of the dissertation.

- **Objective 1: Develop a matricial model that allows analytical treatment of network coding for lossy networks. The model should be applicable at any layer of the protocol stack.**
  - We develop a matricial model for the case without re-encoding in the network. This allows us to compare MDS codes with SNC when used as FEC only.
- **Objective 2: Semi-analytical investigation of achievable throughput and reliability for line networks, a simple yet useful conceptual network model.**
  - We derive the minimum distance of SNC and show that SNC can provide reliability very close to the MDS code. Our simulation results show that SNC guarantees 100% reliability when the code rate is smaller than the capacity whereas the 100% reliability is never guaranteed by simply routing.
- **Objective 3: Develop practical network coding schemes for line networks that significantly outperform state-of-the-art purely FEC-based schemes.**
  - We simulate practical applications at application layer of the protocol stack. First, it is shown that by using progressive decoding SNC outperforms RS codes. Second, an optimal bandwidth distribution for network coding rate is obtained while applying SNC at application layer in band-limited networks. Our simulation results show that by using the proposed network coding rate optimization solution up to 80% gain in code rate is achievable as compared to the case when the network coding rate is not optimized.

3.1.2 Outline of the chapter

This chapter is organized as follows. In Section 3.2, we will present the matricial system model for the one-hop lossy networks. In Section 3.3, we will study the systematic network coding scheme and in Section 3.4, we will have semi-analytical investigation on the reliability in one-hop lossy networks. In Section 3.5, the application of network coding schemes in the upper layers of the protocol stack is studied. We present our simulation results in Section 3.6 and conclude this chapter in Section 3.7.

3.2 Matricial model

Let us consider that a source node has K data packets to send to a sink node. Each packet is a column vector of length M over a finite field $\mathbb{F}_q$. The set of the data packets is denoted by the matrix,

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & . & . & . & \mathbf{s}_K \end{bmatrix}$$

where $\mathbf{s}_t$ is the t^{th} data packet. The source and the sink are connected with one intermediate link. This link is modeled as a delay-free memoryless erasure channel. A packet sent across this link is either erased with the probability of ε or received without error. The capacity of this link and the capacity of this one-hop network is therefore $1-\varepsilon$.

Due to our requirement of low-delay, we assume that there is no feedback from the sink in the network. We also consider that packet transmissions occur at discrete time slots so that the source node can transmit one packet per time slot. In the next section, we will discuss different coding schemes for transmitting the data packets from the source to the sink over the one-hop lossy networks. We will assume that all the coding schemes run for a total of N time slots and the source transmits a packet in each time slot $t = 1, 2, ..., N$.

The complete encoding and decoding operations in the network can be modeled with a linear operator channel, using which an output unit at the sink can be expressed as a linear transformation of the input unit at the source. Let $\mathbf{Y} \in \mathbb{F}_q^{M\times N}$ be the output unit with N columns representing N received packets in N time slots. If the sink does not receive any packet in time slot t then the t^{th} column of $\mathbf{Y}$ should be considered as a zero column. We have,

$$\mathbf{Y} = \mathbf{XH} = \mathbf{SGH} \tag{3.1}$$

where $\mathbf{H} \in \mathbb{F}_q^{N\times N}$ is the transfer matrix for the one-hop lossy network, $\mathbf{G} \in \mathbb{F}_q^{K\times N}$ is the generator matrix and $\mathbf{X} = \mathbf{SG}$ is a generation of N coded packets transmitted from the source. The outer code is defined by $\mathbf{G}$ with code rate $\rho = \frac{K}{N}$. A transfer matrix can be further expressed in terms of matrices representing network operations. For the one-hop lossy networks, the transfer matrix is given by,

$$\mathbf{H} = \mathbf{D} \tag{3.2}$$

where $\mathbf{D}$ is an $N \times N$ diagonal matrix representing erasures in the link such that the diagonal component of $\mathbf{D}$ is zero with probability ε and is one with probability $1-\varepsilon$.

3.3 Coding scheme: One-hop network

3.3.1 Encoding at the source node

The SNC encoder sends K data packets in the first K time slots (systematic phase) followed by $N-K$ random linear combinations of data packets in the next $N-K$ time slots (non-systematic phase). Here $\mathbf{X} = \mathbf{SG}$ represents K systematic packets and $N-K$ coded packets transmitted by the SNC encoder during N consecutive time slots. The generator matrix $\mathbf{G} = \left[\begin{array}{cc} \mathbf{I}_K & \mathbf{C} \end{array}\right]$ consists of identity matrix $\mathbf{I}_K$ of dimensions K and $\mathbf{C} \in \mathbb{F}_q^{K\times N-K}$ with elements chosen randomly from a finite field $\mathbb{F}_q$. The code rate is given by $\rho = \frac{K}{N}$.

3.3.2 Decoding at the sink node

The output at the sink is $\mathbf{Y} = \mathbf{SGH}$ where $\mathbf{H} = \mathbf{D}$ represents the transfer matrix of the network. We assume that the coding vectors are attached in the packet headers so that the matrix $\mathbf{GH}$ is known at the sink. The decoding is progressive using gaussian jordan algorithm as in [27]. In the progressive decoding, the sink uses Gauss Jordan algorithm [**?**] and starts decoding as soon as it receives the first packet. All the K data packets are recovered when K innovative packets are received at the sink, i.e., $rank(\mathbf{GH}) = K$.

3.4 Semi-analytical analysis

3.4.1 Minimum distance

The erasure correction performance of any code depends on the construction of its generator matrix. Now, for a code to be MDS, any $(K \times K)$ sub-matrix from $\mathbf{G}$ should have full rank K [28]. The MDS code will achieve the highest possible minimum distance (d_{MDS}) in the singleton bound and can correct up to $d_{MDS} - 1 = N - K$ erasures. It is known that the RS code is the MDS code [1]. We will compare now the minimum distance of the MDS code and SNC code.

Let us denote the minimum distance of SNC, d_{SNC}, as a random variable, which takes values in $\{1, 2, ..., d_{MDS}\}$. The difference between d_{MDS} and the actual minimum distance of the SNC code is known as degradation of the code [29]. We define degradation δ of SNC as $\delta = d_{MDS} - d_{SNC}$, which means that with $\delta = 0$ $(d_{SNC} = d_{MDS})$, SNC performs exactly as the MDS code and can correct up to $N-K$ erasures. With the coding parameters (N,K) and the field size q, the probability of $d_{SNC} = d_{MDS} - \delta$ is given in Appendix I. Table 3.1 shows the SNC performance with $(N,K) = (256,128)$. It is shown that SNC with $q = 256$ behaves exactly like the MDS code and can correct up to $N-K = 128$ erasures with 99.61% probability. If the degradation of $\delta = 2$ is allowed, SNC can correct up to $N-K-2 = 126$

Degradation (δ)	$q = 128$	$q = 256$
$\delta = 0$ (MDS)	99.21%	99.61%
$\delta = 1$	99.9938%	99.9984%
$\delta = 2$	99.999995%	99.999999%

Table 3.1: Minimum distance of the Systematic random linear code

erasures with 99.999999% probability and so on. These results show that SNC with higher field size can achieve erasure correction performance very close to the MDS codes.

3.4.1.1 Capacity achieving property of SNC

In general, it is known that RNC is a capacity achieving code for a line network [11] (and so this property is valid for one-hop networks as well). In this part, we will present a brief proof to illustrate that SNC is also a capacity achieving code for one-hop lossy networks. We will assume packet losses as Bernoulli-distributed random variables. Let us denote L as the number of coded packets received by the sink. As the erasure events follow Bernoulli distribution, L can be written as a summation of individual erasure events,

$$L = \sum_{i=1}^{N} a_i \tag{3.3}$$

where $\{a_i\}$is a sequence of i.i.d Bernoulli random variables with $Pr(a_i = 0) = \varepsilon$ and expected value $\mathbb{E}[a_i] = Pr(a_i = 1) = 1 - \varepsilon$. It is shown in the previous subsection, that SNC behaves like the MDS code with very high probability. Hence, in order to decode and successfully recover all the data packets, the sink should receive at least K coded packets. Therefore, for the successful decoding using SNC, we should have, $L \geq K$ and using equation (3.3), we have,

$$\sum_{i=1}^{N} a_i \geq K \tag{3.4}$$

Now, using equation (3.4), we have the following lemma on the capacity achieving property of SNC.

Lemma 1. *For the memoryless erasure channel where the erasure events are represented by* a *sequence of i.i.d Bernoulli random variables, SNC asymptotically achieves the capacity when N approaches to the infinity.*

Proof. Let us re-write the equation (3.4) as,

$$\left(\frac{\sum_{i=1}^{N} a_i}{N} \right) N \geq K \tag{3.5}$$

Now, if $\{a_1, a_2, ..., a_N\}$ is a sequence of i.i.d random variables drawn from distributions of expected values given by $1-\varepsilon$ then by the law of large numbers, as $N \to \infty$, the average of these random variables converges to the expected value; i.e., $\underset{N\to\infty}{lim}\left(\frac{\sum_{i=1}^{N} a_i}{N}\right) = \mathbb{E}[a_i] = 1-\varepsilon$. Hence, using the constraint of N approaches to infinity in equation (3.5) we have,

$$\underset{N\to\infty}{lim}(1-\varepsilon)N \geq K$$

$$\underset{N\to\infty}{lim}(1-\varepsilon) \geq \rho \tag{3.6}$$

□

This lemma proves that SNC is asymptotically a capacity achieving code. SNC can provide arbitrarily small erasure probability when code rate approaches the capacity of the network.

3.4.2 Reliability

In this section, we investigate the reliability in one-hop lossy networks. The reliability is defined as $1-\eta$ where η is defined as an effective erasure rate for the data packets that is achieved after the overall coding and decoding operations. For example, if all the data packets are recovered then $\eta = 0$ and if nothing is recovered then $\eta = 1$. Based on the values of K, N and q, η can be between 0 and 1. The residual erasure rate of the SNC is given by the following proposition.

$$\phi_1 = \varepsilon\left[Pr\left(\sum_{i=1}^{N-1} a_i \leq K-1\right)\right] \tag{3.7}$$

$$\phi_2 = \varepsilon\left[Pr\left(\sum_{i=1}^{N-1} a_i \geq K\right) \cap Pr(rank(\mathbf{GH}) < K)\right] \tag{3.8}$$

Proposition 2. *Given erasure rate ε, with (N,K) as coding parameters and q as a finite field size, the residual erasure rate (η) for SNC in one-hop lossy networks is given by,*

$$\eta = \phi_1 + \phi_2 \tag{3.9}$$

where ϕ_1 and ϕ_2 are given in equation 3.7 and equation 3.8 respectively.

Proof. If a systematic packet is lost by probability ε, then it is not recovered if the sink does not able to decode. There could be two possible events during which the sink is not able to perform the decoding.

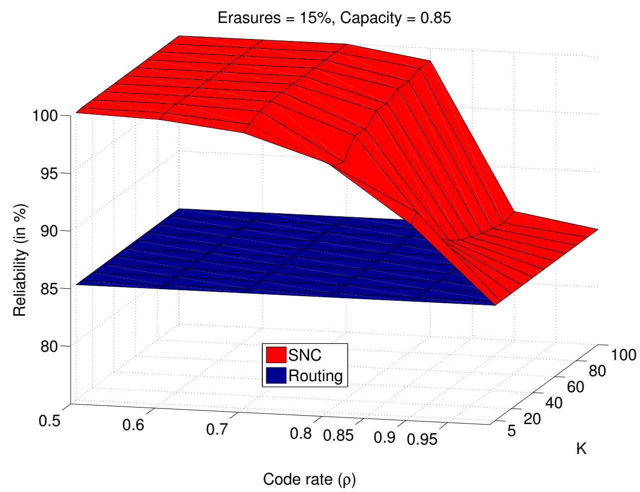


Figure 3.1: Reliability advantage of network coding over routing for one-hop lossy networks

First, if the sink receives less than K coded packets then all the data packets are not recovered. ϕ_1 represents the residual erasure rate due to these events. In particular, ϕ_1 is also equivalent to the residual erasure rate of any MDS code. For the MDS codes, there is only one possibility when the sink is not able to recover all the data packets; i.e., when it receives less than K coded packets.

Second, if the sink receives more than K coded packets but these K coded packets are not independent enough for the sink to recover all the data packets. This is because due to the random encoding and random selection of the coefficients some of the coded packets may be dependent. ϕ_2 calculates the residual erasure rate in SNC due to these events. Note that when the field size is high, then the probability that all the coded packets are independent is also very high. In that case ϕ_2 tends to zero and η converges to ϕ_1. □

Figure 3.1 shows the results on reliability gain of network coding over routing for one-hop lossy networks. Following are the key conclusions from this figure.

- **SNC provides higher reliability than simply routing.** This is because SNC is using outer code at the source as a channel coding solution to counter packet losses. By recovering the lost packets at the sink, SNC provides higher reliability than simply routing.

- **SNC provides 100% reliability when the code rate is smaller than the capacity.** This is because when the code rate is higher than the capacity, there are not enough packets at the sink to decode the complete block. In this case, only systematic packets are recovered. When code rate is smaller than the capacity, the sink starts decoding the complete block and therefore SNC can achieve 100% reliability.

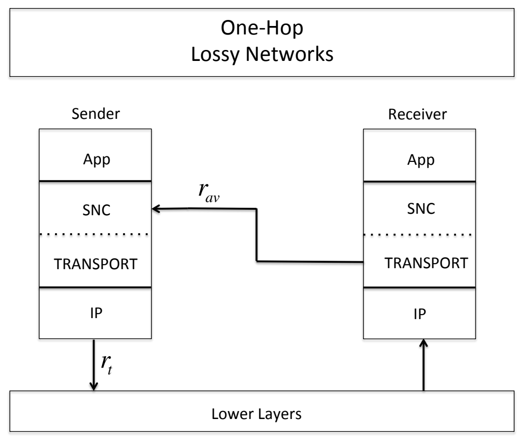


Figure 3.2: Proposed implementation of network coding in the SNC-sublayer of the transport layer

3.5 Practical application

In this section, we will present the application of network coding for the real-time video streaming over the one-hop lossy networks. We consider the application of network coding in the upper layers of the protocol stack. In particular, SNC is implemented as a sub-layer of the transport layer as shown in Fig. 3.2. The choice of sublayer for SNC implementation is crucial. Our proposed implementation of SNC is in contrast with some related work as in [30], [31], where SNC is implemented in lower layers. In this work, however, we choose SNC at the sublayer of the transport layer as our focus is on the applications development over wireless network maintained by network operators.

We use SNC for an efficient streaming of video over the one-hop lossy networks. The video data is usually delivered in video generations known as group of pictures (GoPs). These GoPs from the application layer are segmented into the transport layer packets. Such transport packets are then injected into the network to be transmitted with rate r_t (bits/sec). Transport layer packets are accessible to the application developer and do not require any operating system access for making modifications. Therefore, SNC is implemented as a sublayer of the transport layer, where message packets are encoded and forwarded to the IP layer in order to inject them into the network where the available rate for the transmission is r_{av} $(bits/sec)$. This available rate is either estimated from transport-layer feedback or shared with adaptive mechanisms for adapting to congestion [32]. However, in this work we leave out congestion avoidance mechanisms and assume r_{av} is known and available for the network coding.

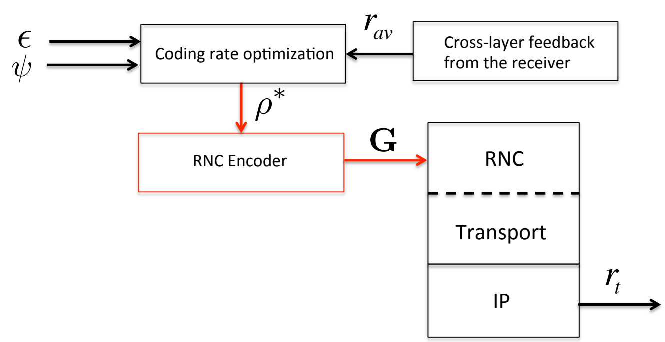


Figure 3.3: Network coding rate optimization using cross-layer optimization framework

Now, in the video model, each GoP is segmented into several data packets. The codec outputs each GoP in a fixed time T_{GoP} with the codec rate r_s. Therefore, the source block size is $K = \left\lfloor \frac{r_s \times T_{GoP}}{M \times 8} \right\rfloor$. The selection of code rate is based on the the available rate known from the lower layers such that network coding does not result in any congestion in the network. We select the code rate to be $\rho = \frac{r_s}{r_{av}}$ such that the overall rate of transmission after SNC does not exceed the available rate provided by the network.

Note that there is a tradeoff between the rate assigned for the source coding (r_s) and the code rate (ρ) assigned for the network coding. The higher source rate can be utilized by the source to ensure the better quality of multimedia data. On the other hand, the smaller code rate will result into a higher protection of data packets and better reliability. In the next subsection, we will present the cross-layer framework of optimization to select the optimal source rate and code rate.

3.5.1 Proposed framework for network coding rate optimization

The optimization of the network coding rate is done by a cross layer optimization block as shown in Fig. 3.3. In addition to the r_{av}, the optimization block uses an erasure rate ε and a target residual erasure rate ψ as the input for the optimization. The optimization is done to satisfy a target ψ at the sink, the values of which are specified in standards like 3GPP specification [33] for real-time video streaming. In the next proposition, we present the optimization problem.

