Universitat Politècnica de Catalunya (UPC)
Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports de Barcelona (ETSECCPB)
Departament d' Infraestructura del Transport i del Territori (ITT)

MODELING OF TAXI CAB FLEETS IN URBAN ENVIRONMENT

Josep Maria Salanova Grau

Memòria presentada per optar al títol de Doctor
Enginyer de Camins, Canals i Ports
ETSECCPB-UPC

Director de la tesi: Dr. Miquel Àngel Estrada Romeu

Barcelona, Novembre de 2013

Acta de calificación de tesis doctoral

Curso académico:

Nombre y apellidos

Programa de doctorado

Unidad estructural responsable del programa

Resolución del Tribunal

Reunido el Tribunal designado a tal efecto, el doctorando / la doctoranda expone el tema de la su tesis doctoral titulada ____
____.

Acabada la lectura y después de dar respuesta a las cuestiones formuladas por los miembros titulares del tribunal, éste otorga la calificación:

☐ NO APTO ☐ APROBADO ☐ NOTABLE ☐ SOBRESALIENTE

(Nombre, apellidos y firma) Presidente/a	(Nombre, apellidos y firma) Secretario/a

(Nombre, apellidos y firma) Vocal	(Nombre, apellidos y firma) Vocal	(Nombre, apellidos y firma) Vocal

______________________, _______ de __________________ de _______________

El resultado del escrutinio de los votos emitidos por los miembros titulares del tribunal, efectuado por la Escuela de Doctorado, a instancia de la Comisión de Doctorado de la UPC, otorga la MENCIÓN CUM LAUDE:

☐ SÍ ☐ NO

(Nombre, apellidos y firma) Presidenta de la Comisión de Doctorado	(Nombre, apellidos y firma) Secretaria de la Comisión de Doctorado

Barcelona a _______ de ____________________ de __________

“Life is what happens to you while you’re busy making other things”

adapted from John Lenon (Beautiful boy 1980)

Acknowledgments

I would like to thank Miquel Angel Estrada for his technical and psychological support during the preparation of this thesis, he was always there with a blank sheet, a pen and a lot of ideas.

I would also like to thank my parents who provided me with the necessary tools for getting a grade, a master and a doctoral degree, but most important, for teaching me how to be a good, responsible and hard-working person.

Magda, thank you for having always your best smile when I was working on my thesis until late at night or during the weekends.

Friends in first place and colleagues in second place, Evangelos and Iraklis. You created the right atmosphere for working on my thesis, you love your job and teach me to love mine.

A great thank also to the Hellenic Institute of Transport, which supported me during the elaboration of this thesis, but also during my master studies.

I would like also to thank the Center for Innovation in Transport for the data provided, especially to Carles Amat and again Miquel Estrada.

Friends, colleagues, teachers, reviewers, project partners, thank you also for your support and contributions.

ABSTRACT

Taxis are a necessary mean of transport in cities for meeting the mobility needs of its citizens. The impact of the taxi market in the daily life is quite important, concerning both financial (e.g. taxi drivers' salaries) and societal issues (e.g congestion, travel demand satisfaction). Therefore, it is crucial to develop models and tools in an effort to help decision makers control the taxi market and meet the customers' requirements for an adequate level of service. In this direction, various methodologies have been developed aiming at quantifying the impacts and evaluating the performance of the taxi sector, initially from an aggregated and economic equilibrium perspective and recently from an operational and more realistic one.

The main objective of this thesis is to develop models that analyze the provision of taxi services, emphasizing on the optimal number of taxis that are in order to provide a reasonable waiting time for customers and assure a minimum profitability to the taxi drivers. For this purpose, two models are developed, an analytical economic one and an agent-based simulation one. The first model is able to analyze the taxi market in a macroscopic way, using average values for the taxi drivers' cost and performance in the area of study for obtaining the optimum number of taxis for achieving the minimum unitary system cost and the related waiting time to this optimum fleet. The second model is able to analyze the operational characteristics of the taxi sector in detail, simulating taxi trips in a road network and recording all the performance indicators related to the operational efficiency of the sector. Observed data from the city of Barcelona are used for calibrating the developed models and obtaining the results of their application to the city of Barcelona.

The developed models perfectly complement each other. Their combination can prove to be a valuable tool for decision makers when analyzing the performance of the taxi sector or when meeting policy related decisions. At the same time, local operation issues can be analyzed in detail when designing taxi policy related issues since all actors are included within the models and can be taken into account when optimizing this performance. The models are used for analyzing the impact of pricing policies in the taxi market, evaluating the demand for taxi trips related to various fare levels and fleet sizes. Policy issues related to the taxi shifts are also taken into account through the development of an optimization problem for matching the optimum supply levels to the ones provided by a shift-based taxi policy, which is the most common around the world.

Optimum ranges of demand are obtained for the hailing, the dispatching and the stand operation modes. Dispatching mode appears to be more efficient at low demand levels, while the hailing mode present lower unitary costs for high demand levels. Aggregated models tend to underestimate the costs of both the customers and the system between 15% and 30% if the demand distribution is uniform. For non-uniform demands, the aggregated models overestimate costs between 20% and 300%, depending on the GINI value of the demand distribution.

RESUM

Els taxis són un mode de transport necessari per satisfer les necessitats de la població relacionades amb la mobilitat. L'impacte del mercat del taxi a la vida quotidiana és molt important, tant des del punt de vista econòmic (per exemple, els salaris dels taxistes) com social (per exemple, la congestió o la satisfacció de la demanda de desplaçaments). S'han desenvolupat diversos models amb l'objectiu de quantificar els impactes i avaluar el rendiment del sector del taxi, inicialment des d'un punt de vista agregat i econòmic i, recentment, des d'un punt de vista operacional i més realista.

L'objectiu principal d'aquesta tesi és el de desenvolupar models d'estudi dels serveis oferts pel sector del taxi, fent èmfasi en el dimensionament de la flota òptima de taxis necessària per satisfer la demanda oferint un temps d'espera raonable per als clients i assegurant al mateix temps una mínima rendibilitat als taxistes. Per a aquest propòsit, s'han desenvolupat dos models, un econòmic analític i un de simulació basada en agents. El primer model és capaç d'analitzar el mercat dels taxis des d'un punt de vista macroscòpic mitjançant l'ús de valors mitjans representatius de tota la regió per obtenir el nombre òptim de taxis que satisfarà les necessitats de la població amb uns costos unitaris mínims, obtenint també el temps d'espera relacionat a aquesta flota òptima. El segon model és capaç de reproduir les característiques operacionals del sector del taxi en detall, simulant viatges en taxi a la xarxa viaria i enregistrant tots els indicadors de rendiment relacionats amb l'eficiència operacional del sector del taxi. S'utilitzen dades reals de la ciutat de Barcelona per al calibratge dels models proposats i per a l'obtenció dels resultats de l'aplicació d'ambdós models al sector del taxi de la ciutat de Barcelona.

Els dos models desenvolupats es complementen perfectament, la seva combinació pot ser una eina valuosa per als responsables de la regulació del sector del taxi, ja que els permetrà analitzar els serveis del taxi i prendre les decisions correctes per millorar els servies oferts. Al mateix temps, els problemes operacionals poden ser analitzats en detall durant la fase de disseny de la política de taxis, ja que tots els actors estàn inclosos dins dels models i es tenen en compte alhora d'optimitzar el rendiment del sector del taxi. Els models s'utilitzen per analitzar l'impacte de les polítiques tarifaries en el mercat del taxi. També es te en compte la política de torns, per la qual s'ha plantejat un problema d'optimització que fa coincidir el nombre òptim de vehicles obtingut dels models amb el real proporcionat per la política de torns, que és la més comú a tot el món .

Es presenten rangs òptims de demanda per als tres modes operatius. Els sistemes de taxi basats en centraletes telefòniques es mostren més eficients per baixes concentracions de demanda, mentre que els sistemes de taxi al carrer presenten menors costos unitaris per altes concentracions de demanda. Els models agregats tendeixen a subestimar els costos, tant dels clients com del sistema entre el 15% i el 30% si la distribució de la demanda és uniforme. Per demandes no uniformes, els models agregats sobreestimen els costos entre un 20% i 300%, depenent del valor de GINI de la distribució de la demanda.

RESUMEN

Los taxis son un modo de transporte necesario para satisfacer las necesidades de la población relacionadas con la movilidad. El impacto del mercado de taxis en la vida cotidiana es muy importante, tanto desde el punto de vista económico (por ejemplo, los salarios de los taxistas) como social (por ejemplo, la congestión o la satisfacción de la demanda de viajes). Varios modelos han sido desarrollados con el objetivo de cuantificar los impactos y evaluar el rendimiento del sector del taxi, inicialmente desde un punto de vista agregado y económico y, recientemente, desde un punto de vista operacional y más realista.

El objetivo principal de esta tesis es el de desarrollar modelos que se ocupen de los servicios del sector del taxi, dando énfasis en el número óptimo de taxis necesarios para satisfacer la demanda ofreciendo un tiempo de espera razonable a los clientes y asegurando una mínima rentabilidad a los taxistas. Para este propósito se han desarrollado dos modelos, uno económico analítico y uno basado en simulación de agentes. El primer modelo es capaz de analizar el mercado del taxi desde un punto de vista macroscópico mediante el uso de valores medios para toda la región y de obtener el número óptimo de taxis que satisfará las necesidades de transporte a un coste unitario mínimo, obteniendo también el tiempo de espera relacionado a esta flota óptima. El segundo modelo es capaz de reproducir las características operacionales del sector del taxi en detalle, simulando viajes de taxi en una red viaria y registrando todos los indicadores de rendimiento relacionados con el funcionamiento eficiente del sector del taxi. Se utilizan datos reales de la ciudad de Barcelona para la calibración de los modelos propuestos y la obtención de los resultados de la aplicación de ambos modelos al sector del taxi de la misma ciudad.

Los dos modelos desarrollados se complementan perfectamente. Su combinación puede ser una herramienta valiosa en el análisis de los servicios de taxi o cuando se toman decisiones políticas en un esfuerzo para mejorarlos. Al mismo tiempo, problemas de funcionamiento locales pueden ser analizados en detalle durante el diseño de las cuestiones relacionadas con la política de taxis ya que todos los agentes se incluyen en los modelos y se pueden tomar en cuenta a la hora de optimizar el rendimiento del sistema. Los modelos se utilizan para analizar el impacto de las políticas tarifárias en el mercado del taxi. Tambien se tiene en cuenta la política de turnos, para la qual se ha planteado un problema de optimización que hace coincidir el numero óptimo de taxis obtenido por los modelos con los proporcionados por una política de turnos, que es la más común en todo el mundo.

Se presentan rangos óptimos de demanda para los tres modos de operación. El sistema de taxis basado en reservas telefónicas parece ser más eficiente en bajas concentraciones de demanda, mientras que el sistema de taxis en la calle presenta menores costes unitarios para altas concentraciones de demanda. Los modelos agregados tienden a subestimar los costes tanto de los clientes como del sistema entre el 15 % y el 30 % si la distribución de la demanda es uniforme. Para demandas no uniformes, los modelos agregados sobreestiman los costes entre un 20% y 300%, dependiendo del valor de GINI de la distribución de la demanda.

ΠΕΡΙΛΙΨΗ

Τα ταξί αποτελούν ένα απαραίτητο μέσο μεταφοράς για την ικανοποίηση των αναγκών μετακίνησης των πολιτών. Οι επιπτώσεις της αγοράς ταξί στην καθημερινή ζωή είναι επομένως πολύ σημαντικές, τόσο σε ότι αφορά οικονομικά (π.χ. οι μισθοί των οδηγών ταξί) όσο σε κοινωνικά θέματα (π.χ. συμφόρηση ή ικανοποίηση της ζήτησης για μετακίνηση). Έτσι, είναι σημαντικό να αναπτυχθούν μοντέλα και εργαλεία σε μια προσπάθεια να ελεχθεί η αγορά των ταξί από τους αντίστοιχους φορείς και να ικανοποιηθούν οι ανάγκες των χρηστών. Διάφορα μοντέλα έχουν αναπτυχθεί με στόχο την ποσοτικοποίηση των επιπτώσεων και την αξιολόγηση των επιδόσεων του κλάδου των ταξί, αρχικά από μια πιο συγκεντρωτική και οικονομική άποψη και, πιο πρόσφατα, από μια λειτουργική και πιο ρεαλιστική άποψη.

Ο κύριος στόχος αυτής της διδακτορικής διατριβής είναι η ανάπτυξη μοντέλων που ασχολούνται με την ανάλυση των υπηρεσιών ταξί, με έμφαση στον βέλτιστο αριθμό των ταξί που απαιτούνται για την ικανοποίηση της ζήτησης παρέχοντας ένα εύλογο χρόνο αναμονής και εξασφαλίζοντας έναν ελάχιστο κέρδος στους οδηγούς ταξί. Για το σκοπό αυτό, δύο μοντέλα έχουν αναπτυχθεί, ένα συγκεντρωτικό οικονομικό μοντέλο και ένα μοντέλο προσομοίωσης. Το πρώτο μοντέλο μπορεί να αναλύσει την αγορά ταξί από μια μακροσκοπική άποψη χρησιμοποιώντας μέσες τιμές για το σύνολο της περιοχής για τον καθορισμό του βέλτιστου αριθμού ταξί που θα παρέχει υπηρεσίες ταξί με το ελάχιστο μοναδιαίο κόστος, παρέχοντας επίσης το αντίστοιχο χρόνο αναμονής για αυτό το βέλτιστο στόλο. Το δεύτερο μοντέλο μπορεί να αναλύσει τα λειτουργικά χαρακτηριστικά του κλάδου των ταξί με κάθε λεπτομέρεια, προσομοιώνοντας μετακινήσεις με ταξί σε ένα οδικό δίκτυο και καταγράφοντας όλους τους δείκτες απόδοσης της λειτουργίας του κλάδου των ταξί. Χρησιμοποιούνται πραγματικά δεδομένα από την πόλη της Βαρκελώνης για τη βαθμονόμηση των προτεινόμενων μοντέλων και τον καθορισμό των αποτελεσμάτων της εφαρμογής των δύο μοντέλων στην πόλη της Βαρκελώνης.

Τα δύο μοντέλα που αναπτύχθηκαν είναι συμπληρωματικά και ο συνδυασμός τους μπορεί να αποδειχθεί ένα πολύτιμο εργαλείο για τους φορείς λήψης αποφάσεων κατά την ανάλυση της απόδοσης των υπηρεσιών ταξί ή για την λήψη αποφάσεων για τη βελτίωσή τους. Τα τοπικά ζητήματα λειτουργίας μπορούν να αναλυθούν λεπτομερειακά κατά το σχεδιασμό της πολιτικής της αγοράς ταξί. Τα μοντέλα χρησιμποιούνται για την ανάλυση των επιπτώσεων τιμολογησης της αγορας ταξί, αξιολογώντας τη ζήτηση για ταξί που συσχετιζεται σε διάφορα επιπέδα τιμολόγησης με τον αριθμό οχημάτων. Περιλαμβάνονται επίσης θέματα διαχείσης της αγοράς ταξί, όπως η χρήση βαρδιών, που είναι το πιο συνηθισμένο, για την παροχή οχηματοορών υπηρεσίας σε επίπεδα αντίστοιχα της βέλτιστης λύσης.

Έχουνε διατυπωθεί βέλτιστα διαστήματα ζήτησης για τις τρεις αγορές ταξί, παρά την οδό, με τηλεφωνική κράτηση και σε στάση. Η αγορά με τηλεφωνική κράτηση φαίνεται να είναι η πιο αποδοτική για χαμηλά επίπεδα ζήτησης, καθώς και η παρά την οδό να είναι η πιο αποδοτική για ψηλά επίπεδα ζήτησης. Τα συγκεντρωτικά μοντέλα υπο-εκτιμούν τα κόστη των χρηστών και του συστηματος μεταξύ 15% και 30% εαν η κατανομή της ζήτησης είναι ομοιομόρφη. Για μη ομοιομόρφες κατανομές ζήτησης, τα συγκεντρωτικά μοντέλα υπερ-εκτιμούν το κόστος μεταξύ 20% και 300%, βάσει της τιμής του συντελεστή GINI της κατανομής της ζήτησης.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

1.1. Subject and motivation

Nowadays, the vast majority of the population is concentrated in urban cities. Various studies point out that in 2030 more than 80% of the population will live in urban areas (United Nations 2007). While the demand for trips and mobility is continuously growing, the capacity of a city's road network to accommodate the incremental vehicle flows is limited, and must be optimized. The construction of new infrastructures could contribute to the generation of even more trips, so that the new infrastructure will be congested rapidly. The high cost of building new roads and the continuous increase for travel demand highlights the necessity for well-planned, efficiently operated and cost-effective Transportation System Management strategies (TSM).

The important aspects of the concept of urban mobility are sustainability, congestion and accessibility. In accordance to these, last years' tendencies are shifting person trips from private transport to public transport, increasing the later's share significantly. At a European level, this shift is estimated at 55% (European Commission, 2011b). Mass Transport Systems, such as the subway, the tram and the bus are the most commonly used. This kind of transport usually has a centralized management, which uses Intelligent Transportation Systems (ITS), for an optimal operation of the service. Unfortunately, non-reliability, inflexibility, long total travel time and insufficient service coverage of Mass Transport systems can cause a low usage of them in some metropolitan areas. The European Commission recently published the results of a survey related to the transport sector, which identifies the reasons why most of the private car users do not use Public Transport. These reasons are nonblack of convenience (71%), lack of connections (72%), low frequency (64%), non-reliability (54%), cost (49%) and lack of scheduling information (49%) (European Commission, 2011a). On the contrary, the taxi is a more convenient transport mode due to its speed, door-to-door services, privacy, comfort, 24 hour operation and lack of parking fees, being the optimal transport mode for low-demand situations. The great inconvenience is the lack of centralized management; each taxi driver takes his own decisions, with a weak intent of control by the policy mechanisms of each city, such as license control or distributing the working days of the taxi vehicles.

A taxi is a mean of public transport with some benefits of both the private and the public transport, which can be exploited in various ways, from individual use and variable routes to collective use and fixed routes. It overcomes the limitations of public transport due to time constraints associated with fixed routes and scheduling. Also, it is an appropriate and flexible transport service for those who are not able to drive its own car due to vehicle availability, parking restrictions or personal capabilities. Therefore, depending on the existing public transport network but also on cultural issues, there is always a market share for taxis, ranging from 1% in most European cities to more than 10% in some large urban metropolises located in China. The number of vehicles dedicated to the taxi market is significant, but there is also a high number of stakeholders and third party services associated to the taxi industry, such as dispatching centers, taxi companies and the competent authorities responsible for the regulation of all involved actors in the taxi market.

A significant percentage of private vehicles in the daily traffic flow is attributed to taxis. Two examples are the 40 % of the flow in Hong Kong (Yang et al. (1998)) or the 8-10% (i. e. 1-1.3 million trips per day) of the total trips in Taipei metropolitan area (Chang et al. 2010). Most of the time, these taxis are empty, e.g. the vacancy rate of taxi riding is close to 57% in the Taipei Metropolitan area (Chang et al. 2010). This phenomenon is affecting both taxi drivers, for whom higher vacant kilometers means lower benefits, and citizens, who are subject to increasing congestion and environmental pollution. There is a need for a balance between the taxi customers level of Service (LoS), the taxi drivers benefits and the externalities caused to the society. The lower benefits of taxi drivers is a situation constantly rising given the current economic recession, which is modifying the market equilibrium: demand is decreasing due to the lower income of the population and supply is growing due to the increasing number of taxi drivers coming from other sectors. It is noteworthy to mention that the number of taxi driver licenses, which grows constantly, is ususally higher than the number of taxi licenses, which remains stable in many cities, which means that the number of double shifts is increasing. Market equilibrium in terms of balance between demand and supply cannot be achieved within this market because of the external control such as market entry and fee restrictions. This is a vicious circle, since vacant hours are increasing and taxi drivers need to work for longer time periods in order to earn the same income, which means lower income per hour. In this situation, taxi drivers tend to stop at taxi stands and wait for a customer, without spending fuel in vacant trips, saturating the taxi stands and creating congestion in the zones near the taxi stands. If the network of taxi stands is not well designed, this situation will create a decrease in the Level of Service (LoS) of the customers and thus cause a decrease of the demand.

Cities around the world have different policies with regard to the regulation of the taxi sector. These regulations aim at controlling the number of taxis in the cities in order to reduce their economic, social and especially environmental impacts. The taxi sector is responsible for an important part of the CO_2 emissions in urban areas since more than 90% of pollution comes from gas emissions by motorized vehicles, with a high proportion of taxi trips (Chang et al. 2010). The European Union (EU) is working in this direction by promoting green policies, which are presented in the chapter 1.2.

1.2. Policy and research frameworks

Several papers, reports and studies have been elaborated by the European Commission during the last years in relation to the transport sector: The new White Paper (2011: Roadmap to a Single European Transport Area – Towards a competitive and resource efficient transport system). The Green Paper (2007: Towards a new culture for urban mobility) and the Action Plan on urban Mobility (2009).

The new White Paper on Transport highlights the necessity of reducing CO_2 emissions by 80-95% below 1990 levels by 2050. One of the ten goals proposed for achieving this reduction is the increase of the efficiency of transport using information systems and developing innovative mobility patterns. For the urban environment, more efficient public transport services must be developed, while the preparation of Integrated Urban Mobility Plans is encouraged for medium and large cities. The paper proposes the internalization of the cost of externalities (such as noise, air pollution and congestion) to the producer, eliminating tax distortions and unjustified subsidies, showing the real cost of non-sustainable modes of transport. Finally, seamless multimodal door-to-door mobility must be achieved, creating the framework conditions for promoting the development and use of new technologies and intelligent systems for interoperable and multimodal scheduling, real time information, online reservation systems and smart ticketing.

The Green Paper recognizes urban mobility as an important factor in growth and employment, identifying urban traffic as responsible for 40% of CO_2 emissions and 70% of emissions of other pollutants. Mobility management is proposed for influencing travel behavior before it starts, shifting travelers towards more sustainable modes. It promotes the idea of better information for better mobility, providing citizens with user-friendly, adequate and interoperable multi-modal trip information for planning a journey with the right choice on mode and time for travel. The paper estimates the increase in capacity due to the use of ITS technologies and dynamic management in 20-30%.

The Action Plan on Urban Mobility proposes different actions related to the urban transport sector: the acceleration of the development of Sustainable Urban Mobility Plans (SUMP), the creation of healthy urban environments, the internalization of external costs, the upgrade of data and statistics, setting up an urban mobility observatory and the use of ITS technologies in urban mobility.

New technologies and platforms, such as smartphones and social networks have an enormous potential for the implementation of new ideas related to all fields, especially to the transportation field, since all smartphones are equipped with GPS and associated software for running various applications. Google© or TomTom© are exploiting these possibilities for real-time information of traffic conditions. Many new projects are developing new software applications based on new technologies, such as the Optitrans project or the Taxi Beat initiative and other taxi-customer meeting applications developed for smartphones in many cities around the world. The Optitrans project, funded also under the FP7, developed a web-based application, providing a multimodal routing planner for Android, Java and IPhone devices. The project is running in two pilot sites, Madrid and Athens, providing travelers with information related to their trip, such as mode availability, travel time, cost and detailed directions for reaching their destination. Other projects, developed under private initiatives, are developing new platforms in the taxi market. Applications such as Taxi beat, Fast-taxi, Go-taxi, My-Taxi developed an independent taxi reservation center platform, where customers can ‘call’ taxi drivers using their smartphones in various European cities. When the customer ‘asks’ for a taxi, the software uses the integrated GPS for finding the available taxis near the customer. When a taxi is assigned to the customers, the situation of the taxi is displayed in real-time in the customer's phone, until the taxi reaches the customer.

1.3. Objectives and originality of thesis

The present doctoral thesis aims at identifying the optimal taxi fleet size and policy in order to minimize the system unitary cost of the provision of taxi services. These values are related to the socio-economical charateristics of each region, such as demand for taxi trips, area extension, Value of Time (VoT) or operational costs of the taxi fleet. The three actors of the taxi market identified in the study and used in the models are the customers, the drivers and the city (represented by other drivers and the citizens). A compact methodology for modeling the taxi market has been developed and is used for the study of its different variables and their relation. The methodology matches the supply with the demand for taxi services, reducing the number of vacant circulated hours and optimizing the income of taxi drivers, while maintaining an acceptable LoS for customers and reducing their externalities. Various operational modes are modeled and validated through an agent-based simulation model in order to define the most suitable mode for each city.

Three basic exploitation models are considered: stand market, where taxis await customers at predefined meeting points or vice versa; hailing market, where customers stop a cruising taxi on the street; and dispatching center markets, where customers reserve taxis calling a dispatching center. Evaluation methodologies for each exploitation model are developed, focusing principally in the three actors named above: the customers (the demand), the taxi drivers (the active supply) and the city (the passive supply). Key Performance Indicators (KPIs) are obtained for each exploitation model and focus group. Several tools using geometric probability, continuous approximations, microeconomic and production functions and discrete-event simulations are developed for the evaluation of the sector, taking into account the performance and the cost of the service.

Finally, the models proposed herein are calibrated and tested using real data from Barcelona, presenting significant results in terms of minimum and optimum fleet sizes for the city. The model can be used by authorities for assessing decision making related to the regulation of the taxi market but also by taxi companies for optimizing their operation. For the study of the variables, a tool for simulating the results of the implementation of the different exploitation models and policies in the taxi sector is developed. Guideilnes are proposed for assessing the management of policies and regulations of the taxi sector, such as pricing, shifts or fleet size restrictions.

1.4. Publications related to the research done in the thesis

The contents of the present doctoral thesis have been published in the following journals:

- A review of the modeling of taxi services. Procedia - Social and Behavioral Sciences, Vol 20, pp 150-161, 2011.
- Agent Based Modeling for Simulating Taxi Services. Journal of Traffic and Logistics Engineering (JTLE) (ISSN: 2301-3680), Vol. 1 No. 2, June 2013. pp. 159 – 163, 2013.
- Aggregated taxi services modeling. Transport Research Part B (under review).

1.5. Structure of the thesis

In section 2, the different models developed in relation to the taxi cab market are reviewed focusing on the formulations presented in the most operational models. The state-of-the-art analysis is followed by the state-of-the-practice, presenting the taxi markets in various cities, with different policies and configurations implemented over the last years. A comparative assessment of the theory and the practice is performed, assessing the results obtained in the theoretical models with the reality of the different taxi markets throughout the world.

The proposed aggregated model methodology and an analytical approach of the formulation are developed together with the detailed definition of the problem, including the variables and the formulation, which are presented in section 3.

The agent-based simulation model is presented and applied to a small test network in section 4. A comparison of the results obtained with measure provided by the application of the aggregated model is also presented.

Section 5 presents the application of the models to the city of Barcelona, calibrating the models with real data obtained from the taxis and presenting the different KPIs together with the optimum fleet size for each city.

The conclusions and further research conclude the thesis with the most valuable points of the research conducted herein and the definition of guidelines for further research in the modeling of taxi services.

1.6. References

Chang S. K. J., Wu C. H., Wang K. Y. and Lin C. H. (2010). Comparison of Environmental Benefits between Satellite Scheduled Dispatching and Cruising Taxi Services; *Proc. of the 89th Annual Meeting of the Transportation Research Board,* Washington D.C.

European Commission (2007) *Towards a new culture for urban mobility* (available at http://eur-lex.europa.eu/LexUriServ/site/en/com/2007/com2007_0551en01.pdf).

European Commission (2009) *Action Plan on urban Mobility 2009* (available at http://ec.europa.eu/transport/themes/urban/urban_mobility/doc/com_2009_490_5_action_plan_on_urban_mobility.pdf).

European Commission (2011a) *Roadmap to a Single European Transport Area – Towards a competitive and resource efficient transport system* (available at http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=COM:2011:0144:FIN:EN:PDF).

European Commission (2011b) *Future of transport: Analytical report*.

United Nations 2007, *World Urbanization Prospects: The 2007 Revision*.

Yang H. and Wong S. C., (1998) A network model of urban taxi services. *Transportation Research Part B* **32** (4) 235 – 246.

2. LITERATURE REVIEW

This section presents an overview of the models presented in the literature, discussing the most important characteristics and providing their respective formulations in detail.

2.1. Introduction

Taxis are an individual transport mode used for public transport services providing door to door personal transport services. They can be divided into three broad exploitation categories: stand, hailing and dispatching market. Taxi stops are designated places where a taxi can wait for customers and vice versa. Taxis form queues and customers must reach the nearest taxi stand to their origin and be served according to a First-in-First-out (FIFO) rule. In the hailing market, customers hail a cruising taxi on the street. In this case, there is higher uncertainty for both the drivers, which do not know the percentage of vacant kilometers, and the customers, which do not know the waiting time and the quality/fare of the service they will find. In the dispatching market, customers call a dispatching center requesting for an immediate taxi service. Only in this kind of market consumers can choose between different service providers or companies. At the same time, companies can fidelize customers by providing good quality services or billing advantages. The market in this case is competitive since companies with larger fleets can offer lower waiting times. However, it is observed that each policy performs better depending on the social characteristics of the city as well as the potential demand to be served.

The taxi sector has been traditionally regulated basically with fixed fares and entry control barriers. The objective of this regulation is to correct the weaknesses of the taxi sector, such as externalities (congestion and pollution), low level of service provided and the imperfection of the competitive behavior of the market. Various authors proposed different models for reproducing different entry market conditions and fares in order to simulate real taxi markets, characterizing them in terms of fares and fleet size. A fundamental distinction in the types of taxi regulations is made among quantity, quality and market behaviour regulation. Quantity regulations include fare regulation and entry restriction, which means that both the applied fares and the issue of new taxi licenses are regulated by the competent authority. When entry is regulated, the control in fares limits the possibility of monopoly in the market. Also fares are regulated in some open-entry markets. Quality regulation includes the standards of vehicles (age, type of vehicle, maintenance), driver and operator standards (uniform, route knowledge). This type of regulation is a safety oriented regulation rather than competitiveness related. Market conduct regulation includes rules regarding pick up of customers or affiliation to a dispatching center.

Restrictions on taxi market entry have been applied by many cities around the world. The first city that restricted the number of taxis was New York City in 1937; nowadays most of the cities regulate the number of vehicles providing taxi services. When this kind of decisions are taken without studying and analyzing the actual situation, or having an implementation plan, entry restrictions and fare regulations affect significantly the taxi sector, leading to important welfare losses. As a result of this, the price of the licenses in markets where taxi licenses are tradable are higher (e. g. Paris €125,000, Sydney $300,000, Melbourne $500,000, New York $600,000 (OECD 2007)). In these cases, as revealed in various studies, they are rising up constantly due to the exploitation of their owners. Reforms have often been opposed to reduce the incomes of drivers, which are normally low, and restrictive conditions have been applied in this direction, but there is no evidence that taxi incomes are higher in markets with regulated entry conditions. License owners are beneficiated by these measures and not the drivers. One example is Melbourne, where taxi licenses valuated at approximately $500,000, but drivers' income is estimated at $8 – $14 per hour (OECD 2007). Many cities are deregulating their taxi markets due to the negative consequences of regulation. Deregulation has most of the times positive impacts, nowadays many cities have removed entry restrictions, resulting in lower waiting times, increased customer satisfaction and fare reduction. In relation to fares, deregulation is very difficult to implement due to the fact that customers must have the possibility of comparing prices, which means that the customers should know the price of the service prior to hail the taxi (not feasible) or choose a taxi in the stand market (opposed to the FIFO rules). Market liberalization is an interesting challenge, but in cities where high supply restrictions have been applied, there will be a strong opposition to reform proposals from the license-owners. Arguments support that license-owners must be compensated in that case: one approach first used in Ireland is to give the additional licenses to each license-owner, ensuring that the new monopoly will remain in their hands; alternatively the new license can be given to taxi drivers without taxi license. One example can be found in Melbourne, where a 12 year program is adding to the stock of licenses a number of licenses equal to the yearly demand growth. On the other side, externalities such as congestion or pollution should be taken into account when increasing the number of vehicles.

The above highlight the necessity of developing modeling tools for decision makers, which will use them for evaluating the impact of the policies applied to the taxi market.

2.2. Extensive scientific literature review on the modeling of taxi services

Since the 1970s, many studies have been published in relation to the taxi sector. The first studies (1970-1990) were related to the profitability of the sector and the necessity for regulation using aggregated microeconomic models. These models were built by using average values of the modeling variables such as demand, waiting time, travel time, whithout taking into account their spatial distribution. Later studies (1990-2010) implemented more realistic models: from the most simple model of Yang and Wong (1997) for a small taxi fleet to the most sophisticated model of Yang and Wong (2009) being able to account congestion, demand elasticity, multiple user classes, congestion externalities and non linear cost calculation.