Proposition 3. *Given the available rate r_{av}, the erasure rate ε and the field size q, optimal coding rate ρ^* to achieve the target erasure probability ψ, is given by,*

$$\rho^* = \max \rho \quad s.t. \tag{3.10}$$

$$\rho \leq 1 \text{ and } \eta \leq \psi$$

where η is the residual erasure rate and ψ is the target residual erasure rate.

Proof. The coding rate ρ is defined as the ratio $\rho = \frac{r_s}{r_{av}}$, hence maximizing the coding rate will result into maximizing the transmission rate r_s. The higher transmission rate can be utilized by the source to ensure the better quality of multimedia data and hence higher quality of service for the end user. □

After optimization is performed, the optimized coding rate ρ^* is sent to the SNC sublayer for network coding. Using the optimization framework, we could adjust the source rate and the network coding rate such that target erasure rate is achieved and the source video quality is maximized. In the next subsection, we will present the network coding solution for content-aware protection to further enhance the quality of the video streaming.

3.5.2 Simulation results

In this section, we will present our simulation results. We will first define the different performance metrics used in this chapter to evaluate the performance of various coding schemes.

3.5.2.1 Performance metrics

- Per-packet delay: The per-packet delay is measured as the average time needed per packet in recovering all the source data packets. If a packet $\mathbf{s}_t$ is recovered at the sink at time $t_r \geq t$ then packet $\mathbf{s}_t$ incurs delay δ_t where, $\delta_t = t_r - t$. For the block of K packets, the average value of per-packet delay is given as, $\triangle = \frac{\sum_{t=1}^{K} \delta_t}{K}$. Note that the delay is evaluated only for the packets which are recovered at the sink.

- Optimal code rate: The optimal code rate ρ^* is the output of the optimization framework of Section 3.5.1. We will use the optimal code rate to illustrate the advantage of the proposed network coding rate optimization framework in this chapter.

3.5.2.2 Simulation setup

We implement the different coding schemes and perform simulations on MATLAB. In the simulation setup, we consider different cases of one-hop lossy networks with erasure rates between 0 to 0.5. We have observed some of these values of erasure rates in several literature work to evaluate the performance of different coding schemes [19], [17], [25]. We assume that the source generates packets of length 1500 bytes. We conduct experiments for two field sizes with $q = 2$ and $q = 256$. We also consider different transmission bit

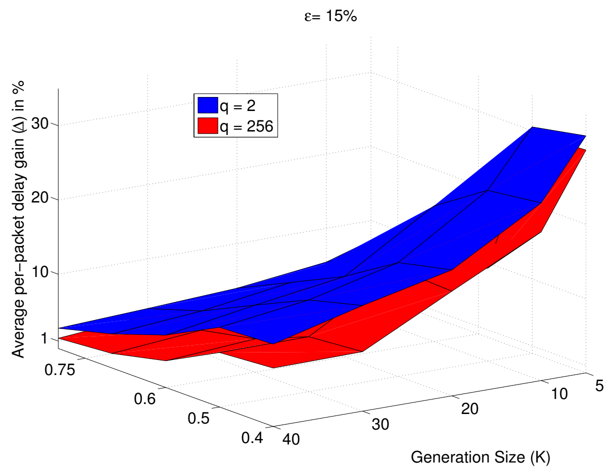


Figure 3.4: Per-packet advantage of network coding over RS coding for one-hop lossy networks

rates of the source with the codec rates varying from 50 kbps to 500 kbps. We consider state-of-the-art video codecs with video frames grouped into Groups of Pictures (GoPs) with $T_{GOP} = 2$ seconds. The source blocks size, corresponding to codec rates of 100 kbps to 500 kbps, vary approximately from $K = 5$ to $K = 100$ respectively. Several values of code rates are considered for comparison. The size of coded block i.e., N varies with the change in code rate. In each case, we conduct 10^6 experiments and take the average to evaluate different performance metrics.

3.5.2.3 Results

In this part, we will present the simulation results showing the advantage of using SNC and our proposed network coding frameworks for coding rate optimization and content-aware protection.

Result 1: Per-packet delay: Advantage of network coding over RS coding: Fig. 3.4 shows the results on per-packet delay gain of SNC over RS for one-hop lossy networks. Following are the key conclusions from this figure.

- **SNC outperforms the RS codes in terms of per-packet delay**. This is because SNC uses progressive method for decoding. The progressive decoding is possible due to the inherent random structure of SNC code. Therefore, in SNC, the decoding starts as soon as the sink receives the first packet. The receiver does not wait for all the packets to start the decoding process and hence the average per-packet delay is smaller than in the RS codes.

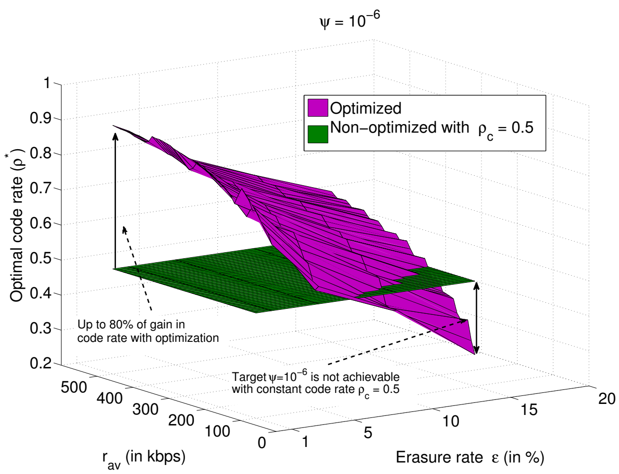


Figure 3.5: Benefits of the proposed network coding optimization framework

- **SNC with field size $q = 2$ outperforms SNC with field size $q = 256$ in terms of per-packet delay**. This is because the use of smaller field size facilitates the progressive decoding. When the field size is $q = 2$, there is a higher probability that a coded packet received is used instantly for the progressive decoding as compared to $q = 256$. This results into a smaller per-packet delay when the smaller field size is used. Note that the higher field size results into a better reliability. Therefore, there also exists a delay/reliability tradeoff in using different field sizes for SNC.

Optimal code rate - Benefits of the proposed network coding optimization framework: In Fig. 3.5, we illustrate the advantage of using the cross-layer optimization to optimize the network coding rate. The comparison of the optimized case is done with the non-optimized case where a constant code rate of $\rho_c = 0.5$ is considered.

- **Our proposed solution provides a higher coding rate than the non-optimized case when there are smaller erasures**. This is because only a small percentage of the available rate is needed to achieve the given target of $\psi = 10^{-6}$. Therefore, the rest of the available rate is assigned for the source coding. In particular, our proposed solution could provide up to 80% gain in code rates as compared to the non-optimized solution. This helps in saving unnecessary waste of the available bandwidth.

- **Our proposed solution always satisfy the required targets whereas non-optimized solution may not satisfies the required targets in some cases.** This is because when there are higher erasure rates, then the non-optimized solution gives residual erasure rates higher than the target erasure rates. On the other hand, our proposed solution always guarantees residual erasure rates smaller than the target erasure rates.

Our proposed optimization framework can also be used for real-time video services over different satellite networks. Next, we will present the different service requirements in the different satellite systems. We perform several simulations following the different requirements of different satellite systems and provide the optimal code rates for them.

Service requirements: Quality of Service (QoS) requirements for multimedia services have been covered by different standardization groups like ITU, ETSI or 3GPP [33, 34, 35, 36] Applications like audio streaming requires a residual erasure rate which should be lower than 10^{-3} for a telephone-quality audio stream. For the video transmission, there are different requirements for the network transmissions with different codecs. Applications like video conferencing, that enables real time communication by allowing two or more people to communicate with each other, can tolerate packet loss of order 10^{-4}. For a video streaming, where a video is sent from a server to a client, a admissible loss rate is around 10^{-5}. In addition, for the HDTV quality, a very small threshold of loss rate around 10^{-6} is required. For all these services with different requirements we evaluate the optimal coding rate based on the available rates of the satellite systems.

Representative satellite systems: Satellite service providers are providing different audio and video related services and targeting different users like industry, home, portable etc. The main difference remains in the maximum available rate provided by the different satellite services. We consider three different satellite systems, which offer different available rates, and we identify the optimal code rate required for different services in these different systems. Firstly, we consider Iridium satellite systems which are based on LEO constellations and have an advantage of smaller propagation delay of 40 ms. They offer up to 64 kbps of available rate. Secondly, Inmarsat's BGAN (Broadband Global Area Network) system is considered which a global satellite network with telephony using portable terminal. It offers up to 492 kbps for the personal, mobile and portable terminal users. Finally, we consider KA-SAT launched by Eutelsat satelites system, considered to be first European high throughput satellite and offers the available rate up to 10 MB/s to home users. Table 3.2 shows optimal code rates ρ^* for different satellite systems (different maximum r_{av}) with different service requirements (different ψ). Erasure rate of ε = 15% is considered for all the cases. We give the optimal code rates for two cases when RNC is used with the field size $q = 2$ and $q = 256$. These values of code rates can be used by the system designers for different satellite systems to provide different service requirements.

Satellite Internet Network		Audio Streaming ($\psi = 10^{-3}$)		Video Conference ($\psi = 10^{-4}$)	
		SNC ($q = 256$)	SNC ($q = 2$)	SNC ($q = 256$)	SNC ($q = 2$)
Satellite	r_{av}	ρ^*	ρ^*	ρ^*	ρ^*
Iridium	64 kbps	.56	.09	-	-
INMARSAT	492 kbps	.77	.71	.71	.62
K-SAT	10 Mbps	.85	.85	.83	.83

Satellite Internet Network		Video Broadcast ($\psi = 10^{-5}$)		HDTV quality ($\psi = 10^{-6}$)	
		SNC ($q = 256$)	SNC ($q = 2$)	SNC ($q = 256$)	SNC ($q = 2$)
Satellite	r_{av}	ρ^*	ρ^*	ρ^*	ρ^*
Iridium	64 kbps	-	-	-	-
INMARSAT	492 kbps	.66	.53	.62	.46
K-SAT	10 Mbps	.82	.81	.81	.80

Table 3.2: Optimal code rates for different satellite systems satisfying different service requirements.

3.6 Conclusions

In this chapter, we have presented the analysis and application of SNC in the one hop lossy networks. By thorough theoretical study of minimum distance and residual erasure rate of SNC, it is shown that SNC provides similar erasure correction performance than RS coding. Additionally, it is also shown using simulation results that SNC outperforms RS codes in terms of per-packet delay due to the use of progressive decoding. Both of these factors show that SNC could be a potential replacement to RS codes for these networks. Furthermore, in this chapter application of SNC in the upper layers of the protocol stack is extensively studied. Specifically, network coding solutions are proposed in the applications layer that can be used for the efficient multimedia delivery by optimizing the network coding rate and using available bandwidth optimally. Finally, the work in this chapter leads to the following publications.

Journals

1. M. A. Pimentel-Niño, **P. Saxena** and M. A. Vázquez-Castro, "Multimedia delivery for situation awareness provision over satellite" submitted in the special issue of Hindawi on recent advances in streaming multimedia content delivery, October 2014.

Book Chapter

1. **P. Saxena** and M. A. Vázquez-Castro, "Network coding advantage over MDS codes for multimedia transmission via erasure satellite channels", Lecture notes of the institute for computer sciences, social informatics and telecommunications engineering, (Springer 2013), Volume 123, 2013, pp 199-210, ISBN: 978-3-319-02761-6

Conferences

1. **P. Saxena** and M. A. Vázquez-Castro, "Network coding advantage over MDS codes for multimedia transmission via erasure satellite channels", The 5th International conference on personal satellite services (PSATS 2013), Tolouse (France), June 2013
2. **P. Saxena** and M. A. Vázquez-Castro, "RNC advantage over MDS codes for adaptive multimedia communications", in International conference on random network codes and design over GF(q), Ghent (Belgium), September 2013
3. M. A. Pimentel-Niño, **P. Saxena** and M. A. Vázquez-Castro, "QoE driven adaptive video with overlapping network coding for best effort erasure satellite links" accepted in 31st AIAA international communications satellite systems conference, Florence (Italy), October 2013

Chapter 4

Systematic network coding for two-hop lossy networks

4.1 Contributions and Outline

In this chapter, we will focus on the systematic network coding for the two-hop lossy networks. Currently, FEC schemes like RS codes or Raptor codes are used in these networks to combat packet losses. These coding schemes are used only at the source while routing is done at the intermediate node. However, as routing is not the optimal solution, both the reliability and the achievable rates can be increased by employing network coding at the intermediate node. In this chapter, for designing solutions for practical scenarios, our focus is on the satellite communication systems. These systems are the straightforward case of the two-hop lossy networks where the source transmits the information to the sink via the intermediate node which is satellite. The application of network coding for satellite communication is relevant in this case as one intermediate node (which can be a gateway or other) fits in the satellite scenario.

We consider the application of network coding for reliability and throughput improvement in Digital Video Broadcasting via Satellite-Second Generation (DVB-S2)[37]. DVB-S2 is a standard for transmission over satellite for which optional extensions already exist (DVB-S2X). The standard includes forward erasure correction at the link layer (LL-FEC) to countermeasure losses which cannot be coped with by the lower layers. LL-FEC makes use of Reed-Solomon (RS) or Raptor codes [38] and [39] and operate as channel coding on an end-to-end basis. In this work we present a possible extension based on systematic random network coding, which we term as LL-SNC. We develop LL-SNC framework over Generic Stream Encapsulation (GSE) protocol [40]. GSE has been introduced recently to allow efficient encapsulation of IP and other network layer packets directly over the DVB-S2 generic stream. By using network coding over variable size GSE packets, a significant reduction in overhead by a factor 2 to 3, with respect to Muti Protocol Encapsulation (MPE) introduced in the first generation of DVB systems [41], is achievable.

4.1.1 Contributions of the chapter

- **Objective 1: Develop a matricial model that allows analytical treatment of network coding for lossy networks. The model should be applicable at any layer of the protocol stack.**
  - We extend the matricial model for the networks with one intermediate node. This allows us to understand the mathematical structure behind the systematic network coding when used at the intermediate node as well.
- **Objective 2: Semi-analytical investigation of achievable throughput and reliability for line networks, a simple yet useful conceptual network model.**
  - We study the reliability and achievable rate in the line networks with one intermediate node. Using the semi-analytical approach, we study and characterize the reliability and achievable rate as a function of network coding rate and capacity of the network. Our results show that when the code rate is smaller than the capacity LL-SNC can provide 100% reliability. The reliability provided by LL-SNC quickly approaches 100% as compared to the reliability provided by LL-FEC. This is because due to re-encoding in LL-SNC, the sink will receiver a higher number of innovative packets which will help to attain 100% reliability quickly.
- **Objective 3: Develop practical network coding schemes for line networks that significantly outperform state-of-the-art purely FEC-based schemes.**
  - We simulate a practical application at link layer of Digital Video Broadcasting via Satellite-Second Generation (DVB-S2). We propose an architectural and encapsulation framework so that network coding can be used over the state-of-the- art protocols at link layer of DVB-S2. Our simulation results show that our proposed LL-SNC achieves significantly higher transmission rates and reliability than LL-FEC for the different frame lengths available in the standard. Specifically, LL-SNC can provide up to 158% higher maximum achievable rates than LL-FEC while complexity is kept low due to the use of systematic encoding.

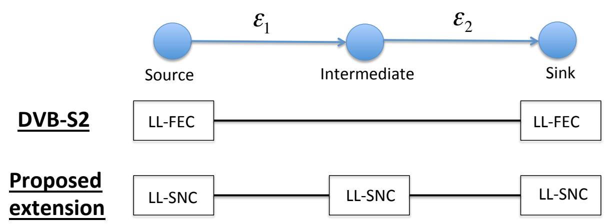


Figure 4.1: System Model

4.1.2 Outline of the chapter

This chapter is organized as follows. In Section 4.2, we present the system model for the two-hop lossy networks. In Section 4.3., we discuss LL-SNC coding scheme for the two-hop lossy networks. In Section 4.4, semi-analytical analysis in two-hop lossy networks is presented and in Section 4.5, we present the proposed the application of network coding in DVB-S2. We present our simulation results in Section 4.6 and conclude this chapter in Section 4.7.

4.2 Matricial model

Let us consider that a source node has K data packets to send to a sink node. Each packet is a column vector of length M over a finite field $\mathbb{F}_q$. The set of the data packets is denoted by the matrix,

$$\mathbf{S} = \left[\begin{array}{ccccc} \mathbf{s}_1 & \mathbf{s}_2 & . & . & . & \mathbf{s}_K \end{array} \right]$$

where $\mathbf{s}_t$ is the t^{th} data packet. The source and the sink are connected with the two intermediate links as shown in Figure 4.1. The first link is from the source to the intermediate and the second link is from the intermediate to the sink. These links are modeled as a delay-free memoryless erasure channel. A packet sent across the link $i, i \in \{1,2\}$ is either erased with the probability of ε_i or received without error. The capacity of the link i is therefore $1-\varepsilon_i$ and the capacity of the two-hop lossy network joining the source to the sink is $\min\{1-\varepsilon_1, 1-\varepsilon_2\}$.

Due to our requirement of low-delay, we assume that there is no feedback from the sink or from the intermediate node in the network. We also consider that packet transmissions occur at discrete time slots so that the each node can transmit one packet per time slot. In the next section, we will discuss different coding schemes for transmitting the data packets from the source to the sink over the two-hop lossy networks. We will assume that all the coding schemes run for a total of N time slots and every node (except the sink) transmits a packet in each time slot $t = 1, 2, ..., N$.

The complete encoding and decoding operations in the network can be modeled with a linear operator channel (LOC), using which an output unit at the sink can be expressed as a linear transformation of the input unit at the source. Let $\mathbf{Y} \in \mathbb{F}_q^{M\times N}$ be the output unit with N columns representing N received packets in N time slots. If the sink does not receive any packet in time slot t then the t^{th} column of $\mathbf{Y}$ should be considered as a zero column. We have,

$$\mathbf{Y} = \mathbf{X}\mathbf{H} = \mathbf{S}\mathbf{G}\mathbf{H} \tag{4.1}$$

where $\mathbf{H} \in \mathbb{F}_q^{N\times N}$ is the transfer matrix for the two-hop lossy network, $\mathbf{G} \in \mathbb{F}_q^{K\times N}$ is the generator matrix and $\mathbf{X} = \mathbf{S}\mathbf{G}$ is a generation of N coded packets transmitted from the source. The outer code is defined by $\mathbf{G}$ with code rate $\rho = \frac{K}{N}$. A transfer matrix can be further expressed in terms of matrices representing network operations at the intermediate node and erasures in different links.. For the two-hop lossy networks, the transfer matrix is given by,

$$\mathbf{H} = \mathbf{D}_1\mathbf{T}\mathbf{D}_2 \tag{4.2}$$

where $\mathbf{D}_1$ and $\mathbf{D}_2$ are the $N\times N$ diagonal matrix representing erasures in each link such that the diagonal component of $\mathbf{D}_i$ is zero with probability ε_i and is one with probability $1-\varepsilon_i$. The operation at the intermediate node is given by the upper triangular matrix $\mathbf{T} \in \mathbb{F}_q^{N\times N}$. In particular, $\mathbf{T}$ defines the inner code used for network coding which is used to increase the transmission rates.

4.3 Coding scheme: two-hop network

4.3.1 Encoding at the source node

The SNC encoder sends K data packets in the first K time slots (systematic phase) followed by $N-K$ random linear combinations of data packets in the next $N-K$ time slots (non-systematic phase). Here, $\mathbf{X} = \mathbf{S}\mathbf{G}$ represents K systematic packets and $N-K$ coded packets transmitted by the SNC encoder during N consecutive time slots. The generator matrix $\mathbf{G} = \begin{bmatrix} \mathbf{I}_K & \mathbf{C} \end{bmatrix}$ consists of identity matrix $\mathbf{I}_K$ of dimensions K and $\mathbf{C} \in \mathbb{F}_q^{K\times N-K}$ with elements chosen randomly from a finite field $\mathbb{F}_q$. The code rate is given by $\rho = \frac{K}{N}$.

4.3.2 Re-encoding at the intermediate node

The SNC re-encoder performs re-encoding operations in every time slot and sends N packets to the sink node. Let $\mathbf{X}_I = \mathbf{X}\mathbf{D}_1\mathbf{T}$ represents N packets transmitted by the SNC re-encoder during N consecutive time slots . The re-encoding matrix $\mathbf{T}$ is modeled as follows.

During the systematic phase, if a packet $\mathbf{s}_t$ is lost i.e., $\mathbf{D}_1(t,t) = 0$ then the non-zero elements of the t^{th} column of matrix $\mathbf{T}$ are randomly selected from $\mathbb{F}_q$. This represents that if the systematic packet is lost from the source node to the intermediate node, then the intermediate node transmits a random linear combination of the packets stored in its buffer. If a packet $\mathbf{s}_t$ is not lost; i.e., $\mathbf{D}_1(t,t) = 1$ then the t^{th} column of matrix $\mathbf{T}$ is the same as the t^{th} column of identity matrix $\mathbf{I}_N$. This represents that the intermediate node forwards this systematic packet to the sink. During the non-systematic phase, the intermediate node sends a random linear combination of the packets stored in its buffer and all the non-zero elements of last $N-K$ columns of $\mathbf{T}$ are chosen randomly from the finite field $\mathbb{F}_q$.

4.3.3 Decoding at the sink node

The output at the sink is $\mathbf{Y} = \mathbf{SGH}$ where $\mathbf{H} = \mathbf{D}_1\mathbf{T}\mathbf{D}_2$ represents the transfer matrix of the network. We assume that the coding vectors are attached in the packet headers so that the matrix $\mathbf{GH}$ is known at the sink. The decoding is progressive using gaussian jordan algorithm as in [27]. In the progressive decoding, the sink uses Gauss Jordan algorithm [**?**] and starts decoding as soon as it receives the first packet. All the K data packets are recovered when K innovative packets are received at the sink, i.e., $rank(\mathbf{GH}) = K$.

4.4 Semi-analytical analysis

4.4.1 Reliability

The reliability is defined as $1-\eta$ where η is as an effective erasure rate for the data packets that is achieved after the overall coding and decoding operations. For example, if all the data packets are recovered then $\eta = 0$ and if nothing is recovered then $\eta = 1$. The figure will be included here. The general conclusions on the reliability in the two-hop lossy networks are as follows.

- When the code rate is smaller than the capacity, both network coding and RS coding can achieve 100% reliability. However, network coding achieves 100% reliability with a higher code rate as compared to RS coding. This results into the saving of bandwidth and eventually a higher achievable rate as shown in the next subsection.

4.4.2 Achievable rates

It is defined as $R = \frac{K(1-\eta)}{N} = \rho(1-\eta)$ where R is in packets per time slots, $(1-\eta)K$ is the number of data packets recovered after decoding and $(1-\eta)$ is the reliability provided by the coding scheme. R is upper bounded by the capacity of the line network which is

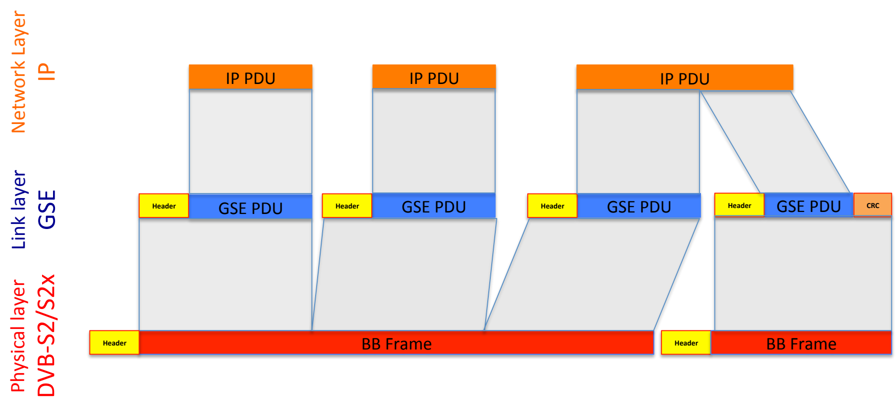


Figure 4.2: Encapsulation in link layer GSE protocol

$\min_i \{1 - \varepsilon_i\}$ packets per time slot. The figure will be included here. The general conclusions on the achievable rates for the two-hop lossy networks are as follows.

- Network coding achieves the maximum rate only when the code rate is smaller than the capacity. Due to the use of re-encoding at the intermediate node, the maximum achievable rate by network coding is always higher than the maximum achievable rate by RS coding.
- When the erasure rate increases, the gain of network coding w.r.t RS coding increases. This is because when there are higher erasures, the intermediate node has more opportunities to encode and so gain of network coding w.r.t RS coding.
- Finally, as the block length increases, network coding provides higher maximum rates. The maximum achievable rates from network coding are closer to the capacity when the block length increases. This is in agreement with the information results reported in e.g. [11].

4.5 Practical application

4.5.1 Link layer GSE protocol

GSE [40] is the state-of-the-art link layer protocol which is used for the efficient encapsulation of network layer (IP) protocol data units (PDUs) over the DVB-S2 generic stream. As shown in Fig. 4.2 each IP PDU can be encapsulated in a single or multiple link layer GSE PDUs. The variable size of GSE PDUs is to match the variable size of the incoming IP PDUs. If an IP PDU is encapsulated in multiple GSE PDUs then a Cyclic Redundancy

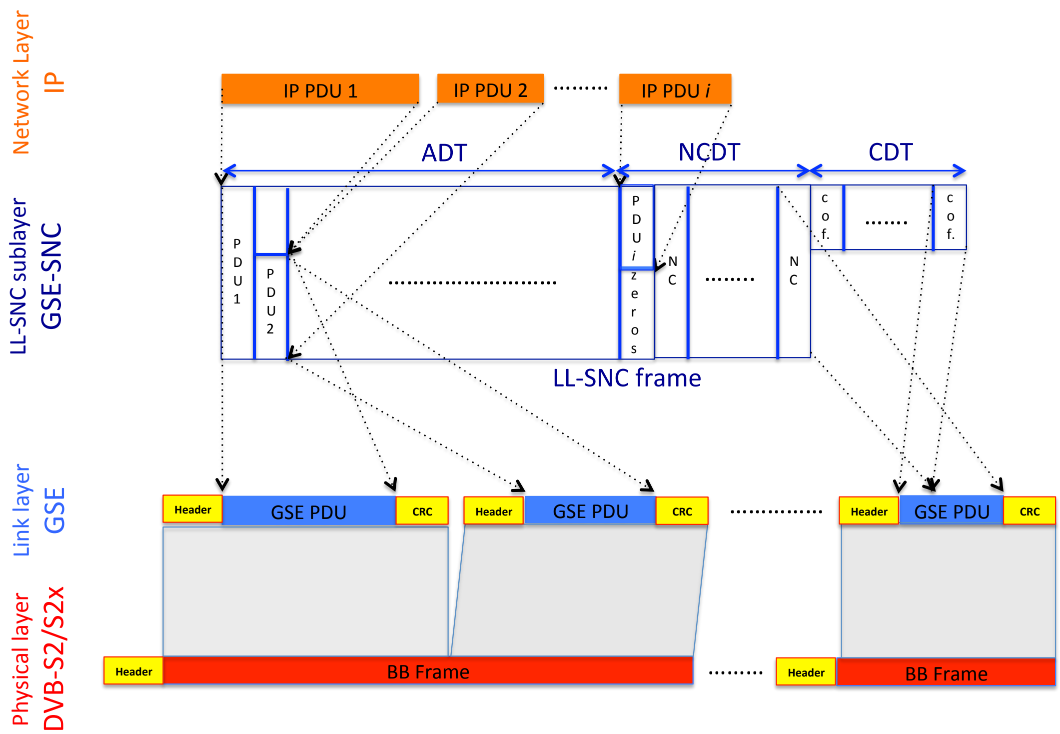


Figure 4.3: LL-SNC encapsulation for DVB-S2 with the proposed LL-SNC frame

Check (CRC)-32 is appended to the last fragment. It is used for the verification of the reassembly operation of these GSE PDUs at the receiving nodes. The GSE PDUs are then encapsulated into variable size BB frames. The size of BB frames changes dynamically on a frame-to-frame basis due to use of adaptive coding and modulation (ACM) in DVB-S2. Each BB frame is then encoded and modulated with the assigned DVB-S2 MODCOD.

The existing LL-FEC framework with GSE [42] enables FEC for the network layer packets. LL-FEC technique is based on the utilization of LL-FEC frame arranged as a matrix which is composed of two parts. In the first part, IP PDUs are filled and in the second part FEC parity data is filled. The data from the LL-FEC frame is then encapsulated in GSE PDUs. The current LL-FEC framework, however, operates only end-to-end and does not utilize the coding opportunities at the intermediate node. In this section, we propose an architectural and encapsulation framework to enable LL-SNC at the source and at the intermediate node for DVB-S2. Our proposal does not need modification of the standardized LL-FEC over GSE [42]. We logically map the proposed LL-SNC framework to the existing LL-FEC framework such that it complies with all the constraints of the standard.

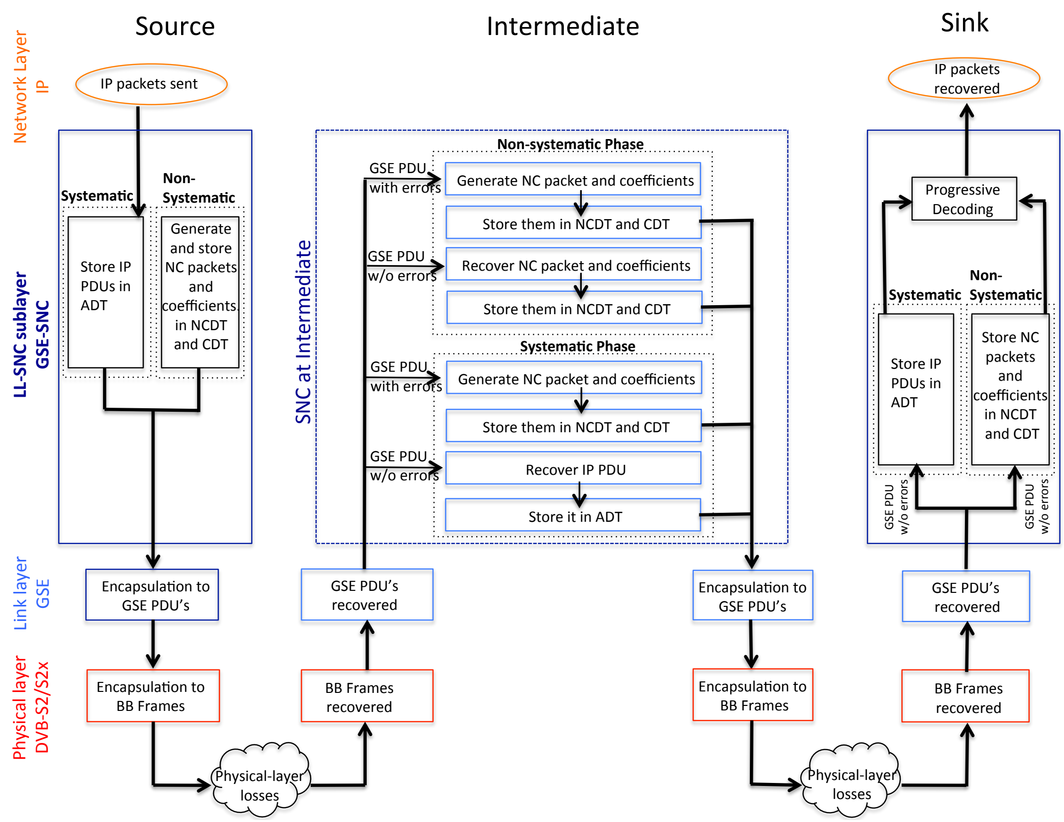


Figure 4.4: Flow diagram with LL-SNC architecture and LL-SNC encapsulation for DVB-S2

4.5.2 Proposed LL-RNC architecture

In the proposed LL-SNC architecture, an LL-SNC frame is introduced both at the source node and at the intermediate node. The LL-SNC frame consists of an application data table (ADT) to store IP PDUs, a network coding data table (NCDT) table to store network coded packets and a coefficient data table (CDT) to store coding coefficients (Figure 4.3). The structures of these tables are as follows.

ADT is filled with IP PDUs. It has K columns and M rows. IP PDUs are arranged column wise starting from the upper left corner. If an IP PDU does not fit in one column, it continues at the top of the following column and so on. If ADT is not completely filled then the zero-padding bytes are inserted in last column to fill it completely.

NCDT is filled with network coded packets. The size of NCDT depends on its location. At the source node, NCDT has $N-K$ columns and M rows whereas at the intermediate node, NCDT can have up to N columns and M rows. This is because the source will transmit $N-K$ coded packets but the intermediate node can transmit variable number of coded packets which will depend on the erasures from the source to the intermediate node. The maximum frame length (maximum value of N) is 256 as specified in the existing LL-FEC framework [43].

CDT is filled with coding coefficients. It has K rows. The number of columns is equals to the number of columns in NCDT. Each column of CDT contains coding coefficients corresponding to coded packets in NCDT. The CDT is filled as follows. The first column of CDT contains K coding coefficients used to generate the first column of NCDT. The second column of CDT contains K coding coefficients used to generate the second column of NCDT and so on.

4.5.3 Proposed LL-RNC encapsulation

The proposed LL-SNC encapsulation is as follows. At the source node, each IP PDU from ADT is encapsulated in a single or multiple GSE PDUs as explained in the section 4.5.1. Each coded packet from NCDT and the corresponding coding coefficients from CDT are encapsulated in one GSE PDU. The first K bytes of GSE payload in GSE PDU contain K coding coefficients followed by the corresponding NCDT column. The value of K is signaled through no_adt_columns specified in GSE format [40]. GSE PDUs are then encapsulated in DVB-S2 BB frames as explained in the section 4.5.1. Note that CRC is used with every GSE PDU (specified in clause 5.3.2.1.2,[42]) to detect errors in GSE PDUs at the receiving node. The GSE PDUs, which are in error, are considered as erased packets at the receiving node. Figure 4.3 shows the complete encapsulation process at the source.

At the intermediate node, the payload of correctly received GSE PDUs is stored in the LL-SNC frame. The IP PDUs are stored in the ADT of the intermediate node. They are stored at the same position as they were in the ADT of the source node. This exact position within the LL-SNC frame is signaled through the GSE header of GSE PDU. The coded packets and the corresponding coefficients are stored in NCDT and CDT of the LL-SNC frame. At the intermediate node coding is performed as explained in Section **??**. When the intermediate node receives GSE PDU without error, it sends the GSE PDU to the sink node and also stores it in the LL-SNC frame. When the intermediate node receives GSE PDU with errors, it discards the GSE PDU and generates new coded packet and coding coefficients as explained in Section **??**. These new coded packets and the corresponding coefficients are stored in NCDT and CDT at the intermediate node. They are encapsulated to GSE PDUs in the same way as explained in the previous paragraph.