Douglas (1972) developed the first aggregated taxi model using economic relationships from other sectors (goods and services). Many authors (de Vany (1975), Beesley (1973), Beesley and Glaster (1983), Schroeter (1983), Manski and Wright (1976), Arnott (1996) and Cairns and Liston-Heyes (1996)) used the model proposed by Douglas for developing their models and applied them to the different market configurations. Yang and Wong (1997 – 2010) developed accurate models, taking into account the spatial distribution of demand and supply with traffic assignment models. Latest models proposed by Wong and Yang (2005 and 2010), assume a bi-directional function considering the willingness of customers to pay for the service. Simulation models have been developed during the last years, most of them based on agents and discrete event methodologies. The three type of models mentioned above are presented in detail in the following subsections.

2.2.1. Introductory note

In order to facilitate the comprehension of the literature review, various economic definitions are presented below:

Demand and supply of a product: The microeconomic theory depicts the demand and supply of a product as two continuous functions, one monotonically decreasing for the demand and one monotonically increasing for the supply. The demand curve represents the relationship between the price of a certain product and the amount of it that consumers are willing to purchase at that price. The supply curve represents the relation between the price of a product and the quantity that suppliers are willing to provide at that price. In a competitive market, the equilibrium of demand and supply is the intersection point of both curves.

Producer income: This term is the income of the providers, which is equal to the product between the demand for taxi services and the price of each trip (see Figure 2-1).

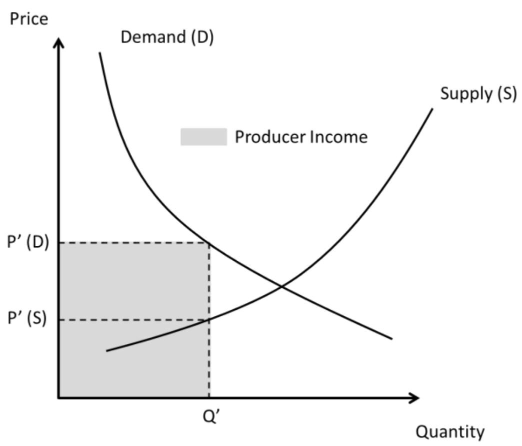

Figure 2-1 Producer income

Total cost: This term is the cost of producing all the units demanded, which is equal to the area under the supply curve for the quantity demanded (see Figure 2-2).

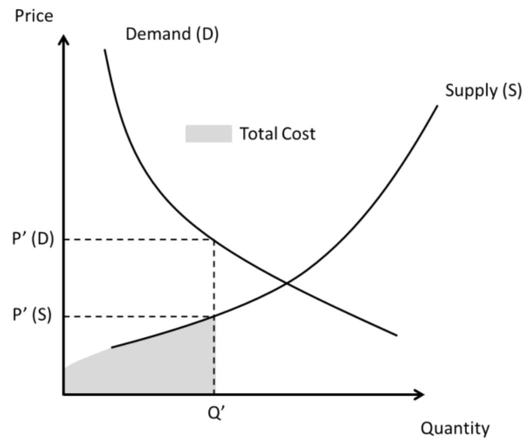

Figure 2-2 Total cost

Producer surplus: This term is the benefit of the producers, which is equal to the difference between the producer income and the total cost (see Figure 2-3).

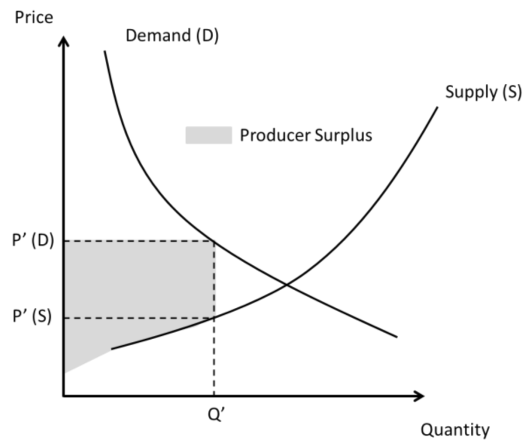

Figure 2-3 Producer surplus

Consumer surplus: This term is the difference between the willigness to pay of the customers and the real cost they pay (see Figure 2-4).

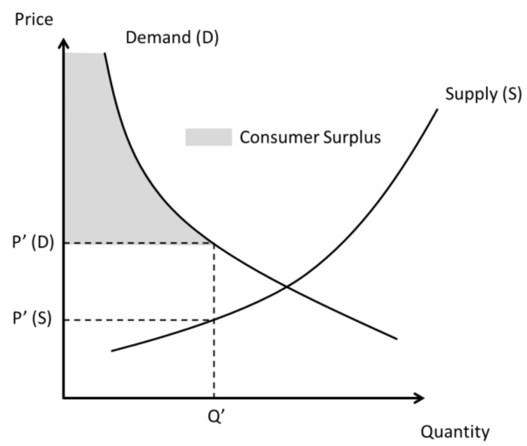

Figure 2-4 Consumer Surplus

Social Welfare: This term refers to the overall benefit of society due to the provision of a service. In taxi modeling it refers to the total revenues of the producer and the surplus of the consumer (see Figure 2-5).

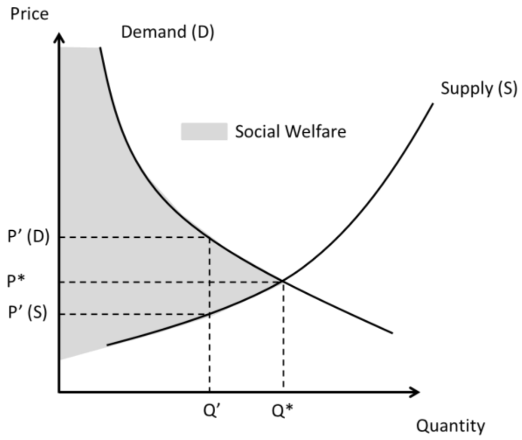

Figure 2-5 Social Welfare

First best solution: This term refers to the optimal social solution that maximizes social Welfare when providing any kind of service. The price in this case is equal to the marginal cost of producing each unit. In the taxi market, this solution is not feasible due to the fact that the benefits of taxi drivers are negative, since vacant hours and fixed costs are not covered by the income.

Second best solution: This term refers to the sub-optimal solution adopted when one or more optimality conditions cannot be satisfied. The price in this case is equal to the average cost of producing each unit. In taxi modeling, second best solution refers to the optimal solution where taxi benefits are not negative (zero profit).

2.2.2. Aggregated models

Douglas (1972) was the precursor of the first studies related to the taxi sector. His work considered a taxi market where taxicabs can be engaged anywhere along the city streets, with scheduled (by a regulatory authority) fares (P), and a free market entry. Demand (Q) and the Production Costs (T_C) are also considered. Market Equilibrium between the demand and the supply for taxi services was studied. Douglas (1972) formulated the demand as a decreasing function of two variables: the trip price (P multiplied by the duration of a trip) and a proxy for service quality (T) (as presented in Equation (2.1))

$$Q=f(P,T)\ ,\ \frac{\partial Q}{\partial P}<0\ ,\ \frac{\partial Q}{\partial T}<0 \qquad (2.1)$$

The price is calculated for an average trip duration using constant average values for the trip length and the distribution of speed during the route. The service quality can be estimated with the expected delay when looking for a taxi. Other variables such as prices in alternative modes, incomes and other quality measures are held constant. Douglas (1972) formulated the production costs (T_C) as a perfectly elastic function of the cost per hour of service time (c), assuming independency of the division of service time between “occupied” (O) and “vacant” (V) (as presented in Equation (2.2)).

$$T_C = c \cdot (O+V) \quad (2.2)$$

It is assumed that the delay function (T) depends on the density of taxis (D) and average speed of vacant taxis (S), presenting a decreasing tendency.

$$T = g(D, S), \quad \frac{\partial T}{\partial D} < 0 \; , \; \frac{\partial T}{\partial S} < 0 \quad (2.3)$$

The Market Equilibrium is formulated without any entry barriers and exogenously set price level (P = P*) assuming that all taxis will achieve the same occupancy ratio over time (Equation (2.4)).

$$P \cdot Q = f(O + V) \quad (2.4)$$

Douglas proposed a specific formulation (Equations (2.5) and (2.6)) for the demand and delay functions for solving the proposed set of Equations, obtaining a set of price solutions P*.

$$Q = A + B(P + rT)^{\alpha} \quad (2.5)$$

$$T = K/V \quad (2.6)$$

where,
Q is the demand
P is the price level
V is the vacant time
T the delay function
K, A, B, r and α are parameters

The occupancy rate and the waiting time are related to the value of P* as shown in Figure 2-6, where P_Q is the price corresponding to the maximum demand and P_N is the price that produces the maximum vehicle-hours in service.

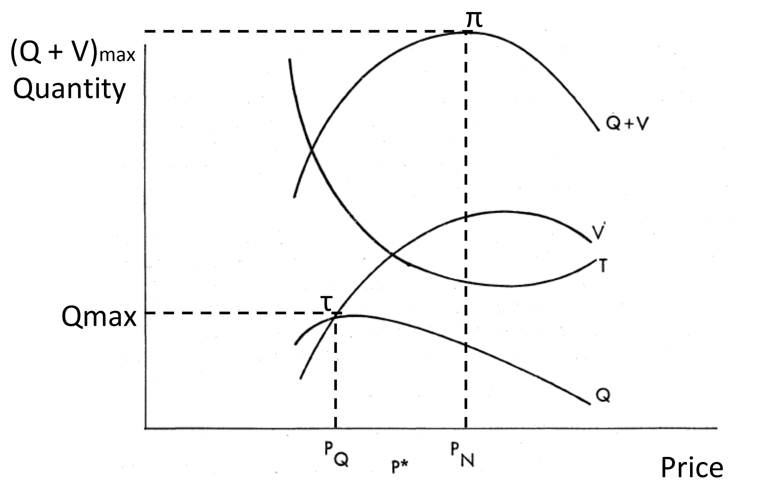

Figure 2-6 Feasible equilibrium points of the system proposed by Douglas (1972).

Douglas (1972) concluded that the point π is the maximum revenue to the industry, which occurs at the point where the number of taxi hours in service (Q + V) is maximized and where Q is less than Q_{max} (point τ). Douglas (1972) characterized social welfare as an efficient but unfeasible equilibrium, as it it generates deficit, expressing it as (2.7):

$$W=\int P(Q,T)dQ - c(Q+V) \tag{2.7}$$

Finally, Douglas (1972) discussed the importance of the value of time (VoT) parameter pointing that the problem of the regulation is to find a unique price for different groups of customers, with different levels of willingness to pay for a better service.

The formulation of Equation 2.7 has been used as a reference model by all authors. De Vany (1975) uses the full price demand function (π) proposed by Douglas, adding an index of the full prices of all other goods (π_0), while introducing the value of time of customers and the waiting time (B – demand shift parameter) for a cab in the demand assumptions.

$$Q=Q(\pi,\pi_0,B) \tag{2.8}$$

Solutions for various types of markets are proposed: the monopoly market (with regulated entry and fares) and the competitive market (with free entry but regulated fares). In the monopoly solution, the objective of the taxi firm proposed by De Vany (1975) is to maximize total benefits as shown in Equation (2.9).

$$\text{Max } p \cdot Q\,(p,\, VoT(H)\,) - c \cdot H \tag{2.9}$$

where,
Q is the demand
p is the price
VoT is the value of time
H is the total supplied hours
c is the hourly cost

It is stated that unitary elasticity (e=1) represents a zero profit point, while e>1 a negative profit zone. For this reason the elasticity must be less than 1.

In the competitive solution, the owners' objective is to maximize their own benefits as it is shown in Equation (2.10)

$$\text{Max } p \cdot Q(p,\, vt(H))\frac{h_i}{H} - C\,H \tag{2.10}$$

where,
h_i is the hours supplied by driver i

De Vany (1975) also solved the problem for a market with limited entry but unconstrained price and studied the efficiency of pricing, postulating that *Q* is maximized subject to a zero-profit constraint. His work was in line with that of Douglas (1972) in that the efficient price minimizes output and observed that a comparable increase in the regulated price will be more likely to expand capacity under competition than under monopoly. Conclusions obtained by De Vany (1975) are very useful, characterizing the competitive and the monopoly markets in terms of capacity and output in relation to price. It is shown that price is higher and capacity is lower for monopoly market in comparison to competitive market.

Beesley (1973) and Beesley and Glaister (1983) also investigated the different markets and their characteristics, trying to establish guidelines for decision makers using a model for simulating relevant inferences in the taxi market. The important elements and the problems of regulation (monopoly rights, entry conditions and fare control) are identified, introducing the external cost (congestion produced by taxi cabs) and testing the regulation effects with the taxi related data obtained from London, Liverpool, Manchester and Birmingham. The conclusion of their work was that any elasticity larger than 1 is only possible in a regulated market (free markets have elasticity lower than 1).

Manski and Wright (1976) provided a structural model of a taxi stand, assuming Poisson customer arrivals, negative exponential service time and FIFO queue rule, in opposition to previous models that used aggregated values and assumed a decreasing function of expected waiting time for the demand and increasing returns to scale, concluding that over a certain range, increasing the number of licenses will decrease the expected waiting time and increase the expected utilization rate. In their work, a diagram with the main elements and relations between demand and supply in a taxi market (Figure 2-7) is presented.

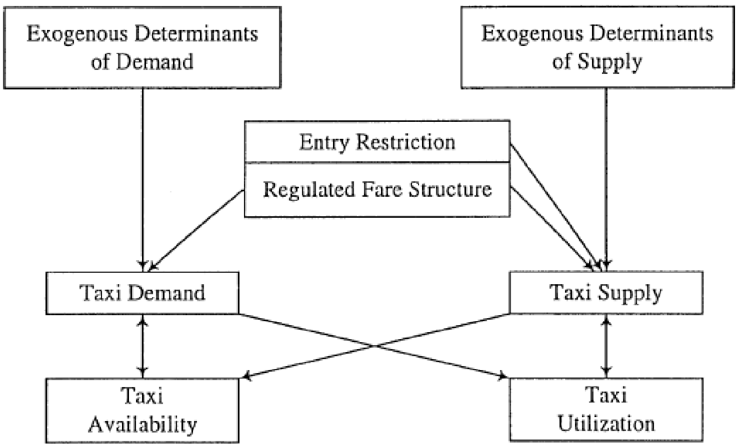

Figure 2-7 Demand-availability-utilization-supply relation in a taxi market. Source: Manski and Wright (1976)

Schroeter (1983) developed a theoretical model in a regulated market where dispatching and airport taxi stand are the primary modes of operation. The principal difference between this model and the models of Orr (1969), De Vany (1975) and Douglas (1972) is the use of the early work of Manski and Wright (1976), creating a dynamic and probabilistic model. Schroeter (1983) proposed equilibrium in terms of income between the taxi cabs waiting in the airport and the taxi cabs in the city waiting for a call from the radio dispatching center, which means that both modes will have the same benefits. This methodology was applied to the Minneapolis taxi sector concluding that any growth in the number of taxis decreases the waiting time, and increases the demand, but reduces the benefits for each taxi (in opposition to the scale economies announced by Manski and Wright, 1976).

Foerster and Gilbert (1979) studied the effects of regulation within a framework of eight regulatory scenarios involving different prices, entry policies and type of industry concentration factors. The 8 scenarios are presented in Figure 2-8.

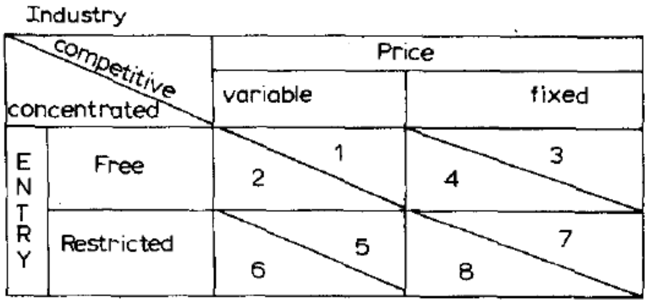

Figure 2-8 Scenarios for taxi regulations analyzed by Foerster and Gilbert (1979).

For each combination the equilibrium situation is analyzed, concluding that

- In an unorganized industry, price will not be regulated by the market; it will tend to rise without any countervailing down pressure, decreasing the utilization rate.
- If prices are fixed, monopoly will produce a lower level of output in relation to the level produced by the competitive industry.
- Entry control has the same effects related to price increases in both types of industry.

Finally various guidelines for policy makers were proposed and the necessity of empirical data for documenting and demonstrating regulatory impacts highlighted.

Cairns and Liston-Heyes (1996) redefined the demand proposed by Douglas (1972) and its relation to the supply and assume uniform demand within the day, which decreases with the increase of the waiting time, while analyzing the monopoly market (maximizing the benefits of the industry), the social optimum or first best solution (maximizing the sum of the social and industrial benefits) and the second best solution (non-negative profits). The maximization of total profits in a monopoly market as is formulated as:

$$\pi = p \cdot f(p, w(N \cdot h - 24 \cdot t \cdot Q)) - N \cdot c(h)/24 \qquad (2.11)$$

Where,
h are the working hours
p is the fare
w is the waiting time
N is the number of taxis
t is the duration of the trip
Q is the number of trips demanded per hour
c is the trip cost

For obtaining the social optimum, the calculation of welfare (W) as it is shown in Equation (2.12) is proposed, where x is the variable fare.

$$W=\int f(x,w)dx+\ pf(p,w)\ -Nc(h)/24 \tag{2.12}$$

It is therein observed that profits are zero when taxis are used at their optimal intensity and obtained the second best solution and concluded that regulation is needed for achieving second best solution: “controlling entry, welfare and benefits will increase, regulating fares, demand-supply differences will not lead to bargaining the price”.

Arnott (1996) analyzes the shadow cost of taxis in the first best solution, proposing subsidization for covering these costs in the vacant trips of taxis. Various authors focused their work on the second best solution, treating the first best solution as an unattainable idealistic situation, while Arnott focused his work in the first best solution, trying to decentralize it. The social surplus (SS) for obtaining the first best solution at its maximum value is derived in his work:

$$SS=\int_0^Q P\big(Q',W(V)\big)dQ'\ -m(Q+V) \tag{2.13}$$

$$Q{:}P\big(Q,W(V)\big)-m=0 \tag{2.14}$$

where
Q is the number of occupied taxi-hours
V is the number of vacant taxi-hours
P is the price per occupied taxi hour
W is the expected waiting time
m is the cost per taxi-hour

The social surplus (social welfare) provided in Equation 2.13 is calculated as the difference between the drivers’ benefit and the cost and proposed in Equation 2.14 that the marginal cost of the occupied taxis is equal to the willingness of customers to pay (first best solution). Arnott (1996) presented a model which attempts to treat explicitly the technological and informational aspects of the taxi problem, considering dispatching rather than hailing taxi services. A uniform customer demand distribution over a spatially homogenous two-dimensional city is considered in his work, and a dispatching center supply where waiting time is calculated as proportional to the square of the density of stopped taxis (in a hailing market, this is indirectly proportional to the square of the density of free taxis). For this reason dispatching centers are used in small cities, while in large cities hailing markets are more frequent. His work concluded that subsidization is necessary, justifying it with the decentralization of the social optimum, observing that the shadow cost is covered only when a taxi is occupied.

Daganzo (2010a) was the first to study the travel and waiting time as physical variables. In his work the optimal size of the taxi fleet using the queue theory is studied and a region (R) where dispatching services are offered to customers (distributed with uniform density λ) is considered. Daganzo (2010a) analysed the three states of the dispatching taxies (idle, assigned and servicing), and the number of taxis in each state and formulated the problem as in Equation 2.15:

$$m=\frac{R}{(2vT_0)^2}+\lambda R(T_0+\frac{l}{v}) \qquad (2.15)$$

where,
m: optimal fleet size
R: region area
v: average speed
T_0: customer waiting time
λ: demand density
l: average trip length

The minimum LoS is presented in Equation 2.16:

$$T_0=0.8\lambda^{1/3}v^{2/3} \qquad (2.16)$$

while the minimum fleet size for this LoS is presented in Equation 2.17:

$$m=1.2R\lambda^{2/3}v^{-2/3}+\lambda R\frac{l}{v} \qquad (2.17)$$

In his work, costs and time are combined, obtaining the lowest generalized cost as:

$$\left(\frac{z_T^*}{\beta}-\frac{l}{v}\right)=T_0^*+T_s+m^*\frac{\gamma}{\beta\lambda R}\approx\left(0,8+1,2\frac{\gamma}{\beta}\right)\lambda^{-1/3}v^{-2/3}+\frac{\gamma l}{\beta v} \qquad (2.18)$$

where,
Z_T* is the generalized cost
γ is the taxi cost per unit of time
Ts is the customer waiting time
B is the value of time

It is stated that the extra cost is at least l/v, even for λ=0, concluding that taxis do not have significant economies of scale (as observed by Schroeter (1983)).

Chang and Huang (2003) expanded the research of Douglas (1972) optimizing the vacancy rate and fares and specified a log-nonlinear function to simulate the demand of taxi trips, failing in obtaining the consumer surplus under elasticities between (-1,0).

Chang and Chu (2009) continued the work of Chang and Huang (2003) using a more generalized model with the welfare maximization objective for avoiding the elasticity constraint and used a log-linear demand function for rewriting the Equations proposed in the literature, while proposing the formulation presented in Equation 2.19 and Equation 2.20:

$$Q = A_1 P^{\alpha_1} w^{\beta_1} \tag{2.19}$$

$$w = A_2 V^{\alpha_2} \tag{2.20}$$

where,
Q is the daily occupied distance in the market (km/day)
V is the total daily vacant distance (km/day)
P is the fare rate (dollars/taxi day)
w is the waiting time
α_1 is the price elasticity of taxi demand
α_2 is the vacant distance elasticity of waiting time
β_1 is the waiting time elasticity of taxi demand
A_1 is the constant term of demand function
A_2 is the constant terms of waiting time function

Their model can analyze and optimize the vacancy rate and fares subsidization in a first-best environment and uses the definition of consumer surplus (CS) and welfare (W) shown in Equation 2.21 and Equation 2.22, where PS is the producer surplus.

$$CS = \int_0^Q P(X)dX - PQ \tag{2.21}$$

$$W = CS + PS \tag{2.22}$$

Equation 2.23 to Equation 2.26 calculate the maximized vacant distance (V*), occupied distance (Q*), vacancy rate (R*) and fare (P*) according to the authors

$$V^* = A_1^{\frac{1}{1-\alpha_2\beta_1}} A_2^{\frac{\beta_1}{1-\alpha_2\beta_1}} c^{\frac{\alpha_1}{1-\alpha_2\beta_1}} \left(-\frac{\alpha_2\beta_1}{1+\alpha_1}\right)^{\frac{1}{1-\alpha_2\beta_1}} \tag{2.23}$$

$$Q^* = A_1^{\frac{1}{1-\alpha_2\beta_1}} A_2^{\frac{\beta_1}{1-\alpha_2\beta_1}} c^{\frac{\alpha_1}{1-\alpha_2\beta_1}} \left(-\frac{\alpha_2\beta_1}{1+\alpha_1}\right)^{\frac{\alpha_2\beta_1}{1-\alpha_2\beta_1}} \tag{2.24}$$

$$R^* = \frac{V^*}{V^* + Q^*} \tag{2.25}$$

$$P^* = \left(\frac{Q^*}{A_1 A_2^{\beta_1} (V^*)^{\alpha_2\beta_1}}\right)^{\frac{1}{\alpha_1}} = c \tag{2.26}$$

The maximum social benefit (Social Welfare - SW) for the demand Q is the balance between the total revenues (TR - producer incomes), consumer surplus (CS - benefit for the customer) and total costs (TC). The maximum SW is obtained from Equation 2.27.

$$\frac{dSW}{dQ} = \frac{d(TR + CS - TC)}{dQ} = 0 \tag{2.27}$$

Or from Equation 2.28

$$\frac{d(TR+CS)}{dQ} = \frac{dTC}{dQ} \quad (2.28)$$

If the demand (D) is a function of the price (Q) (Equation 2.29), Equation 2.30 and Equation 2.31 are valid.

$$D = P(Q) \quad (2.29)$$

$$TR + CS = \int_0^Q P(Q)\,dQ \quad (2.30)$$

$$\frac{d(TR+CS)}{dQ} = P(Q) \quad (2.31)$$

From Equation 2.30 and Equation 2.31 it is shown that the maximum welfare is equivalent to Equation 2.32, where price is equal to the marginal cost (C_m).

$$P(Q) = \frac{dTC}{dQ} = C_m \quad (2.32)$$

The maximum social willingness-to-pay (P(q)) is calculated by using the log-linear demand function of the form showed in Equation 2.33, where A and u are parameters of the model.

$$P(q)=Aq^{\frac{1}{u}} \quad (2.33)$$

Finally, the relationship between the demand function (D), average cost (AC), marginal cost (MC) and subsidy (S) is described as it is shown in Figure 2-9.

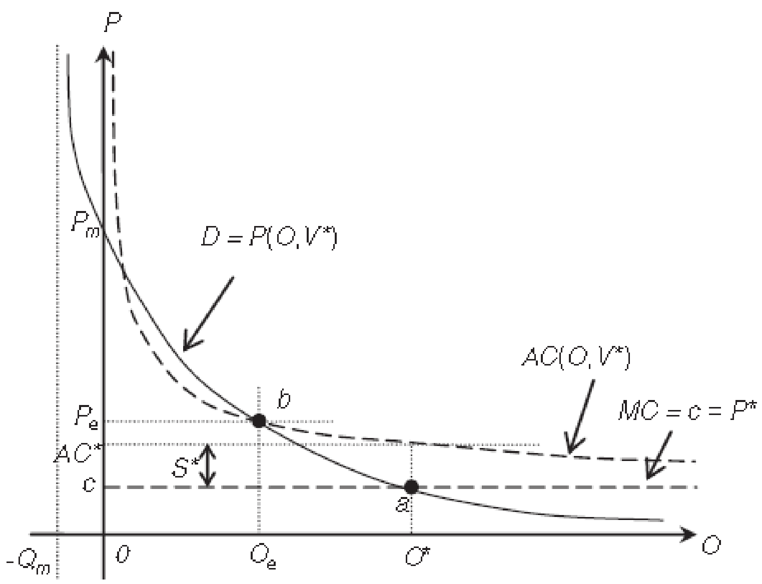

Figure 2-9 Relationship between demand function, average cost, marginal cost and subsidy. Source: Chang and Chu (2009)

As it can be observed in Figure 2-9, subsidization can reduce the actual cost to the marginal cost (virtual equilibrium between cost and demand).

Daniel (2003) modeled a taxi market in which fare and entry are regulated and tested using the data obtained by Schaller (2007) and found an inelastic relationship between vacant taxicabs and demand. In his work, a demand function (Q) is used, depending on the price of the service (P) and the number of vacant taxi cabs (V).

$$Q=P^{\beta}V^{\gamma}X_D \quad (2.34)$$

Where γ and β are the elasticities of price and demand respectively and X_D is a function of exogenous variables. The individual supply function (T) has the form:

$$T=\left(\alpha/_{A}\right)^{\alpha}P^{\alpha}Q^{\alpha}N^{1-\alpha}X_S \quad (2.35)$$

Where X_S is a function of exogenous variables and A>0, α<1 (the model assumes increasing marginal costs of operation).

Massow and Canbolat (2010) developed a model for simulating the taxi behavior in a dispatching market where taxis are assigned to virtual queues generated in each zone, and also in high demand points and conclude that taxis will wait in the borders between zones and proposed the creation of super zones for increasing the level of service to customers.

Fernandez et al. (2006) studied the characteristics of the hailing taxi market, defining demand as a function of generalized price, and proving that a unique equilibrium exists for a deregulated market corresponding to a monopolistic equilibrium. Their assumptions are:

- Only hailing services are considered.
- Geographic and time restrictions for taxis.
- Vehicles operate independently.0
- Average travel length and time.
- Same number of trips per taxi.

In their work, it is also analyzed the relations between the free market, the best social or first best solution and the second best solution. Their work agrees with most of the authors in that the social optimum fare produces losses to taxi operators, but at the same time it is shown that economies of scale are produced by externalities (reducing waiting time and operating cost). Their work concludes that entry regulations are redundant with fare regulations and that they produce worse industry conditions. They analyze the problem firstly from the short run point of view (one cycle), calculating the individual demand of each vehicle and the costs (taxi, industry and customer cost), and secondly from the long run point of view, enveloping the family of short run average cost functions. They presented the expressions for the total system cost (TSC – Equation 2.36), average system cost (ASC – Equation 2.37) and marginal system cost (MSC – Equation 2.38):

$$TSC(Q)=2\,(\theta\kappa Qc)^{1/2}+Qt(c+\varphi) \quad (2.36)$$

$$ASC(Q)=2\left(\theta\kappa c/Q\right)^{1/2}+t(c+\varphi) \qquad (2.37)$$

$$MSC(Q)=\left(\theta\kappa c/Q\right)^{1/2}+t(c+\varphi) \qquad (2.38)$$

where,
Q is the number of runs produced by taxis during the analysis period
κ is a calibration parameter for the specific area considered
θ is the waiting time value
c are the operation costs
t is the average run duration
φ is the customer value of time

They obtained a similar representation to the one presented by Chang and Chu (2009) (Figure 2-9), proving the presence of increasing returns to scale in the taxi services.

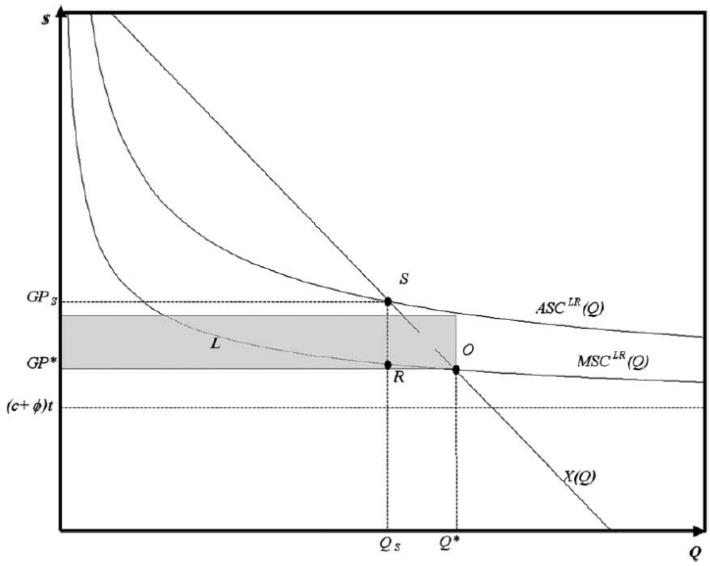

Figure 2-10 Long run average and marginal system cost functions. Social optimum or first best and second best solutions. Source: Chang and Chu (2009)

As it can be observed in Figure 2-10, social optimum (O) is the intersection of the marginal cost and the demand functions. Second best solution (S) is the intersection of the demand and the average cost functions. Fernandez et al. (2006) agree with most authors when affirming that social optimum does not fully cover taxi operating cost, while second best solution covers it exactly, maximizing social welfare subject to non-negative profits of taxi drivers. The triangle area SRO represents the social loss generated in the second best solution with respect to the social optimum. They observed that, for a supply of services where many small operators exist, the returns to scale make it impossible to obtain the social optimum without subsidy (area L in Figure 2-10), coinciding with Cairns and Liston-Heyes (1996) and Arnott (1996). They also calculate the waiting time externality as a product of the derivative of the average waiting time and the supply. They conclude that the need for regulation should be carefully considered case by case, due to the fact that the difference between the second best solution and the unregulated free market equilibrium depends on the specifications of each case.

2.2.3. Equilibrium models

The above studies were mostly realized by economists and examined extensively both price and entry controls in the taxi market. Models proposed in section 2.2.1 are mostly based in aggregate demand and supply models and related to various markets configurations (monopolistic and competitive). The principal assumptions are:

- Waiting time of customers related to the total number of vacant taxi hours.
- Demand based in fares and waiting time of customers.
- Constant operating cost per hour.

Equilibrium or network models take into account the spatial distribution of demand and supply and the characteristics of the network where these trips take place. Yang and Wong presented a series of models between 1997 and 2010 studying the taxi market in the network of Hong Kong. Their spatial models are more realistic than the aggregated ones presented in the previous literature since they take into account the spatial distribution of supply and demand.

Yang et al. (1997) developed a macroscopic simultaneous Equations model of urban taxi services using data from a simple modal split model of taxis developed in Hong Kong for transport studies and the annual taxi service surveys conducted since 1986. They used this survey for the evaluation of the taxi services and government decision-making with respect to the number of taxis and fares.

Yang and Wong (1998) stated that relationships for taxi services are much more complicated than the corresponding for many goods and services used as examples in classical economic analyses. The principal differences are the role of the involved variables (e. g. customer waiting time) of taxi availability and the taxi utilization through which the demand and supply are interrelated. They used the demand model proposed by Douglas for developing a model describing how vacant and occupied taxi will cruise in a road network searching for customers. Due to the fact that demand and supply in the taxi sector are distributed in the city streets, they propose the use of a detailed road network structure and a customer origin-destination matrix solving a conventional network traffic assignment. The model attempts to provide information to decision-makers related to taxi regulations on the basis of different system performance measures at equilibrium. They assume the following:

- Stationary taxi movements and customer demand.
- The demand between each origin and destination is fixed and given for each instant of the day (demand elasticity is not considered).

- Travel time on links is constant (traffic congestion is not considered), what means that travel times via shortest path between each origin and destination are constant. What means that taxis in the network will follow an "all-or-nothing" routing behavior
- Each taxi tries to minimize its travel time when searching for a new customer.
    - o The expected search time in each zone is identically distributed following a Gumbel density function.
    - o The probability of a vacant taxi in a zone j to meet a customer in a zone i follows the logit model as defined in Equation (2.39).

$$P_{i/j}=\frac{e^{-\theta(h_{ji}+w_i)}}{\sum_{m\in I} e^{-\theta(h_{jm}+w_m)}}\ \forall i,j \tag{2.39}$$

where,
θ is the calibration parameter (θ>0)
h_{ij} is the travel time between zones i and j
w_k is the taxi waiting time at zone k

The value of θ reflects the degree of information of the taxi drivers concerning the location of waiting customers. If θ tends to infinite, the model leads to a deterministic case where perfect information concerning searching time is available.

Yang and Wong (1998) proposed a methodology for obtaining the minimum taxi fleet size that ensures the existence of a stationary equilibrium state. They related the characteristics of the market with θ and w_k, (e. g. in a dispatching monopolistic taxi market w_k must be zero because drivers have complete information about the customers) and obtained the minimum fleet size in relation to the value of θ for a simple numerical example proposed by the authors.

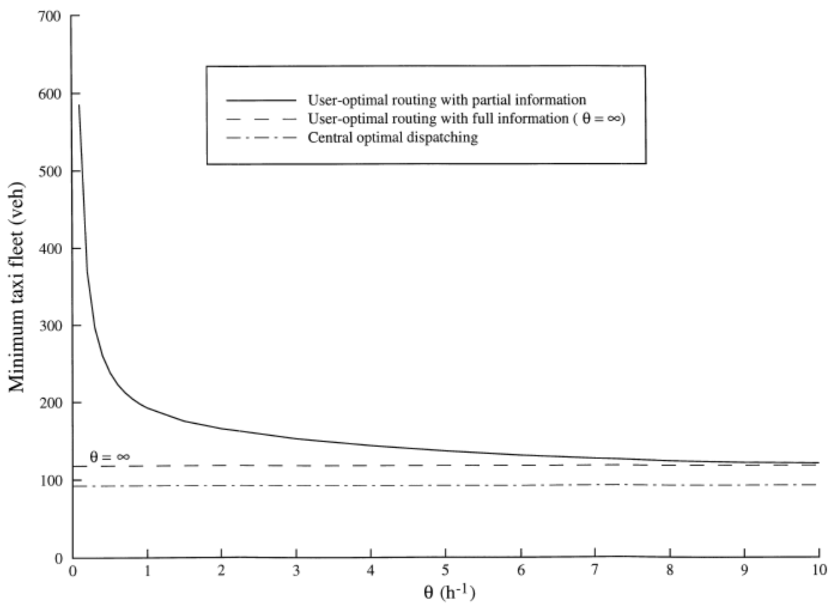

Figure 2-11 Minimum taxi fleet size vs. uncertainty parameter. Source: (Yang and Womg 1998)

As observed in Figure 2-11, better knowledge of the network (where to look for customers) results in smaller minimum fleet. They concluded that taxi fleet and information of taxicabs must be regulated in order to achieve better taxi utilization, while maintaining a certain level of service. Meanwhile, a better real time information service about the market will reduce unnecessary vacant taxi movements, reducing environmental impacts and customer waiting time.

The fixed-point algorithm proposed by Yang and Wong (1998) cannot guarantee convergence in large-scale applications. For solving this, Wong and Yang (1998) proposed an improved algorithm and applied it to the taxi traffic problem. They used the same assumptions as in the first model Yang and Wong (1997), but they used an optimization model for the entire movement of all vacant and occupied taxis, from which a gravity-type distribution model of vacant taxis was derived. The new algorithm proposed by Wong and Yang (1998) is better because the iterative balancing procedure is convergent and can be easily obtained; the procedure is also more robust and stable over the fixed-point algorithm.

With the improved algorithm of Wong and Yang (1998), the congestion of the network can be added to the model. Indeed Wong et al. (2001) extended the work of Wong and Yang (1998) including congestion effects in the network and customer demand elasticity. With congestion effects, the problem evolved into a bi-level optimization problem, and the development of a new solution algorithm was necessary. The lower-level problem is a combined network equilibrium model describing simultaneously movement of vacant and occupied taxis, while the normal traffic follows a user-optimal behavior for reaching their destinations. The upper-level is a set of linear and non linear Equations ensuring that the relation between demand and supply is satisfied. The lower level problem is solved by the conventional multi class combined trip distribution and assignment algorithm, whereas the upper level problem is solved by a Newtonian algorithm with line search. Wong et al. (2001) consider separate demand functions for each OD pair (i, j), depending on customer waiting time (W_i), trip price (F_{ij}) and travel time (h_{ij}). They use an exponential function for expressing customer demand from zone i to zone j as it is shown in Equation (2.40).

$$D_{ij} = \widetilde{D}_{ij} \cdot e^{-\gamma(F_{ij}+\nu_1 h_{ij}+\nu_2 W_i)} \qquad (2.40)$$

where,
$\widetilde{D}_{ij}$ is the potential demand from zone i to zone j
ν_1, ν_2 are the monetary value of travel time and waiting time respectively
γ is the scaling parameter

The scaling parameter (γ) indicates the sensitivity of demand to full trip price. They analyze the waiting time (both the customer waiting time and the taxi waiting time), the demand and the taxi utilization rate in relation to the number of taxis. Figures below show their findings.

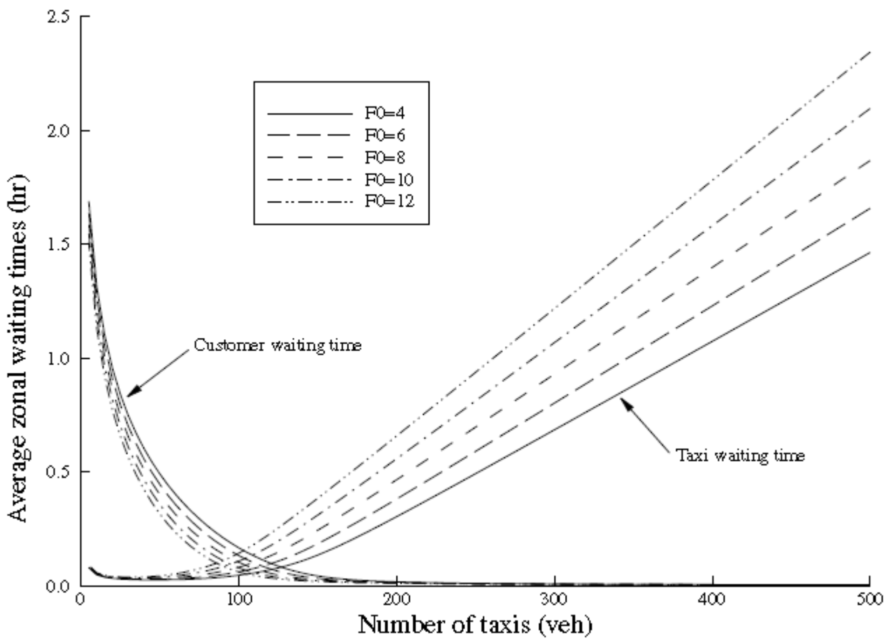

Figure 2-12 Average waiting time vs. taxi fleet size and fare. Source: Wong and Yang (1998)

Figure 2-12 shows that taxi waiting time has almost a linear relationship with the number of taxis, while customer waiting time has a quadratic shape. From Figure 2-12, the optimum number of taxis can be obtained, where both the taxi waiting time and customer waiting time have low values.

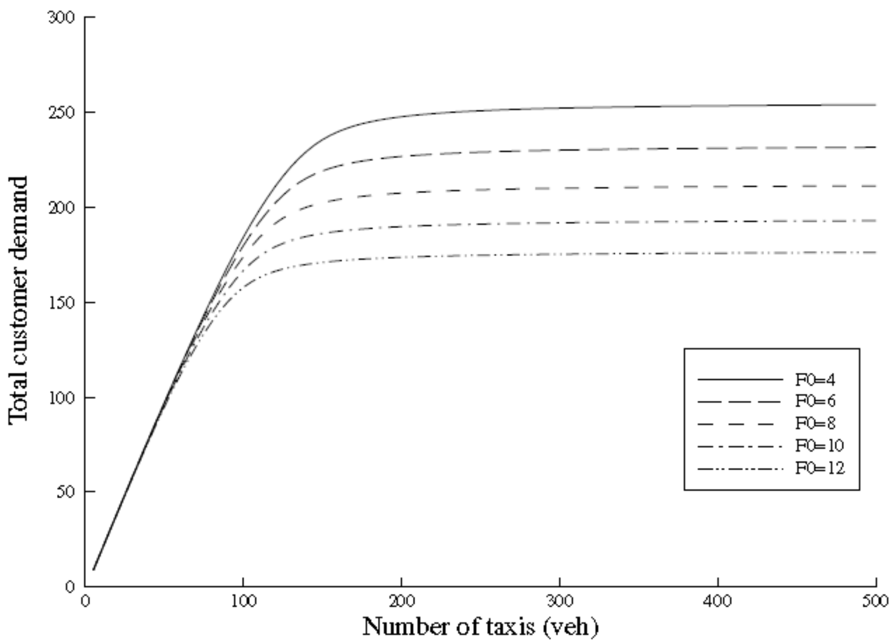

Figure 2-13 Total customer demand vs. taxi fleet size and fare. Source: Wong and Yang (1998)

Figure 2-13 shows how demand reaches a maximum, and its linear relation to the number of taxis in markets with few taxis.

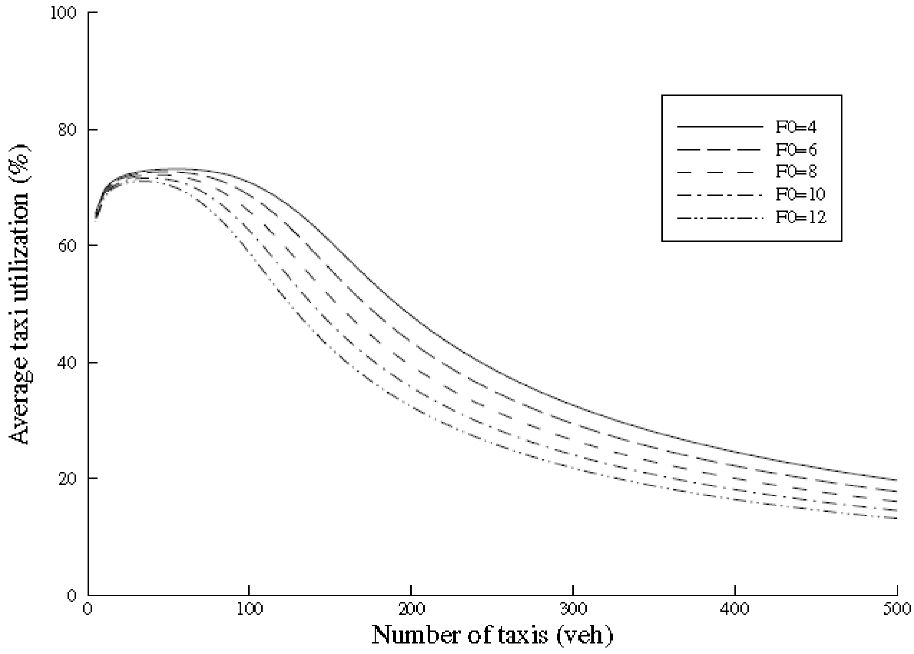

Figure 2-14 Average taxi utilization vs. taxi fleet size and fare. Source: Wong and Yang (1998)

Figure 2-14 shows the increase of taxi utilization in small markets with the increase of the number of taxis (this increase rapidly changes into decrease when the market grows significantly).

They agreed with Manski and Wright (1976), Schroeter (1983) and Arnott (1996) in the fact that an increase in the number of taxis will be beneficial for both, customer and drivers, but only in a small taxi fleet (this is an unstable situation, and it rarely emerges in a realistic taxi market). They conclude that their model is useful in Hong Kong, because taxis make up a large portion of the traffic stream in the city, having an important role in the travel time of each link. In another city, with a lower number of taxis, the impact of taxis flow on congestion can be neglected.

Yang et al. (2002) continued the development in the line of their earlier work focusing on the impacts of alternative regulatory constraints on the market equilibrium by investigating the social surplus (SS), firm profit (R) and customer demand at various levels of fares and fleet sizes in the regulated, competitive and monopoly markets. They studied the equilibrium conditions for the monopoly and competitive markets, first best and second best solutions. They applied their findings to the city of Hong Kong. They proposed to maximize the functions presented in Equaiton 2.41, Equation 2.42, Equaion 2.43 and Equation 2.44:

For the monopoly market

$$R(N^m) = \sum_{i \in I} \sum_{j \in J} F_{ij} D_{ij} - cN^m \tag{2.41}$$

For the competitive market

$$c = \frac{\sum_{i \in I} \sum_{j \in J} F_{ij} D_{ij}}{N^c} \tag{2.42}$$

First-best social optimum

$$SS(N^f) = \sum_{i \in I} \sum_{j \in J} \int_0^{D_{ij}} F_{ij}\big(\omega, W_i(N^f)\big) d\omega - c\, N^f \tag{2.43}$$

Second-best social optimum

$$SS(\tau^s, N^s) = \sum_{i \in I} \sum_{j \in J} \int_0^{D_{ij}} F_{ij}(\omega, W_i(N^s)) d\omega - c\, N^s \tag{2.44}$$

Where
D is the demand
F is the expected fare
W is the expected waiting time
c is the cost per taxi hour of service time
N is the solution of taxi fleet for each configuration
N^m is the monopoly solution of taxi fleet
N^c is the competitive solution of taxi fleet
N^f is the first-best solution of taxi fleet
N^s is the second-best solution of taxi fleet

Figure 2-15 and Figure 2-16 show the iso-social, iso-surplus, iso-profit and iso-demand contours of their findings.

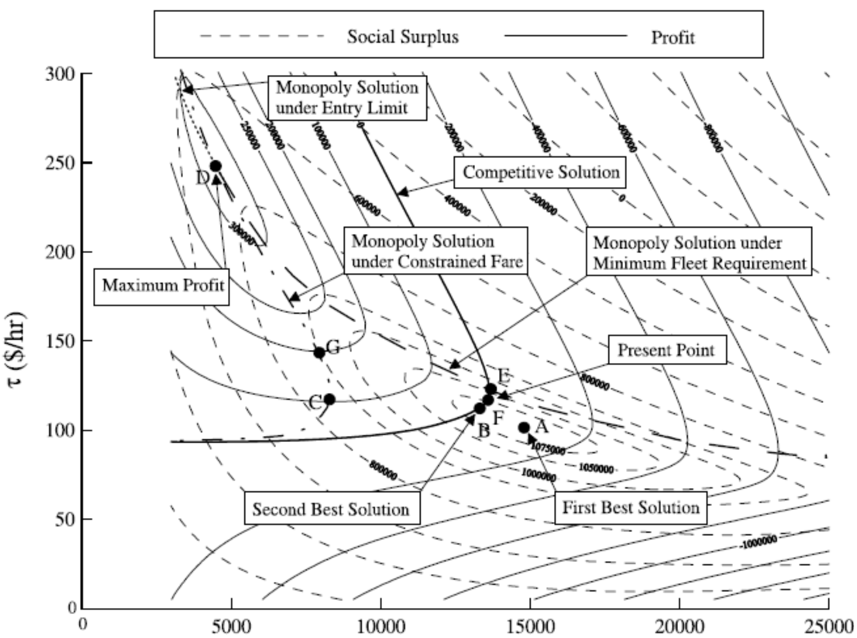

Figure 2-15 Social surplus and profit of the taxi market in a fleet-fare system. Source: Yang et al. (2002)

Figure 2-15 shows the surplus and profit for each combination of fare (τ) and number of vehicles (N), that are the two principal regulation parameters of the taxi market. It is noteworthy to mention that:

- D is the maximum profit, with a market characterized by a small taxi fleet and the high fare (monopoly behavior, where only a few customers will use the taxi due to the high cost – selective market).
- A is the maximum social surplus, the first best solution.
- B is the second best solution, with smaller fleet and higher fare than A (the difference between A and B is very small and can be achieved by subsidizing the taxi market).

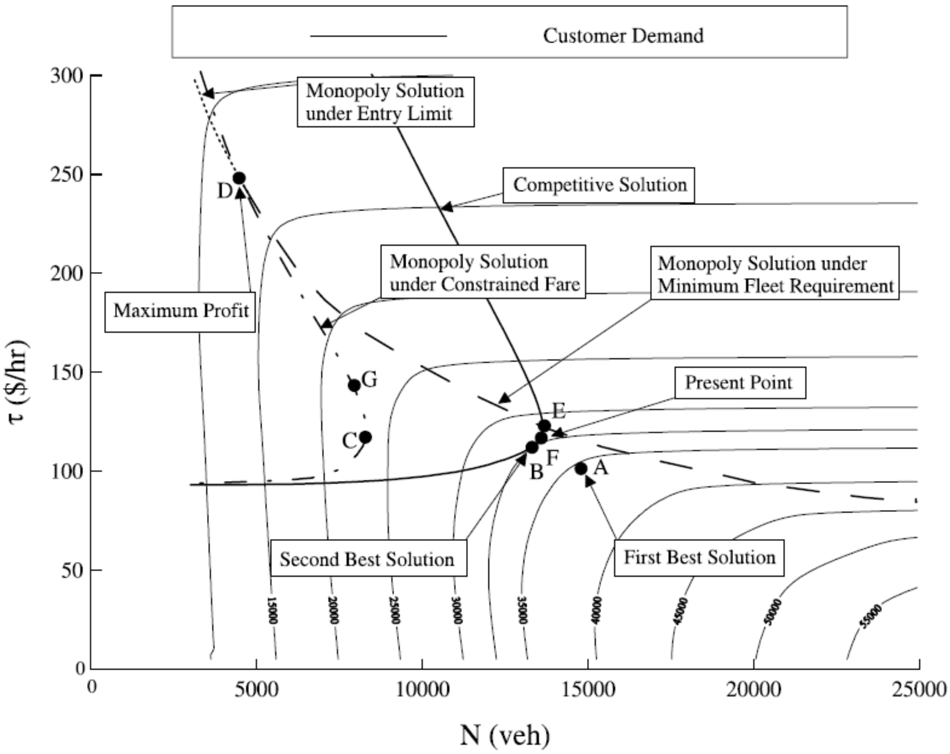

Figure 2-16 Customer demand of the taxi market in a fleet-fare system. Source: Yang et al. (2002)

Figure 2-16 shows how demand varies in the same conditions of fare and fleet, as shown in Figure 2-15.

Yang et al. (2001) studied the relations between the endogenous and the exogenous variables of their model (Figure 2-17).

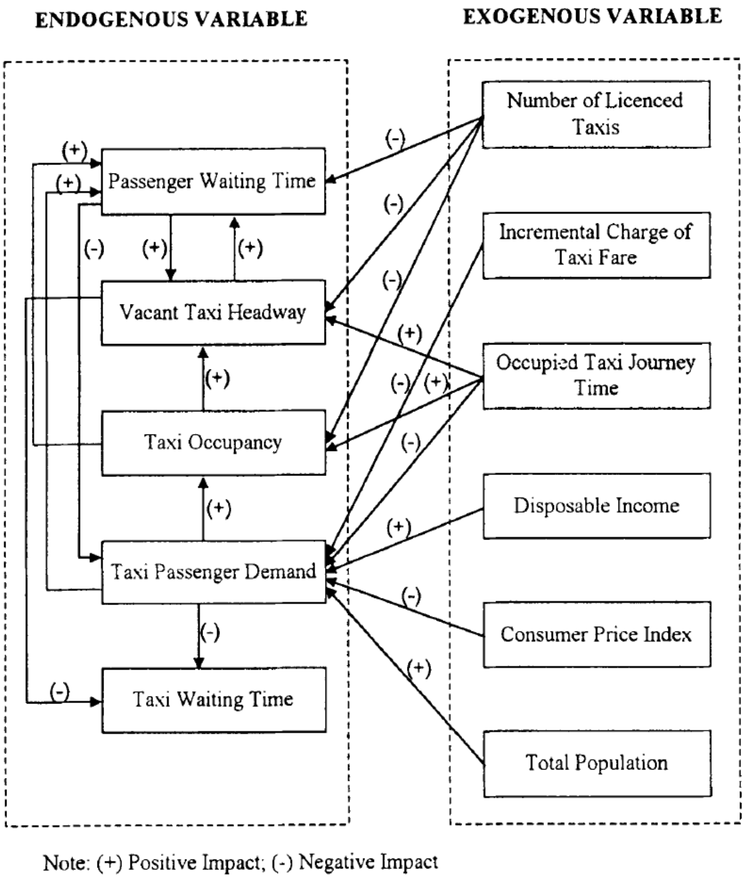

Figure 2-17 Relationships between endogenous and exogenous variables in the simultaneous Equation model. Source: Yang et al. (2001)

Figure 2-17 shows many of the assumptions listed above, such as the negative relationship between the number of taxis and the customer waiting time, or the negative relation between demand and fares. Other relations are obtained from the Equations presented above, such as the positive relation between taxi occupancy and vacant taxi headway.

The quasinewton method used in Wong et al. (2001) is not efficient when applied to large networks. For this reason Wong and Wong (2002) developed a more efficient solution algorithm for the taxi model with congestion and elastic demand, in which the computation of the Jacobean matrix in the lower-level takes into account the characteristics of the upper-level using a sensitivity-based solution algorithm.

Yang et al. (2005) developed a model for analyzing the monopoly, the social optimum and the stable competitive solutions of hailing taxi services in the presence of congestion externalities. They postulated that a profitable first-best social optimum emerges in a severely congested taxi market, where the entry of additional taxis into the market has a large marginal congestion effect (and thus the entry should be highly controlled at the social optimum). They postulated that in an effort to extract as much profit as possible, the monopolist would charge a price in excess of marginal cost per ride by an amount equal to the consumer´s marginal net willingness-to-pay for a ride. In the competitive solution case, they concluded that the maximum competitive taxi fleet size and hence revenue occurs at the unit elasticity of the customer demand, at which the increase in the revenue due to a higher fare is cancelled out with the loss in the customer demand. The function to maximize in the competitive solution is the same than the one used in the first-best social optimum, but the solution is a weighted average of the fist-best markup price and the monopoly markup price. Comparing the solutions, they observed that the second-best markup price is a portion of the marginal consumer surplus. In contrast to that, the monopoly markup price is exactly equal to the marginal consumer surplus in the absence of congestion externality. They concluded that in the competitive market, the second-best solution leads to a more efficient use of taxis, with a higher demand served by a smaller fleet and lower fare. They developed a new diagram with the relations between the endogenous and the exogenous variables of their model (Figure 2-18), including the internal loop of the demand generation.

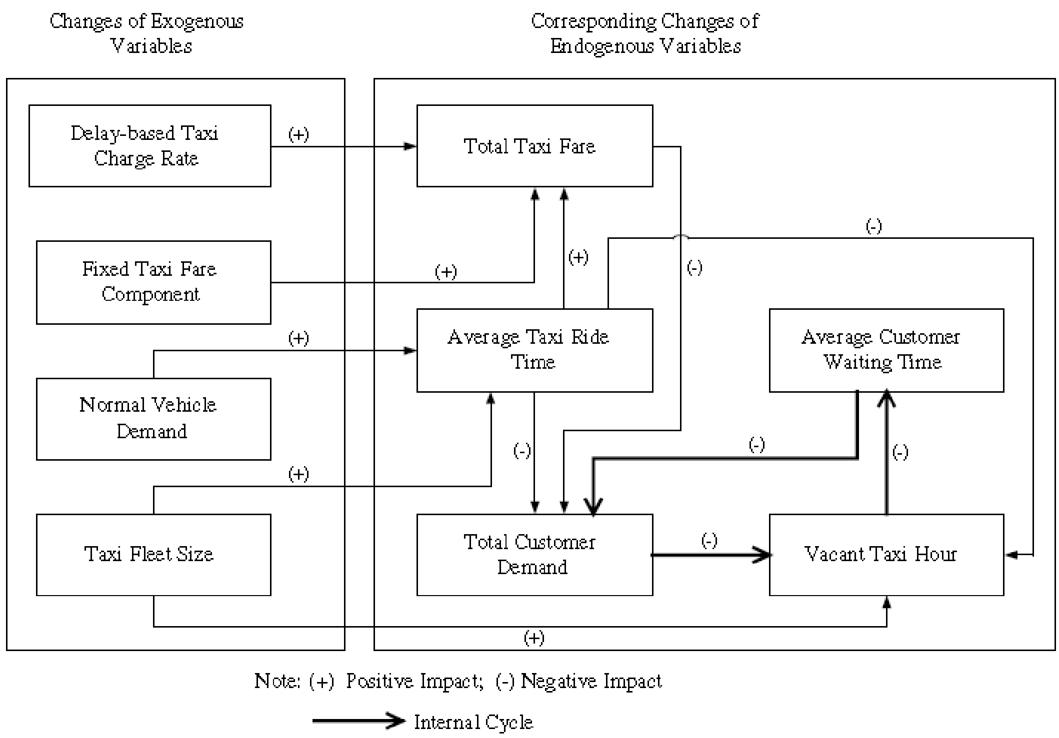

Figure 2-18 Relationships among endogenous and exogenous variables in the simultaneous Equation model. Source: Yang et al. (2005)

In the new relationships diagram the following differences with the diagram shown in Figure 2-17 can be observed: Many exogenous variables related to demand are not in the model (total population, consumer price index, disposable income). The normal vehicle demand is given as exogenous variable. Taxi fares (fixed and variable terms) are given exogenously, but the final fare is calculated endogenously using the average ride time. Taxi occupancy and waiting time are not included in the model. An internal loop in the equilibrium is presented (customer waiting time – customer demand – vacant taxi hours). They obtained a new mapping of the different market configurations based on the taxi fleet size and the applied fare for various levels of congestion (Figure 2-19).

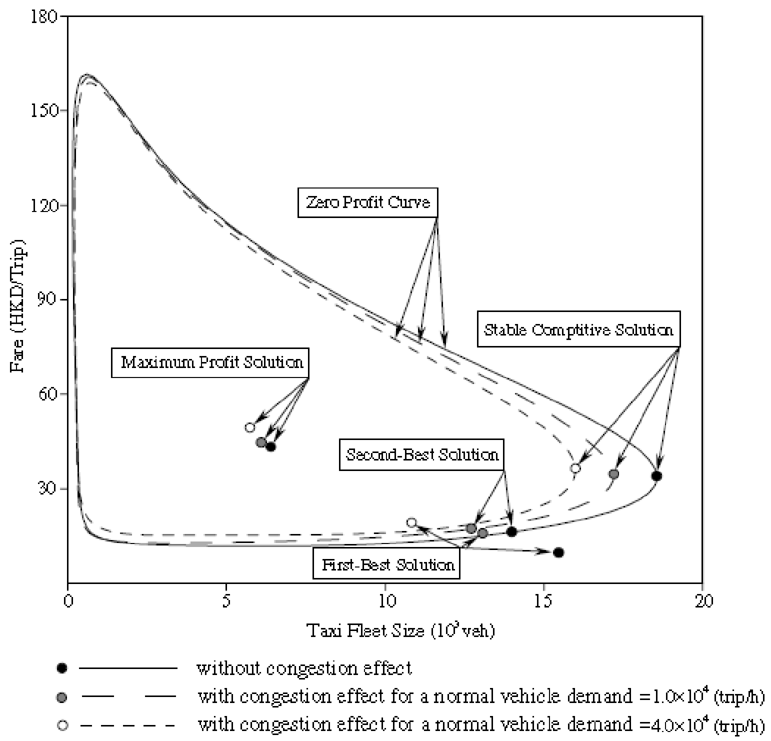

Figure 2-19 Taxi fleet size vs. fare. Source: Yang et al. (2005)

Figure 2-19 shows the effect of congestion externalities in various configurations of fares and fleet. As it can be observed, the effects of congestion externalities are the reduction of the number of taxis and a small increase in the fare.

Wong et al. (2004) developed a combined distribution and hierarchical mode choice and assignment network with multiple customers and mode classes, extending their single-class network model presented in Wong et al. (2001). They included multiple customer classes, multiple taxi modes and the hierarchical modal choice of customers for taxi services.

All the models presented above use a linear taxi fare calculation, making long-distance (from/to the airport) trips more profitable, creating over-supply in airports and wasting many taxi service hours in the airport's queue. Schaller (2007) proved that a free entry to the market in the USA and Canada had the consequence that taxi drivers would only accept the most profitable trips, offering a very low level of service to customers. In order to diverge excess taxi supply from the airport to other areas, increase the utilization of the taxi capacity and the quality of the service, Yang et al. (2010b) included a nonlinear taxi pricing of taxi services in their model. They identified the win-win situation (surplus for both producer and consumer) created by a Pareto-optimal improving situation, allocating more efficiently the taxi services in the region.

Hyunmyung et al. (2005) added demand stochasticity developing a stochastic modeling approach in a dynamic transportation network. They simulated taxi drivers' learning process implementing the day-to-day evolution approach introduced by Horowitz (1984), Vythoulaks (1990) and Cascetta and Cantarella (1991). They applied their model in a test network, generating demand at each node based on the demand rate at each peak period and the trip distribution pattern, proving drivers capacity in predicting customer queues at nodes. They also investigated the effectiveness of taxi information systems in reducing unnecessary trips, proving that using information systems is equivalent to increasing the number of taxis by 20% in regard to the quality of the service.

2.2.4. Simulation models

The concept of agent based models was developed in 1940, but it was in 1990 when the advances in computation procedures allowed them to widespread. The first use of the word agent as it is currently used today was done by Miller (1991).

Kikuchi et al. (2002) examined the link between the agent-based modeling and the current transportation problems and presented the definitions of agent and agent-based modeling as well as their attributes and structure. His work concluded with a study presenting various applications of agent-based modeling to the traditional transportation theory. Teodorovic (2003) and Chen (2010) continued the work started by Kikuchi and extended the review of the use of agent-based in the transportation field.

Only a few simulation based models for the taxi services have been presented. Bailey and Clark (1987) investigated changes of performance in the dispatching market related to the number of vehicles, concluding that the waiting time is relatively insensitive to changes in demand but highly sensitive to changes in the number of taxi cabs. Bailey and Clark (1992) used a discrete event method to simulate dispatching taxi services, obtaining a linear relation between total distance and fleet size.

Kim et al. (2005) developed a simulation based stand taxi services, which includes a knowledge building process. They proved that the use of information technologies could improve the quality of the service by 20%.

Song and Tong (2006) and Song (2006) presented dynamic taxi demand models using the simulation model approach of the taxi stand market. They studied the time-dependent taxi demand patterns, the assumption of imperfect information, the learning process and the effects of non-equilibrium in the taxi market due to entry control. They highlighted the limitations of traditional aggregated models (time-dependent patterns, imperfect information, learning convergence and non-equilibrium in taxi market due to regulation) and tested the effects of Advanced Transport Information Systems (ATIS) in this specific market. They proved that the learning behaviors benefit both the drivers and the customers, reducing the vacant time and the waiting time respectively.