At the sink node, the correctly received GSE PDUs are stored in the LL-SNC frame. The IP PDUs are stored in the ADT of the sink node. They are stored at the same position as they were stored in the ADT of the source node. Note that an IP PDU may form a part of the column or the complete column. If it is lost, then the corresponding part of the column or the complete column is also lost. The objective is to fill completely (or partially) lost columns of ADT in order to recover all the IP PDUs. The coded packets and the coding coefficients are stored in NCDT and CDT respectively. The progressive decoding is performed and lost columns (or lost part of columns) in ADT are filled with the recovered data. IP PDUs are recovered and then passed to the upper layers. Figure 4.4 shows the complete information flow with LL-SNC architecture and LL-SNC encapsulation in DVB-S2.

4.5.4 Simulation results

In this section, we will present our simulation results. We will first define the different performance metrics used in this chapter to evaluate the performance of various coding schemes.

4.5.4.1 Performance metrics

- Reliability and achievable rates: They are defined in Section 4.4.
- Per-packet delay: The per-packet delay is measured as the average time needed per packet in recovering all the source data packets. If a packet $\mathbf{s}_t$ is recovered at the sink at time $t_r \geq t$ then packet $\mathbf{s}_t$ incurs delay δ_t where, $\delta_t = t_r - t$. For the block of K packets, the average value of per-packet delay is given as, $\triangle = \frac{\sum_{t=1}^{K} \delta_t}{K}$. Note that the delay is evaluated only for the packets which are recovered at the sink.

4.5.4.2 Simulation setup

We implement the different coding schemes and perform simulations on MATLAB. In the simulation setup, we consider realistic scenarios with links having light rainfall (erasure

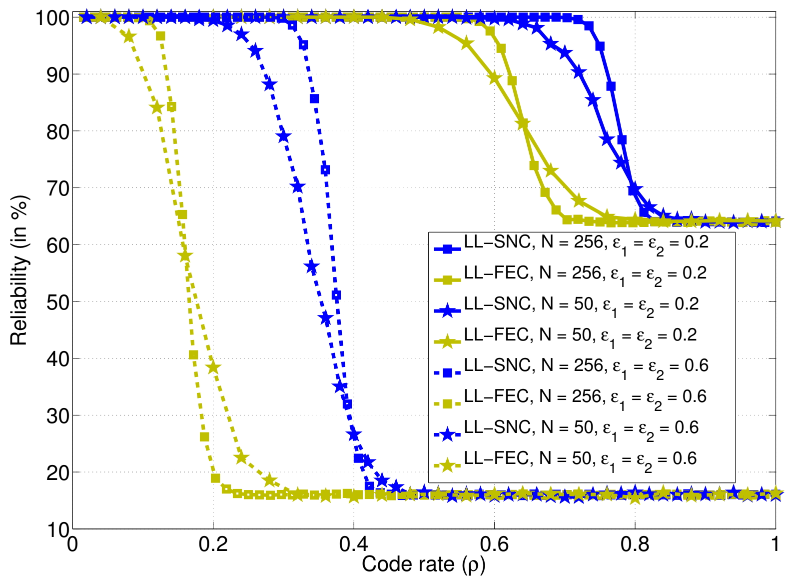


Figure 4.5: Reliability for symmetric links with light and heavy rainfall

rate of 0.2) and/or heavy rainfall (erasure rate of 0.6) [44]. In each case, we compare LL-SNC with LL-FEC. We assume IP PDUs of length 1500 bytes. Each IP PDU is mapped to a column in application data table (ADT) of LL-SNC frame. Two LL-SNC frame lengths, $N \in \{50, 256\}$ [43] and several values of code rates are considered for comparison. The size of ADT; i.e., K changes with the code rate. We set the physical layer symbol rate of $B_s = 27.5$ Mbaud/s, $\varsigma = 2$ as the modulation constellation and $r_{phy} = 1/2$ as the physical coding rate. Such that the bit rate is $B_s \varsigma r_{phy} = 27.5\ Mbps$. The transmission delay is set to be 250 ms. In each case, we average over 1000 experiments for every performance metric.

4.5.4.3 Simulation results

Reliability and achievable rates: Figure 4.5 and Figure 4.6 show the results on reliability and achievable rates respectively. Table 4.1 shows the results on maximum achievable rates. We will present the key conclusions on the application part which are in line with the general conclusions pointed out in Section 4.4.

- When the code rate is smaller than the capacity LL-SNC can provide 100% reliability. The reliability provided by LL-SNC quickly approaches 100% as compared to

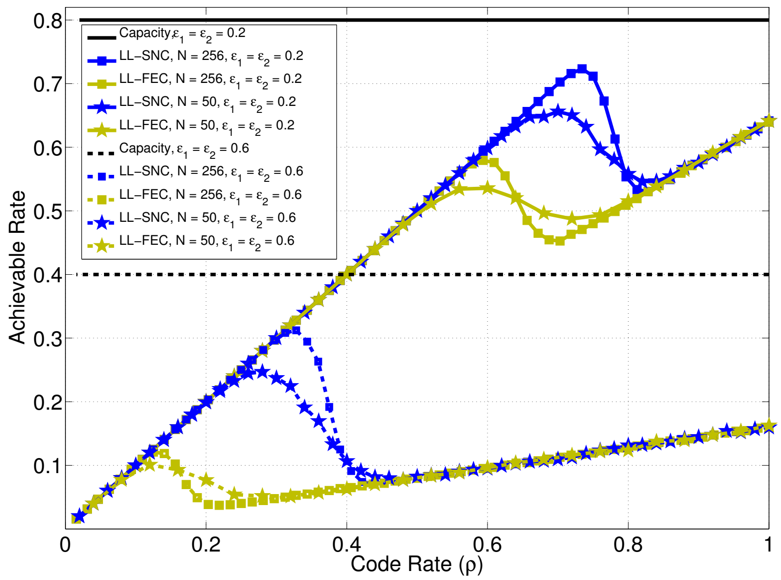


Figure 4.6: Achievable rates for symmetric links with light and heavy rainfall

	N	Capacity	LL-SNC	LL-FEC	Gain
$\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$	256	0.8000	**0.7233**	0.5794	24.87%
$\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.6$	256	0.4000	**0.3122**	0.1209	158.23%
$\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.2$	256	0.4000	**0.3387**	0.2640	28.30%
$\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.6$	256	0.4000	**0.3366**	0.2640	27.50%
$\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$	50	0.8000	**0.6561**	0.5357	22.48%
$\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.6$	50	0.4000	**0.2469**	0.1309	147.90%
$\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.2$	50	0.4000	**0.2945**	0.2312	27.38%
$\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.6$	50	0.4000	**0.2735**	0.2312	18.30%

Table 4.1: Maximum achievable rates for symmetric and non-symmetric links

the reliability provided by LL-FEC. This is because due to re-encoding in LL-SNC, the sink will receiver a higher number of innovative packets which will help to attain 100% reliability quickly.

- LL-SNC provides higher reliability and higher achievable rates than LL-FEC. As the erasure rate increases, the gain of LL-SNC w.r.t LL-FEC increases. Specifically, LL-SNC can provide up to 158% higher maximum achievable rates (Table 4.1) when there is a heavy rainfall in both the links. This is because when there are higher erasures, the intermediate node has more opportunities to re-encode in LL-SNC.

- As the frame length increases, SNC provides higher maximum rates. LL-SNC provides higher maximum achievable rates closer to capacity and in agreement with reported in e.g. [11]. In particular, for the light rain situation, when $\varepsilon_1 = \varepsilon_2 = 0.2$, the maximum achievable rate by SNC, with $N = 50$, is 0.6561 (17.99% less than the capacity) and with $N = 256$, it is 0.7233 (9.59% less than the capacity). Similarly for the heavy rain situation, when $\varepsilon_1 = \varepsilon_2 = 0.6$, the maximum achievable rate by SNC, with $N = 50$, is 0.2469 (38.28% less than the capacity) and with $N = 256$, it is 0.3122 (21.95% less than the capacity).

Per-packet delay: LL-SNC provides smaller per-packet delay than LL-FEC as shown in Figure 4.7. This is because LL-SNC allows progressive decoding and it also helps the sink to receive a higher number of innovative packets during the same coding window of N time slots which facilitates faster recovery of data packets as compared to LL-FEC. Note that when the code rate decreases per-packet delay decreases due to decrease in the block size K. When the block size is small, the waiting time for decoding is small which results into a smaller per-packet delay.

Tradeoff: The two main tradeoffs of using LL-SNC instead of LL-FEC are (i) the encoding complexity at the intermediate node and (ii) the overhead due to the coding coefficients. However, using SNC, the intermediate node is involved in encoding process mainly when it does not receive the packet from the sink and most of the time it just forwards the correctly received packets. This reduces the encoding complexity. Furthermore, because of systematic coding in SNC, our results (4.8) show that the loss because of overhead due to coding coefficients is only between 0%-5% as compared to 0%-25% in RNC.

4.6 Conclusions

In this chapter, we have presented the analysis and application of SNC in the two-hop lossy networks. In this chapter, we have extensively studied the use of SNC in the lower layers of the protocol stack. We have proposed the LL-SNC framework, its architecture

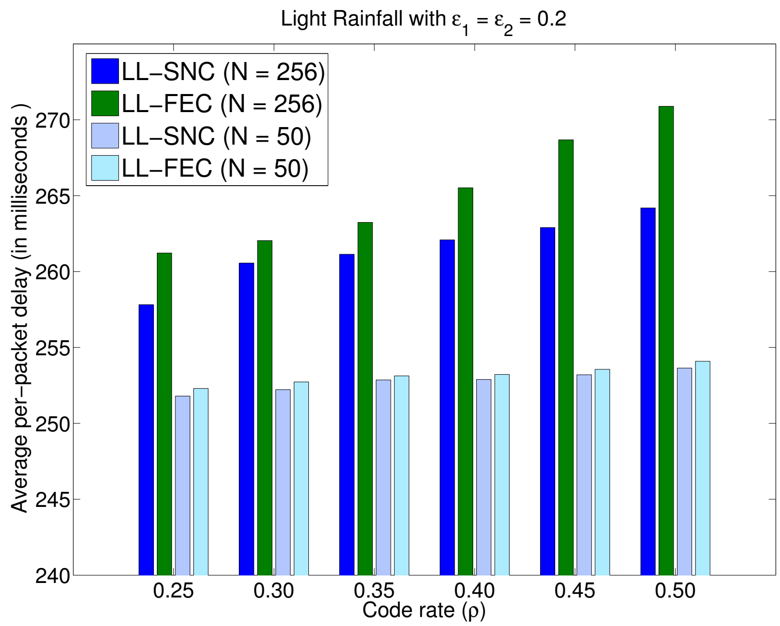


Figure 4.7: Per-packet delay in symmetric links with light rainfall

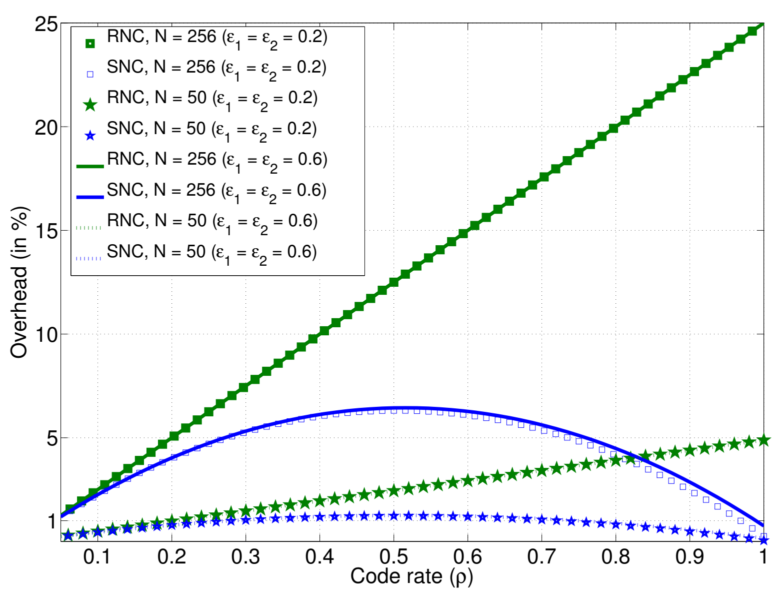


Figure 4.8: Coding coefficients overhead in the two-hop lossy network with light and heavy rainfall

and encapsulation to enable network coding in DVB-S2. Our simulation results have shown that LL-SNC can provide significantly higher rates and reliability than the existing LL-FEC schemes. In addition, LL-SNC also provides smaller per-packet delay which is useful as satellite systems already incurs large transmission delay. Although there are two tradeoffs, encoding complexity and overhead, but they are kept low due to the use of systematic coding. We conclude that by providing several benefits, LL-SNC can serve as a possible extension of LL-FEC framework in DVB-S2. Finally, the work in this chapter leads to the following publications.

Journals

1. **P. Saxena** and M. A. Vázquez-Castro, "Link layer random network coding for DVB-S2X/RCS" under review in IEEE wireless communications letters, 2014.

Conferences

1. **P. Saxena** and M. A. Vázquez-Castro, "Random Linear Network Coding over Satellite" in Conference on algebraic approaches to storage and network coding, Barcelona, Feb 2014

Chapter 5

Systematic network coding for lossy line networks

5.1 Contributions and Outline

In this chapter, we will focus on the network coding solutions for the multiple-hop lossy line networks. In the previous chapters 3 and 4, we have shown SNC to be an efficient practical network coding solution for the one-hop and two-hop lossy networks respectively. However, when there are several nodes in the network, SNC losses its advantages over RNC. When there are several lossy links, many systematic packets are lost during the systematic phase. The sink receives fewer systematic packets and therefore all the advantages of SNC over RNC diminishes.

In order to overcome limitations of SNC in the multiple-hop lossy line networks, we propose SS-SNC as a low-delay, low-complexity and low-overhead network coding strategy based on the systematic concatenation of outer and inner codes. In the proposed SS-SNC scheme, the design of outer code is based on the SNC structure and the design of inner code is based on the smart scheduling at the intermediate nodes. To this end, we propose a specific solution for efficient scheduling based on the weighted round-robin selection of the packets at the intermediate nodes. The proposed SS-SNC solution provides smaller delay, smaller complexity and smaller overhead than SNC over a general line network (with fewer nodes or with several nodes).

Some recent work on the concatenation of outer and inner codes include BATS codes [17], FUN codes [25] and Fulcrum network codes [26]. BATS codes and FUN codes are based on dividing the source blocks into batches. Our work is different from [17] and [25] as we focus on low-delay and low-complexity solution for real time multimedia streaming like video streaming which usually have small block sizes. Hence dividing this small block into batches may add to unwanted complexity and delay. Fulcrum network codes are designed to provide multimedia delivery to heterogeneous receivers with different processing capabilities with the coding design based on the concatenation of two separate finite fields. SS-SNC is different from Fulcrum network codes as it does not add design complexity nor sacrifices achievable rates w.r.t routing while minimizes delay, complexity and overhead which are the key ingredients for efficient multimedia streaming.

5.1.1 Contributions of the chapter

- **Objective 1: Develop a matricial model that allows analytical treatment of network coding for lossy networks. The model should be applicable at any layer of the protocol stack.**
  - Towards the first objective, we extend the matricial model for the network with several intermediate nodes. This allows us to understand the mathematical framework of mapping communication entities to mathematical entities at different intermediate nodes of the network.
- **Objective 2: Semi-analytical investigation of achievable throughput and reliability for line networks, a simple yet useful conceptual network model.**
  - We analyze semi-analytical reliability, achievable rates, delay and complexity of network coding schemes. Specifically, we show that as the block length increases, SS-SNC provides higher maximum achievable rates. These results are in coherence with the information theoretical results that the random linear network coding strategy can achieve the capacity of the line network when block length tends to infinity.
- **Objective 3: Develop practical network coding schemes for line networks that significantly outperform state-of-the-art purely FEC-based schemes.**
  - We develop a smart re-encoding network coding scheme, SS-SNC, which includes packet scheduling at the intermediate nodes. Our simulation results confirms that the proposed SS-SNC provides overall smaller delay, smaller complexity and smaller overhead than SNC. Moreover, the proposed SS-SNC scheme achieves higher transmission rates and higher reliability than routing.

5.1.2 Outline of the chapter

This chapter is organized as follows. In Section 5.2, we present the system model for the multiple-hop lossy line networks. In Section 5.3., we proposed SS-SNC as a practical network coding solution for lossy line networks. Section 5.4 presents the semi-analytical investigation in lossy line network. We present our simulation results in Section 5.5 and conclude this chapter in Section 5.6.

5.2 Matricial model

Let us consider that a source node has K data packets to send to a sink node. Each packet is a column vector of length M over a finite field $\mathbb{F}_q$. The set of the data packets is denoted by the matrix,

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & . & . & . & \mathbf{s}_K \end{bmatrix}$$

where $\mathbf{s}_t$ is the t^{th} data packet. The source and the sink are connected with $n-2$ intermediate nodes, resulting into $n-1$ links joining the source to the sink. The links are modeled as delay-free memoryless erasure channels. A packet sent across the link $i, i \in \{1,2,...,n-1\}$ is either erased with the probability of ε_i or received without error. The capacity of the link i is therefore $1-\varepsilon_i$ and the capacity of the line network joining the source to the sink is $\min_i \{1-\varepsilon_i\}$.