Chen (2009) presented a dispatching architecture for the increase of the customers' satisfaction by concurrently dispatching multiple taxis to the same number of customers in the same geographical region. They proposed the definitions for the agents presented in Figure 2-20, Figure 2-21 and Figure 2-22:

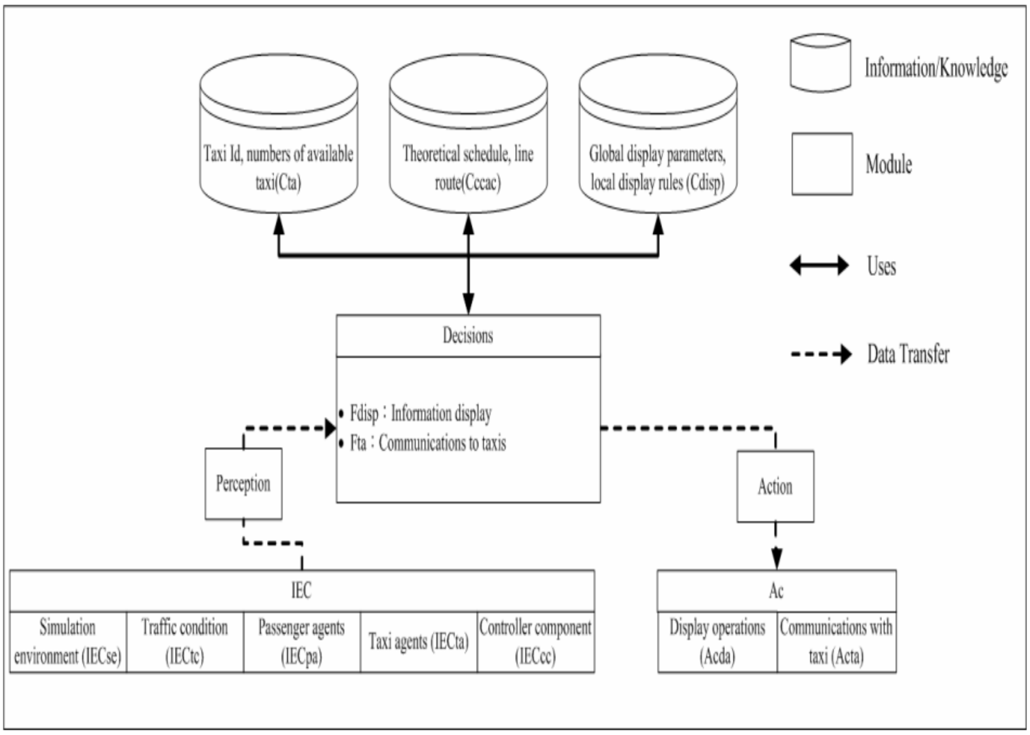

Figure 2-20 Definition of a Control Center agent. Source: Chen (2009)

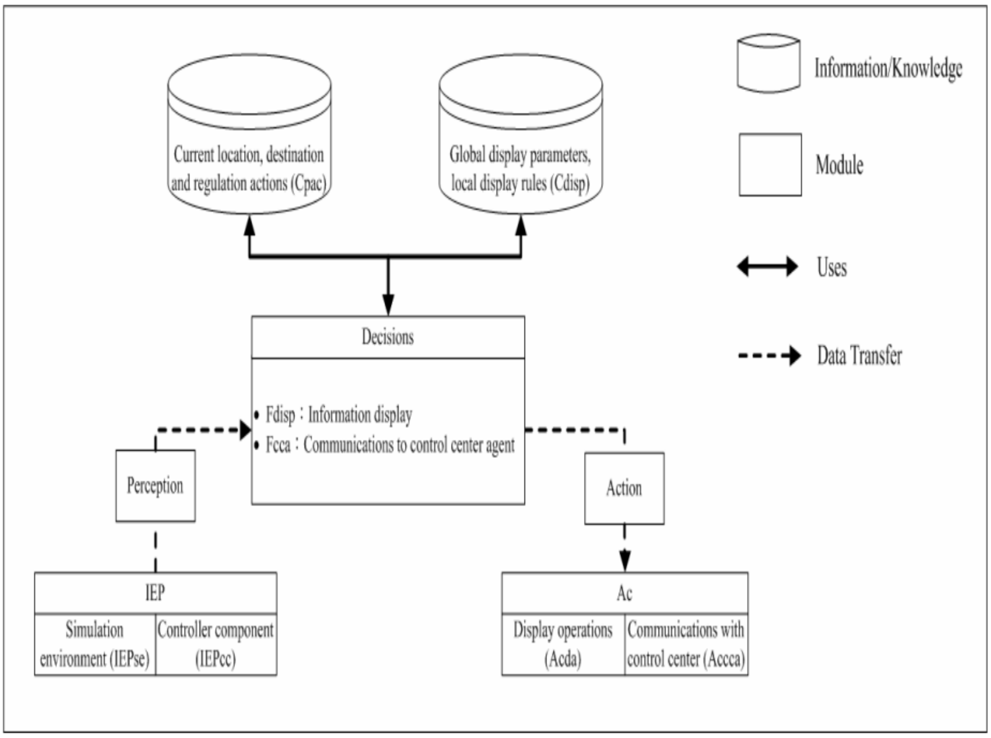

Figure 2-21 Definition of a customer agent. Source: Chen (2009)

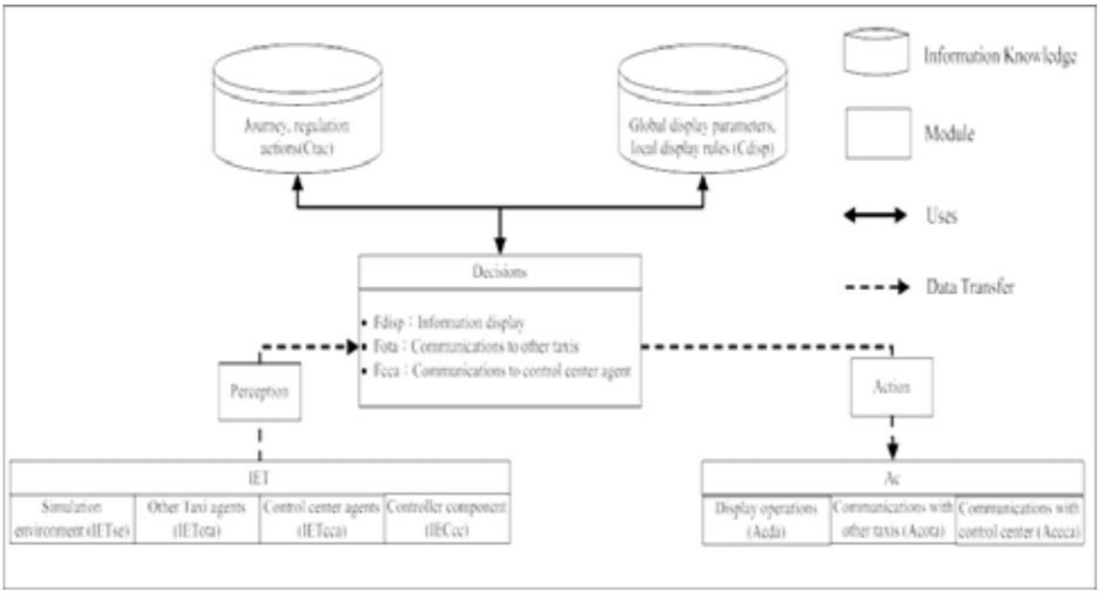

Figure 2-22 Definition of a taxi agent. Source: Chen (2009)

Recently, Lioris et al. (2010) developed a discrete-event simulation model for reproducing the real taxi on demand market conditions. They proposed a two level decision problem where the design and dimensioning of the fleet size are handled at the first level, while the real time operating stage is handled at the second level. They tested the model for three types of customers: customers hailing taxis at the road side, customers calling to a dispatching center for reserving a trip and a mixed mode where both type of customers are handled simultaneously. The architecture of the simulator is presented above:

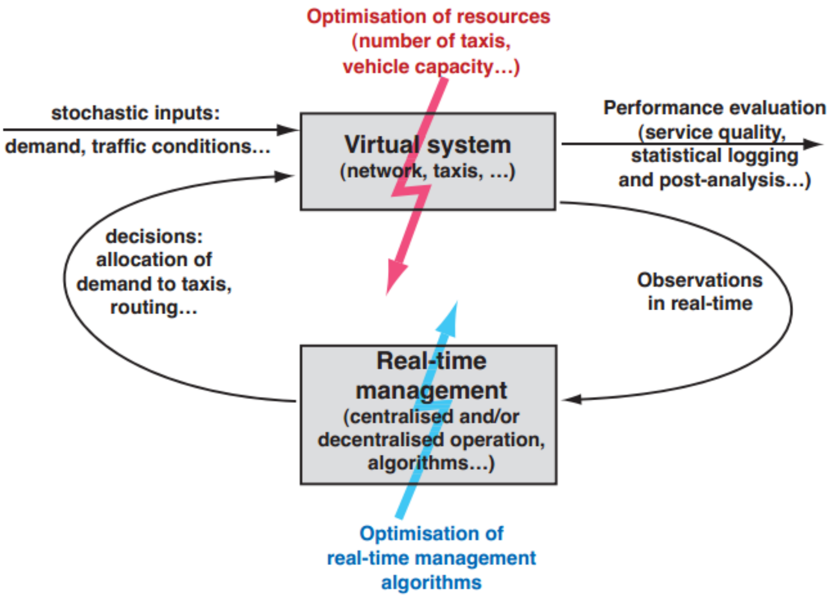

Figure 2-23 Architecture of the simulator. Source: Lioris (2010)

The proposed demand follows a Poisson process where the time between arrivals at node i follows an exponential law of parameter λ_i, while the probability of wanting to go to a node j is M_{ij}, where M is the OD matrix. They examined various demand geometries and intensities and tested it in the Paris Plan network (288 nodes and 674 links) using a total of 3.744 5-place taxis by using the model for answering the "what-if" questions since the mathematical models are out of reach for such a complex multi-agent system (taxis, customers, network or stochasticity).

Shi (2010) presented an event-based simulation stand taxi model for analyzing the customer-searching behavior of individual drivers and its influence on the performance of the system. They use a varying demand pattern where taxi drivers adopt different strategies for finding customers. The model is applied to a case study composed by a linear city with 20 residential zones and a single city center.

Kim et al. (2011) developed a time-dependent agent-based taxi simulation model and tested it with various customer patterns in order to provide policy-related guidelines for improving the service performance when the demand pattern is asymmetric. The model is composed by two types of agents, the taxi drivers and the customers. The travel times are considered to be constant with a small influence of the taxi flow. They divided the day into three periods, the Morning and Evening peak periods and the Off-peak period. They tested their model in a network consisting of 5 nodes and 16 links, where the central node was considered to be the Central Business District. As expected, as the vacant time of drivers was reduced the waiting time of customers was increased.

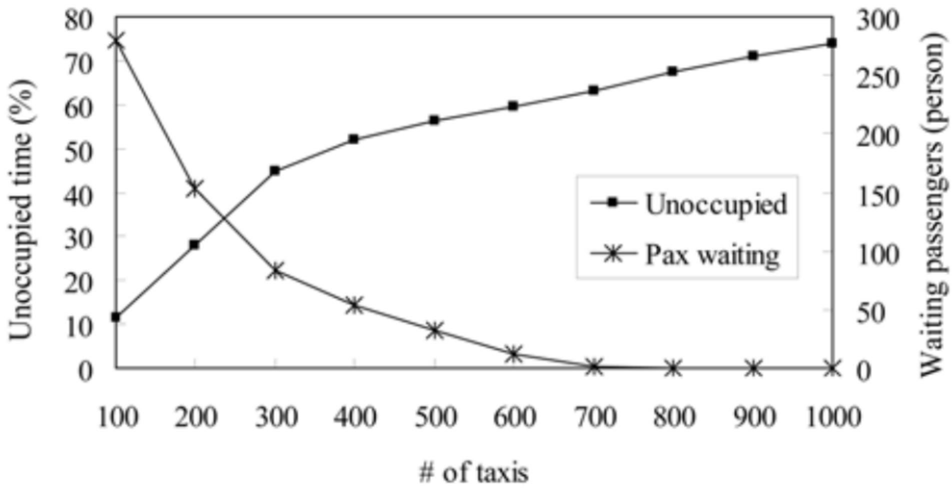

Figure 2-24 Change in Unoccupied Rate and Customer Waiting Time to the Number of Vehicles. Source: Kim et al. (2011)

Cheng and Nguyen (2012) proposed a massive multiagent simulation platform for investigating interactions among taxis and customers. They incorporated real-world driver's behaviors and validated the model in a real-world case study. They proposed the system presented in Figure 2-25:

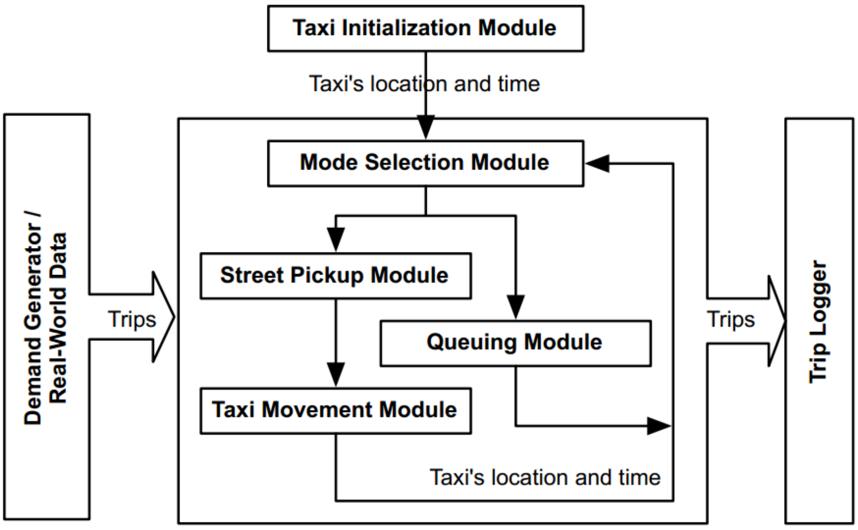

Figure 2-25 Agent-centric view of the system. Source: Kim et al. (2011)

2.3. Critical assessment of scientific literature review

The literature review presented in section 2.2 is summarized below, highlighting the important factors presented and discussed in the above models, unifying conclusions and identifying debilities and gaps.

Mathematical formulations using endogenous and exogenous variables for calculating demand, supply, waiting time, social welfare and other important indicators that explain the performance of the different taxi markets are proposed by most of the authors. New parameters have been introcuded in the modeling of taxi services, making it more realistic, such as the model developed by Yang and Wong (1997) and applied to the city of Hong Kong. They started with a simple model, adding congestion, demand elasticity, multiple customer classes and nonlinear taxi pricing. New technologies applied to the taxi market such as GPS, GIS and GPRS were also simulated in various models, proving their benefits and justifying their use. Many of the models developed have been applied to various cities around the world. Beesley (1973) and Beesley and Gaister (1983) studied the data obtained from questionnaires in different cities in the UK, especially from London. Schroeter (1983) was the first to use data from taximeters in his model, using the data from a taxi company in Minneapolis (USA). Yang and Wong (1997) use the data obtained from questionnaires in the city of Hong Kong in 1992. Schaller (2007) uses interviews and questionnaires from taxi agents and customers in different cities of the USA.

2.3.1. Regulation

Historically, most of the taxi markets were regulated (basically controlling entry and fares). Fares are regulated by fixing a flag down price and a fee regulating the way fares are applied to customers (per time, per distance, baggage supply, airport supply…). Most of the entry regulations were done by freezing the number of taxi licenses, without supporting in any way why the current number of taxis was optimal, or simply good. Most of the cities maintained the number of taxis at 1980 levels. On the contrary, only certain cities increased timidly their number of licenses following the GDP value or other economic indexes. Due to the characteristics of the taxi sector, each modification in the law related to taxi was very difficult to approve or implement due to their strong reactions to regulation measures. This situation created in many cities a suboptimal or inefficient taxi market, with more or less taxis than the optimum. Many authors support that the moment of the regulation has enormous influence in the results of the regulation, and the market situation must be studied in the moment of the regulation for justifying each measure adopted, from the number of taxis until the value of fares, therefore the starting point of the market is crucial in the success of the regulation policies. Fernandez et al. (2006) affirm that the two regulations must not be applied simultaneously, entry regulations are redundant with fares regulation, and the effect of entry regulation is negative on markets where fares are regulated (and vice versa).

There are economic and non-economic arguments in favor of entry control in taxi markets. The economic argument is basically the social welfare achievable with entry control, avoiding market failures. Non-economic arguments are potentially cross-modal competition, congestion and pollution issues. Moore & Balaker (2006) declared recently that most of the economic opinions favor open entry to the taxi industry. OECD (2007) identifies arguments against free entry and arguments against control entry, (resumed in Table 2-1). Also conclusions of CENIT (2004) are listed in the Table 2-1.

Table 2-1 Arguments against free entry and entry control. Source: own elaboration from OECD (2007) and CENIT (2004)

Arguments	Against free entry	Against entry control
Productivity arguments	Excess of capacity “Diversion” of demand from PT Most efficient use of resources	Augment of demand
Impact on congestion / pollution	More taxis than the optimum Congestion reduction	Less Private Vehicles
Distributional arguments and competitiveness	Preserve the income position of incumbent laborers	Reduction of development of new products (rivalry) Reduction of fares Reduction of waiting time
Impacts on service quality and information	Reduced standards of taxi services	Absence of information, tools and rules for regulators

CENIT (2004) also identifies arguments in favor and against fare control, presented in Table 2-2.

Table 2-2 Arguments in favor and against fare control. Source: own elaboration from CENIT (2004)

Arguments	Against free fares	Against fare control
Operational arguments	No feasible in the hailing and stand markets	Free market will give better solutions to each operation mode
Consumer protection	Higher fares in low demand zones	Uncertainty fixing the right fares No fare abuses
Competitiveness		Car renting has no regulation

2.3.2. Demand and supply

Various authors developed models for analyzing the effects of economic and non-economic regulations in the taxi market. They proposed mathematical formulations for calculating demand and supply, simulated various types of markets and obtained different results for each regulation scheme. Aggregated models calculated total demand and supply using different parameters; Douglas (1972) used the price of the trip and the expected waiting time for calculating the demand, and a flat cost rate for the supply and stated that if different customers have different willingness to pay, the regulator must find a price p for maximizing all global benefits; De Vany (1975) added an index of the full prices to the calculation of the demand; Cairns and Liston-Heyes (1996) supposed uniform demand within the day, which decreases as waiting time increases; Chang and Huang (2003) and Chang and Chu (2009) used log-nonlinear and log-linear functions respectively for simulating demand; Daniel (2003) used a demand function depending on the number of vacant taxis and the price; Fernandez et al. (2006) used the generalized price for obtaining the demand; Manski and Wright (1976) assumed a Poisson process of customer arrivals in a FIFO queue discipline for the stand market. Equilibrium models took into account the spatial distribution of demand and supply; Arnott (1996) considered a uniform demand distribution over a spatially homogenous two-dimensional city; Yang and Wong (1998) used the model of Douglas (1972) in an origin-destination matrix, where demand is fixed for each pair OD; Wong et al. (2001) considered separate demand exponential functions for each OD pair, depending on waiting time, travel time and trip price, adding in this way elasticity to the demand function; Wong et al. (2004) include multiple customer classes and taxi models; Yang et al. (2010b) use a non-linear taxi pricing for treating long-distance trips; Hyunmyung et al. (2005) use stochastic demand. The simulation models use OD matrices in order to generate the trips. They assign weights to each node and the trips between each pair of nodes are generated based on these weights. Various simulated models have tested both uniform and non-uniform demand patterns in order to evaluate the impact of the spatial distribution of the demand in the performance of the taxi services. The time dimension can also be taken into account by using various OD matrices along the day in order to evaluate the performance of the system at peak and non-peak hours.

The above models have focused exclusively on the taxi availability for calculating the customer waiting time, and therefore the demand resulted. Schroeter (1983) was the first in presenting a matching function between taxi availability and taxi demand. Cairns and Liston-Heyes (1996) used a model for simulating drivers and riders searching for each other. Wong et al. (2005) examined bilateral searching and meeting using the stochastic micro searching behaviors of both, taxi drivers and customers. Matsushima and Kobayashi (2006) modeled a single taxi stand where a double-queue system simulated waiting and meeting between taxis and customers. Yang et al. (2010a and 2010c) modeled a network bilateral searching and meeting function between taxis and customers where a taxi driver searches for a customer taking into account searching time and ride revenue, while the customer searches for a ride trying to minimize the full trip price. They use a logit model for calculating the probability of a customer to meet a taxi with the meeting function (presented in Equation (2.45)).

$$m_k^{c-t} = M_k(w_k^c Q_k^c, w_k^t T_k^{vt}) \quad (2.45)$$

where,
m_k^{c-t} is the meeting rate between customers and taxis at location k
T_k^{vt} is the arrival rate of vacant taxis at location k
Q_k^c is the customer arrival rate at location k
w_k^t is the taxi searching and waiting time at location k
w_k^c is the customer waiting time at location k

Yang et al. (2010a and 2010c) investigated the properties of an aggregate taxi service model using bilateral searching and meeting functions (considering a specific form of the Cobb-Douglas type production function) for characterizing the meeting frictions between vacant taxis and customers. They examined the market profitability at social optimum, finding that taxi services should be subsidized only when there are returns to scale in the meeting function (same conclusion obtained and supported by Fernandez et al. (2006)).

Some authors developed models for obtaining the optimum number of taxis. Ho (1993) proposed a model for determining the optimum fare and number of taxicabs, proposing to the government to sell the calculated optimum number of licenses in relation to the annual optimum fee (rather than perpetual licenses). Schaller (2007) conducted a regression analysis on seven variables, concluding that the taxi demand is generated by households without private cars or trips to the airport. Daganzo (2010a) proposed a formulation for obtaining the optimum fleet size in relation to the maximum waiting time of customers.

Elasticity of demand has been an important issue: De Vany (1975) proved that unit elasticity represents zero profit, and elasticity higher than one is related to a negative profit, concluding that elasticity must be less than one. Daniel (2003) obtained an inelastic relationship between vacant taxis and demand. Yang et al. (2005) conclude that the unitary elasticity achieves the maximum competitive taxi fleet size.

2.3.3. Operation modes

The operation modes presented in the literature review are hailing, stand and dispatching. The stand market normally works with a FIFO queue systems, where customers cannot choose the taxi vehicle, which means that price has no effects on customer choice. Information in the points where taxis are waiting is better than information in the hailing market, because customers and taxi drivers know from their experience if there will be taxi/customer waiting. In the hailing market, each customer can choose a taxi vehicle, but there is no information about the price or the waiting time for the next vehicle, and once the customer rejected an offer, he/she cannot find it again if the next offer is higher (in this case risk is high). The third operation mode is the most favorable for customers, as they can choose the dispatching center, taking into account price, waiting time or conditions of the taxis. At the same time, dispatching centers will try to have the best characteristics for fidelizing their customers.

Farrell (2010) explored patterns of taxi engagement and relationships between generated trips and taxi stand locations for optimizing the taxi stand distribution in relation to the demand patterns in a 3 level (county, town and stand) model. She applied her findings to the Ireland taxi market, and realized a comparative cost benefit analysis, identifying benefits and disbenefits resulting from developing new taxi stands. She obtained a cost-benefit ratio of 1 to 11 for the construction of a new stand, and 1 to 3 for the relocation of an existing stand.

Gilbert et al. (1993) reported that dispatching times decrease by 50% to 60% with the use of dispatching centers. Rawley and Simcoe (2009) estimate that average fleet utilization increases by 15% to 20% with the use of dispatching centers.

Schaller studied the taxi market composition in many American cities, plotting some of them in the below three-axis graph.

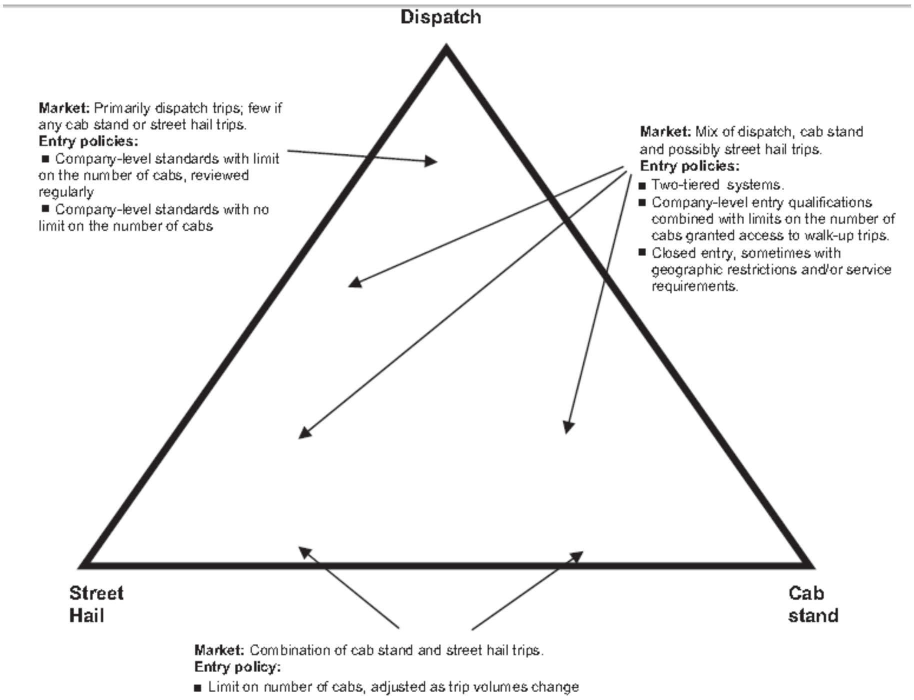

Figure 2-26 Typical entry policies for taxi customer market segments. Source: Schaller (2007)

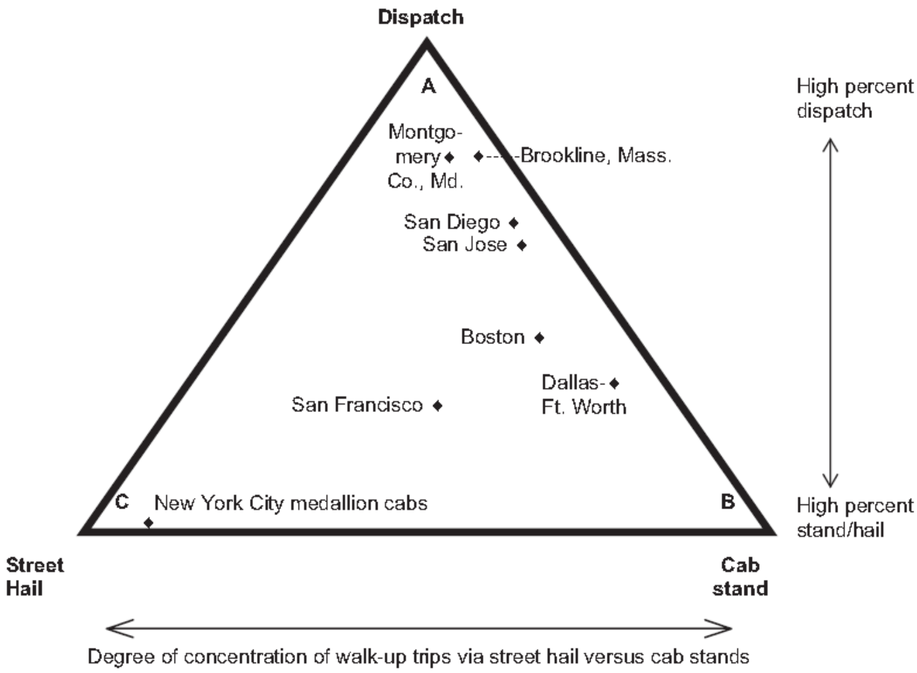

Figure 2-27 Schematic diagram of taxi customer market segments. Source: Schaller (2007)

The densities of population of the cities of Figure 2-27 are presented in Table 2-3. As seen, dispatching and cab stand markets predominate in cities with lower density. As density is growing, the percentage of cab stands is growing, while hailing markets predominate only in cities with a high density.

Table 2-3 Data from the cities represented by Schaller (2007).

City	Population	Area	Density	Market composition estimated by the authors based on the visual representation of Schaller (2007)		
				Street Hail	Cab stand	Dispatch
Montgomery	201.998	404	500	Inexistent	Inexistent	Dominant
Dallas	1.254.236	887	1.414	Inexistent	Dominant	Coexistent
San Diego	1.359.132	840	1.618	Inexistent	Coexistent	Dominant
San Jose	964.695	452	2.134	Inexistent	Coexistent	Dominant
Brookline	58.732	18	3.318	Inexistent	Inexistent	Dominant
Boston	645.169	125	5.144	Inexistent	Coexistent	Coexistent
San Francisco	808.976	121	6.686	Coexistent	Coexistent	Coexistent
New York city	8.175.133	789	10.356	Dominant	Inexistent	Inexistent

Schaller (2007) concludes that there is a strong need for numerical control in hailing and stand markets, and a relatively relaxed entry control in dispatching markets.

2.3.4. Model evolution

From the first models developed in 1970 to the last ones in 2012, many improvements have been added, trying to make models as realistic as possible. Figure 2-28 shows the evolution of the models, highlighting the most important models and their innovative characteristics and summarizing the literature review presented above.

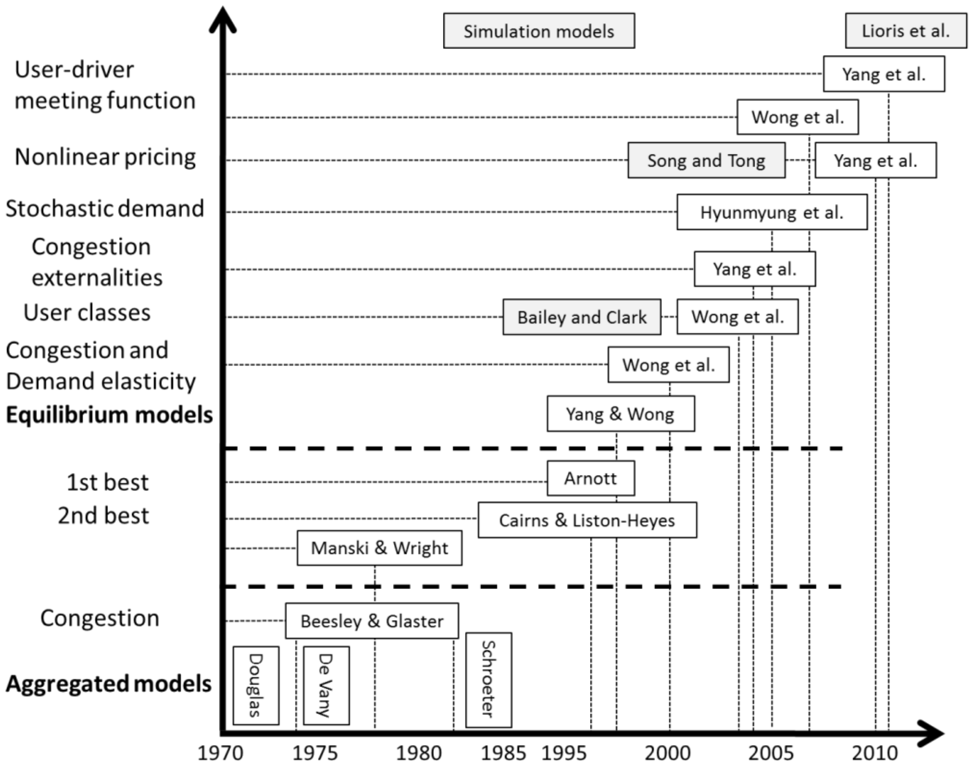

Figure 2-28 Evolution of the taxi models

2.4. Review of the formulations of the operational models

Formulations of various operational models are presented in this section. In order to facilitate the reader, a common variable definition is used throughout this section aiming at presenting all the formulations without repeating the variable definition in each formulation and using a common representation in the presented formulations. Each model uses different decision variables and formulations. All the variables and parameters are presented below.