Due to our requirement of low-delay, we assume that there is no feedback from the sink or from the intermediate nodes in the network. We also consider that packet transmissions occur at discrete time slots such that each node can transmit one packet per time slot. In the next section, we will discuss different coding schemes for transmitting the data packets from the source to the sink over the lossy line networks. We will assume that all the coding schemes run for a total of N time slots and every node (except the sink) transmits a packet in each time slot $t = 1,2,...,N$.

The complete encoding and decoding operations in the network can be modeled with a linear operator channel (LOC), using which an output unit at the sink can be expressed as a linear transformation of the input unit at the source. Let $\mathbf{Y} \in \mathbb{F}_q^{M\times N}$ be the output unit with N columns representing N received packets in N time slots. If the sink does not receive any packet in time slot t then the t^{th} column of $\mathbf{Y}$ should be considered as a zero column. We have,

$$\mathbf{Y} = \mathbf{XH} = \mathbf{SGH} \tag{5.1}$$

where $\mathbf{H} \in \mathbb{F}_q^{N\times N}$ is the transfer matrix for the line network, $\mathbf{G} \in \mathbb{F}_q^{K\times N}$ is the generator matrix and $\mathbf{X} = \mathbf{SG}$ is a generation of N coded packets transmitted from the source. The outer code is defined by $\mathbf{G}$ with code rate $\rho = \frac{K}{N}$. The outer code is used as a channel coding solution to recover the packet losses. A transfer matrix can be further expressed in terms of matrices representing network operations at every intermediate node and erasures in different links. For the line network, with $n-1$ links, the transfer matrix of the network is,

$$\mathbf{H} = \mathbf{D}_1\mathbf{T}_1...\mathbf{D}_i\mathbf{T}_i...\mathbf{D}_{n-1} \tag{5.2}$$

where $\mathbf{D}_i$ is an $N \times N$ diagonal matrix representing erasures in link i such that the diagonal component of $\mathbf{D}_i$ is zero with probability ε_i and is one with probability $1-\varepsilon_i$. The operation at the intermediate node i is given by the upper triangular matrix $\mathbf{T}_i \in \mathbb{F}_q^{N\times N}$. In particular, $\mathbf{T}_i$ also defines the inner code used for network coding to increase the transmission rates.

Now, the different coding schemes described below undergoes different encoding operations at the source (represented by $\mathbf{G}$) and at the intermediate nodes (represented by $\mathbf{T}_i$). In the following, we will discuss different coding schemes and the design of matrices $\mathbf{G}$ and $\mathbf{T}_i$ corresponding to these coding schemes. We assume that the coding vectors are attached in the packet headers so that the decoding matrix, $\mathbf{GH}$, is known at the sink [5]. The decoding is only possible when K innovative packets are received by the sink, i.e., $rank(\mathbf{GH}) = K$.

5.3 Coding scheme: line network

5.3.1 Encoding at the source node

The SS-SNC encoder sends K data packets in the first K time slots (systematic phase) followed by $N-K$ random linear combinations of data packets in the next $N-K$ time slots (non-systematic phase). Here, $\mathbf{X}_1 = \mathbf{SG}$ represents K systematic packets and $N-K$ coded packets transmitted by the SNC encoder during N consecutive time slots. The generator matrix $\mathbf{G} = \begin{bmatrix} \mathbf{I}_K & \mathbf{C} \end{bmatrix}$ consists of identity matrix $\mathbf{I}_K$ of dimensions K and $\mathbf{C} \in \mathbb{F}_q^{K\times N-K}$ with elements chosen randomly from a finite field $\mathbb{F}_q$.

5.3.2 Re-encoding at the intermediate nodes

The SNC re-encoder at every intermediate node performs re-encoding operations in each time slot and sends N packets to the next node. Let $\mathbf{X}_{i+1} = \mathbf{X}_i\mathbf{D}_i\mathbf{T}_i$ represents N packets transmitted by the SNC re-encoder of i^{th} node (first node is the source node) during N consecutive time slots where $\mathbf{D}_i \in \mathbb{F}_q^{N\times N}$ represents erasures from the i^{th} node to the $(i+1)^{th}$ node and $\mathbf{T}_i \in \mathbb{F}_q^{N\times N}$ represents re-encoding operations at $(i+1)^{th}$ intermediate node. The re-encoding matrix $\mathbf{T}_i$ is modeled as follows. Each node i is linked with a row vector $\mathbf{f}_i$ which contains K elements. The t^{th} column of vector $\mathbf{f}_i$ contains the number of times packet $\mathbf{s}_t$ has been transmitted from the node i. The priority for transmission is given to the packet with the smallest value in $\mathbf{f}_i$. As soon as the systematic packet s_t is transmitted from the node i, the t^{th} column of $\mathbf{f}_i$ is updated. Now, during the systematic phase, if a systematic packet $\mathbf{s}_t$ is lost in link j and in time slot t; i.e., $\mathbf{D}_j(t,t) = 0$ then the t^{th} column of matrices $\mathbf{T}_i, i < j$ is the same as the t^{th} column of identity matrix $\mathbf{I}_N$ (which represents that the systematic packet is being forwarded before it is lost in link j) and the t^{th} column of matrices $\mathbf{T}_i, i \geq j$ is the l^{th} column of identity matrix $\mathbf{I}_N$. Here l is the index of the column which is selected from one of the first t non-zero columns of matrix $\mathbf{B}_i = \mathbf{D}_1\mathbf{T}_1 \ldots \mathbf{T}_{i-1}\mathbf{D}_i$ where $\mathbf{SGB}_i$ represents the packets stored in the buffer at node i. The packet corresponding to column l in $\mathbf{B}_i$ should have the smallest weight in vector $\mathbf{f}_i$. If there are several columns with packets with the same smallest value in $\mathbf{f}_i$, then one of them is selected randomly. Hence, by using matrix $\mathbf{B}_i$ and vector $\mathbf{f}_i$, we are able to model SS-SNC where the intermediate nodes can use weighted round-robin scheduling and send the systematic packets to the next node. If a systematic packet $\mathbf{s}_t$, transmitted by the source in time slot t, is not lost in any intermediate links, then the t^{th} column of all the matrices $\mathbf{T}_i$ is the same as the t^{th} column of $\mathbf{I}_N$. For the non-systematic phase, SS-SNC behaves similar to RNC, and all the non-zero elements of last $N-K$ columns of $\mathbf{T}_i$ are chosen randomly from a finite field $\mathbb{F}_q$.

5.3.3 Decoding at the sink node

The output at the sink is $\mathbf{Y} = \mathbf{SGH}$ where $\mathbf{H} = \mathbf{D}_1\mathbf{T}_1...\mathbf{D}_i\mathbf{T}_i...\mathbf{D}_{n-1}$ represents the transfer matrix of the network. We assume that the coding vectors are attached in the packet headers so that the matrix $\mathbf{GH}$ is known at the sink. The decoding is progressive using gaussian jordan algorithm as in [27]. In the progressive decoding, the sink uses Gauss Jordan algorithm [**?**] and starts decoding as soon as it receives the first packet. All the K data packets are recovered when K innovative packets are received at the sink, i.e., $rank(\mathbf{GH}) = K$.

5.4 Semi-analytical analysis

5.4.1 Reliability

The reliability is defined as $1-\eta$ where η is as an effective erasure rate for the data packets that is achieved after the overall coding and decoding operations. For example, if all the data packets are recovered then $\eta = 0$ and if nothing is recovered then $\eta = 1$. The figure will be included here. The general conclusions on the reliability for the lossy line networks are as follows.

- When the code rate is smaller than the capacity, network coding can achieve 100% reliability in lossy line networks. However, the gap, between the capacity and the exact code rate when the 100% reliability is achieved, increases with the increase in number of links in the line network.

- Irrespective of the value of code rate (whether it is smaller or higher than the capacity), SNC always provides reliability higher than the routing. This is not valid in the case of RNC which provides almost zero reliability when the code rate is higher than the capacity. This is because when the code rate is higher than the capacity then the sink does not receive sufficient packets to decode the complete block. Hence, nothing is recovered in RNC. In SNC, the correctly received systematic packets are always recovered and so SNC provides higher reliability than the routing.

5.4.2 Achievable rates

It is defined as $R = \frac{K(1-\eta)}{N} = \rho(1-\eta)$ where R is in packets per time slots, $(1-\eta)K$ is the number of data packets recovered after decoding and $(1-\eta)$ is the reliability provided by the coding scheme. R is upper bounded by the capacity of the line network which is $\min_i \{1-\varepsilon_i\}$ packets per time slot. The figure will be included here. The general conclusions on the achievable rates for the lossy line networks are as follows.

- In general, network coding provides maximum achievable rates higher than the rates offered by routing in the line networks. As the number of links increases, the gain increases. This is because several packets are lost when there are many intermediate lossy links. As routing does not employ any erasure correction scheme, achievable rates are very small as compared to the capacity of these networks.
- Finally, as the block length increases, network coding provides higher maximum rates in the line networks. The maximum achievable rates from network coding are closer to the capacity when the block length increases. This is in agreement with the information results reported in e.g. [11].

5.5 Practical application

In this section, we will present the practical application of SS-SNC. It is applicable at different layers of the protocol stack. In this chapter, we will investigate the application of SS-SNC at the application layer of the protocol stack. The coding parameters are selected accordingly. These parameters are specified later in Section 5.5.1.1.

5.5.1 Simulation results

To evaluate the results, we will first define the different performance metrics used in this chapter to evaluate the performance of various coding schemes.

- Reliability and achievable rates: They are defined in Section 5.4.
- Per-packet delay: The per-packet delay is measured as the average time needed per packet in recovering all the source data packets. If a packet $\mathbf{s}_t$ is recovered at the sink at time $t_r \geq t$ then packet $\mathbf{s}_t$ incurs delay δ_t where, $\delta_t = t_r - t$. For the block of K packets, the average value of per-packet delay is given as, $\triangle = \frac{\sum_{t=1}^{K} \delta_t}{K}$. Note that the delay is evaluated only for the packets which are recovered at the sink.
- Decoding complexity: The decoding complexity [22] is measured by the average number of operations that are needed per packet in recovering all the source data

packets. The number of multiplications (n_{α}^{GE}) and the number of additions (n_{β}^{GE}) required using Gaussian Elimination are the functions of systematic packets reaching the sink [45]. We compare the decoding complexity in different coding schemes by comparing the average number of multiplications per packet, given by $\frac{n_{\alpha}^{GE}}{KM}$. Similar conclusions are valid for number of additions as well.

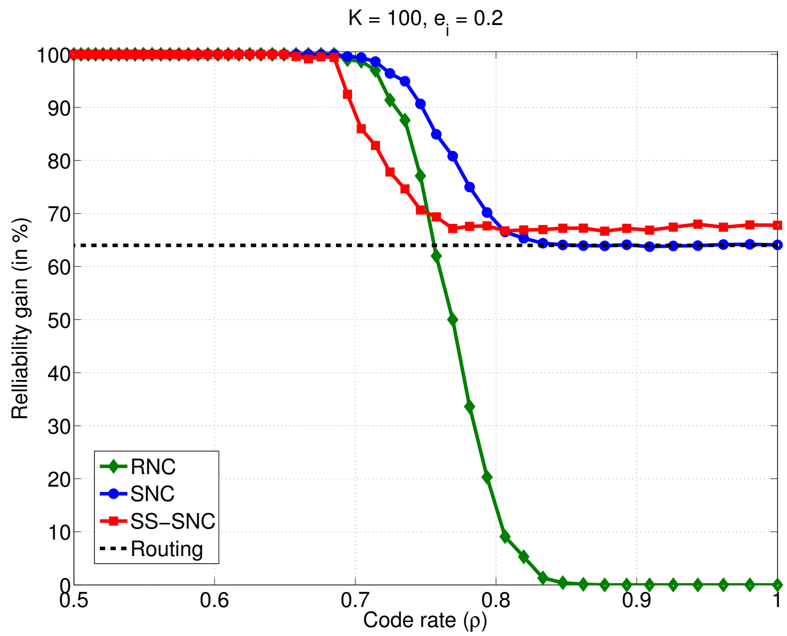
- Overhead: The overhead is due to the coding coefficients attached with the packet [5]. In SS-SNC, each node transmits only systematic packets in the first K time slots and only coded packets in the last $N-K$ time slots. The overall overhead is given by $\frac{K(N-K)(n-1)}{MN(n-1)} = \frac{\rho N(1-\rho)}{M}$ where $N(n-1)$ are the total packets transmitted from $(n-1)$ nodes during N time slots and $(N-K)(n-1)$ are the coded packets transmitted during non-systematic phase of $N-K$ time slots. Each coded packet consists of the coding coefficients of K bytes.

5.5.1.1 Simulation setup

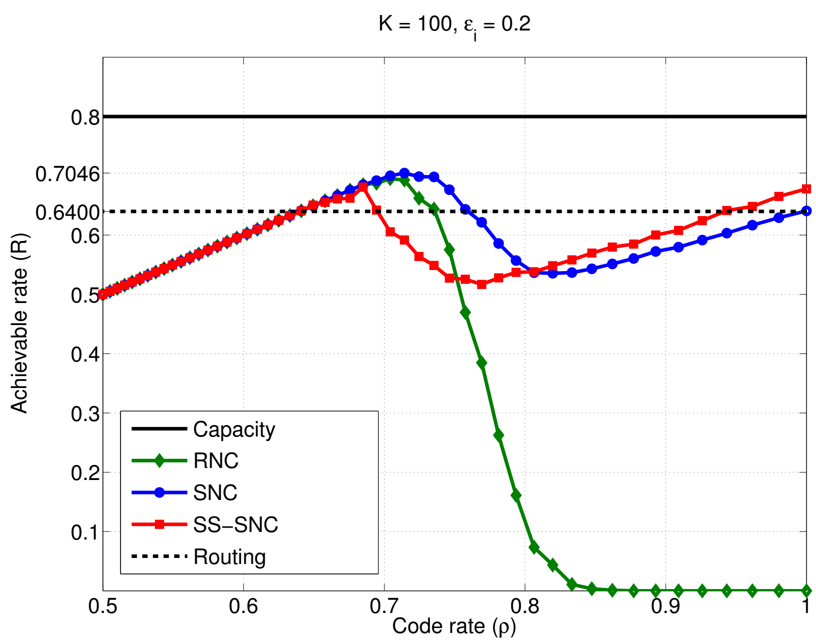
We implement the different coding schemes and perform simulations on MATLAB. In the simulation setup, we consider different cases of line networks with erasure rates of 0.2 and 0.6. We have observed these values of erasure rates in several literature work to evaluate the performance of different coding schemes [19], [17], [25]. In each case, we compare following three coding schemes: (i) SS-SNC (ii) RNC and (iii) SNC. We assume that the source generates packets of length $M = 1500$ symbols over $GF(2^8)$, where each symbol corresponds to 1 byte. We set the transmission bit rate of the source as $r_s = 200kbps$ such that the packet rate is $B_p = 16.66$ packets/seconds. We consider state-of-the-art video codecs with video frames grouped into Groups of Pictures (GoPs). Each GoP contains several packets. The codec outputs each GoP in a fixed time T_{GoP} such that the source block $K \approx \frac{r_s \times T_{GoP}}{M \times 8}$. Two configurations of codecs are considered with $T_{GOP} = 3$ seconds and $T_{GOP} = 6$ seconds. The source blocks corresponding to these configurations are $K = 50$ and $K = 100$ respectively. Several values of code rates are considered for comparison. The size of coded block; i.e., N varies with the change in code rate. In each case, we conduct 1000 experiments and take the average to evaluate all the four performance metrics.

5.5.1.2 Results

Reliability and achievable rates: Figure 5.1 and Figure 5.2 show the results on reliability and achievable rates for symmetric line network with different number of links. Table 5.1 and Table 5.2 show the results on the maximum achievable rates for symmetric and non-symmetric cases respectively. We will present the key conclusions on the application part which are in line with the general conclusions pointed out in Section 5.4.