The costs of the different stakeholders:
Z_d is the cost of the drivers (€) - z_d is the unitary cost of the drivers (€)
Z_u is the cost of the customers (€) - z_u is the unitary cost of the customers (€)
Z_c is the additional cost for the city (€) - z_c is the unitary cost for the city (€)
G is the cost of the infrastructure (€) - g is the unitary infrastructure cost (€)

The variables:
λ_u is the hourly demand for taxi trips (trips per hour and area)
λ_d is the taxi hourly supply (vehicles per hour and area)
T_A is the access time (min)
T_W is the waiting time (min)
T_{IV} is the in-vehicle time (min)
ε is the relation between travel time with and without customer

$\bar{c}$ is the average trip cost[1] (€)
$\bar{d}$ is the average distance of the trip (km)
$\bar{t}$ is the average time of the trip (min)
$\bar{v}$ is the average speed of the trip (km/h)
V_D is the vacant distance (km)
H_v is the vacant taxi headway (min)
A is the area of the region (km^2)
s is the number of taxi stands

The parameters:
VoT is the value of time (€/hour)
C_h is the hourly cost of the moving taxis (€/hour)
C'_h is the hourly cost of the stopped taxis (€/hour)
C_E is the emission unitary cost for all vehicles (€/kg of CO_2)
r is the area and network parameter
T is the waiting time for central dispatching center (min)
OM_C is the trip characteristic of the other modes
P_0 is the customer willingness to pay when the waiting time is 0 (€/hour)
u is the communication cost (€, only in the dispatching market)

2.4.1. The demand for taxi trips

The demand for taxi trips depends on both the socioeconomic characteristics of the population and the comparison between the characteristics of the taxi services and the services provided by alternative transport modes. Various formulations can be found in the literature, using in most cases the expected waiting time or the number of vacant taxis and the relative or absolute cost of the trip. The most recent models propose the use of searching and meeting functions between customers and taxi drivers, where the taxi driver searches for a ride taking into account the searching time and the ride revenue while the customer searches for a taxi ride trying to minimize the generalized cost of his/her trip. Following the proposed models (Yang et al. (2001), Yang et al. (2007), Schroeter (1985)), the demand can be obtained using Equation 2.46:

$$\lambda_u = f(T_W, T_A, T_{IV}, \alpha_W, \alpha_A, \alpha_{IV}, VoT_u, \bar{t}, OM_c{}^{2}) \qquad (2.46)$$

[1] Note that the term $\bar{c}$ does not affect the global objective function of the stakeholders since it will appear in both the users' cost and the drivers' cost with opposite signs. However it is an important factor when the profitability of each particular stakeholder is analyzed.

[2] Using a modal split will make more complex the model since detailed data for each OD pair is needed from the taxi services and all the alternative modes. Two examples of the modal split calculation are presented in Lo et al. (2004), where the authors applied a Multinomial Logit Model to the transit services, and in Wong et al. (2005b).

An example of the above formulation is the demand formulation proposed by Chang and Chu (2009), where the demand rate ($\lambda_u A$ trips per hour and area of service) is calculated as:

$$\lambda_u A = \frac{D\bar{c}^{\alpha} {T_W}^{\beta}}{\bar{d}} \qquad (2.47)$$

where D, α and β are calibrated parameters of the model.

The demand level can be related to the peak hour or to the whole day. From Daganzo (2010b) it is assumed that the demand per hour in the peak hour is 2,5 times the demand per hour of the whole day.

2.4.2. The taxi supply

The supply for taxi services depends on the expected benefit when offered to the customers. There exists an opportunity cost for both the license holder and the taxi driver. The license holder has invested money in the license and expects high revenues from his investment, while the taxi driver invests his own time in exchange for a salary. In macroeconomic terms, the supply depends on the revenues and the salary, related to revenues and salaries from alternative activities, however in most of the taxi models, the supply depends on the revenues as an absolute value and not in relation to the other economic sectors. Different formulations can be found in the literature, using mostly flat rates for the cost and variable incomes, depending on the average number of trips, the unitary fees and the average distance/time of the trips. An important variable for the supply side is the vacant distance; in the stand market it is equal to the distance between the destination of the customer and the nearest stand. In the dispatching market the vacant distance is equal to the distance between the stand and the customer´s origin, and between the customer´s destination and the nearest taxi stand (both can be supposed to be equal). Different formulations for calculating this distance are presented in Equation 2.48 and Equation 2.49. In the hailing market the vacant distance can be approximated by the difference between the distance travelled by the taxis during one hour ($\lambda_d A\bar{v}$) and the distance travelled by the customers during one hour ($A\lambda_u\bar{d}$).

$$V_D = \lambda_d A\bar{v} - A\lambda_u\bar{d} \qquad (2.48)$$

Chang et al. (2010) also propose the formulation presented in Equation 2.49 for the estimation of the vacancy mileage per taxi in the dispatching market, where s is the number of taxi stand and θ the stand allocation coefficient:

$$V_D = \frac{A^{1/2} r}{2s^{\theta}} \qquad (2.49)$$

Yang et al. (2005) relate the fleet size (vacant and occupied) to the demand and the travel time, where the occupied taxi fleet is calculated from the total travelling time of all customers.

Daganzo (2010a) proposes the formulation presented in Equation 2.50 for the equilibrium fleet calculation in the dispatching market:

$$\lambda_d = \frac{1}{(2\bar{v}T_w)^2} + \lambda_u (T_w + \frac{\bar{d}}{\bar{v}}) \tag{2.50}$$

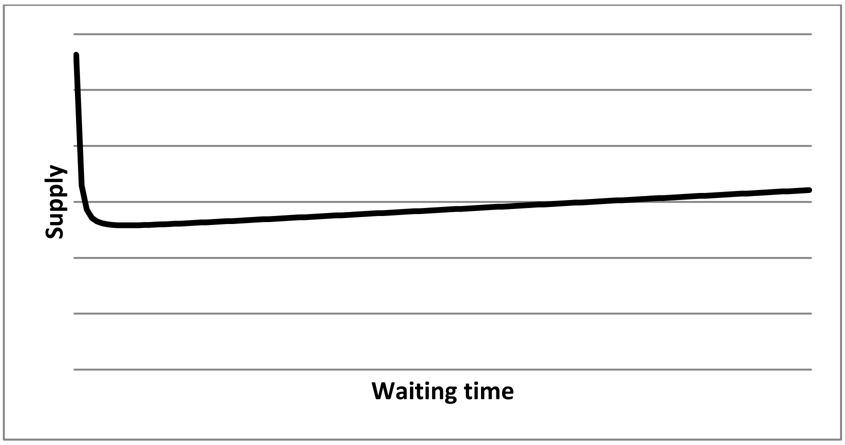

Figure 2-29 Equilibrium supply versus waiting time by Daganzo (2010a).

As shown in Figure 2-29, the optimum fleet size initially decreases when the waiting time increases, but after the minimum value, it increases as the waiting time increases. This minimum fleet size ensures the minimum LoS within the desired region (larger waiting times are undesirable). The formulation proposed by Daganzo (2010a) can be obtained by considering a taxi dispatching service in a region of area A, demand λ_u, average trip distance $\bar{d}$ and average speed $\bar{v}$, where the taxi fleet ($\lambda_d A$) can be divided into three categories: idle taxis (m_i), assigned taxis (m_a) and servicing taxis (m_s). If the system is in equilibrium, the three changing rates between the categories will be equal (the calls rate is λ_uA). Taking Little´s formula, the rate can be approximated by the rate between the number of taxis and the travel time in each category, providing Equation 2.51:

$$\lambda_u \mathrm{A} = \frac{m_a}{T_w} = \frac{m_s}{T_{IV}} \tag{2.51}$$

From Equation 2.51, the assigned taxis ($m_a = \lambda_u \mathrm{A} T_w$) and the servicing taxis number can be obtained ($m_s = \lambda_u \mathrm{A} \frac{\bar{d}}{\bar{v}}$).

The travel time can be approximated by the ratio between the expected distance and the average speed. Considering that the region is a square, the expected waiting time of the customer can be approximated by the ratio between the expected distance to the nearest free taxi ($\overline{d_i}$) and the average speed ($\bar{v}$):

$$T_w = \frac{\bar{d_\iota}}{\bar{v}} = \frac{1}{2\bar{v}}\sqrt{\frac{A}{m_i}} \tag{2.52}$$

From Equation 2.52 the number of idling taxis is obtained ($m_i = \frac{A}{(2\bar{v}\mathrm{T_w})^2}$).
The equilibrium taxi fleet is the one presented in Equation 2.53:

$$\begin{aligned}\lambda_d A = m_a + m_s + m_i = \lambda_u \mathrm{A} T_w + \lambda_u \mathrm{A}\frac{\bar{d}}{\bar{v}} + \frac{A}{(2\bar{v}\mathrm{T_w})^2} \\ = \frac{\mathrm{A}}{(2\bar{v}\mathrm{T_w})^2} + \lambda_u \mathrm{A}(\mathrm{T_w} + \frac{\bar{d}}{\bar{v}})\end{aligned} \tag{2.53}$$

The value of the waiting time for the minimum taxi fleet is obtained from Equation 2.53:

$$T_w = (2\lambda_u \bar{v}^2)^{-1/3} \tag{2.54}$$

Corresponding to the minimum fleet size presented in Equation 2.55:

$$\lambda_d = (2^{-\frac{4}{3}} + 2^{-\frac{1}{3}})\bar{v}^{-2/3}{\lambda_u}^{2/3} + \lambda_u \frac{\bar{d}}{\bar{v}} \tag{2.55}$$

Chang (2010) proposes that the total profit can be calculated as the difference between the total revenue and the total cost and states that the total revenue can be estimated as the product between the total demand and the trip price and that the total cost can be calculated as the product between the total distance and the cost per kilometer (in this case the cost per kilometer includes the hourly cost and the kilometric cost).

$$Total\, profit = \bar{c}\lambda_u A - C'_{km}\lambda_d A\bar{v} \tag{2.56}$$

Introducing the price estimation ($\bar{d}\tau_{km}$) and the waiting time proposed by Chang (2010) in Equation 2.56, the optimum fleet size obtained is presented below, where φ is the opportunity cost of having a taxi reserved place (€/hour) and δ represents the tolerance of customers to the waiting time.

$${\lambda_d}^* = \frac{L}{\bar{v}}\left[\left(\frac{120^{\varphi}\cdot VoT\cdot\delta\cdot\lambda_u}{C_{km}A^{\varphi+1}}\right)^{\frac{1}{\varphi+1}} + \lambda_u \bar{d}\right] \tag{2.57}$$

2.4.3. In-vehicle travel time

The in-vehicle travel time is the same for the three operational modes. It can be expressed by using the average distance between two interior points within the zone and the average speed, as shown in Zamora (1996), where the distance is estimated between two random points within a region of area A as proportional to the root of the area. Smeed (1975) and Holroyd (1965) investigated this proportionality factor (relation between Euclidean distance and network distance) in different network geometries, providing different values for each type of network (direct, radial, ring road, rectangular). The expected travel time is the factor between this expected distance and the average speed.

$$T_{IV} = \frac{rA^{1/2}}{2\bar{v}} \qquad (2.58)$$

Smeed (1975) and Holroyd (1965) investigated this proportionality factor (relation between Euclidean distance and network distance) in different network geometries, providing different values for each type of network (direct, radial, ring road, rectangular). They calculated different r values for different network configurations, presented in Table 2-4.

Table 2-4 Network parameters proposed by Smeed and Holroyd. Source: Zamora (1996)

Network	Network parameter		
	Smeed	Holroyd	r
Direct distance		0.905	1.00
Radial		1.333	1.47
External ring		2.237	2.47
Internal ring		1.445	1.59
Radial arc		1.104	1.21
Rectangular	0.78 – 0.97	1.153	1.27
Triangular		0.998	1.1
Hexagonal		1.153	1.27
Irregular	0.80 – 1.06		

The network geometries comtained in Table 2-4 are presented in Figure 2-30:

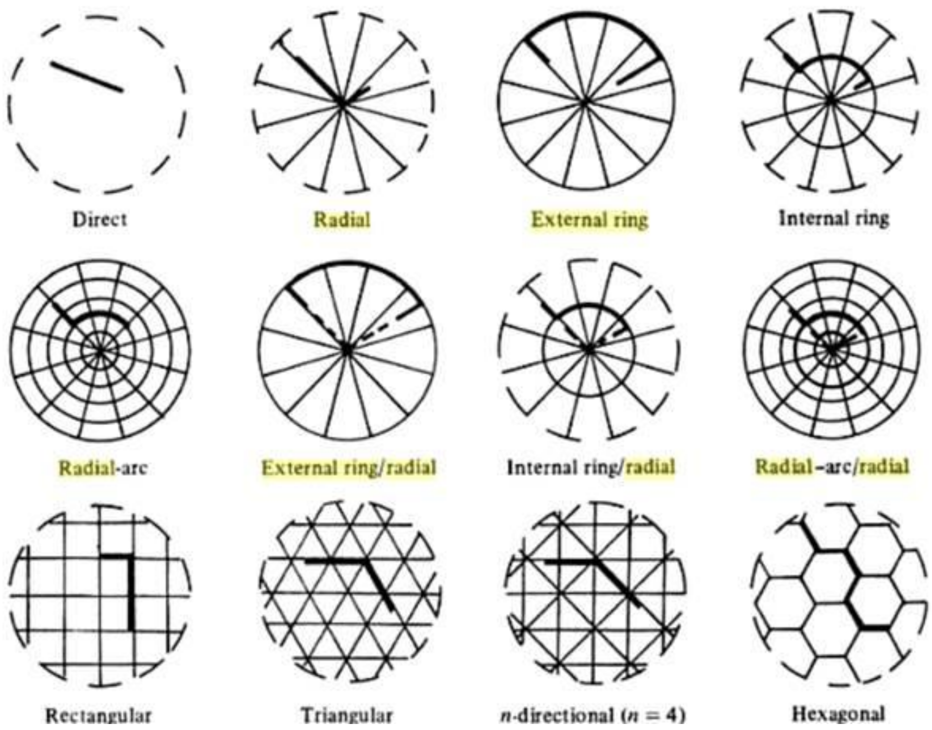

Figure 2-30 Network geometries. Source: (Urban Spatial Traffic Patterns 1987)

They also calculated the network distance in relation to the area, as presented in Figure 2-31.

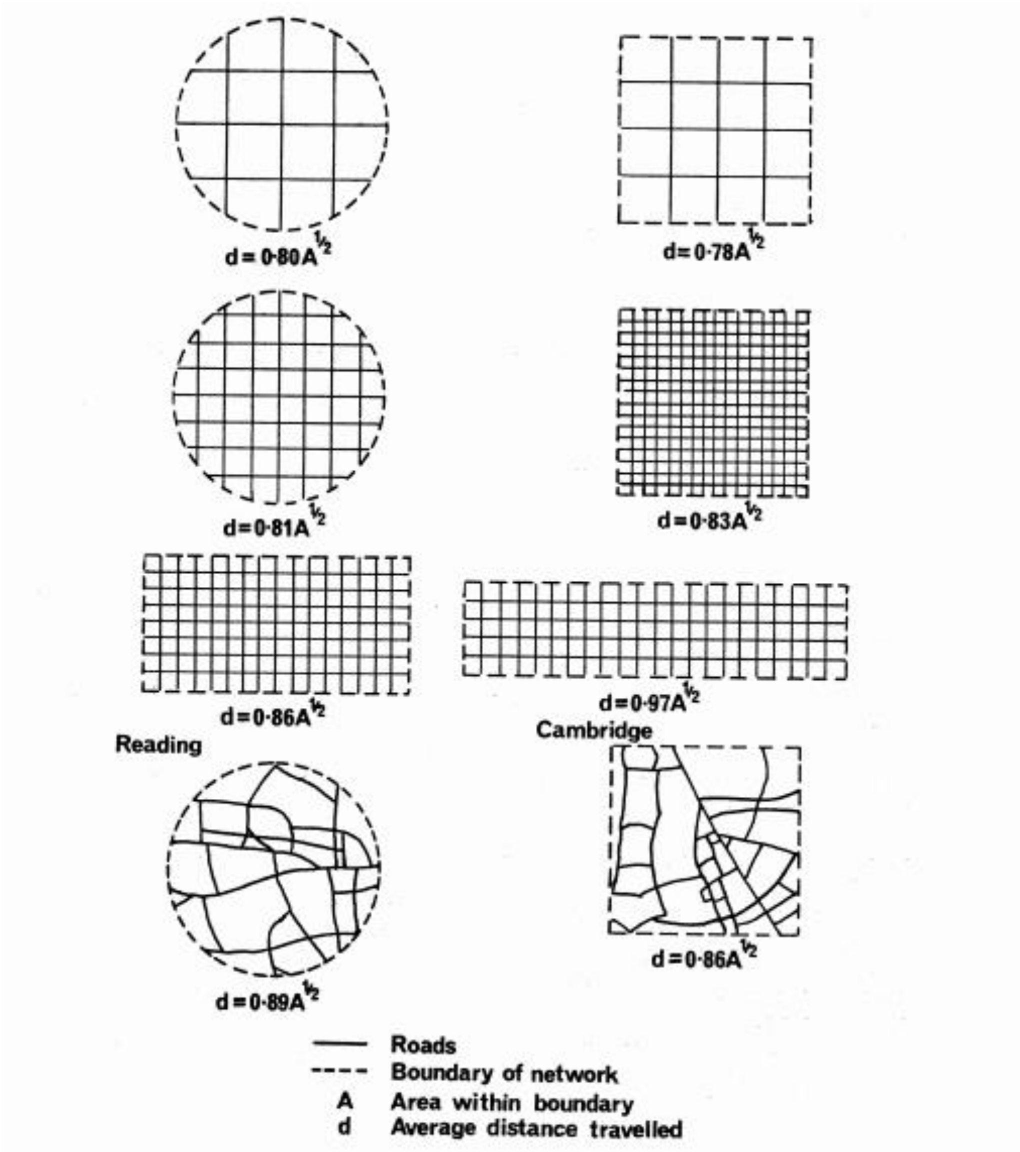

Figure 2-31 Network average distance depending on the network geometry. Source: (Urban Spatial Traffic Patterns 1987)

2.4.4. Access and waiting time

In the hailing and dispatching markets the access time is either 0 or very small while in the case of the stand market it can be approximated by the surface covered by each taxi stand. The models of Yang (2005) and Chang (2009) use the number of available taxis for obtaining the customers waiting time, but other models have developed more accurate formulations for each type of market.

In the dispatching market, the average waiting time can be expressed in terms of reaction time (negligible) and access time. The waiting time is the average travel time between the customer and the nearest vehicle, related to the density of free taxis in the area as proposed in Zamora (1996).

Chang (2010) proposes the following methodology for estimating the waiting time in the dispatching market. With the hypothesis that in the dispatching market the taxis are waiting for a call in taxi stands (half of the vacancy distance is generated when searching for the assigned customer and the other half when going to the nearest taxi stand and waiting for the next call), the waiting time can be estimated as the ratio between half of the vacant distance ($\mathrm{V_D}/2$) and the average speed, adding a fixed time (T) representing the waiting time for the control service center dispatching the service.

$$T_w = \frac{V_D/2}{\bar{v}} + T \qquad (2.59)$$

According to the assumption that the distance from the taxi stands to the customer's origin and the distance between the customer's destination and the next taxi stand are equal, the vacant distance can be estimated as two times the distance between the taxi stand and the origin (as proposed in Equation 2.59).

Daganzo (2010a) proposes a similar formulation for the waiting time of a customer in the dispatching market and considers a square region of area A with a grid of streets and m_i taxis waiting at taxi stands. In his work the probability of a disk of diameter with length 2x (centered in the customer) is calculated containing zero taxis (i.e. the distance between the nearest free taxi and the customer is higher than x) as:

$$P(\bar{d_\iota} > x) = \left(1 - \frac{2x^2}{A}\right) \qquad (2.60)$$

Meyer and Wolfe (1961) presented a very detailed formulation for the estimation of the waiting time in the dispatching market. They presented and analyzed the complex case where customers have to wait until the first occupied taxi is available, increasing the waiting time and reducing the LoS. They proposed the formulation presented in Equation 2.61:

$$T_w = \frac{3}{2}\sqrt{\frac{1}{\pi\lambda_u\bar{v}^2} + \frac{1}{2}(\bar{t} - \frac{\lambda_d}{\lambda_u})} \qquad (2.61)$$

In the hailing market, Wong et al. (2008) used the number of vacant taxis and the size of the area of service for calculating the waiting time (they suppose a continuous taxi distribution). Using a more statistical approach, the waiting time is equal to the time the customer must wait until the first free taxi reaches the customer, starting from a random moment when the customer decides to take a taxi.

Another approach is presented in Bautista (1985). The probability of taking the n-passing taxi is the probability of passing (n-1) occupied taxis before the first free taxi, which can be expressed as $w^{n-1}(1-w)$, where w is the proportion of occupied taxis in the whole traffic composition. The associated waiting time is then the addition of the waiting time for the first occupied vehicle (u=0.5c) and (n-1) intervals of duration 1/c, where c is the rate of vehicles passing by (hypothesis of constant interval between taxis). The expected waiting time is expressed in Equation 2.62:

$$\begin{aligned} T_w &= 0.5c^{-1} + \sum_{n=1}^{\infty} \frac{(n-1)w^{n-1}(1-w)}{c} = \\ &= 0.5c^{-1} + (1-w)c^{-1}\left(\sum_{n=1}^{\infty} nw^{n-1} - \sum_{n=1}^{\infty} w^{n-1}\right) = \\ &= 0.5c^{-1} + (1-w)c^{-1}\left((1-w)^{-2} - (1-w)^{-1}\right) = \\ &= c^{-1}(1-w)^{-1} - 0.5c^{-1} = \\ &= (1+w) \big/ (2c(1-w)) \end{aligned} \qquad (2.62)$$

Using the rate of all vehicles joining the streets, Equation 2.62 can be rewritten as (the value of w becomes w/λ_v and c becomes cλ_v):

$$T_w = \frac{(\lambda_v + w)}{(2c\lambda_v(\lambda_v - w))} \qquad (2.63)$$

In a similar way, if the interval between taxis is exponentially distributed with median $1/c$, Equation 2.64 is obtained using a Poisson process with value $c * (1-w)$

$$T_w = \frac{1}{(c(\lambda_v - w))} \qquad (2.64)$$

Finally, with the hypothesis that taxis move into groups of cars (due to the traffic lights), Equation 2.65 is obtained using a Poisson distribution of the number of taxis in each group (the pass interval of the groups is h, which can be approximated by the average cycle time of the traffic lights):

$$T_w = \frac{h}{2} \frac{1 + exp(-hc(\lambda_v - w))}{1 - exp(-hc(\lambda_v - w))} \qquad (2.65)$$

Fernandez et al. (2008) presented Equation 2.66 for the calculation of the waiting time in the hailing market, where K is a parameter of the model:

$$T_w = \frac{K}{\lambda_d - \lambda_u \bar{t}} \qquad (2.66)$$

Douglas (1972) used Equation 2.67 for the approximation of the parameter K:

$$K = \frac{A}{\bar{v}} \qquad (2.67)$$

The waiting time of the taxi driver in the hailing market (searching time) is approximated by Fernandez et al. (2008) using the Equation 2.68:

$$t_{cr} = \frac{1}{q} - t \qquad (2.68)$$

Where q is the number of trips and t the duration of the trips.

Chang et al (2010) proposed Equation 2.69 for the calculation of the driver waiting time in the hailing market, where s is the number of taxi stands and θ the stand allocation coefficient:

$$t_{cr} = \frac{240A^{1/2}s^{\theta}}{r} \qquad (2.69)$$

In the stand market, the waiting time can be estimated by applying queue theory to a double queue, where vehicles and customers meet each other (Yang et al (2010a and 2010b) and Matsushima and Kobayashi (2006 and 2010)). Matsushima and Kobayashi (2010) proposed Equation 2.70 and Equation 2.71 for the calculation of the waiting time of both the driver and the customer, where η and μ are the arrival and service rates of taxis at taxi stands:

$$T_w(dr) = \frac{\rho^{M+1}/1-\rho}{\lambda} \qquad (2.70)$$

$$T_w(us) = \frac{M - \rho/1-\rho\,(1-\rho^M)}{\mu} \qquad (2.71)$$

where M is the maximum number of vehicles in the stands and $\rho = \lambda/\mu$

Yang et al (2000) proposed the empirical formulation presented in Equation 2.72 for the waiting time at a taxi stand:

$$T_w = B\left(\frac{1}{H_v}\right) + C\left(\frac{\lambda_u}{\lambda_d}\right) \qquad (2.72)$$

where B and C are parameters of the model, calibrated with data from the taxi fleet of Hong Kong. The vacant taxi headway is estimated by using the percentage of occupied taxis (w).

2.4.5. Trip fare

The characteristics of the trip (average distance, average duration and average speed) depend on the size and topology of the city. The average distance has been presented in 0; the average duration depends on the average distance and the average speed; the average speed depends on the policy applied to the taxi sector (if they can use the bus lanes) and the congestion level of the city for each time interval and zone. Other characteristics, such as unitary fees or emissions pricing depend on the policy applied by the city to the transport sector. Chang (2010) proposed the formulation presented in Equation 2.73 for estimating the willingness of taxi customers to pay:

$$P = P_0 - VoT \cdot {T_w}^{\delta} - u \qquad (2.73)$$

In accordance to Equation 2.73, customers have a maximum willingness to pay (P_0), which is reduced with the increase of the waiting time at a rate controlled by δ. An interesting discussion on different customer classes with different values of time can be found in Wong et al ((2004) and (2008)).

The value of time of the taxi customers is estimated in order to quantify the economic cost of the total travel time. Many studies have obtained specific values for the VoT of the citizens by trip purpose, trip length, income and others. For the value of time, Small (1992) proposes the 50% of the average hourly salary, while Daganzo (2010a) assumed it to be 20$/hour.

2.4.6. Costs

In the dispatching market the unitary driver cost depends on the number of taxis stopped at taxi stands and the number of vehicles hailing (Zamora (1996)). The total operation costs of the hailing taxis is the product between the total offered vehicle-hours ($\lambda_d A$) and the hourly operation cost of a hailing taxi (C_h). The ratio between these costs and the total demand (λ_uA) is the unitary operation cost ($\mathrm{z_d}$).

$$z_d = \frac{\lambda_d}{\lambda_u} C_h \qquad (2.74)$$

Zamora (1994) proposes the estimation of C'_husing the hourly costs of the driver and the benefits of the license holder, while forC_h, they propose the addition of the operation cost to C'_h. In order to separate the kilometric costs and the hourly costs, Equation 2.75 is proposed instead of the calculation of the unitary costs of the moving vehicles in the hailing market.

$$z_d = \frac{\lambda_d}{\lambda_u} C_h + \frac{\lambda_d \bar{v}}{\lambda_u} C_{km} \qquad (2.75)$$

In the other markets, where taxis wait at taxi stands, the second term of Equation 2.75 will apply only to the distance travelled when assigned or occupied, obtaining the formulation presented in Equation 2.76:

$$z_d = \frac{\lambda_d}{\lambda_u} C_h + \frac{r A^{1/2}}{2\bar{v}} \varepsilon \, C_{km} \qquad (2.76)$$

For the dispatching market, Chang (2010) presents a detailed cost formulation for each actor (customer, driver and operator). The customer cost is composed by the cost of communications, the product between the waiting time (half vacant time) and the VoT and the time for the control centre dispatching vehicle.

$$Z_u = u \cdot \lambda_u \cdot A + \frac{VoT \cdot V_D}{2\bar{v}} + VoT \cdot \lambda_u \cdot A \cdot T \qquad (2.77)$$

The driver cost is composed by both the occupied and vacant distances:

$$Z_d = C_{km} \cdot \lambda_u \cdot A \cdot \bar{d} + C_{km} \cdot V_D \qquad (2.78)$$

The operator cost (G) is calculated as the marginal cost of each taxi stand (b) multiplied by the number of taxi stands (supposed as non linear).

$$G = sb\left(\frac{\lambda_d}{s}\right)^{\gamma} \tag{2.79}$$

where γ is the incremental operating cost coefficient of taxi stand (0< γ <1)

The average stand cost can be obtained as the sum of the fixed cost (negligible) and the variable costs. The variable costs are a function of the number of offered places. It is shown in Zamora (1996) that the optimum number of places per stand is one, and the only question is to find the optimum number of stands. The stand cost is calculated as the product between the number of places and the opportunity cost of each place (if it was used as a private parking place). The fixed term due to the stand infrastructure is considered null or very small along the cycle of life of the stand. The total number of places is calculated as the number of occupied places and the number of free places. For estimating the number of free places, the total cost, composed by the traveled time between the destination of the last customer and the nearest taxi stand ($\frac{0{,}4\mathrm{r}}{\mathrm{v}\sqrt{\Psi}}$, where Ψ is the density of free places) and the cost of the taxi stands is minimized. The results are presented in Equation 2.80:

$$s^* = \lambda_d A - \lambda_u A \frac{rA^{1/2}}{2\bar{v}} \varepsilon + \left(\frac{H_1}{\varphi}\right)^{2/3} \tag{2.80}$$

$$H_1 = \frac{0.5(C_h - C'_h) r 0.4 \lambda_u A^{2/3}}{\bar{v}} \tag{2.81}$$

It is observed in Equation 2.80 that the first term corresponds to the occupied places, while the second term correspond to the free places, indirectly proportional to the opportunity cost of having a reserved place and directly proportional to the difference between the operation cost of hailing taxis and the operation cost of stopped taxis $(C_h - C'_h)$.

2.4.7. Generalized cost

The generalized system cost is the total cost in monetary or time terms of the trip, taking into account the access time, the waiting time and the in-vehicle time, but also the trip cost, the operating costs and the infrastructure costs for providing the taxi service. As commented above, the optimum fleet is normally obtained as the fleet that minimizes the formulated generalized cost, and consequently, the minimum cost is obtained by introducing this optimum fleet in the formulation. Using the optimum fleet size presented in 2.4.2, the optimum generalized cost for the dispatching fleet can be formulated. The proposed formulation in Zamora (1996) is:

$$Z_d = \frac{rA^{1/2}}{2\bar{v}} + \frac{0.4r\beta}{\bar{v}\sqrt{K_{EI}}} + \alpha \frac{\frac{rA^{1/2}}{2\bar{v}}\epsilon}{\bar{v}} + \frac{\alpha K_{EI}}{\lambda_u} + \frac{\varphi s}{\lambda_u A} \tag{2.82}$$

where α is C_h for vacant vehicles and C'_h + ϕ for waiting vehicles (€/hour)

Daganzo (2010a) combines costs and time obtaining the lowest generalized cost for the optimum dispatching fleet. The generalized cost is the addition of the total time (waiting and travel time) of all customers and the operation cost of the taxi fleet expressed in terms of time.

$$\frac{Z_D^*}{VoT}=(T_W{}^*+T_{IV})\lambda_u A+\lambda_d{}^*\frac{C_h}{VoT} \qquad (2.83)$$

Using the formulation of the optimum fleet and the associated waiting time to this optimum fleet, the unitary generalized cost is obtained dividing the total system cost by the demand:

$$\frac{z_D^*}{VoT}=T_W{}^*+T_{IV}+\lambda_d{}^*\frac{C_h}{VoT\lambda_u A}\approx T_{IV}+\left(0{,}8+1{,}2\frac{C_h}{VoT}\right)\lambda_u{}^{-1/3}\bar{v}^{-2/3}+\frac{C_h\bar{d}}{VoT\bar{v}} \qquad (2.84)$$

2.5. State of practice

2.5.1. Technologies applied to the taxi market

Initially, taxicabs worked independently from each other, cruising while looking for a ride with no collaboration or any type of organization. In the 1940s, two-way radios were introduced to taxi markets, connecting demand and supply through central dispatching centers. Computers were introduced into taxis in 1970s, but automated data dispatching systems did not arrive until the 1980s. In the 1980s computer terminals were inserted into taxis, allowing dispatchers to automatically locate the nearest taxi. In 1990s, computerized dispatching systems were adopted, with central computers and on-board units. Most simple systems, called “partially automated”, required drivers to send a signal to the central computer, indicating their position, and human dispatchers to announce ride allocations. More advanced systems, called “fully automated”, used devices with a two-way communication capability for communicating directly with onboard computers in taxicabs. The first approach was to divide the city in zones, creating virtual queues and assigning trips to the first vehicle on the queue (giving the opportunity to search for a ride or being in a physical queue while waiting in the virtual queue). Vehicle terminals allowed drivers to see the queue in each zone and choose a new zone with small queue. The most advanced computerized dispatching systems are GPS-based, tracking a vehicle´s exact location at all time and precluding cheating behaviors, logging cars into specific zones (improving at the same time safety of taxi drivers). Currently, many countries use GPS-based systems for dispatching taxi services. Taxi companies in Singapore use the GPS Automated Vehicle Location and Dispatch System (AVLDS), consisting of Autocall, Dial-a-Cab, Fax-a-Cab, PCDial, Hot Button and Taxi order terminals in which customers can reserve a taxi service via phone, fax, internet or using automatic taxi-calling machines. New platforms are being developed, taking advantage of new technologies and providing services for smartphones, where a directly driver-customer communication channel is established, reducing the necessity of a dispatching control center, reducing the reaction time and increasing the level of service.