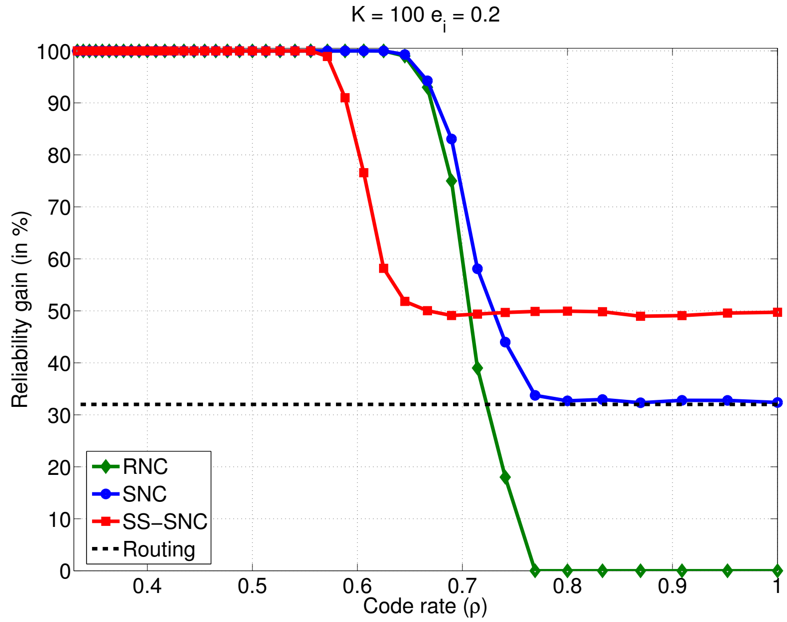


(a) Reliability

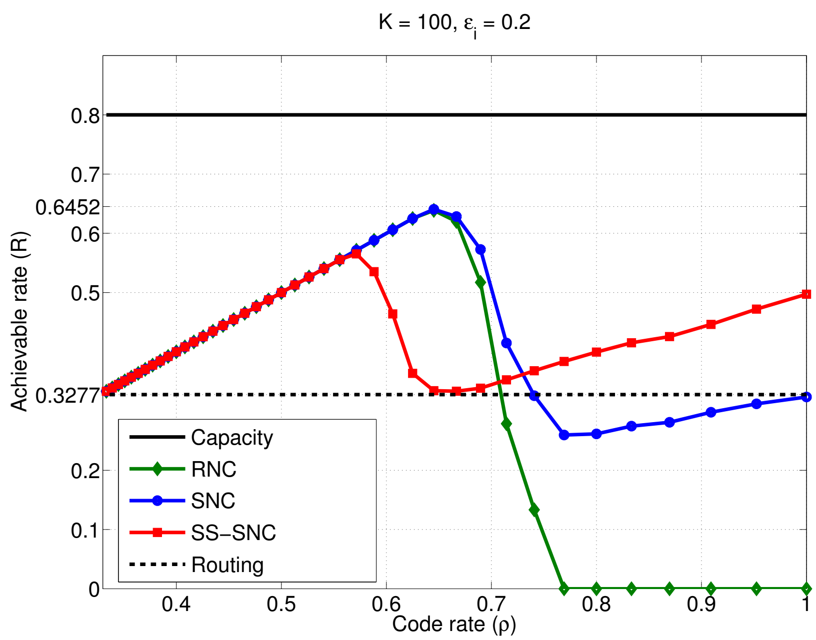


(b) Achievable rates

Figure 5.1: Reliability and achievable rates for the line network with two intermediate links



(a) Reliability



(b) Achievable rates

Figure 5.2: Reliability and achievable rates for the line network with five intermediate links

Capacity = 0.8

K	Links	Routing	RNC	SNC	SS-SNC
50	2	**0.6400**	0.6611	0.6739	**0.6599**
100	2	**0.6400**	0.6951	0.7046	**0.6810**
50	5	**0.3277**	0.6037	0.6055	**0.5407**
100	5	**0.3277**	0.6387	0.6452	**0.5653**

Table 5.1: Maximum achievable rates for symmetric line network

Capacity = 0.4, K = 100

Links	Erasures	Routing	RNC	SNC	SS-SNC
2	$\varepsilon_1 = 0.6, \varepsilon_2 = 0.2$	**0.3200**	0.3379	0.3395	**0.3354**
2	$\varepsilon_1 = 0.2, \varepsilon_2 = 0.6$	**0.3200**	0.3362	0.3376	**0.3338**
5	$\varepsilon_1 = 0.6, \varepsilon_i = 0.2$	**0.1638**	0.3345	0.3361	**0.3331**
5	$\varepsilon_i = 0.2, \varepsilon_5 = 0.6$	**0.1638**	0.3333	0.3338	**0.3285**

Table 5.2: Maximum achievable rates for non-symmetric line network

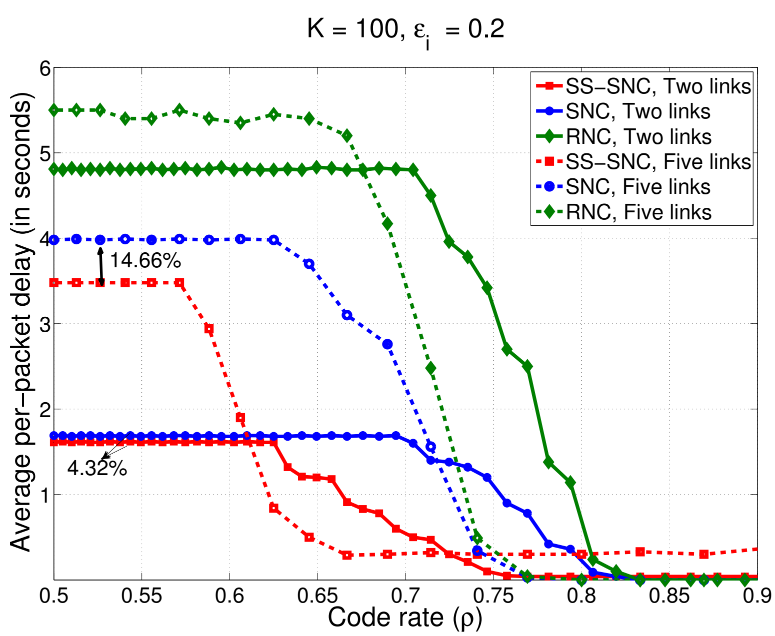


Figure 5.3: Per-packet delay in line network with different number of links

- **SS-SNC provides maximum achievable rates higher than the rates offered by routing. As the number of links increases, the gain increases (Table 5.1).** This is because several packets are lost when there are many intermediate lossy links. As routing does not employ any erasure correction scheme, achievable rates are very small as compared to the capacity of these networks. These rates are improved when SS-SNC is used at the intermediate nodes. In particular, for $K = 100$, SS-SNC provides up to 6.41% higher rates than routing when there are two intermediate links and up to 72.51% higher rates than routing when there are five intermediate links.

Remark 4. **An additional good feature of SS-SNC is that when the code rate is higher than the capacity, SS-SNC provides higher rates than both RNC and SNC. Furthermore, in several instances, SS-SNC provides rates higher than routing whereas both SNC and RNC performs worst than the routing when code rate is higher than the capacity (**Figure 5.1 and Figure 5.2**).** This is because when the code rate is higher than the capacity, the sink does not receive sufficient packets for decoding the complete block. In case of RNC, nothing is recovered and therefore both rates and reliability is zero. In case of SNC and SS-SNC, finite rates and reliability are still achievable due to the flow of systematic packets. In particular, a higher flow of systematic packets is guaranteed in SS-SNC, which results into higher rates and higher reliability than SNC.

- **As the block length increases, SS-SNC provides higher maximum achievable rates (Table 5.1).** Our results show that the maximum achievable rates from SS-SNC are closer to the capacity when the block length increases. These results are in coherence with the information theoretical results of [**?**], which state that the random linear network coding strategy can achieve the capacity of the line network when block length tends to infinity. In particular, for the line network with two intermediate links, the maximum achievable rate by SS-SNC is 0.6599 (17.51% less than the capacity) when K =50 and the maximum achievable rate by SS-SNC is 0.6810 (14.87% less than the capacity) when $K = 100$.

- **For the non-symmetric cases, the maximum achievable rates offered by SS-SNC are slightly higher when the first link has higher erasures (Table 5.2).** This is because when the first link has higher erasures, there are more re-encoding opportunities at the intermediate nodes which facilitates the recovery at the sink node. However, if the first link has smaller erasures and the last link has higher erasures, then there are fewer opportunities of re-encoding at the intermediate nodes. In this case, most of the packets are lost in the last link and hence, we can observe slightly smaller maximum achievable rates.

Per-packet delay, decoding complexity and overhead:

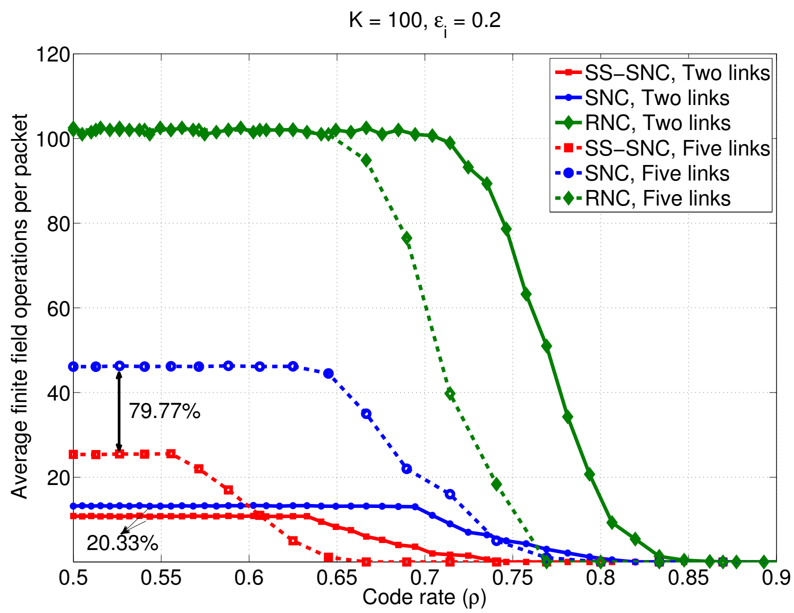


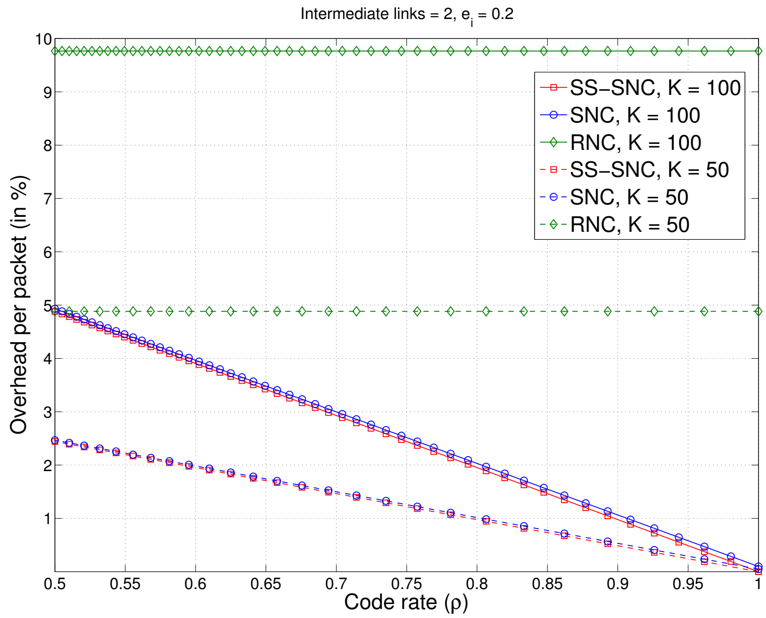
Figure 5.4: Decoding complexity in line network with different number of links

Figure 5.3, Figure 5.4 and Figure 5.5 show the results on per-packet delay, decoding complexity and overhead for the line networks. Following points are concluded from these figures:

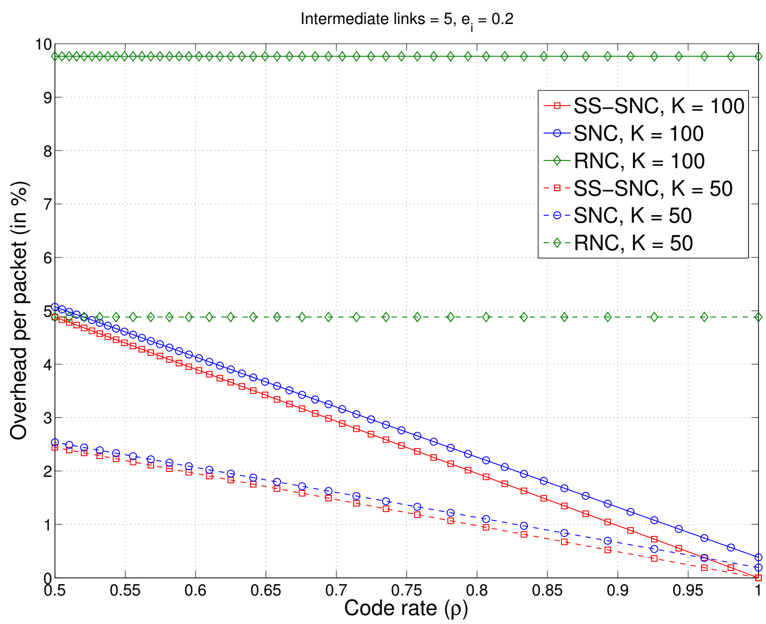
- **When the code rate is higher than the capacity, there is no per-packet delay in any coding scheme.** This is because the sink is not receiving enough packets for decoding and the delay is evaluated only for the packets which are recovered. In case of RNC, nothing is recovered. In case of SNC and our proposal, only systematic packets are recovered. These systematic packets are recovered instantly and hence the per-packet delay is almost zero.

- **When the code rate is smaller than the capacity, the per-packet delay converges to a constant value.** This is because when the code rate is slightly smaller than the capacity, the sink starts receiving sufficient packets for decoding. Now, if the code rate is further reduced, then the extra coded packets due to the decrease in the code rate will not be used for decoding and the sink will simply discard them. Therefore, the sink will not wait for these extra packets in order to start decoding and they will not add any value to the per-packet delay. Hence, when code rate is smaller than the capacity, the per-packet delay converges to a constant value.

- **SS-SNC achieves smaller delay than both RNC and SNC.** This is because our proposal guarantees a higher flow of systematic packets than both RNC and SNC



(a) Line network with two intermediate links



(b) Line network with five intermediate links

Figure 5.5: Overhead of attaching coding coefficients in the line network with two and five intermediate links

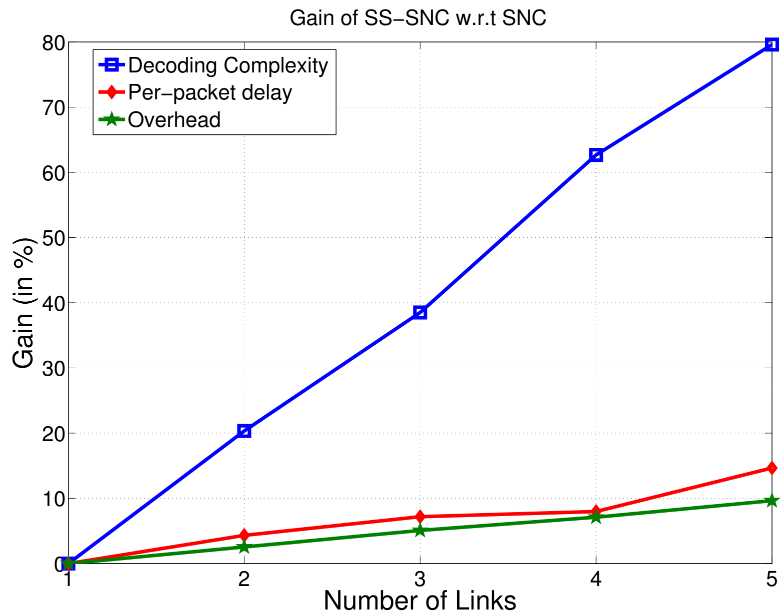


Figure 5.6: Gain of SS-SNC w.r.t SNC for different performance metrics

schemes. These packets are recovered instantly and therefore overall per-packet delay is minimum from our proposal. In particular, it is shown in the figures, that our proposal could provide up to 15% smaller per-packet delay than SNC.

- **Similar conclusions have been drawn for the decoding complexity** in Figure 5.4: (i) complexity is almost zero when the code rate is higher than the capacity, (ii) when the code rate is smaller than the capacity, its converges to a constant value and (iii) SS-SNC achieves smaller complexity than both RNC and SNC. In particular, when there are five links, SS-SNC is shown to have 79.77% fewer finite field operations than SNC.

- **SS-SNC also provides smaller overhead than SNC in sending coding coefficients** (Figure 5.5). Note that to reduce this overhead, the another way is to use pseudo random network coding (PRNC) [46] which sends a seed of a pseudo-random generator that produces a sequence of network coding coefficient. It is an efficient solution when there is only the source and the sink. However, when there are several intermediate nodes, the extension of PRNC requires a synchronization of all the nodes and need of large look-up tables which could lead to high computational complexity. Nevertheless, using the proposed coding scheme, overhead due to sending coefficients could be reduced up to 1%-2%.

- **Furthermore, when there are several links, advantages of SNC diminishes due to the smaller number of systematic packets reaching the sink while SS-SNC still guarantees higher number of systematic packets and provide smaller decoding complexity**, **smaller per-packet delay and smaller overhead**. As a consequence, as shown in the Figure 5.6, the gain, in terms of both complexity and overhead, achieved by SS-SNC w.r.t SNC increases linearly with increasing number of links. It is shown that SS-SNC can provide up to 79.77% smaller decoding complexity, 15% smaller per-packet delay and 9.64% smaller overhead than SNC.

5.6 Conclusions

In this chapter, we have proposed the novel SS-SNC scheme that can overcome limitations of SNC in general lossy line networks. SS-SNC is shown to provide smaller delay, smaller complexity and smaller overhead than SNC. In addition, it is also shown that SS-SNC provides higher throughput and higher reliability than routing. Finally, the proposed simple smart scheduling at the intermediate nodes makes SS-SNC as a robust network coding solution that can be applied across different layers of the protocol stack and can be used to recover packets losses in different wireless systems. We conclude that in the networks with several intermediate nodes, where both RNC and SNC are not practically useful as they incur high delay and complexity, SS-SNC provides a comprehensive practical network coding solution for the future wireless networks. Finally, the work in this chapter leads to the following publication.

Journals

1. **P. Saxena** and M. A. Vázquez-Castro, “DARE: DoF-Aided Random Encoding for Network Coding over Lossy Line Networks” under review in IEEE wireless communications letters, 2014.