All these technologies have been modeled, tested and evaluated by several authors. Massow and Canbolat (2010) examined taxi protocols and proposed a model for evaluating the impact of driver decisions on waiting time. They proposed centering zones on high demand points, generating incentives to move to the center of the zones. They also proposed the creation of hyperzones when high demand points do not exist.

Hyunmyung et al. (2005) investigated the effectiveness of taxi information systems. The learning process implemented in their model shows that taxi drivers could predict traffic congestion quite well from their experience, but this capability may not improve their operational efficiency (under normal conditions). They showed that taxi information systems improve the effectiveness of the system. They also showed that benefits of taxi information systems decrease with the increase of the penetration rate (% of informed drivers). New technologies applied to the taxi sector are capable of evaluating taxi drivers' operation behavior and skills, or mobility intelligence as named by Liu et al. (2010).

Chang et al. (2010) evaluated the environmental benefits of the GIS-GPS-based scheduling service in relation to the conventional hailing services of taxi market concluding that dispatching models reduce traffic accidents, pollution and congestion compared to conventional hailing models. They proposed formulations for calculating fare paid and waiting time in both operation modes. Chang et al. (2010) evaluated the environmental benefits of the GIS-GPS based scheduled dispatching services compared to the conventional hailing services of taxi market. They calculated the fare paid by customers as a function of the maximum willingness to pay, the average waiting time and the tolerable coefficient of waiting time in both cases (in the dispatching model the fare includes the commutation cost for each trip). They also calculated the waiting time in each market, and tested their models using 2008 data obtained from the Taipei metropolitan area. They concluded that the dispatching model has comparative advantages in relation to hailing models in terms of decreased external costs and environmental sustainability aspects.

Rawley and Simcoe (2009) examined how ICT influences the taxi sector. They used data from the 1992 and 1997 Economic Census including every Taxi firm in the US with at least one employee and showed that taxi ownership increased by 12% when they adopted new computerized dispatching systems. They observed that the percentage of companies using new technologies in the US taxi market increases with the fleet size, as shown in Figure 2-32.

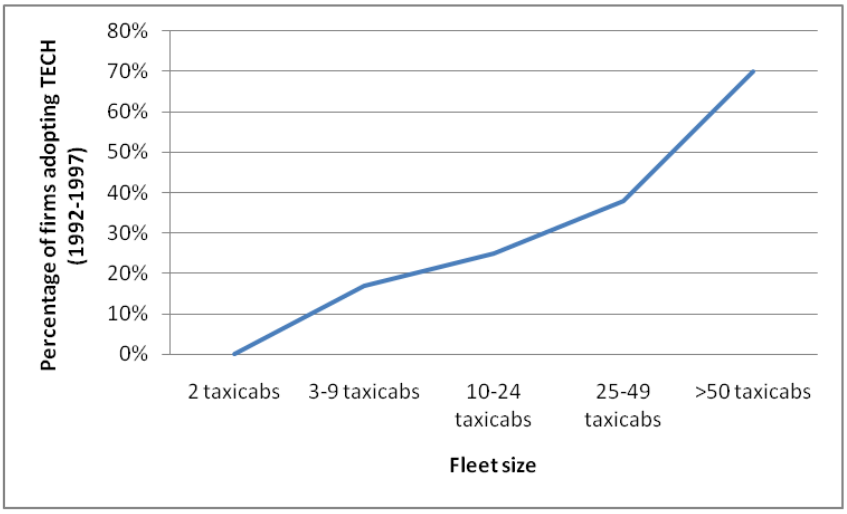

Figure 2-32 Fleet size versus TECH utilization. Source: (Own elaboration from Rawley and Simcoe (2009))

Hyunmyung et al. (2005) proposed a model for analyzing the effect of Taxi Information Systems on the hailing taxi market. They proved that ITS applied to the taxi sector may not further improve the operational efficiency of the taxi market due to their travel experience predictions.

2.5.2. Taxi markets in other countries

Most of the cities in the world have regulated taxi markets, controlling the number of vehicles circulating in the streets and the respective applied fares. There are various methods applied in the U. S. cities to set the number of taxi licenses. Two of them are described below:

- Freezing the number of taxis in operation at the moment the decision is made (arbitrary measure). Adopted in Boston, Chicago, New York and other major cities during the 1930s.
- Periodic reviews of the Public Convenience and Necessity (PCN) of increasing the number of taxis.

The main consequences of regulated markets are the increase of the prices and the reduction of the number of licenses. Various examples are pesented below:

- In Brisbane (Australia), the number of taxis per 10,000 inhabitants decreased from 19.8 in 1960 to 9.8 in 1999. In Melbourne (Australia), the corresponding number declined from 12.3 in 1951 to 9.6 in 1995. In the same city, price of the license has increased by 76% in the period 1989-2004, due to the near-zero releases of new licenses by the regulator.
- In Dublin, the price of licenses increased by 25 in the 20 years prior to the deregulation of the Irish taxi industry.
- In New York, the number of taxi medallions is almost 1.400 fewer than in 1937. Prices in 2000 were 250,000 $, in 2005 had reached 379,000 $, and 600,000 $ in 2007.
- In Hong Kong, medallions have increased from 200,000 HK$ in 1980 to 1.500,000 HK$ in 1987.

Each city has its own regulation for the taxi market. Table 2-5 shows the regulation characteristics of some cities along the world.

Table 2-5 Regulation issues in different cities around the world. Source: Own elaboration from OECD (2007).

Country - Zone/city	Fare regulation	Entry regulation	Period	Restrictions	Characteristics
Belgium - Flamande	yes	yes	5 years	1 vehicle per 1,000 hab	personal and intransferable
Belgium - Brussels	yes	yes	7 years	1 vehicle per 1,000 hab	personal and intransferable
Belgium - Wallon	yes	yes	10 years		personal and intransferable
Czech Republic	yes	no			intransferable
Denmark	yes	yes	10 years		intransferable
France – Paris	yes	yes		100 new licenses per year	
Germany	yes	no	5 years	license subjected to a quota	
Hungary	yes				
Ireland	yes	no (2000)		license subjected to a fee and quota	
Italy	yes	yes		4,5 per 10,000 hab / 1 lic per person	
Japan	yes	no (2002)			
Korea	yes	yes			
Netherlands	yes (2004)	no (2002)			
Norway	depending on the city	yes			not tradable, not transferable
Sweden		no (1990)			
Switzerland	depending on the city	yes	3 years		not tradable, not transferable
United States - Seattle	no (1979)	no (1979)			
Romania	yes	yes		4 vehicles per 1,000 hab	

As shown in Table 2-5, a few countries have deregulated the taxi market; from their experience some deregulation effects are collected in OECD (2007):

- Sweden (1990): Larger taxi fleet, better accessibility for customers, reduction of waiting time, various types of new available vehicles.
- Ireland (2000): quadriplication of the number of licenses, fare and quality regulation needed for avoiding overcharging and uncompetitive operation of the market (uncertainty of waiting for another taxi and price competition unfeasible at taxi stands).
- Japan (2002): 8.4% and 9.7% increase in the number of companies and taxis respectively. Introduction of a large variety of fares, discounts and flat rates.
- United States (Seattle 1979): 5% reduction in fares (taxi-stand raised while radio-dispatching fell); increase in service at the airport, generating queues, but without price reduction due to the FIFO queuing system applied.
- United States (Indianapolis 1994): Increase in the number of taxicabs and companies, fare reductions, service level improvements and reduction in customer complaints.
- United States: fare control is needed for controlling the appropriate level of entry; use of contracts between firms and hotels/airport authorities for avoiding queues at those locations where waiting times are always low.
- Taiwan: over-supply and high vacancy rate, resulting in poor service, unhealthy competition and law-breaking behaviors.
- Ireland: the number of taxis in Dublin increased by 216% in the two years after deregulation.

- New Zealand: the number of taxies increased by almost 200% following deregulation.
- Sweden: the number of taxies was doubled in the first two years after deregulation, but simultaneously, significant innovative taxi schemes had been developed for encouraging taxi use in off-peak periods.

A liberalization of the market will increase the taxi fleet and level of service of customers, but a fare regulation is needed (indeed, most of the US markets that deregulated entry control continued to regulate fares). As exposed by Fernandez et al. (2008), a fare regulation is sufficient for controlling the taxi market as derived by the USA example. The example of deregulation in the United States confirmed the results of Schaller (2007); taxi drivers will create over-supply in airports due to the higher trip cost.

It is important to highlight that effects of deregulation depend on the pre-deregulation situation. In markets where regulation kept supply close to free entry equilibrium levels and low license values, there will be no changes. In markets where the number of taxis is very low due to the strict applied regulation, supply will increase significantly after deregulation, as shown in the examples listed above. This entry of new supply will lead to low incomes, high fares and business failure (short terms results), while the adaptation of consumers will occur over long term time periods.

2.5.3. The taxi industry in Europe

Detailed data from European cities is presented in this sub-section. Table 2-6 presents general data of 19 European cities.

Table 2-6 General data related to the taxi market of different European cities. Source: CENIT (2004)

City	Average trip fare*	Taxi vs Bus**	Taxi vs oil***	Monthly benefits ****	Monthly trips	Population	Urban population density	GDP per inhabitant	Number of taxis	Taxis per thousand inhabitants
Amsterdam	14.8	14.1	2.2	1404	95	850,000	57.3	34100	1.504	1.77
Athens	7.4	23.1	1.6	256	35	3,900,000	65.7	11600	15.249	3.91
Barcelona	8.5	17.7	1.8	594	70	4,390,000	74.7	17100	11.765	2.68
Berlin	11.3	9.8	1.8	1199	106	3,390,000	54.7	20300	6949	2.05
Brussels	14.9	19	2.5	1495	100	964,000	73.6	23900	1.243	1.29
Budapest	6.2	22.5	1.1	201	32	1,760,000	46.3	9840	5.596	3.18
Copenhagen	14.9	13	2.3	2661	179	1,810,000	23.5	34100	2.805	1.55
Dublin	6.2	8.5	1.2	919	148	1,120,000	25.9	35600	1.993	1.78
Lisbon	5.3	5.6	0.8	441	83	2,680,000	27.9	17100	4.529	1.69
London	12.5	11.4	1.8	1286	103	7,170,000	54.9	36400	55.997	7.81
Madrid	9.8	15.8	2	594	61	5,420,000	55.7	20000	14.471	2.67
Milan	9.9	17.6	1.6	997	101	2,420,000	71.7	30200	4.573	1.89
Oslo	18.8	9.7	2.5	1570	84	981,000	26.1	42900	2.148	2.19
Paris	8.7	12	1.3	1095	126	11,100,000	40.5	37200	17.538	1.58

Prague	6.8	15.2	1.5	314	46	1,160,000	44	15100	3.978	3.43
Rome	8.3	19	1.3	997	120	2,810,000	62.6	26600	5.816	2.07
Stockholm	11.9	7.8	3.5	1879	158	1,840,000	18.1	32700	5.207	2.83
Vienna	15.7	17.6	3	914	58	1,550,000	66.9	34300	4.433	2.86
Warsaw	4.2	12.8	0.8	241	57	1,690,000	51.5	13200	5.999	3.55

*2002 prices, 5 km trip, day fares, inside the city
** Taxi cost per km/bus cost per km
*** Cost per km/cost of 1 liter of oil
****National average

Conclusions obtained from Table 2-6 are:

- Average trip fare is higher in cities with higher GDP. A relation between monthly income, cost of taxi and cost of fuel also exists.
- The relation of taxi cost versus bus cost grows with the density of the city due to the economies of scale of the Mass Public Transport.
- The number of taxis has a very strong relation with the population of the city (excluding London, as it can be seen in graph Figure 2-33). This relation is between 1.3 and 4 times the number of taxis per thousand inhabitants (in London this relation is 8 taxis per 1000 inhabitants).

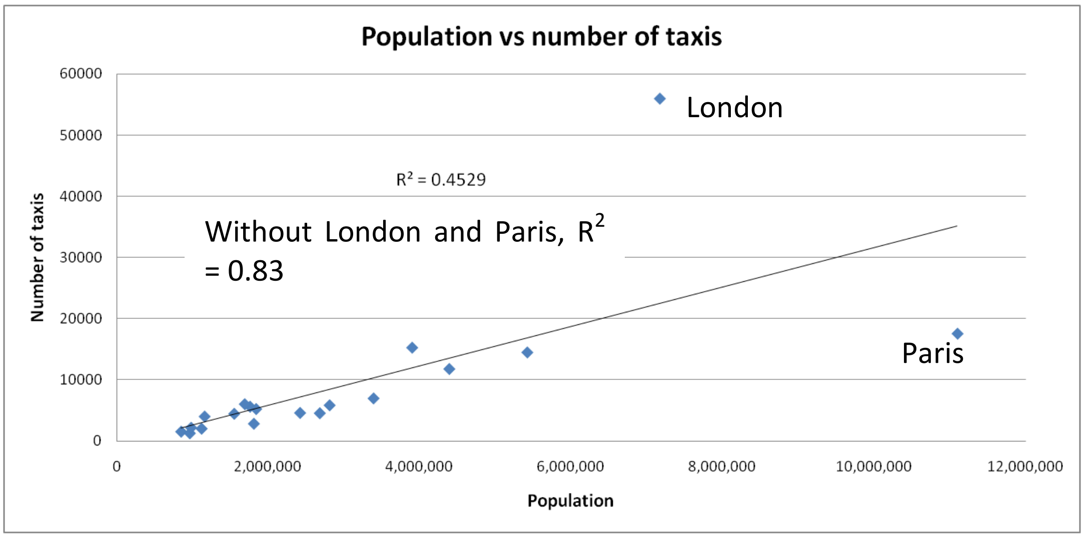

Figure 2-33 Fleet size versus population. Source: (Own elaboration from CENIT (2004))

2.6. Conclusions

The first taxi models developed used aggregated techniques for explaining the relation between the most significant variables of the taxi market. Such models do not consider the fact that taxi markets operate in an urban network, sharing the street network with other transport modes. Later models introduced this spatial dimension, along with other hypotheses for simulating the real taxi market, such as the network knowledge, while calculating the customer trip generation-distribution and assignment. Other hypotheses include the customer-driver search function, which increases the reality of the simulation of the finding process between a taxi and a customer and the day-to-day learning process. The third generation models are based on simulation and are thus the most appropriate models for analyzing the operational issues of the taxi market.

There are detailed formulations in the literature for estimating the different parameters of the taxi market. All authors agree in the most important variables for modelling the taxi market, such as the waiting time, the generalized cost of the system and the optimum fleet. Each author defines his own generalized cost and the related optimum fleet. Most authors have studied the dispatching market, where taxis wait at taxi stands for a call, but the stand and the hailing market have not been modeled at the same detail level. There is a need for research in the other two modes of operation. In addition, combined markets must be modelled, with a heterogeneous taxi fleet composed by the three mentioned modes.

All models so far have investigated the taxi market from the taxi driver (income) and the customer (waiting time, level of service, total cost) point of view, but few models have taken into account the consequences of the market regulations in the city (contamination, congestion). It is therefore important to add environmental considerations as a determinant factor in the future models since in most of cities, taxi flows have negative consequences for the overall health of the citizens.

Models proposed in the literature are characterized by significant data requirements due to the high number of determinants in the demand and supply of taxi services. The use of GPS and GIS has enabled an easier data collection process, but the reluctance of the taxi sector to share this data remains an important barrier.

Regulaiton has been extensively discussed by most authors. Most of them agree on the statement that regulation of entry and fares must not act simultaneously; deregulation of access to the taxi market must be implemented in most cities, which will increase the supply and the level of service of customers. Entry deregulation must be accompanied by new regulations, such as fare regulation and special regulations on high-demand generation points, such as airports, train stations or hotels.

Both aggregated and simulation based modeling approaches are useful, each one in its respective scale. Aggregated models can analyze major variations in the taxi market using fewer variables, simulating fare and entry regulations easily and obtaining clear results. More detailed models can simulate the taxi market in a more efficient way, taking into account the spatial characteristics of the demand and supply, the different types of operation modes that work together in the same city, the exogenous and endogenous factors that are generating the demand, and the congestion.

Data availability is an important parameter for modeling the taxi market. As models become more detailed, the accuracy of results increases, but the necessary data is more difficult to collect; on the other hand, aggregate models need fewer quantity and quality of data, but results are not as analytical as they can be in a more detailed model. With new developed ITS technologies, a lot of data can be recorded, and more detailed and complex models can be developed.

2.7. References

Arnott R., (1996) Taxi Travel Should Be Subsidized. *Journal of Urban Economics* **40**, 316 – 333.

Bailey, W. A. and Clark, T. D. (1987) A simulation analysis of demand and fleet size effects on taxicab service rates. *Proc. of the 19th conference on Winter simulation*, 838-844.

Bailey, W. A. and Clark, T. D. (1992) Taxi management and route control: a systems study and simulation experiment. *Proc. of the 24th conference on Winter simulation*, 1217-1222.

Bautista (1985) Models de distribució del temps d'espera del taxi. *Tesina final de Carrera ETSEIB,* Barcelona.

Beesley M. E. and Glaister S. (1983) Information for regulating: the case of taxis. Royal economic society. *The economic journal* **93** (371), 594 – 615.

Beesley M. E. (1973) Regulation of taxis. Royal economic society. *The economic journal* **83** (329), 150 - 172.

Cairns R. D., Liston-Heyes C. (1996) Competition and regulation in the taxi industry. *Journal of Public Economics* **59**, 1 – 15.

Cascetta E. and Cantarella G. E. (1991) A Day-to-day and Within-Day Dynamic Stochastic Assignment Model. *Transportation Research Part A* **25**, 277 – 291.

CENIT (2004) Metodologia per a l'establiment de les tarifes del taxi a l'AMB i la seva revisió. *Informe final per a l'Institut Metropolità del Taxi*.

Chang S. K. and Chu-Hsiao Chu (2009) Taxi vacancy rate, fare and subsidy with maximum social willingness-to-pay under log-linear demand function. *Transportation Research Record: Journal of the Transportation Research Board* **2111**, 90 – 99.

Chang S. K. and Huang S. M. (2003) Optimal fare and unoccupancy rate for taxi market. *Transportation Planning Journal* **32** (2), 341 – 363.

Chang S. K. J., Wu C. H., Wang K. Y. And Lin C. H. (2010) Comparison of Environmental Benefits between Satellite Scheduled Dispatching and Cruising Taxi Services. *Proc. of the 89th Annual Meeting of the Transportation Research Board,* Washington D.C.

Chen B. and Cheng H. H. (2010) A Review of the Applications of Agent Technology in Traffic and Transportation Systems. *IEEE transactions on intelligent transportation systems* vol. **II** (2) pp. 485 - 497.

Chen Y. M. and Wang B. Y. (2009) Towards Participaroty Design of Multi-agent Approach to Transport Demands. *DCSI International Journal of Computer Science Issues* **4** (1) pp. 10 - 15.

Cheng S. F. and Nguyen T. D. (2012) Taxisim: A Multiagent Simulation Platform for Evaluating Taxi Fleet Operations. *AAMAS Workshops*, LNAI **7068**, 359-360.

Daganzo C. F. (2010a) Lesson notes (available at http://www.ce.berkeley.edu/~daganzo/index.htm).

Daganzo C. F. (2010b) Structure of competitive transit networks. *Transportation Research Part B* **44**, 434-446.

Daniel F.G. (2003) An Economic Analysis of Regulated Taxicab Markets. *Review of Industrial Organization* **23,** 255 – 266.

De Vany A. (1975) Capacity Utilization under Alternative Regulatory Restraints: An Analysis of Taxi Markets. Chicago Journals. *The journal of Political Economy* **83** (1), 83 – 94.

Der-Horng L., Hao W., Ruey L. C. and Siew H. T. (2004) Taxi Dispatch System Based on Current Demands and Real-Time Traffic Conditions. *Transportation Research Record: Journal of the Transportation Research Board* **1882**, 193 – 200.

Douglas G. (1972) Price Regulation and optimal service standards. *Journal of Transport Economics and Policy*, May, pp. 116-127.

Estrada M., F. Robusté, C. Amat, H. Badia and J. Barceló (2011) On the optimal length of the transit network with transit performance simulation. Application to Barcelona. *Proc. of the 90th Annual Meeting of the Transportation Research Board,* Washington D.C.

Farrell S. (2010) Identifying demand and optimal location for taxi ranks in a liberalized market. *Proc. of the 89th Annual Meeting of the Transportation Research Board,* Washington D.C.

Fernandez L. J. E., de Cea Ch. J. and Briones M. J.(2006) A diagrammatic analysis of the market for cruising taxis. *Transportation Research Part E* **42**, 498 – 526.

Fernandez L. J. E., de Cea Ch. J. and Malbran R. H. (2008) Demand responsive urban public transport system design: Methodology and application. *Transportation Research Part A* **42**, 951 – 972.

Foerster J. F. And Gilbert G. (1979) Taxicab deregulation: economic consequences and regulatory choices. *Transportation* **8**, 371 – 378.

Geroliminis N. and Daganzo C.F. (2008) Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B* **42** (9), 759-770.

Gilbert G., Nalevanko A., Stone J. R. (1993) Computer Dispatch and Scheduling in the Taxi and Paratransit Industries: An Application of Advanced Public transportation Systems. *Transportation Quarterly, Vol.* ***94*** Is. 2, pp. 173.

Ho, LS. (1993) An Optimal Regulatory Framework for the Taxicab Industry. *Department of Economics, Chinese University of Hong Kong, Department of Economics,* working paper no. 19.

Holland, J.H.. Miller, J.H. (1991). Artificial Adaptive Agents in Economic Theory. *American Economic Review* **81** (2), 365-71.

Holroyd E. M. (1965) The optimum bus service: a theoretical model for a large uniform urban area. In L. C. Edie, R. Herman, and R. Rothery (Eds.), Vehicular Traffic Science. *Proc of the 3rd International Symposium on the Theory of Traffic Flow*. New York: Elsevier.

Horn M. E. T. (2002) Fleet scheduling and dispatching for demand-responsive passenger services. *Transportation Research Part C* **10,** 35 – 63.

Horowitz J. L. (1984) The Stability of Stochastic Equilibrium in a Two-Link Transportation Network*. Transportation Research Part B* **18B** , pp 13 – 28.

Hyunmyung K., Oh J. S. and Jayakrishnan R. (2005) Effect of Taxi Information System on Efficiency and Quality of Taxi Services. *Transportation Research Record: Journal of the Transportation Research Board* **1903**, 96 – 104.

Lee K.T., Lin D.J. and Wu P.J. (2005) Planning and Design of a Taxipooling Dispatching System. *Transportation Research Record: Journal of the Transportation Research Board* **1903**, 86 - 95.

Kikuchi S., Rhee J. and Teodorovic D. (2002) Applicability of an agent-based modeling concept to modeling of transportation phenomena. *Yugoslav Journal of Operations Research* **12** (2), 141-156.

Kim H., Yang I. and Choi K. (2011) An Agent-based Simulation Modelfor Analyzing the Impact of Asymetric Passenger Demand on Taxi Service. *KSCE Journal of Civil Engineering* **15** (1), 187-195.

Kim, H., Oh. J. D., and Jayakrishnan, R. (2005). Effect of taxi information system on efficiency and quality of taxi services. *Transportation Research Record*, *Journal of the Transportation Research Board* **1903**, 96-104.

Liu L., Andris C., Biderman A. and Ratti C. (2010) Uncovering Taxi Driver's Mobility Intelligence through His Trace. *Environment and Urban Systems.*

Lioris J. E., Cohen G., and La Fortelle A. (2010) Evaluation of Collecvite Taxi Systems by Discrete-Event Simulation. Second International Conference on *Advances in System Simulation* pp. 34-39.

Lo H. K., Yip C. W. and Wan Q. K. (2004) Modeling Competitive Multi-modal Transit services: A Nested Logit Apporach. *Transportation Research Part C*. **12C** (3-4), 251-272.

Manski C. F. and Wright J. D. (1976) Nature of equilibrium in the market for taxi services. *Transportation Research Record: Journal of the Transportation Research Board* **619**, 296 – 306.

Massow M. And Canbolat M. (2010) Fareplay: An examination of taxicab drivers' response to dispatch policy. *Expert systems with applications* **37,** 2451 – 2458.

Matsushima K. And Kobayashi K. (2006) Endogenous market formation with matching externality: an implication for taxi spot markets. *Structural Change in Transportation and Communications in the Knowledge Economy*, 313 – 336.

Matsushima K. And Kobayashi K. (2010) Spatial equilibrium of taxi spot markets and social welfare. *12th World Conference of Transport Research*, Lisbon, Portugal.

Meyer, R. F. and Wolfe, H. B. (1961) The organization and operation of a taxi fleet. *Naval Research Logistics* Quarterly **8**, 137–150.

Moore, AT., and Balaker, T. (2006) Do Economists Reach a Conclusion on Taxi Deregulation?. *Economic Journal Watch* **3** (1), 109-132.

Organisation for Economic Co-operation and Development, Policy roundtables (2007) Taxi Services: Competition and Regulation.

Orr D. (1969) The taxicab problem: A proposed solution. *Journal of Political Economy* **77** (1), 141-7.

Rawley E. and Simcoe T. (2009) Information Technology, Capabilities and Asset Ownership: Evidence from Taxicab Fleets. *Danish Research Unit for Industrial Dynamics Working Paper* **10-01**

Schaller B. and Gilbert G. (1995) Factors of production in a regulated industry: New York Taxi Drivers and the Price for Better Service. *Transportation* quarterly **49** (4), 81-91

Schaller B. (2007) Entry controls in taxi regulation: Implications of US and Canadian experience for taxi regulation and deregulation. *Transport Policy* **14,** 490 – 506.

Schroeter J. R. (1983) A model of taxi service under fare structure and fleet size regulation. *The Bell Journal of Economics* **14** (1), 81 – 96.

Shi W. (2010) Dynamic simulation and quantitative analysis of urban taxi services. *Thesis for the degree of Master of Philosophy at the University of Hong Kong*.

Small K. A. (1992) Urban Transportation Economic Volume 4 of *Fundamentals of Pure and Applied Economics* Series **51**, Harwood Academic Publishers, Switzerland.

Smeed R. J. (1975) Traffic studies and urban congestion. *Journal of Transport Economics and policy* **2** No. 1 pp. 33 - 71.

Song, Z. Q. (2006) A Simulation Based Dynamic Taxi Model. *Master thesis at the University of Hong Kong*.

Song, Z. Q. and Tong, C. O. (2006) A simulation based dynamic model of taxi service. *Proc. of Dynamic Traffic Assignment: First International Symposium on Dynamic Traffic Assignment*.

Teodorovic D. (2003) Transport modeling by multi-agent systems: a swarm intelligence approach. *Transportation planning and Technology* **26** (4), 289-312.

Vaughan R. (1987) Urban Spatial Traffic Patterns. ISBN 0 85086 122 5.

Vythoulkas P. C. (1990) A Dynamic Stochastic Assignment Model for the Analysis of General Networks. *Transportation Research Part B* **24**, 453 - 469.

Wong S. C. and Yang H. (1998) Network Model of Urban Taxi Services. Improved Algorithm. *Transportation Research Record: Journal of the Transportation Research Board* **1623**, 27 – 30.

Wong K. I., Wong S. C. and Yang H. (2001) Modeling urban taxi services in congested road networks with elastic demand. *Transportation ResearchPart B* **35**, 819 – 842.

Wong K. I. and Wong S. C. (2002) A sensitivity-based solution algorithm for the network model of urban taxi services. *Transportation and Traffic Theory in the 21st Century,* pp. 23-42.

Wong K. I., Wong S. C., Wu J.H., Yang H. and Lam W.H.K. (2004) A combined distribution, hierarchical mode choice, and assignment network model with multiple user and mode classes. *Urban and regional transportation modeling,* pp. 25-42.

Wong K. I., Wong S. C. Bell M. G. H. and Yang H. (2005) Modeling the bilateral micro-searching behavior for urban taxi services using the absorbing Markov chain approach. *Journal of Advanced Transportation* **39** (1), 81 - 104.

Yang H. Lau Y. W. and Wong S. C. (1997) A simultaneous Equation system of customer demand, taxi utilization and level of services. *Working paper, Department of Civil and Structural Engineering, The Hong Kong University of Science and Technology.*

Yang H. and Wong S. C. (1998) A network model of urban taxi services. *Transportation Research Part B* **32** (4), 235 – 246.

Yang H., Yan Wing Lau, Wong S.C. and Hong K.L. (2000) A macroscopic taxi model for passenger demand, taxi utilization and level of services. *Transportation* **27**, 317-340.

Yang H., Wong K. I. and Wong S. C. (2001) Modeling Urban Taxi Services in Road Networks: Progress, Problem and Prospect. *Journal of Advanced Transportation*, **35** (3), 237 – 258.

Yang H., Wong S. C. and Wong K. I. (2002) Demand-supply equilibrium of taxi services in a network under competition and regulation. *Transportation Research Part B* **36**, 799 – 819.

Yang h., Ye M., Tang W. H. and Wong S. C. (2005) Regulating taxi services in the presence of congestion externality. *Transportation Research Part A* **39**, 17 – 40.

Yang H., Cowina W. Y. L., Wong S. C. And Michael G. H. Bell (2010a) Equilibria of bilateral taxi-customer searching and meeting on networks. *Transportation Research Part B* **44**, 1067 – 1083.

Yang H., Fung C. S., Wong K. I. and Wong S. C. (2010b) Non-linear pricing of taxi services. *Transportation Research Part A* **44**, 337 – 348.

Yang T., Yang H. and Wong S. C. (2010c) Modeling Taxi Services with a Bilateral Taxi-Customer Searching and Meeting Function. *Proc. of the 89th Annual Meeting of the Transportation Research Board,* Washington D.C.

Zamora D. (1996) Modelització dels costos unitaris d'una flota de taxis. *Tesina final de carrera. Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports de Barcelona*.

3. THE AGGREGATED MODEL FOR THE ESTIMATION OF THE TAXI SUPPLY

3.1. Introduction

In this section, an aggregated formulation for the estimation of the optimum taxi fleet size is introduced, presenting the objective function and analyzing the involved variables. The formulation is related to the three operational taxi modes: hailing, stand and dispatching. Most of the variable definitions and calculations apply to the three modes, but in some cases the variables formulation differs between the operation modes.

The proposed model uses the various mathematical formulations presented in the literature (Bautista (1985), Fernandez et al. (2008), Zamora (1996) and Meyer (1961)) for estimating some of the variables. A generalized cost function is proposed and optimized for obtaining the optimum size fleet related to each operational mode, city size and demand level. The correspondent generalized cost and Level of Service (waiting time of customers) are also obtained, comparing the characteristics of the different operational modes for taxi services in the same city. The optimum fleet size and the related unitary cost and waiting time of customers for the dispatching, hailing and stand taxi markets are obtained and compared.

Various approximations and relations have been presented in Chapter 2, resulting in a simple model where the cost and performance components of the system can be obtained from a small number of decision variables (the number of licenses, the demand and the area size). The optimization of the problem can be done by deriving mathematically the generalized cost formulations and by depicting their values in a grid. In order to define the optimum fleet size, all the costs (monetary costs and time costs) of the involved stakeholders are added in a unique function. Various quality constraints are defined and the objective function is optimized, presenting the results in terms of minimum and optimum fleet size for each operation mode, demand level and city size. The objective function captures the “costs” (in terms of time) of the involved actors and the cost of the taxi services infrastructure. In the case of the customers the cost is composed by the total travel time (access time, waiting time and in-vehicle time) and the trip monetary cost. In the case of the city, the costs are composed by the increase of travel time caused to other drivers and the smissions generated by the taxi drivers. Finally, the taxi drivers cost is the difference between the cost of offering the taxi service and the income (in this case the cost is expected to be negative). The cost of the infrastructure varies depending on the operational mode. The estimation of the speed due to the variation of taxi vehicle-kilometers in the network and therefore the external cost calculation is based on the existence of an accurate and not scattered MFD, which existence at region level has been recently proved by Geroliminis and Daganzo (2008). An example of this methodology can be found in Estrada et al (2011), where the impact of new bus lanes in the city of Barcelona by obtaining the MFD of the city is studied.