Chapter 6

Contributions to IRTF

6.1 Contributions and Outline

The Internet Research Task Force (IRTF) includes several research groups working on topics related to internet protocols, applications, architecture, technology etc. IRTF promotes research and development for the evolution of internet and its better performance. Recently, network coding has been studied to improve network's throughput, reliability, efficiency, scalability etc. Inspired by the several benefits provided by network coding, network coding research group (NWCRG) is one of the groups in IRTF that focuses on the research of the network coding methods that can benefit internet communication.

6.1.1 Contributions of the chapter

The main contribution of this chapter is to describe the recent network coding contributions to IRTF. We discuss several recent network coding schemes and discuss briefly their objectives and contributions. We show that the research conducted in this thesis is not undertaken in isolation but in coherence with several other work which share similar objectives as addressed in this thesis.

6.1.2 Outline of the chapter

This chapter is organized as follows. In Section 6.2, we present the existing contributions to IRTF/IETF on FEC using RS codes. We describe recent network coding contributions and network coding architecture for different use cases in Section 6.3. In Section 6.4., we conclude this chapter.

6.2 Existing contributions to IRTF/IETF on FEC using RS codes

The use of network coding for FEC is recent in the internet community. However, the use of application layer erasure codes in Internet Engineering Task Force (IETF) has already been standardized in the RMT [47] and the FECFRAME [48] working groups. In this section, we describe some of the existing contributions to IRTF/IETF on FEC using RS codes.

6.2.1 Application layer FEC with RS codes

RS codes belong to the class of MDS codes and offer optimal erasure correction performance. [49] defines the use of RS codes over the real time transport protocol (RTP) for protecting application layer data units (ADUs). [49] also describes the RTP payload format for using the RS codes. The method described in the document is generic to all media types and provides the sender with the flexibility of deciding if FEC protection is required and if so, how many source packets and FEC packets are to be used in the block.

The encapsulation process of using RS coding for FEC of ADUs over RTP is shown in Figure 6.1. A source block, in the form of table, consists of k ADUs in k columns. The number of rows in the source block is $E+2$ where E is the length of the largest ADU. Zeros are filled in all the columns (except the column containing largest ADU) such that each column is completely filled. Each column can be considered as a source packet. The first two bytes of all the columns in the source block contain the length of the ADU. ADUs are encapsulated into RTP packets. Each RTP packet, that encapsulates ADU, contains RTP header. Note that first two bytes and zero paddings are not sent over the network.

The FEC block contains $n-k$ columns with $n-k$ FEC packets. These FEC packets are generated using RS coding over k source packets. FEC packets are then encapsulated into RTP packets. Each RTP packet contains RTP payload, RTP header and FEC payload ID. This FEC payload ID is used for signaling the coding parameters like source block ID, FEC packet ID, values of k and n etc. At the receiver, the values of coding parameters are extracted from the FEC payload ID. Now, if ADUs are lost then the complete columns are lost. So, if FEC decoding succeeds, the receiver recovers ADUs by filling the erased columns. Initial two bytes are used to remove zero padding from the source packets to recover ADUs.

6.2.2 Comparison of FEC framework at different layers of the protocol stack

Note that the FEC framework can be used in different ways at different layers of the protocol stack. In the previous subsection, we describe the use of RS codes for FEC of application layer data packets. RS codes are also specified for FEC of IP packets in DVB-S2

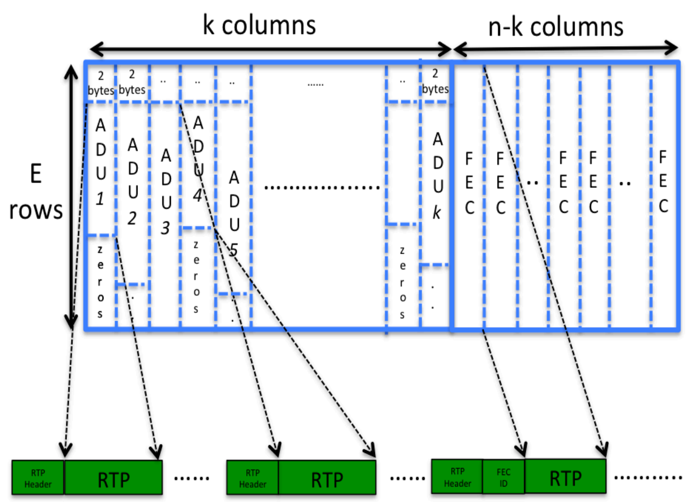


Figure 6.1: Forward erasure correction for application layer data units over RTP protocol

standard over link layer GSE protocol [42]. In chapter 4, we have provided the possible extension of this framework to use the network coding for FEC in DVB-S2 over GSE protocol. The FEC framework in DVB-S2 has some similarities and differences as compared to the FEC framework in application layer which is described in the previous subsection. It is imperative to understand the similarities and differences between the two FEC framework as they both provide different ways of implementing RS codes for erasure protection. The encapsulation process of using RS coding for FEC of network layer packets over GSE has been discussed in Chapter 4. In this section, we will present the corresponding encapsulation process as shown in Figure 6.2. We will provide table 6.1 which list the similarities (in black) and differences (in red) of using FEC frameworks at different layers of the protocol stack.

6.3 Current focus of IRTF on network coding

The NWCRG in IRTF focusses on network coding research to increase the performance and efficiency of the internet network. The interaction of network coding with different layers of the protocol stack and its implications on security, privacy, network usage etc are also the topics of interest in which network coding can benefit the internet communication. In this section, we describe some of the recent network coding contributions to IRTF.

	Link-layer Forward error correction (FEC) with RS codes	Application-layer Forward error correction with RS codes
Losses	Effective in front of random and burst losses	Effective in front of random and burst losses
Re-transmissions	No	No
Layer in the protocol stack	FEC for network layer IP packets over Generic Stream Encapsulation (GSE) protocol	FEC for application layer data units (ADUs) over Real-Time Transport Protocol (RTP) protocol
Flexibility	- Flexibility to decide if FEC protection is required - Flexibility to decide how many source packets and how many FEC packets are required - No unequal protection	- Flexibility to decide if FEC protection is required -Flexibility to decide how many source packets and how many FEC packets are required - Unequal Protection is available
Encapsulation	- A source block consists of network layer IP packets. - IP packets may fill one or more than one column. - M is the number of rows in the table - M is constant from block-to-block - IP packets are encapsulated into GSE packets. Each GSE packet contains GSE header. - Each GSE packet contains CRC to check the errors at the receiving end - A FEC block contains n-k columns with n-k FEC packets - FEC packet is encapsulated into GSE packet with a GSE header - GSE header carries information on FEC packet ID, source block ID, values of k and n.	- A source block consists of application data units (ADUs) - Fist k columns contain k application data units (ADUs). - E is the number of rows in the block - E is the length of largest ADU. It changes from block-to-block - The first two bytes of first k columns contain the length of ADU - ADUs are encapsulated into RTP packets. Each RTP packet contains RTP header. First two bytes and zero padding are not encapsulated into RTP packets. - A FEC block contains n-k columns with n-k FEC packets - FEC packet is encapsulated into RTP packet with RTP header and FEC payload ID - FEC payload ID carries FEC packet ID, source block ID, values of k and n.
Decoding	- If an IP packet is lost then the partial or complete column is lost. - If FEC decoding succeeds, the receiver recovers IP packets by filling completely or partially erased columns.	- If an ADU is lost then the complete column is lost - If FEC decoding succeeds, the receiver recovers recover ADUs by filling completely erased columns. Zero padding is removed by using initial 2 bytes.
Media type registration	- Audio, - Video	- Audio, Video, Text
Security considerations	Yes	Yes

Table 6.1: Comparison of FEC framework at different layers of the protocol stack

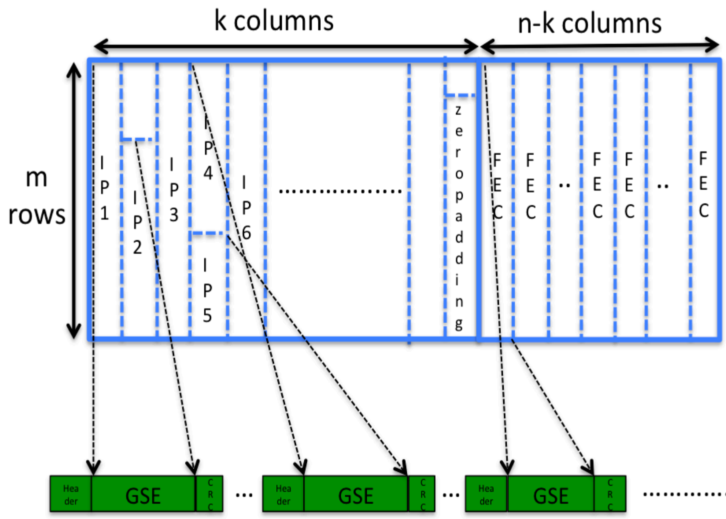


Figure 6.2: Forward erasure correction for network layer IP packets over GSE protocol

6.3.1 Recent network coding contributions to IRTF

6.3.1.1 Random Linear Network Coding (RNC)

RNC [7] is a capacity-achieving network coding scheme for both unicast and multicast connections [11]. RNC provides practical application of network coding in the distributed manner and for the networks whose topologies are not known. Although RNC is a capacity achieving code and provides several benefits, it does not utilize efficiently the computational resources. It has three main limitations: high delay, high complexity and high overhead as described in Chapter 2.

6.3.1.2 Tetrys

Tetrys [50] is on-the-fly network coding protocol to recover packet losses. The main novelty of the Tetrys is the use of elastic window algorithm. In Tetrys, the size of an encoding window may change periodically based on the receiver's feedback. The encoding window contains data packets which are not received and/or acknowledged by the receiver. Tetrys generates coded packets which are the linear combination of all the non-acknowledged data packets in the elastic coding window. Tetrys provides smaller encoding complexity by allowing encoder to reduce the elastic encoding window size removing all the acknowledged data packets. The current Tetrys framework can be used for unicast, multicast and broadcast communications.

However, there are two main limitations of Tetrys. First, Tetrys requires feedback in order to adjust the coding window size. In general, feedback based mechanisms are not efficient for communication networks with long round-trip times like satellite networks. Moreover, feedback may not result into efficient use of resources when delay-sensitive applications like audio/video streaming are used. Second, the current approach of Tetrys is useful only in an end-to-end fashion. Its extension to hop-by-hop network coding is resource-intensive because it requires the management of multiple feedbacks and multiple elastic windows at several intermediate nodes.

6.3.1.3 BATS

BATS [17] codes are used for efficient transmission of large files using chunks (generations or classes) in order to reduce the encoding and decoding complexity. They extend the idea of fountain codes to the realm of networks and utilizes both network coding and the properties of overlapping chunks by using belief propagation decoding where packets from the already decoded batches can help to decode the packets from the other batches.

BATS codes are designed with the objective to reduce the complexity for transmitting a large file by diving a file into batches and performing coding operations within those batches. However, they may not be efficient for streaming applications because multimedia streaming like video streaming usually has small block sizes, hence, dividing these small blocks into batches may further add to unwanted complexity and delay.

6.3.1.4 Structured RLC codes

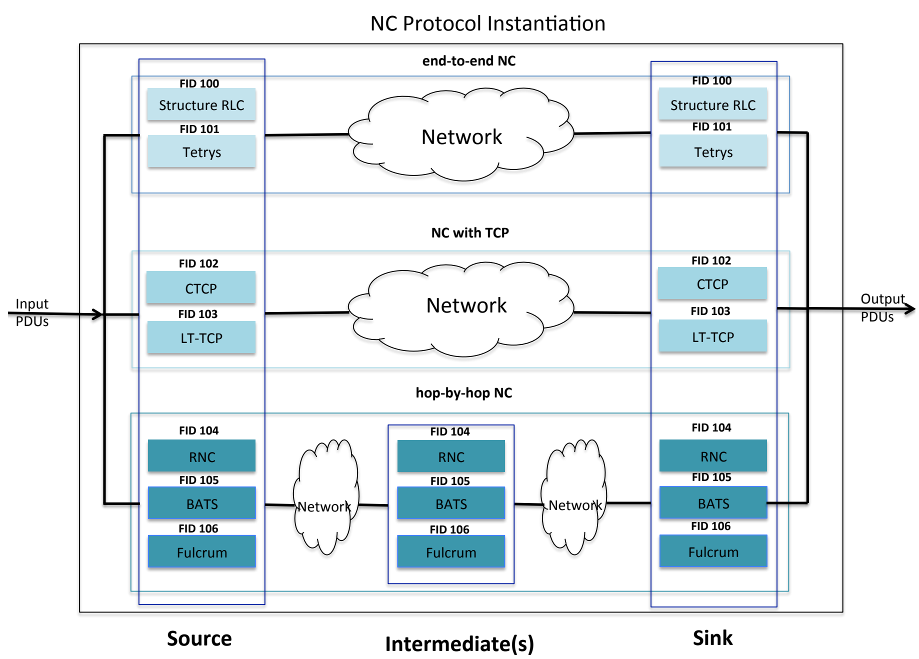
Structured RLC codes [51] are based on the idea of mixing binary and non binary coefficients together. By using binary coefficients, a structured gaussian elimination decoding can be used at the receiver increasing the decoding speed and by using limited non-binary coefficients a good erasure recovery performance is achievable. However, the proposed solution can only used in end-to-end fashion.

6.3.1.5 Fulcrum Network Codes

Fulcrum network codes [26] provide multimedia delivery to heterogeneous receivers with different processing capabilities. Fulcrum network codes design is based on the concatenation of two codes on separate finite fields. These two codes are (i) the outer code which is constructed using finite field of higher size and (ii) the inner code which is constructed using the binary field. The receiver, which decodes in finite field of higher size, achieves higher throughput and higher reliability but it also spends a lot of computational resources due to high decoding complexity. Fulcrum network codes allow receivers with limited computational resources to decode in the binary field in order to reduce the decoding complexity and spend less resources.

Recent contributions to IRTF	Main Novelty	Benefits	Limitations	Systematic Coding
RNC	coding using random coefficients	asymptotically capacity achieving code, Higher throughput than routing, distributed network coding	high decoding complexity, high delay and high overhead	NO
BATS	coding with overlapping batches	smaller decoding complexity, smaller buffer requirement	delay and complexity for video streaming applications (or others) with small sized source blocks	NO
Structured RLC	mix binary and non binary coefficients	smaller decoding complexity	only end-to-end (no in-network) coding	Yes
Tetrys	Elastic coding window	smaller encoding complexity, adaptive network coding	requires feedback, only end-to-end	Yes
Fulcrum Network Codes	Concatenation of two codes based on separate finite fields	multimedia delivery to heterogenous receivers, smaller decoding complexity	smaller reliability and throughput, intermediate nodes are allowed only to encode using binary fields	Outer code can be systematic, Inner code uses RNC
CTCP	Variable block length network coding for erasure correction with modified AIMD algorithm for congestion control	higher throughput than TCP for networks with high link losses and high RTT	requires feedback, end-to-end (no in-network) coding	NO
LT-TCP	Using NC for FEC for link losses, distinguishing congestion and link losses using ECN	higher throughput than TCP for networks with high link losses and high RTT	requires feedback, end-to-end (no in-network) coding	Yes

(a) Table with different NC schemes



(b) NC protocol instantiation

Figure 6.3: Comparison of different network coding schemes

However, the main limitation of Fulcrum network codes is that the receivers, which decode in the binary field, achieve smaller throughput and smaller reliability. Moreover, Fulcrum network codes allow intermediate nodes to code only in the binary fields. This will result into smaller throughput and smaller reliability even for the receivers which have better computational resources and decode using the higher finite field size.

6.3.1.6 Network coding with TCP: Coded TCP (CTCP) and Loss Tolerant - TCP (LT-TCP)

CTCP [52] and LT-TCP [53] investigate the use of network coding with TCP for congestion control and erasure protection. TCP can not distinguish between congestion losses and link losses and performs worst when there are high link losses. Both CTCP and LT-TCP show that higher throughput is achievable by using network coding with TCP. LT-TCP uses explicit congestion notification (ECN). ECN distinguishes losses due to congestion and TCP gives response only to those losses which are due to congestion while NC is used as a FEC coding to recover link losses. CTCP uses variable block length network coding for erasure correction with modified additive-increase/multiplicative-decrease (AIMD) algorithm using feedback loop for congestion control.

Figure 6.3 shows the comparison of the different network coding schemes discussed in the previous subsections. These coding schemes are classified in terms of their main novelty, benefits and limitations. Figure 6.3b also shows different FEC building blocks (BB) with corresponding FEC encoding ID for NC protocol instantiation [54]. NC schemes are identified by an FEC Encoding ID where the FEC Encoding ID allows receivers to select the appropriate decoder/re-encoders. The different NC protocol could be used for different applications and for different purposes. For example, it is shown in Figure 6.3b that different NC protocols for end-to-end NC, hop-by-hop NC or NC with TCP can be instantiated.

6.3.1.7 Several other ongoing contributions to IRTF on network coding

We list some of the ongoing work related to network coding in this subsection.

- Network coding for broadcasting [55].
- Impact of Virtualization and SDN on Emerging Network Coding [56].
- Network coding for distributed cloud storage [57].
- Kodo [58] is a C++ library used for research on implementation of network codes.
- Network coding for bi-directional IP-traffic over transparent satellites using XOR operations [59].