In relation to the externalities, during the last years there have been many attempts to internalize the externalities of road transport in terms of congestion and pollution. The customers that are paying these costs nowadays are the other road drivers (congestion) and the citizens (local Greenhouse Gas emissions). On the one hand, taxis circulate in the road network that is also used by other types of vehicles, such as buses or private cars. The taxi flow has a significant influence in the travel time of the rest of the customers, especially when the percentage of taxis in the daily volume is high[3]. A variation in the number of circulating taxis will affect the travel time of all road customers, changing the corresponding emissions and fuel consumption. Recent studies (Geroliminis and Daganzo (2008) and Estrada et al (2011)) have shown how to quantify the impact of this variation in terms of average speed reduction for a whole zone within the city. Using the MFD, it is possible to estimate the average speed increase/reduction of the whole network due to a decrease/increase in the number of taxis in the network. On the other hand, the environmental issues are gaining importance when developing policies and planning transport systems. In order to quantify the impact of the taxi services on local Greenhouse Gas emissions, an emission unitary cost is applied to the taxi emissions and to the additional emissions from other vehicles caused by the extra travel time caused by the taxis. The fuel consumption and the emission levels are estimated through the travelled distance and the average speed using the formulations[4] proposed in the different environmental models. The impact of the taxi fleet on the average speed of the network can be approximated by the MFD: more taxi vehicle-kilometers will increase the density of the network and therefore reduce speed; oppositely, a reduction in the number of vehicle-kilometers produced by taxis will increase the average speed of the network, reducing fuel consumption and emissions.

The involved stakeholders are, as proposed by Lo et al. (2004), the taxi customers, the taxi drivers or service providers and the city or society in general (building a multiobjective problem). In order to define the optimum fleet size, all the costs of the involved stakeholders are added in a unique function. This unique function takes into account all the costs of the involved parties, monetary costs and time costs. Various quality constraints are defined and the objective function is solved, presenting the results in terms of minimum and optimum fleet size for each demand level and city size.

[3] Taxi volume in Hong Kong represents the 60% of the total volume in the peak hours (Yang et al. (2000).

[4] Most of the models propose a second grade function for estimating the kilometric fuel consumption and the emissions using the average speed.

3.2. The stakeholders

The three involved actors are the customers, the drivers and the city. The customers aim to minimize the total time and cost in order to satisfy the necessity for a trip, the taxi drivers look for trips in order to maximize their benefits and the city is paying for the externalities of the congestion and pollution generated by the circulating taxis.

3.2.1. The customers

The customers are estimated as the quantity of demanded trips per hour and km^2 (λ_u). The most important variables of the customers are the value of time parameter, the fee and the total travel time (composed of access time, waiting time and in-vehicle time). The parameter used for converting the total travel time into monetary costs and viceversa is the customers' value of time.

3.2.2. The taxi drivers

The number of taxi drivers is estimated as the quantity of taxis (vacant and occupied) per hour and km^2 (λ_d). The two important variables for the drivers are the expected income from the provision of taxi services and the cost of providing the services. The income of the taxi driver depends on the number of trips, the average length, the average duration and the fee applied. The cost depends on the mode of operation; if the taxi driver is circulating while waiting for a call or looking for a ride, the cost can be considered as fixed cost per hour of circulation (with or without customer), however if the taxi is not circulating while waiting for a call or looking for a ride, then the cost is only the cost per hour of circulation with customer or the cost of accessing to the nearest taxi stand or taxi waiting zone when waiting for a customer.

3.2.3. The city

This actor represents the externalities of the taxi services in the cities, from the citizens to the other drivers. The externality of the citizens is assumed to be the pollution of the urban environment by both, the taxis and the private cars (the taxis generate congestion to the other cars and therefore these cars pollute more). The externality to the other cars is the congestion, the extra circulating time due to the congestion generated by the taxis. The city is represented by the quantity of other drivers per hour and area of region (λ_v). Other externalities such as noise are not considered in this model.

3.3. The mathematical formulation

3.3.1. The objective function

The objective function consists of the "costs" of the involved actors and the cost of the taxi service infrastructure. In the case of the customers, the cost is composed by the total travel time and the trip cost. In the case of the city, the costs are composed by the travel time increase and fuel consumption of other drivers and the pollution. Finally, the taxi drivers cost is the difference between the cost of offering the taxi service and the income (in this case the expected cost is negative). The cost of the infrastructure varies depending on the operational mode: zero cost in the hailing market; stand construction and space opportunity cost for the stand market; communications, office and personnel costs in the dispatching market case. The proposed objective function is presented in Equation 3.1, while the different components and variables are defined in Table 3-1:

$$Min\ Z = Z_d + Z_u + Z_c + G \tag{3.1}$$

$$Z_u = \lambda_u \cdot A \cdot \left[\alpha_A \cdot T_A + \alpha_W \cdot T_W + \alpha_{IV} \cdot T_{IV} + \frac{\bar{c}}{VoT_u}\right] \tag{3.2}$$

$$Z_d = \frac{\lambda_d \cdot A}{VoT_d}\left[-\bar{n} \cdot \bar{c} + \left(\bar{n} \cdot \bar{d} \cdot C_{km} + C_h\right)\right] \tag{3.3}$$

$$Z_c = \lambda_v \cdot A \cdot (\Delta T_v + \frac{\Delta F_c + C_E \cdot \Delta T_v \cdot E}{VoT_v}) + \frac{\lambda_d \cdot A \cdot C_E \cdot E}{VoT_d} \tag{3.4}$$

3.3.2. The variables

The variables are presented in Table 3-1.

Table 3-1 Variables definition

Variable	
Model outputs	Z is the cost of the system (€) - z is the unitary system cost (€/trip)
	Z_d is the cost of the drivers (min) - z_d is the unitary cost of the drivers (min/trip)
	Z_u is the cost of the customers (min) - z_u is the unitary cost of the customers (min/trip)
	Z_c is the additional cost for the city (min) - z_c is the unitary cost for the city (min/trip)
	G is the cost of the infrastructure (min) - g is the unitary infrastructure cost (min/trip)
	T_W is the waiting time of customers (min)
	T_A is the access time of customers (min)
	$\bar{c}$ is the average trip cost (€)
	$\bar{n}$ is the average number of trips per hour and driver (trips)
	ΔT_v is the increase in the travel time of the other drivers caused by taxis (min)
Decision variables	λ_d is the taxi hourly supply (vehicles per hour and area of service)
	D is the flag-drop charge (€)
	τ_{km} is the taxi fee per unit of distance (€/km)
	τ_{sec} is the taxi fee per unit of time (€/min)
Model inputs (variables)	λ_u is the hourly demand for taxi trips (trips per hour and area of service)
	A is the area of the region (km^2)
	λ_v is the hourly circulating vehicles (vehicles per hour and area of service)
	T_{IV} is the in-vehicle time of customers (min)
	$\bar{d}$ is the average distance of the trip (km)
	$\bar{v}$ is the average speed of the trip (km/h)
	$\bar{v}$ is the average pedestrian speed (km/h)
	E are the hourly vehicle emissions (kg of CO_2)
Model inputs (parameters)	VoT_u is the value of time of the taxi customers (€/min)
	VoT_d is the value of time of the taxi drivers (€/min)
	VoT_v is the value of time of the other drivers (€/min)
	α_A is the customer perception factor of the access time
	α_W is the customer perception factor of the waiting time
	α_{IV} is the customer perception factor of the in-vehicle time
	C_s is the hourly cost of each taxi stand (€/min)
	C_{km} is the operational cost per unit of distance of taxis (€/km)
	C_h is the hourly operational cost of the moving taxis (€/min)
	C_E is the emission unitary cost for all vehicles (€/kg of CO_2)
	r is the area and network parameter (depending on the geometry as proposed in Holroyd (1965) and Smeed (1975))

Three weighting parameters (α_A, α_W and α_{IV}) are proposed in order to use a unique VoT for the taxi customers. The parameters take into account the perception of the time by the customers in each case. There is the need for calibration of the parameters in each city and type of users. Kittelson et al. (2003) proposed the values presented in Table 3-2.

Table 3-2 Relative Importance of TravelTime Components for Work Trips. Source: Kittelson et al. (2003).

Value	In-vehicle time (T_{IV})	Walking time (T_A)	Initial waiting time (T_W)
Average	1.0	2.2	2.1
Range	1.0	0.8-4.4	0.8-5.1

These values should be adapted to each region. Recently, Raveau et al. (2011) have obtained similar values for the metro of the city of London:

- One minute of waiting equal to 1.07 minutes of travel.
- One minute of walking equal to 1.79 minutes of travel.

The trip cost is calculated based only on the distance traveled by the taxi and the fee per unit of distance. Note that the term $\bar{c}$ does not affect the global objective function of the stakeholders since it will appear in both the customers' cost and the drivers' cost with opposite signs. However it is an important factor when the profitability of each particular stakeholder is analyzed.

3.3.3. Externalities

Using the formulation presented above for estimating the speed drop and the approximations to the fuel consumed and emissions, the impact on both parameters can be quantified. The formulations used for the calculation of the fuel consumption and the CO_2 emissions[5] are based on the estimations proposed by Ntziachristos and Samaras (2012). Various speed-depedent formulations depending on the speed range, vehicle class and engine capacity for estimating the fuel consumption (F_c) and the emissions of various pollutants (E_d) are proposed in their work. The formulations used in this thesis are presented in Equation 3.5 and Equation 3.6:

$$F_c = 102.5 - 1.364\,\bar{v} + 0.0086\bar{v}^2 \quad (3.5)$$

$$E_d = 14.653 - 0.220\bar{v} + 0.001163\bar{v}^2 \quad (3.6)$$

[5] Pollution is represented in the model by CO_2 emissions, but emissions of other gases can be taken into account by changing the coefficients of Equation 3.6.

3.3.4. The constraints

The above metrics are expressed in terms of output per hour, analyzing the characteristics and providing the results for the typical peak hour of the market. Longer periods can be also selected if there is homogeneity in their characteristics along time. The constraints presented below must be taken into account when applying the model in order to reflect physical or temporal restrictions of the real world:

$$T_A < T_{Amax} \quad (3.7)$$
$$T_W < T_{Wmax} \quad (3.8)$$
$$Z_d < -B_{dmin} \quad (3.9)$$
$$E_d + E_v < E_{max} \quad (3.10)$$
$$\Delta T_v < \Delta T_{vmax} \quad (3.11)$$
$$G < G_{max} \quad (3.12)$$
$$\lambda_d < \lambda_{dmax} \quad (3.13)$$
$$\lambda_d > \lambda_{dmin} \quad (3.14)$$

The above constraints correspond to the following limitations:

- Access and waiting time of customers lower than T_{Amax} and T_{Wmax} (Equation 3.7 and Equation 3.8).
- Benefit of taxi drivers higher than B_{dmin} (Equation 3.9).
- Emissions lower than E_{max} (Equation 3.10).
- Travel time increase of the other drivers lower than the ΔT_{vmax} (Equation 3.11).
- Infrastructure cost lower than G_{max} (Equation 3.12).
- Number of licenses between minimum λ_{dmin} and λ_{dmax} (Equation 3.13 and 3.14).

The problem is to minimize the objective function while respecting the above constraints, which can be added to the problem formulation by using Lagrange multipliers, converting the constrained problem in an unconstrained problem.

3.4. Application to the dispatching market

3.4.1. Formulation of the trip distance

The trip distance is calculated by considering the region as a square of side a and estimating the expected distance between two random points within the region.

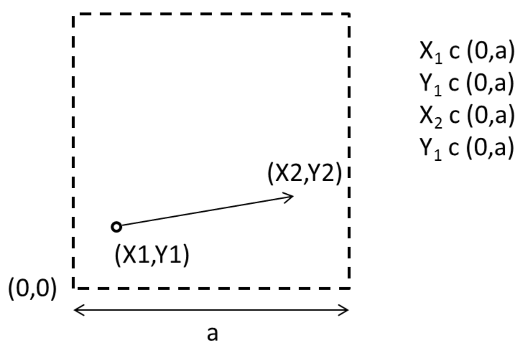

Figure 3-1 Euclidian distance between two random points

Making the hypothesis that the coordinates of the two points are independent, the probability function of the distance between the two points can be expressed as follows:

$$F(d)=Prob((X_1-X_2)^2+(Y_1-Y_2)^2<d^2) \tag{3.15}$$

And the expected value is the one presented in Equation 3.16:

$$E_{dist} = \int_0^{\sqrt{2}a} v g_v(v) dv \tag{3.16}$$

The solution for Equation 3.16 is the one presented in Equation 3.17 for a square region of side a:

$$E_{dist} = \frac{a}{3} ln\left(1+\sqrt{2}\right) + \frac{a}{15}\left(2+\sqrt{2}\right) = 0.52140543a \approx 0.5a \tag{3.17}$$

Daganzo (1984) proposes a similar procedure for calculating the distance but making the hypothesis that the individual x and y coordinates of the origin and the destination are not independent.

The expected travel time is the factor between this expected distance and the average speed.

$$T_{IV} = \frac{rA^{1/2}}{2\bar{v}} \tag{3.18}$$

3.4.2. Formulation of the waiting time

Considering that the taxis are waiting for a call distributed in an homogeneous stand network, this distance depends on the density of free taxis in the zone (Δ), expressed as the ratio between the free taxis and the area of the zone.

$$d(\Delta) = \frac{0,4r}{\sqrt{\Delta}} \tag{3.19}$$

The number of free taxis is calculated as the difference between the total number of vehicle-hours ($\lambda_d A$) and the occupied/assigned taxi-hours. Finally, the occupied number of taxi-hours is calculated as the product between the total demand ($\lambda_u A$) and the average trip time $\left(\frac{rA^{1/2}}{2\bar{v}}\right)$. In order to take the travel time between the taxi stand and the customer (assigned taxi-hours) into account, when the taxi has been called, a factor ε is introduced in the calculation of the occupied taxi-hours. Introducing the density into Equation 3.19, Equation 3.20 is obtained:

$$T_w = \frac{0,4r}{\bar{v}\sqrt{\frac{\lambda_d A - \lambda_u A \frac{rA^{1/2}}{2\bar{v}} \varepsilon}{A}}} \tag{3.20}$$

3.4.3. Optimization of the fleet size

Introducing the formulations of the trip distance and the waiting time into Equation 3.1 and Equation 3.2, Equation 3.21 is obtained:

$$Z_u = \lambda_u \cdot A \cdot \left[\left(\alpha_A \cdot 0 + \alpha_W \cdot \frac{0{,}4r}{\bar{v}\sqrt{\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\varepsilon}} + \alpha_{IV} \cdot \frac{rA^{1/2}}{2\bar{v}} \right) + \frac{D + \frac{rA^{1/2}}{2} \cdot \tau_{km}}{VoT_u} \right] \tag{3.21}$$

$$\begin{aligned} Z_d &= \frac{\lambda_d \cdot A}{VoT_d}\left[\frac{\lambda_u}{\lambda_d} \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \frac{\lambda_u}{\lambda_d} \cdot \frac{rA^{1/2}}{2} \cdot C_{km} + C_h\right] \\ &= \frac{A}{VoT_d}\left[\lambda_u \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \lambda_u \cdot \frac{rA^{1/2}}{2} \cdot C_{km} + \lambda_d \cdot C_h\right] \end{aligned} \tag{3.22}$$

Deriving Equation 3.21 and Equaiton 3.22 with respect to the supply:

$$\frac{\partial Z_u}{\partial \lambda_d} = -\frac{\lambda_u \cdot A\alpha_W 0{,}4r}{2\bar{v}\sqrt{\left(\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\varepsilon\right)^3}} \tag{3.23}$$

$$\frac{\partial Z_d}{\partial \lambda_d} = \frac{A \cdot C_h}{VoT_d} \tag{3.24}$$

And the optimum supply is the one presented in Equation 3.26:

$$A \cdot C_h = \frac{\lambda_u A\alpha_W VoT_d 0{,}4r}{2\bar{v}\sqrt{\left(\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\varepsilon\right)^3}} \tag{3.25}$$

$$\lambda_d = \lambda_u \frac{rA^{1/2}}{2\bar{v}}\varepsilon + \left(\frac{\lambda_u \alpha_W VoT_d 0{,}4r}{2\bar{v}C_h}\right)^{2/3} \tag{3.26}$$

The first term is the minimum fleet size for serving all trips and the second term is the extra fleet needed for providing a better LoS to customers, while maintaining a satisfactory profit to taxi drivers. The constant value of this extra fleet is directly proportional to the VoT and inversely proportional to C_h and $\bar{v}$, which means:

- High VoT of taxi customers implies a higher extra fleet in order to reduce waiting time.
- High r (longer trips due to the complex geometry of the network) values implies more taxis
- High hourly operating cost implies fewer taxis for reducing the vacant distance and time of taxis.
- Higher speeds are related to smaller taxi fleets due to the higher performance of the vehicles.

It is interesting to highlight that the obtained formulation is very similar to the one proposed by Daganzo (2010a) for calculating the optimum supply, where the fix values of the second term are approximated by 1.2 (Equation 2.55).

Figure 3-2 shows the minimum and the optimum fleet sizes obtained by the formulation presented above in relation to the demand level or area size.

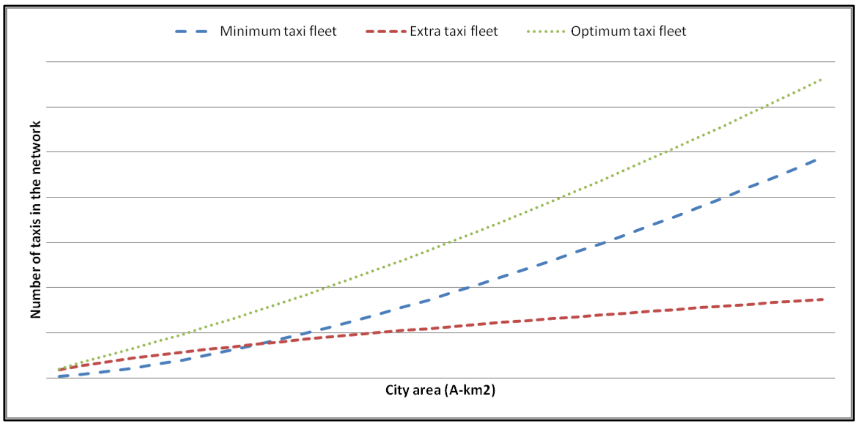

Figure 3-2 Minimum, extra and optimum fleet in relation to the city area

Figure 3-3 shows that small cities need a very small minimum fleet due to the small distance of the trips, but the extra fleet is much larger in relation to this minimum fleet. Larger cities need larger minimum fleets (longer trips), but the extra fleet is smaller in relation to this minimum fleet.

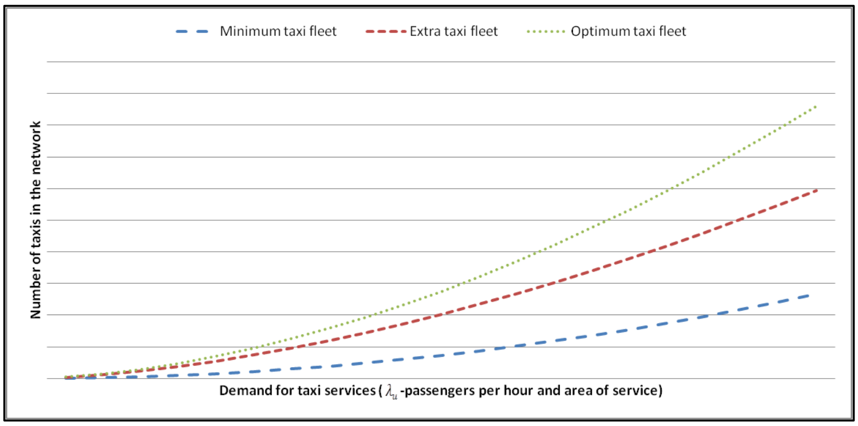

Figure 3-3 Minimum, extra and optimum fleet in relation to the demand for taxi services

Figure 3-3 shows the optimum fleet and its composition (minimum fleet and extra fleet) in relation to the demand. The relation between the minimum fleet and the extra fleet decreases as the demand level increases. Again, low demand levels require larger extra fleet in relation to the minimum fleet.

The associated waiting time to the optimum supply is presented in Equation 3.27:

$$T_W = \frac{0{,}4\mathrm{r}}{\bar{v}\sqrt{\lambda_d - \lambda_u \frac{\mathrm{r}A^{1/2}}{2\bar{v}}\varepsilon}} = \frac{0{,}4\mathrm{r}}{\bar{v}}\left(\frac{2C_h\bar{v}}{\lambda_u \alpha_W VoT_d 0{,}4\mathrm{r}}\right)^{1/3} = \left(\frac{\alpha_W VoT_d}{0{,}32 \cdot C_h \mathrm{r}^2}\lambda_u \bar{v}^2\right)^{-1/3} \quad (3.27)$$

The waiting time is multiplied by a similar factor to the one commented above for the extra fleet (directly proportional to r and C_h and inversely proportional to VoT). Similar conclusions can be obtained:

- Higher VoT or network speed implies less waiting time.
- Higher r or C_h implies higher waiting times.

The obtained result is exactly the same as that proposed by Daganzo (2010a), where the fixed values are approximated by 2 (Equation 2.54).

Finally, by introducing Equation 3.27 and Equaiton 3.26 into the generalized cost function, Equation 3.28 is obtained:

$$Z_u + Z_d = A\left[\frac{\lambda_d{}^{*}C_h}{VoT_d} + \lambda_u \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\varepsilon + \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u \alpha_W T_W{}^{*}\right] \tag{3.28}$$

where the first term corresponds to the fixed cost of the taxi fleet, the second term represents the customer and driver variable cost due to the trips and the final term refers to the waiting time cost of the customers. Rearranging the terms and introducing the formulations of the optimum fleet (Equation 3.26) and minimum waiting time (Equation 3.27), the following actors (Equation 3.29 and Equation 3.30) and system unitary costs (Equation 3.31) are obtained.

$$Z_u = \lambda_u \cdot A \cdot \left[\left(\alpha_W \cdot \left(\frac{\alpha_W VoT_u}{0{,}32 \cdot C_h r^2}\lambda_u \bar{v}^2\right)^{-1/3} + \alpha_{IV} \cdot \frac{rA^{1/2}}{2\bar{v}}\right) + \frac{D + \frac{rA^{1/2}}{2}\cdot \tau_{km}}{VoT_u}\right] \tag{3.29}$$

$$Z_d = \frac{A}{VoT_d}\left[-\lambda_u\left(D + \frac{rA^{1/2}}{2}\cdot\tau_{km}\right) + \lambda_u \cdot \frac{rA^{1/2}}{2}C_{km} + \left(\lambda_u \frac{rA^{1/2}}{2\bar{v}}\varepsilon + \left(\frac{\lambda_u \alpha_W VoT_u 0{,}4r}{2\bar{v}C_h}\right)^{2/3}\right)C_h\right] \tag{3.30}$$

$$\frac{Z_u + Z_d}{\lambda_u \cdot A} = \left(\frac{rA^{1/2}}{2\bar{v}}\varepsilon + \left(\frac{\alpha_W VoT_u 0{,}4r}{2\bar{v}C_h\sqrt{\lambda_u}}\right)^{2/3}\right)\frac{C_h}{VoT_d} + \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\varepsilon + \frac{\alpha_{IV}}{\bar{v}}\right) + \alpha_W VoT_u\left(\frac{\alpha_W VoT_u}{0{,}32 \cdot C_h r^2}\lambda_u \bar{v}^2\right)^{-1/3} = \tag{3.31}$$

$$\frac{C_h}{VoT_d}\frac{rA^{1/2}}{2\bar{v}}\varepsilon + \left(\frac{VoT_d{}^3\bar{v}^2\lambda_u}{0{,}04(\alpha_W VoT_u r)^2 C_h}\right)^{-1/3} + \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\varepsilon + \frac{\alpha_{IV}}{\bar{v}}\right) + \left(\frac{VoT_u\lambda_u\bar{v}^2}{0{,}32 \cdot C_h(\alpha_W r)^2}\right)^{-1/3}$$
$$= \frac{C_h}{VoT_d}\frac{rA^{1/2}}{2\bar{v}}\varepsilon + \left(0{,}926\frac{VoT_d{}^3\bar{v}^2\lambda_u}{(\alpha_W VoT_u r)^2 C_h}\right)^{-1/3} + \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\varepsilon + \frac{\alpha_{IV}}{\bar{v}}\right)$$

Figure 3-4 represents the total costs depending on the demand and the supply for a fixed area size, where the loci of demand and supply points with equal total costs are plotted. Three zones can be observed:

- Zone A: the taxi supply cannot serve all the requested services. In this region the minimum supply constraint is violated.
- Zone B: the driver waiting time is very small; total taxi-hours in service are almost equal to the total customer travel time, providing the customers with a very low LoS and a maximum profit to taxi drivers (maximum utilization of their vehicles).
- Zone C: the number of taxis is higher than the minimum. More taxis imply more waiting time for drivers but less waiting time for customers. In this zone the

optimum fleet size can be observed, providing the optimum number of taxis for each demand level, where the total unitary cost of the system will be minimum.

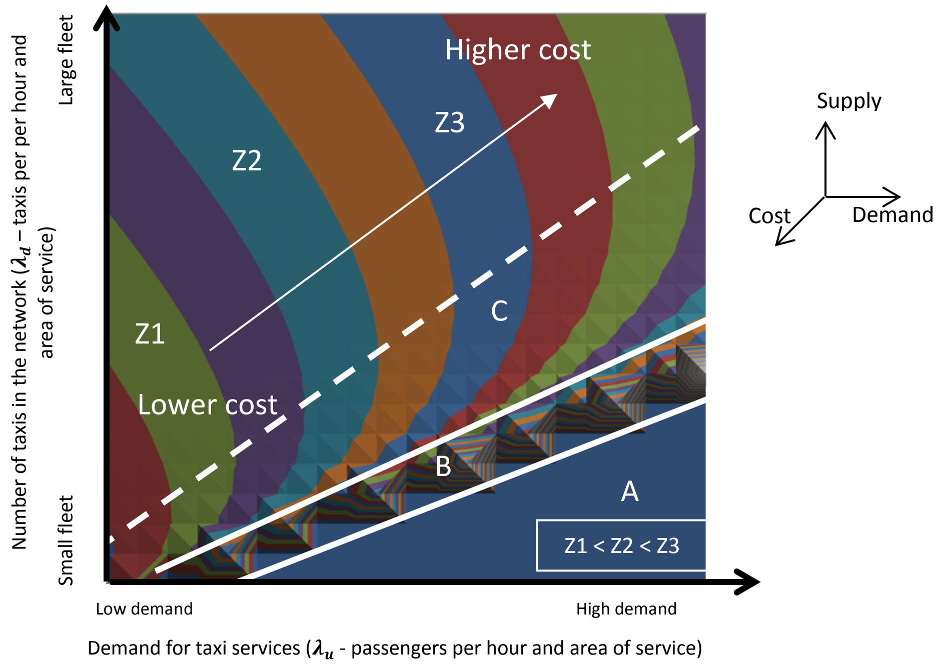

Figure 3-4 System cost (Z) of each demand and supply configurations for the dispatching market

The same figures for the taxi and customers costs are presented in Figure 3-5:

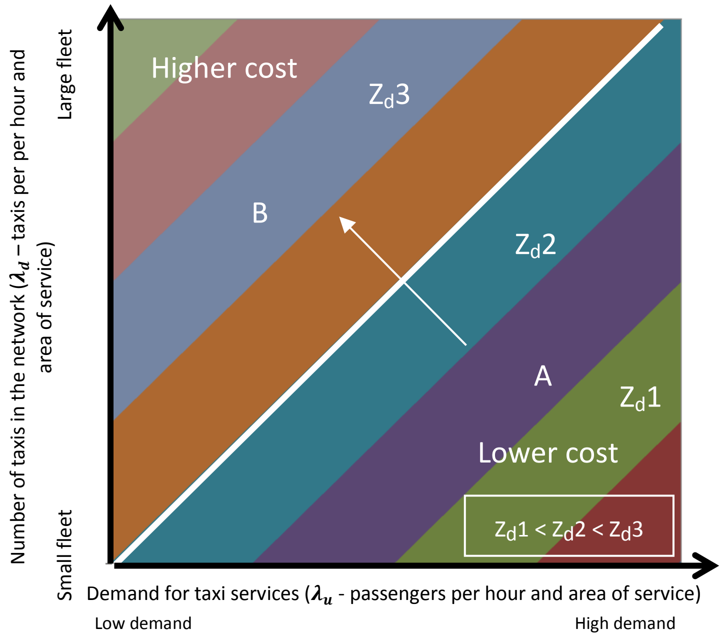

Figure 3-5 Driver cost (Z_d) of each demand and supply configurations for the dispatching market

Figure 3-5 shows the driver costs for each combination of demand and supply. The combinations of demand and supply in region B have negative profits for the drivers, while the combinations in region A present benefits for the drivers. Smaller number of taxis produces higher benefits for the same demand level, but increments the waiting time of customers and reduces the LoS (not shown in the figure). The diagonal lines are regions with fixed benefits, formed by different combinations of demand and supply, but offer the same benefit to the drivers (iso-benefit lines).

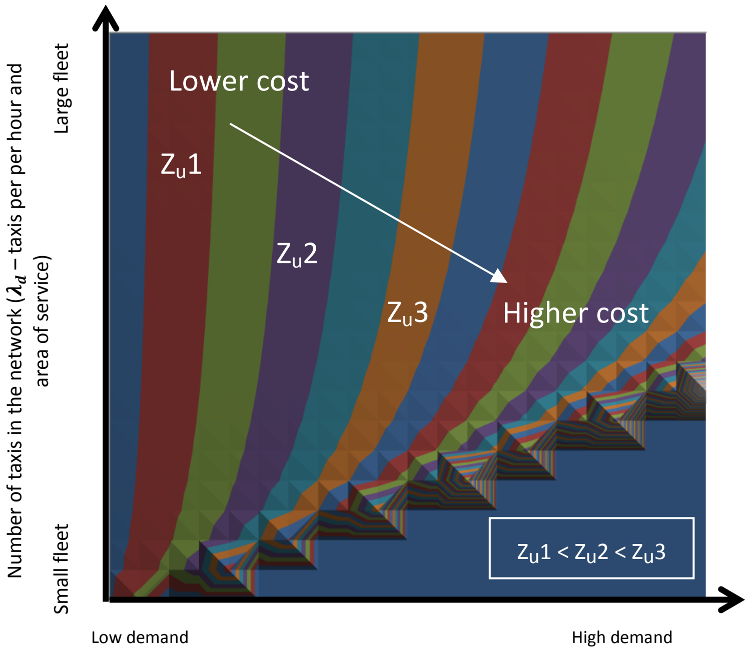

Figure 3-6 Customer cost (Z_u) of each demand and supply configurations for the dispatching market

Figure 3-6 shows the customers' total cost, presenting the same three regions observed in the system costs. It can be observed that for the same demand level, the costs are reduced as the number of taxis is increased (the waiting time is lower with larger fleets). By representing the unitary costs ($^{Z}/_{\lambda_u A}$) instead of the total costs, the results shown in Figure 3-7 are obtained, where the minimum and optimum fleets are represented using the formulations presented in Equation 3.23.

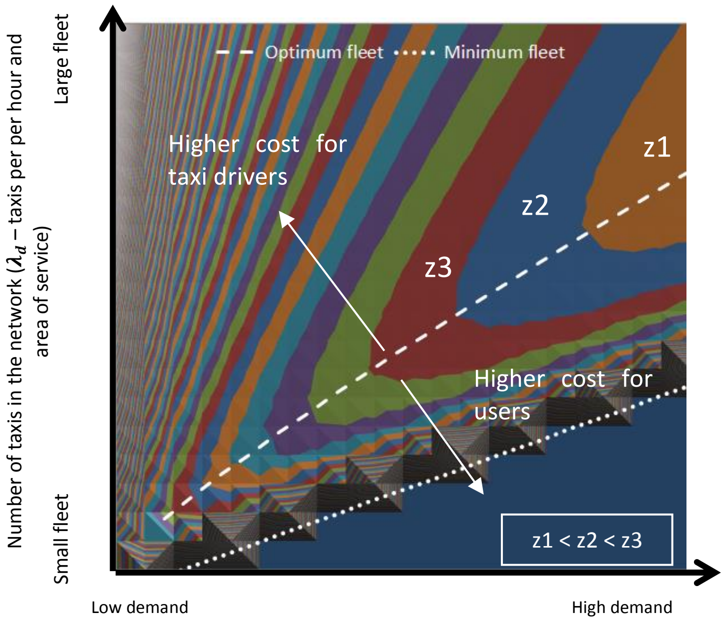

Figure 3-7 System unitary cost (z) of each demand and supply configurations for the dispatching market

Figure 3-8 shows the waiting time and the driver and customer costs in relation to the supply for a fixed demand.