IPR ID	IETF Document	Person	Patent
2325	BATS codes	Raymond W. Yeung	Subset coding for communication systems US 20120128009 A1
2303	Presentation by Tracey Ho in Vancouver on 11/7/13	Tracey Ho	Distributed reed-solomon codes for simple multiple access networks US 20130297990 A1
2296	IRTF meeting in Orlando 3/11/13	Muriel Medard	Method and apparatus providing network coding based flow control US 8130776 B1
2184	**IRTF meeting Orlando on 3/11/13**	**Muriel Medard**	**Randomized distributed network coding US 7706365 B2**
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Secure Network Coding for Multi-Resolution Wireless Video Streaming US 20110243324 A1
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Random Linear Network Coding for Time Division Duplexing US 20120236763 A1
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Traffic Backfilling Via Network Coding In A Multi-Packet Reception Network US 20130107764 A1
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Coding Approach For A Robust And Flexible Communication Protocol, US 20130114481 A1
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Multi-Path Data Transfer Using Network Coding, WO 2013116456 A1
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Minimum-cost routing with network coding , Appl. No.: 11/027,889
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Random linear coding approach to distributed data storage, Appl. No.: 11/026,550
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Feedback-based online network coding US 8068426 B2
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Method and apparatus providing network coding based flow control US 8130776 B1
2183	IRTF meeting Berlin on 7/29/13-7/31/13	Muriel Medard	Network coding for multi-resolution multicast, Appl. No.: 12/846,292

Table 6.2: Network coding related IPR disclosures in IETF

- Network coding for content-based networks [60].

Finally, we have also outline the different intellectual property rights (IPR) in IETF and the corresponding patents in table 6.2.

6.3.2 Network coding architecture

Recently, some ideas are presented in IRTF for the network coding architecture [54]. The NC architecture should accommodate several use cases for practical application of network coding. NC can be used at several layers of the protocol stack as shown in Figure 6.4. The use of NC at application layer is kernel-agnostic and this case is of main interest for the application developers. To implement NC at transport or network layer, kernel access is

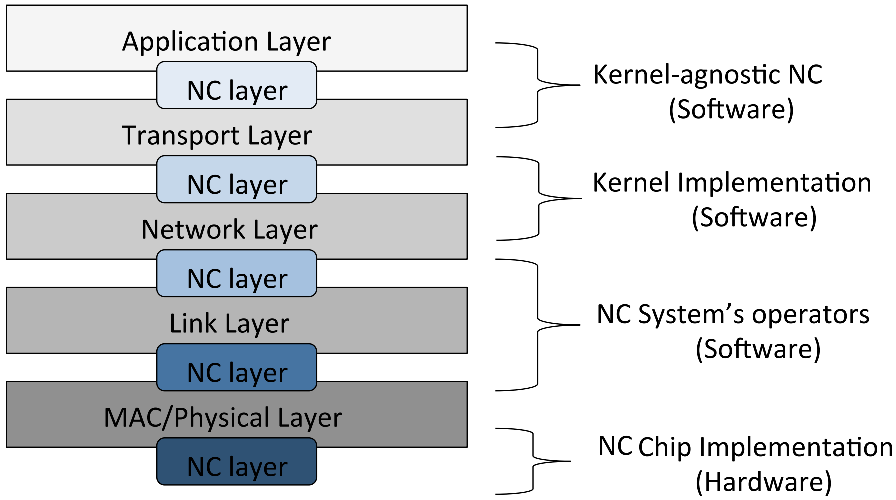
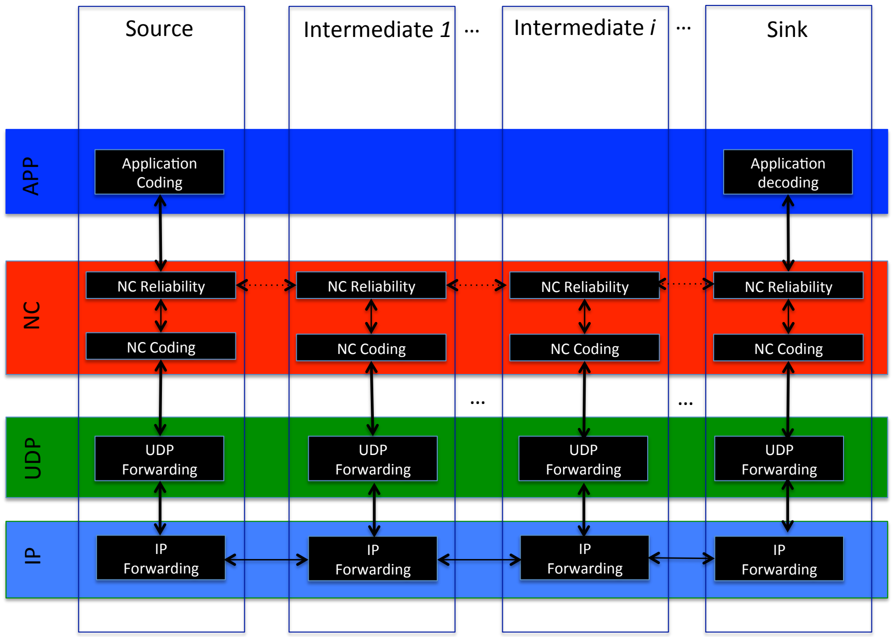
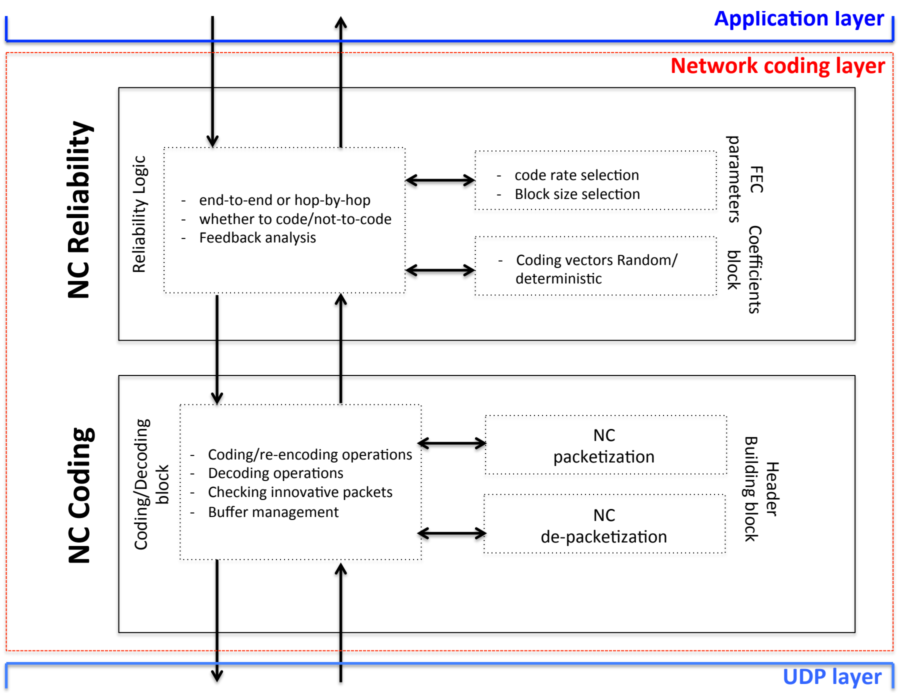


Figure 6.4: Use cases of network coding at different layers of the protocol stack

required. In particular, only these layers are of interest to IRTF which focusses on inernet applications. In [54], specifically five use cases are presented which are summarized in Table 6.3. These use cases can be built using several building blocks (BB). Mainly three building blocks are discussed: (i) NC coding BB which contains different coding operations like encoding, decoding, test for innovative packets etc (ii) NC reliability BB which contains different logical operations like end-to-end coding, hop-by-hop coding, use of feedback and (iii) NC congestion control BB to use NC for congestion control. Figure 6.5 shows the network coding architecture for the erasure correction of application layer data packets. The NC layer has been introduced between the application and transport layers. In this case, NC layer mainly consists of two building blocks: NC reliability and NC Coding [54]. Figure 6.5b shows the functionality of these different building blocks. NC reliability BB consists of reliability logic block which could have several functionalities like deciding on whether the coding will be end-to-end or hop-by-hop, analyzing the feedback to decide whether to code or not-to-code etc. NC reliability logic block functions jointly with FEC parameters block and coefficients block to decide on the code rate, block size, coding vectors etc. The selection of these parameters could be done on-the-fly by using the feedback or offline using other optimization techniques. NC reliability block signals all the information to NC coding block which performs coding/decoding operations. This block also takes care of NC packetization and NC de-packetization while sending/receiving packets at NC layer.



(a) Network Coding for forward erasure correction of application layer data packets



(b) NC reliability and NC Coding building blocks with network coding functional areas

Figure 6.5: Network Coding Architecture

Use Cases	Position of network coding in the protocol stack	Coding	Usage of NC
Use Case 1	between TCP/UDP and IP	end-to-end, in-network re-coding (optional)	Reliability, NC under TCP/UDP
Use Case 2	between App and IP	in-network	Reliability (with congestion control), assisted by multi-path routing
Use Case 3	between APP and UDP/TCP	in-network	Reliability (with congestion control), NC over overlay networks
Use Case 4	between TCP/UDP and IP	end-to-end	Reliability (with congestion control), NC with multi protocol label switching (MPLS)
Use Case 5	(i) between App and TCP and (ii) between TCP and IP	end-to-end	Reliability (with congestion control for each path), NC over disjoint paths

Table 6.3: Use cases for network coding application

6.3.3 Our contributions to IRTF

This thesis investigates practical network coding solutions for lossy line networks. This thesis proposes network coding frameworks and network coding solutions which can also be used for the better performance of internet communication. Following are the key contributions of this thesis to IRTF:

- Chapter 3: The proposed cross layer framework for network coding rate optimization could be used to provide optimal distribution of available bandwidth in IRTF architecture. By using the proposed cross-layer solution, the available bandwidth can be optimally distributed to tackle erasures and congestion in the internet network. Our solution fits for end-to-end NC protocol instantiation (figure 6.3b). However, our solution requires the cross-layer interaction, specifically between the application layer and the transport layer. The five use cases (table 6.3) discussed in IRTF meetings [54] do not include such interaction. In these use cases, the main responsibility for erasure recovery and congestion control are both with the NC block. Our requirement is different as in our solution NC is responsible only for the erasure recovery which will interact with the congestion control algorithm at the application layer. Therefore, to include this case, a room for cross layer interaction should be included in the IRTF architecture. This could accommodate our solution as well as other solutions which require such cross layer interactions.

- Chapter 4: The proposed network re-encoding framework could be used to provide erasure protection to application layer data units. Our proposal of encapsulation includes filling of FEC data table in a different way than the encapsulation in previously proposed FEC frameworks at IRTF/IETF groups. The comparison of two different frameworks is shown in table 6.1 and the encapsulation processes in figures (figure 6.1) and (figure 6.2). Our proposed encapsulation does not include adding of zero bytes for each unfilled column. Therefore, the overhead due to extra padding bytes could be reduced and higher throughput could be achieved. In the current IRTF meetings, the encapsulation process is not discussed. In the future meetings, it is imperative to discuss several possibilities of encapsulation process and to understand their comparison for different use cases. In Chapter 6, we discuss two such possibilities of encapsulation and their comparison which can be utilized by IRTF.

- Chapter 5: The proposed network coding scheme, SS-SNC, is a potential candidate for FEC at different layers in IRTF architecture. It has several benefits that can be utilized for improving internet reliability and capacity. It can provide higher throughput and reliability than routing, smaller delay and complexity than several state-of-the-art network coding schemes. Moreover it does not need any feedback loop, it is distributed and do not need any coordination between different nodes in the network.

Our proposed scheme provides the flexibility as it can be used with different NC protocol instantiations (end-to-end or hop-by-hop as shown in figure 6.3b) and it can be used for different use cases which are discussed in IRTF (table 6.3).

- Chapter 3, Chapter 4 and Chapter 5: The proposed NC architecture in IRTF, which is still in the initial phases, includes several use cases. These use cases are either for hop-by-hop or for end-to-end network coding. The overall work done in this thesis helps to understand the similarities and differences in hop-by-hop and end-to-end network coding. It also helps to understand the limitations and benefits of different network coding schemes for different cases. This will contribute to IRTF to understand and identify the requirements and expected results of network coding schemes for different use cases.

6.4 Conclusions

In this chapter, we describe several recent network coding related contributions to IRTF. We have first identified the similarities and differences on the existing use of RS codes for FEC at different layers of the protocol stack. Table 6.1) shows the comparison of different FEC frameworks based on different aspects like encapsulation, signaling, encoding, decoding etc. We have then identified the similarities and differences on several recent network coding contributions to IRTF. Table 6.3a) is presented where different network coding schemes are classified in terms of their main novelty, benefits and limitations. Finally, we have used the recently proposed building blocks in NWCRG meetings and identified the possible network coding architecture to use NC for protecting application layer data packets.

Chapter 7

Overall Conclusions and Future work

7.1 Conclusions

The overall conclusions of this thesis are based on the work done to achieve the four primary objectives. These conclusions are listed as follows.

- In this thesis, we have developed the matricial model that allowed us to understand the mathematical structure behind the network coding schemes. It was helpful in understanding the mapping of communication entities (which are packets) to mathematical entities (which are matrices) at different nodes of the network. The model developed in this dissertation can be used to analyze network coding application across different layers of the protocol stack.

- In this thesis, semi-analytical investigation is done to characterize several metrics like achievable throughput, reliability, delay and complexity. These metrics are useful for the performance evaluation of network coding scheme when applied in practice. The analysis is done using both theoretical and simulation tools. It is illustrated that the systematic network coding based schemes outperform state-of-the-art coding schemes in different instances of the line network.

- In this thesis, the application of network coding is explored at different layers of the communication protocol stack. The two different regions of the protocol stack are considered based on their usability in the communication systems. First, the application of network coding is considered in the application layer of the protocol stack. The analysis done in this part is useful for the application's developer who has an access to the data flowing in this layer. Second, the application of network coding is considered in the link layer of the protocol stack. The analysis done in this part is usefil for the network operators who have an access to the data flowing in these layers. In both parts, it is shown that network coding can be applied in practice over state-of-the-art communication protocols and outperforms other coding schemes. Finally, the work done in this thesis is in line with IRTF efforts. This work can be used for the future deployment of network coding solutions for better internet and its evolution.

7.2 Future work

In the following we list the future directions to be considered in relation to the contributions of the thesis.

- Extension to complex networks: This dissertation focusses on line networks which are simple yet useful conceptual model. The analysis could be extended to more complex network models including single-source multicasting, broadcasting etc or multiple-source multiple-receiver network model. All these models appear in practice for different communication scenarios and developing practical network coding schemes for these models can lead to better performance of current communication systems.

- Applicability at other layers of the protocol stack: This dissertation focusses on the application of network coding at application layer and link layer of the protocol stack. The work done in this thesis could be extended to analyze the application of network coding over other layers of the protocol stack as well. Several other factors should be considered while applying network coding across different layers of the protocol stack. In particular, whether it is the case of using network coding at transport layer which requires kernel access or using network coding at network layer which is typical the area of interest for systems operators. The work done is this thesis establishes the primarily analysis of investigating network coding in protocol stack.

- Correcting errors in the network: This dissertation focusses mainly on the packet erasure recovery. The packet is considered as erased if errors are found in the packet with some error detection mechanisms. The work done in this thesis can be extended to use network coding for correcting errors in the network. In order to achieve this task, encoding and decoding algorithms should be developed which are computationally efficient and could be used in practice in different communication systems.

Appendix A

Minimum distance of SNC

In this appendix, we present the probability $P(d_{SNC} = d_{MDS} - \delta)$ in (3) for SNC with generator matrix $\mathbf{G} = \left[\begin{array}{cc} \mathbf{I}_K & \mathbf{C} \end{array}\right]$ where $\mathbf{C} \in \mathbb{F}_q^{K \times N-K}$ is a matrix with random coefficients from the finite field $\mathbb{F}_q$. The proof of (A.3) is as follows. Firstly, for SNC to behave exactly like MDS codes, i.e. with degradation $\delta = 0$, any submatrix $(K \times K)$ from $\mathbf{G}$ should have full rank K [28]. There are total $\left(\begin{array}{c} N \\ K \end{array}\right)$ submatrices of dimensions $(K \times K)$ from $\mathbf{G}$. If any submatrix of size $(K \times K)$ has J independent columns from the systematic part and $K-J$ columns from the non-systematic part, then the probability of submatrix $(K \times K)$ to be full rank is given by $\prod_{F=0}^{K-J-1} (1 - q^{F-K+J})$. In (A.3), the summation is taken over J which can vary from 0 to K (K is the maximum number of columns from systematic part). Similarly, for any $\delta > 0$, the submatrix of size $(K \times K + \delta)$ from $\mathbf{G}$ should have full rank K to have the minimum distance $d_{RNC} = d_{MDS} - \delta$ and to correct up to $d_{RNC} - 1 = d_{MDS} - \delta - 1 = N - K - \delta$ erasures. This concludes the proof of minimum distance expression of SNC code in (A.3).

$$P(d_{SNC} = d_{MDS} - \delta) = \frac{1}{\left(\begin{array}{c} N \\ K+\delta \end{array}\right)} \sum_{j=0}^{K} \left[\left(\begin{array}{c} K \\ j \end{array}\right) \left(\begin{array}{c} N-K \\ K-J+\delta \end{array}\right) \prod_{F=0}^{K-J+1} \left(1 - q^{F-K+j-\delta}\right) \right] \tag{A.3}$$

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