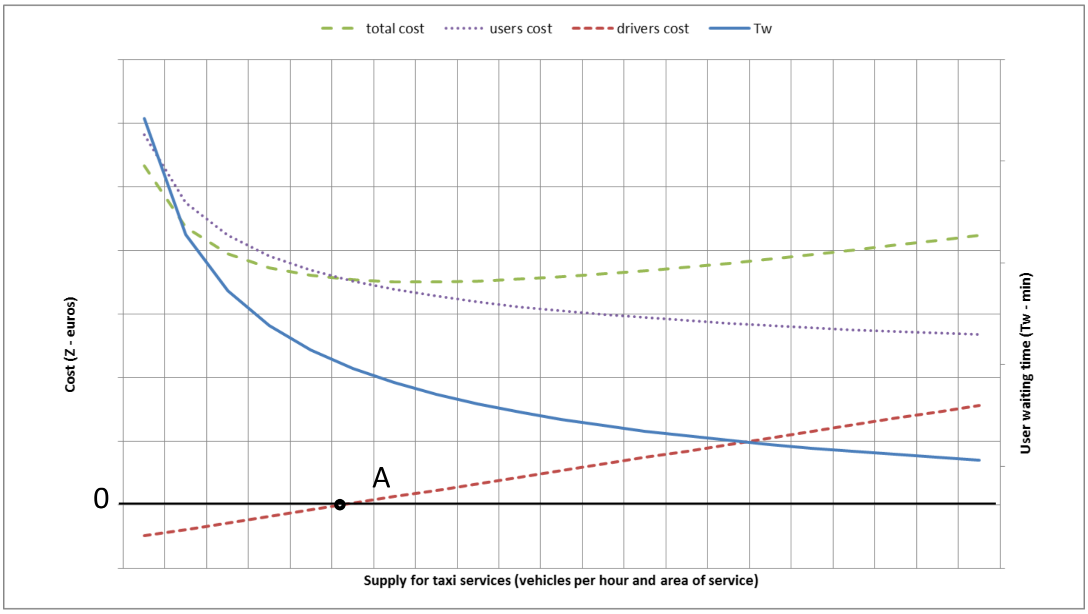

Figure 3-8 Waiting time, customer, driver and system costs of different demand levels for the dispatching market

Point A represents the maximum supply that will provide taxi services at zero profit (refered as second best solution in the literature). In this case all the costs are composed of the customers' costs since the balance between income and costs of taxi drivers is zero. Higher fleet sizes cause low customer cost due to the low waiting time, but a higher cost for drivers, due to the low income. Calculating the unitary costs, the results shown in Figure 3-9 are obtained, presenting a minimum global cost (M-first best solution) and a drivers' minimum fleet (N-second best solution).

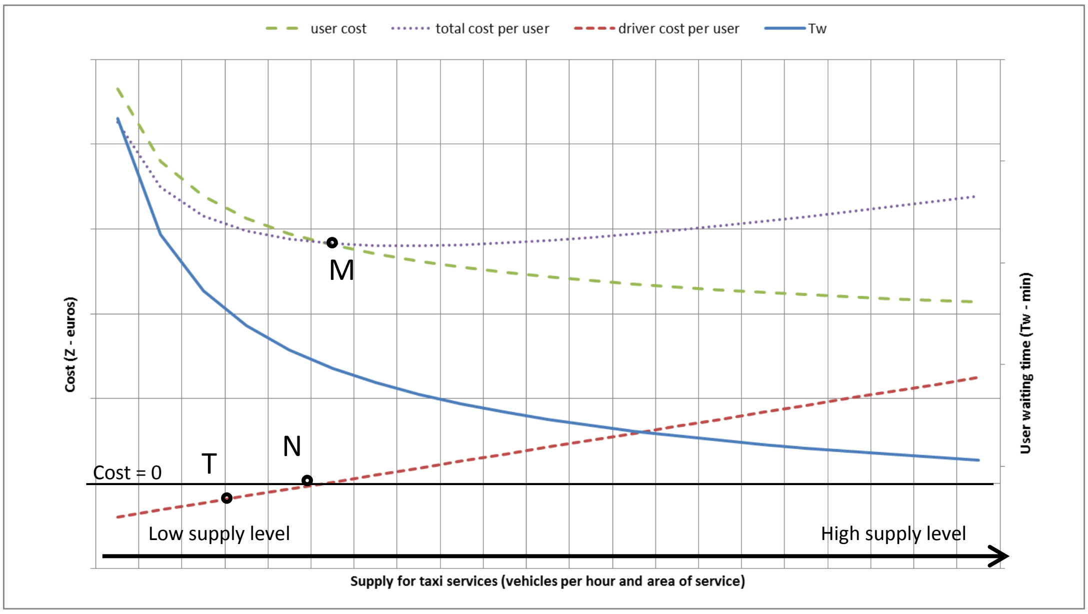

Figure 3-9 Waiting time, customer, driver and system unitary costs of different demand levels for the dispatching market

Each demand level has a fleet size M for which the total unitary cost is minimum (first best). It can be observed that for this supply size the cost of the drivers is positive, which means that the benefits are negative. The second best (N) is the point where the costs of the taxi drivers are zero, which implies an increase in the waiting time. Taking into account the benefit of the taxi license, a third point (T) could be identified, where the benefits for the license holders are equal to the expected benefit (B).

The expected benefit can be calculated as the opportunity cost of the taxi license value in comparison with a more secure investment. A detailed discussion and methodology is presented in CENIT (2013).

The formulation of the second best solution can be obtained by solving the Equation 3.32:

$$Z_d = \frac{A}{VoT_d}\left[-\lambda_u \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \lambda_u \cdot \frac{rA^{1/2}}{2} \cdot \varepsilon C_{km} + \lambda_d \cdot C_h\right] = 0 \quad (3.32)$$

$$\lambda_d = \frac{\lambda_u \left[D + \frac{rA^{1/2}}{2}(\tau_{km} - \varepsilon C_{km})\right]}{C_h} \quad (3.33)$$

The formulation of the point T is presented in Equation 3.34:

$$Z_d = \frac{A}{VoT_d}\left[-\lambda_u \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \lambda_u \cdot \frac{rA^{1/2}}{2} \cdot \varepsilon C_{km} + \lambda_d \cdot C_h\right] = B \quad (3.34)$$

$$\lambda_d = \frac{\frac{B \cdot VoT_d}{A} + \lambda_u \left[D + \frac{rA^{1/2}}{2}(\tau_{km} - \varepsilon C_{km})\right]}{C_h} \quad (3.35)$$

3.5. Application to the stand market

3.5.1. Calculation of the number of taxi stands and the related access time

In the case of the stand market, if the stand location is supposed to be uniform within the region, the access distance can be approximated by $a/2$ (using a L1 metric), where a is the length of the squares generated by an orthogonal stand network, as it is shown in Figure 3-10.

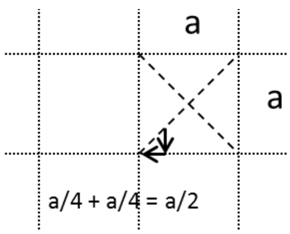

Figure 3-10 Expected access distance to taxi stands.

Therefore the access time can be expressed as presented in Equation 3.36:

$$T_a = \frac{a}{2v_u} \quad (3.36)$$

The number of stands (s) can be approximated by A/a^2, where A is the area of the region and a is the distance between stands (each stand serves an area of a^2). Assuming that the number of stands is related to the number of free taxis in the way that there is always one taxi waiting in every taxi stand, the number of vacant vehicle hours for finding the value of a can be used:

$$s = \frac{A}{a^2} = A\lambda_d - A\lambda_u \frac{rA^{1/2}}{2\bar{v}} \frac{\varepsilon}{2} \quad (3.37)$$

$$a = \frac{1}{\sqrt{\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}} \frac{\varepsilon}{2}}} \quad (3.38)$$

Therefore, the access time can be formulated as:

$$T_a = \frac{1}{2\overline{v_u}\sqrt{\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}} \frac{\varepsilon}{2}}} \quad (3.39)$$

3.5.2. Optimization of the fleet size

Taking into account the defined objective function and the approximations for the different variables presented above, the customer and driver costs can be rewritten in terms of demand (pax/hour), supply (number of taxis) and area of service for the stand market.

$$Z_u = \lambda_u \cdot A \cdot \left[\left(\alpha_A \cdot \frac{1}{2\overline{v_u}\sqrt{\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}} \frac{\varepsilon}{2}}} + \alpha_W \cdot 0 + \alpha_{IV} \cdot \frac{rA^{1/2}}{2\bar{v}}\right) + \frac{D + \frac{rA^{1/2}}{2} \cdot \tau_{km}}{VoT_u}\right] \quad (3.40)$$

$$\begin{aligned} Z_d &= \frac{\lambda_d \cdot A}{VoT_d}\left[-\frac{\lambda_u}{\lambda_d} \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \frac{\lambda_u}{\lambda_d} \cdot \frac{rA^{1/2}}{2} \frac{\varepsilon}{2} \cdot C_{km} + C_h\right] \\ &= \frac{A}{VoT_d}\left[-\lambda_u \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \lambda_u \cdot \frac{rA^{1/2}}{2} \varepsilon/2 \cdot C_{km} + \lambda_d \cdot C_h\right] \end{aligned} \quad (3.41)$$

$$G = \frac{C_s s}{VoT_d} = \frac{C_s}{VoT_d}\left[A\lambda_d - A\lambda_u \frac{rA^{1/2}}{2\bar{v}} \frac{\varepsilon}{2}\right] \quad (3.42)$$

Deriving the Equation 3.40, Equation 3.41 and Equaiton 3.42 formulations with respect to the supply:

$$\frac{\partial Z_u}{\partial \lambda_d} = -\frac{\lambda_u \cdot A \cdot \alpha_{A\cdot}}{4\overline{v_u}\sqrt{\left(\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}} \frac{\varepsilon}{2}\right)^3}} \quad (3.43)$$

$$\frac{\partial Z_d}{\partial \lambda_d} = \frac{A \cdot C_h}{VoT_d} \quad (3.44)$$

$$\frac{\partial G}{\partial \lambda_d} = \frac{A \cdot C_s}{VoT_d} \quad (3.45)$$

And the optimum supply is the one presented in Equation 3.47:

$$A \cdot (C_h + C_s) = \frac{\lambda_u \cdot A \cdot \alpha_{A\cdot} \cdot VoT_d}{4\overline{v_u}\sqrt{\left(\lambda_d - \lambda_u \frac{rA^{1/2}}{2\overline{v}}\frac{\varepsilon}{2}\right)^3}} \qquad (3.46)$$

$$\lambda_d = \lambda_u \frac{rA^{1/2}}{2\overline{v}}\frac{\varepsilon}{2} + \left(\frac{\lambda_u \alpha_A VoT_d}{4\overline{v_u}(C_h + C_s)}\right)^{2/3} \qquad (3.47)$$

where the first term is the minimum fleet size for serving all trips and the second term is the extra fleet needed for providing a better LoS to customers, while maintaining a satisfactory profit to taxi drivers. The constant value of this extra fleet is directly proportional to the VoT and inversely proportional to C_h and $\overline{v_u}$, presenting similar behavior as the one observed for the dispatching market.

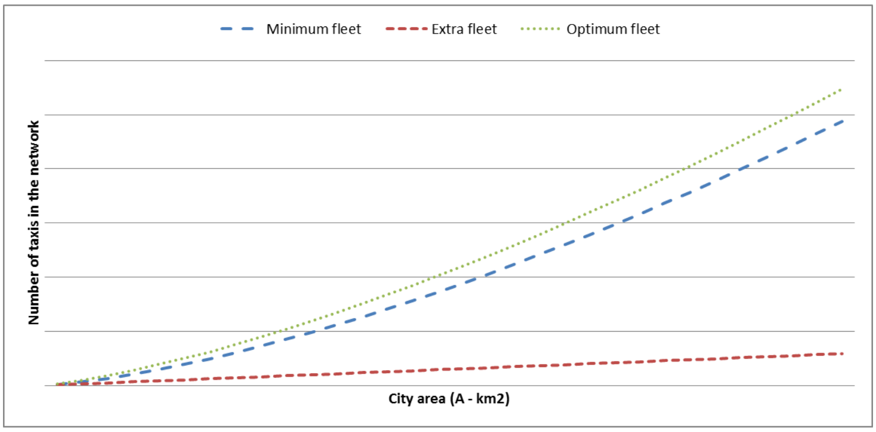

Figure 3-11 Minimum, extra and optimum fleet in relation to the city area

Figure 3-11 shows the minimum and the optimum fleet sizes obtained by the formulation presented above in relation to the area size for a generic case. It shows that the extra fleet grows linearly in relation to the city size, while the minimum fleet grows with a higher exponent. The number of extra taxis in relation to the extra fleet decreases as the area grows.

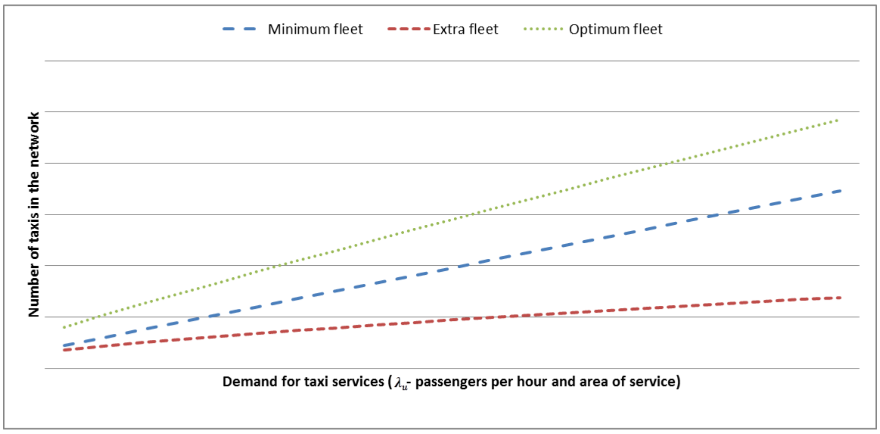

Figure 3-12 Minimum, extra and optimum fleet in relation to the demand for taxi services

Figure 3-12 shows the optimum fleet as the sum of two components, the minimum fleet and the extra fleet, in relation to the demand level. As seen, the relation between the minimum fleet and the extra fleet decreases as the demand level increases.

The associated access time to the optimum supply is presented in Equation 3.48:

$$T_A = \frac{1}{2\bar{v}_u\sqrt{\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}} = \frac{1}{2\bar{v}_u}\left(\frac{4\bar{v}_u(C_h + C_s)}{\lambda_u\alpha_A VoT_d}\right)^{1/3} = \left(\frac{(C_h + C_s)}{2\lambda_u\bar{v}_u^{2}\alpha_A VoT_d}\right)^{1/3} = \left(\frac{2\alpha_A VoT_d}{(C_h + C_s)}\lambda_u\bar{v}_u^{2}\right)^{-1/3} \quad (3.48)$$

The access time is multiplied by the same factor commented above (directly proportional to the VoT and inversely proportional to C_h).

Finally, introducing all Equation 3.47 and Equaiton 3.48 into the generalized cost function:

$$Z_u + Z_d + G = A\left[\frac{\lambda_d{}^{*}C_h}{VoT_d} + \lambda_u\frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u\alpha_A T_A{}^{*}\right] + sC_s \quad (3.49)$$

Where the first term corresponds to the fixed hourly cost of the taxi fleet, the second term represents the customer variable cost due to the trips and the final term is the access time cost of the customers. Rearranging the terms and introducing the formulations of the optimum fleet (Equation 3.47) and minimum access time (Equation 3.48) the following actors (Equation 3.50, Equayion 3.51 and Equation 3.52) and system unitary costs (Equation 3.53) are obtained:

$$Z_u = \lambda_u \cdot A \cdot \left[\alpha_A \cdot \left(\frac{2\alpha_A VoT_d}{(C_h + C_s)}\lambda_u\bar{v}_u^{2}\right)^{-1/3} + \alpha_{IV} \cdot \frac{rA^{1/2}}{2\bar{v}} + \frac{D + \frac{rA^{1/2}}{2}\cdot\tau_{km}}{VoT_u}\right] \quad (3.50)$$

$$Z_d = \frac{A}{VoT_d}\left[-\lambda_u \cdot \left(D + \frac{rA^{1/2}}{2}\cdot\tau_{km}\right) + \lambda_u \cdot \frac{rA^{1/2}}{2}\frac{\varepsilon}{2}\cdot C_{km} + \left(\lambda_u\frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2} + \left(\frac{\lambda_u\alpha_A VoT_d}{4\bar{v}_u(C_h + C_s)}\right)^{2/3}\right)\cdot C_h\right] \quad (3.51)$$

$$G = \frac{C_s s}{VoT_d} = \frac{AC_s}{VoT_d}\left(\frac{\lambda_u\alpha_A VoT_d}{4\bar{v}_u(C_h + C_s)}\right)^{2/3} \quad (3.52)$$

$$\frac{Z_u + Z_d + G}{\lambda_u \cdot A} = \frac{C_h}{VoT_d}\frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2} + \left(1{,}35\frac{VoT_u\bar{v}_u^{2}\lambda_u}{(C_h + C_s)\alpha_A{}^{2}}\right)^{-1/3} + \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}}\right) \quad (3.53)$$

By representing the unitary costs, the results shown in Figure 3-13 are obtained, where the minimum and optimum fleet are represented using the formulations presented in Equation 3.47.

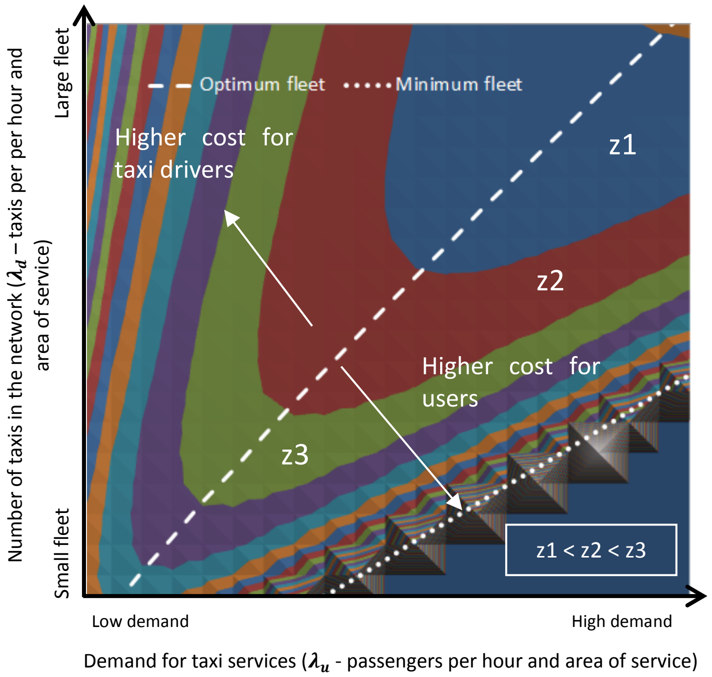

Figure 3-13 System unitary cost (z) of each demand and supply configurations for the stand market

Similar characteristics as the commented in the dispatching application can be observed in Figure 3-13 and Figure 3-14.

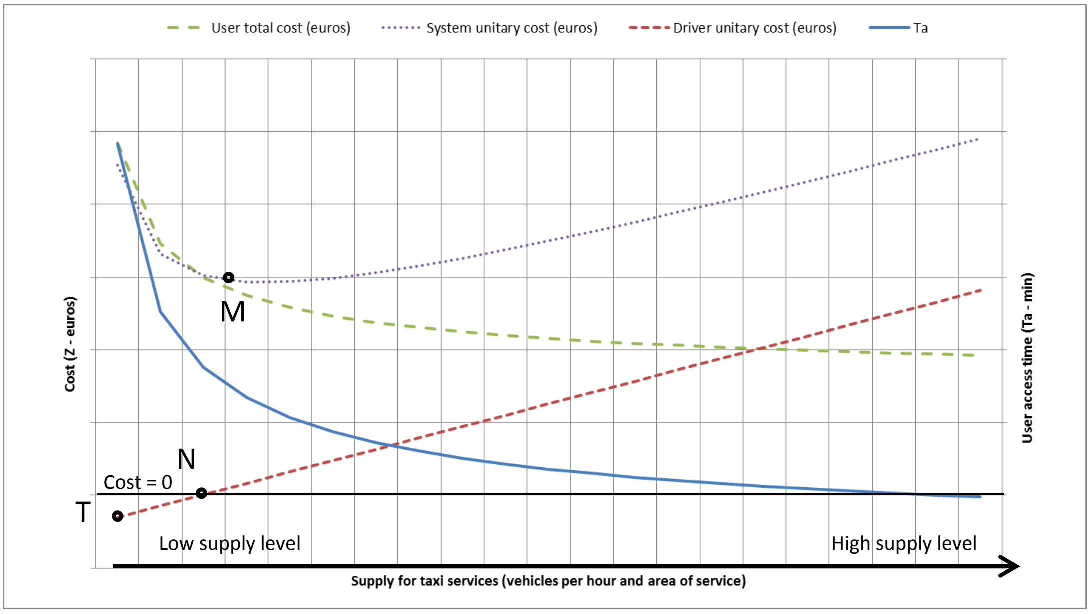

Figure 3-14 Waiting time, customer, driver and system unitary costs of different demand levels for the stand market

The formulation of the second best solution can be obtained by solving Equation 3.54:

$$Z_d = \frac{A}{VoT_d}\left[-\lambda_u \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \lambda_u \cdot \frac{rA^{1/2}}{2} \cdot \frac{\varepsilon C_{km}}{2} + \lambda_d \cdot C_h\right] = 0 \qquad (3.54)$$

$$\lambda_d = \frac{\lambda_u \left[D + \frac{rA^{1/2}}{2} (\tau_{km} - \frac{\varepsilon C_{km}}{2}) \right]}{C_h} \quad (3.55)$$

The formulation of the point T is presented in equation 3.56:

$$Z_d = \frac{A}{VoT_d} \left[-\lambda_u \cdot (D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \lambda_u \cdot \frac{rA^{1/2}}{2} \cdot \frac{\varepsilon C_{km}}{2} + \lambda_d \cdot C_h \right] = B \quad (3.56)$$

$$\lambda_d = \frac{\frac{B \cdot VoT_d}{A} + \lambda_u \left[D + \frac{rA^{1/2}}{2} (\tau_{km} - \frac{\varepsilon C_{km}}{2}) \right]}{C_h} \quad (3.57)$$

3.6. Application to the hailing market

In the case of the hailing market application, environmental issues should be taken into account since taxis are constantly circulating and their impact on the other drivers depend not only on the demand for taxi trips but also on the fleet size. In order to do this, the relation between the density and the average network speed will be used.

3.6.1. External costs

The estimation of the impact of circulating taxis on the average speed is presented below. Geroliminis and Daganzo (2008) demonstrated that a well-defined Macroscopic Fundamental Diagram (MFD) relating vehicle flow and density exists at network level besides the fundamental diagram relating these variables at a corridor level, as stated by Greenshield (1935). The MFD is independent of the demand, but the following characteristics are needed for proving its existence (Geroliminis and Daganzo, 2008):

- Homogeneous congestion and network topology
- Slow variation of the demand
- Redundant network (multiple routes exist between the same pair of zones within the network)

The speed-density relation of the MFD will be a steady decreasing function based on real data, however it can be approximated by the simplification that the relation between speed and density is linear (as proposed by Greenshield 1935):

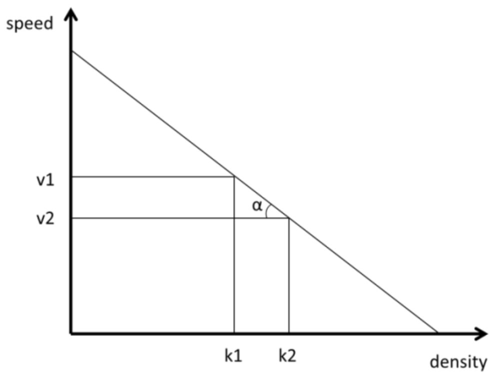

Figure 3-15 Speed-density linear relation

where,
$v1$ is the average speed of the network if there were no taxis
$k1$ is the average density of the network if there were no taxis
$v2$ is the average speed of the network with the presence of taxis
$k2$ is the average density of the network with the presence of taxis
α is the relation factor between speed and density
The relation between the speed and the density can be expressed as follows:

$$v2 = v1 - \alpha(k2 - k1) \tag{3.58}$$

Taking into account that the distance traveled by the hailing taxis during one hour is equal to the speed, and using the time traveled by the cars as expressed in Equation 3.18 divided by the speed, the densities can be expressed as presented in Equation 3.59:

$$k2 - k1 = \lambda_v - \left(\lambda_v + \lambda_d \frac{v2}{0.5 * rA^{1/2}}\right) = \lambda_d \frac{2 * v2}{rA^{1/2}} \tag{3.59}$$

Introducing the Equation 3.59 into the Equation 3.58:

$$v2 = v1 - \alpha\lambda_d \frac{2 * v2}{rA^{1/2}} \tag{3.60}$$

$$v2\left(1 + 2 * \alpha \frac{\lambda_d}{rA^{1/2}}\right) = v1 \tag{3.61}$$

$$v2 = \frac{v1}{1 + 2 * \alpha \frac{\lambda_d}{rA^{1/2}}} \tag{3.62}$$

Therefore, the drop of speed (i. e. the impact of taxis in service on the road network performance) can be expressed as presented in Equation 3.63:

$$\frac{\Delta v}{v} = \left(1 - \frac{1}{1 + \frac{2\alpha\lambda_d}{rA^{1/2}}}\right) \tag{3.63}$$

The increase in the travel time of the other drivers can be calculated in a very compact way, as follows:

$$\Delta t = t2\text{-}t1 = \frac{rA^{1/2}}{2v2} - \frac{rA^{1/2}}{2v1} = \frac{rA^{1/2}}{2}\left(\frac{1}{v2} - \frac{1}{v1}\right) = \frac{rA^{1/2}}{2}\left(\frac{1 + 2 * \alpha \frac{\lambda_d}{rA^{1/2}}}{v1} - \frac{1}{v1}\right)$$
$$= \frac{rA^{1/2}}{2} \cdot \frac{2 * \alpha \frac{\lambda_d}{rA^{1/2}}}{v1} = \alpha \frac{\lambda_d}{v1} \tag{3.64}$$

More realistic formulations for the relation between the speed and the density are presented and developed below, but they will not be used for the costs formulation due to their complexity. The compact expression obtained in 3.64 for the increase of the travel time will be used for the estimation of the optimum fleet size.

Greenberg (1959) proposed a more realistic approximation for the relation between speed and density, which is exponential. Similar developments are presented below in order to estimate the impact of the presence of taxis in the average speed of the network.

$$k = \alpha e^{-\left(v/\beta\right)} \tag{3.65}$$

The speed can be expressed as a function of the density as follows:

$$v = -\beta ln(k/\alpha) \tag{3.66}$$

And the speed variation due to the presence of taxis as follows:

$$v2 - v1 = -\beta\left(ln(k2/\alpha) - ln(k1/\alpha)\right) = \beta\ ln(k1/k2) \tag{3.67}$$

$$v2 = v1 + \beta\ ln(k1/k2) = v1 + \beta\ ln\left[\frac{\lambda_v}{\lambda_v + \lambda_d \frac{v2}{0.5 * rA^{1/2}}}\right] \tag{3.68}$$

$$v1 = v2 - \beta\ ln\left[\frac{\lambda_v}{\lambda_v + \lambda_d \frac{v2}{0.5 * rA^{1/2}}}\right] \tag{3.69}$$

Finally, the increase of the travel time can be expressed as a function of the new speed.

$$\Delta t = t2\text{-}t1 = \frac{rA^{1/2}}{2v2} - \frac{rA^{1/2}}{2v1} = \frac{rA^{1/2}}{2}\left(\frac{1}{v2} - \frac{1}{v2 - \beta\ ln\left[\frac{\lambda_v}{\lambda_v + \lambda_d \frac{v2}{0.5 * rA^{1/2}}}\right]}\right) \tag{3.70}$$

This calculation of the increase in the travel time presents two complications, on the one side it provides a more complex mathematical formulation and, on the other side, it uses the new speed instead of using the original speed, which depends on the number of taxis.

The same results are obtained when using the exponential model presented by Underwood (1961) replacing the logarithm with an exponential.

The environmental costs and fuel consumption can be approximated by Equation 3.5 and Equation 3.6 but, in order to simplify the expressions, they will be considered equal in the situation with and without taxis. The impacts will be calculated as the pollution generated and the fuel consumed during the extra travel time, instead of calculating the new emission factor and fuel consumption for the new average speed.

3.6.2. Optimization of the fleet size

The operating kilometers per hour can be estimated by the product between the number of taxis per hour and the speed ($\lambda_d A\bar{v}$).

Taking into account the defined objective function and the approximations for the different variables presented above, the customer and driver costs can be rewritten in terms of demand (pass/hour), supply (number of taxis) and area of service for the hailing market. Introducing the formulations presented by Fernandez et al. (2008) for the waiting time and the formulations presented for the dispatching model for the average trip distance and duration to the generalized costs presented in the model, costs can be defined as follows:

$$Z_u = \lambda_u \cdot A \cdot \left[\alpha_A \cdot 0 + \alpha_W \cdot \frac{A}{\bar{v}\left(A\lambda_d - A\lambda_u \frac{rA^{1/2}}{2\bar{v}}\right)} + \alpha_{IV} \cdot \frac{rA^{1/2}}{2\bar{v}} + \frac{D + \frac{rA^{1/2}}{2} \cdot \tau_{km}}{VoT_u}\right] \quad (3.71)$$

$$Z_d = \frac{\lambda_d \cdot A}{VoT_u}\left[-\frac{\lambda_u}{\lambda_d}(D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \bar{v} \cdot C_{km} + C_h\right] \quad (3.72)$$

$$Z_c = \lambda_v \cdot A\left[\left(\frac{rA^{1/2}}{2 * v2} - \frac{rA^{1/2}}{2 * v1}\right) + \frac{\Delta F_c + C_E \cdot \left(\frac{rA^{1/2}}{2 * v2} - \frac{rA^{1/2}}{2 * v1}\right) \cdot E_d}{VoT_u}\right] + \frac{\lambda_d \cdot A \cdot C_E \cdot E_d}{VoT_d}$$
$$= \lambda_v \cdot A\left[\frac{\alpha\lambda_d}{v1}\left(1 + \frac{C_E \cdot E_d}{VoT_u} + \frac{F_c}{VoT_u}\right)\right] + \frac{\lambda_d \cdot A \cdot C_E \cdot E_d}{VoT_d} \quad (3.73)$$

Finding the derivative of Equation 3.71, Equation 3.72 and Equaiton 3.73 with respect to the supply:

$$\frac{\partial Z_u}{\partial \lambda_d} = -\frac{\lambda_u \cdot A\alpha_W \cdot \bar{v}}{\left(\bar{v}\lambda_d - \lambda_u \frac{rA^{1/2}}{2}\right)^2} \quad (3.74)$$

$$\frac{\partial Z_d}{\partial \lambda_d} = \frac{A}{VoT_d}[\bar{v} \cdot C_{km} + C_h] \quad (3.75)$$

$$\frac{\partial Z_c}{\partial \lambda_d} = A\left(\frac{\alpha\lambda_d}{v1}\left(1 + \frac{C_E \cdot E_d}{VoT_u} + \frac{F_c}{VoT_u}\right) + \frac{C_E \cdot E_d}{VoT_d}\right) \quad (3.76)$$

And the optimum supply is the one presented in Equation 3.77:

$$\frac{A}{VoT_d}\left[\bar{v} \cdot C_{km} + C_h + \frac{\alpha\lambda_d}{v1}(VoT_d + C_E \cdot E_d + F_c) + C_E \cdot E_d\right] = \frac{\lambda_u \cdot A\alpha_W \cdot \bar{v}}{\left(\bar{v}\lambda_d - \lambda_u \frac{rA^{1/2}}{2}\right)^2} \quad (3.77)$$

$$\lambda_d = \lambda_u \frac{rA^{1/2}}{2\bar{v}} + \left(\frac{\lambda_u \alpha_W VoT_d}{\bar{v}\left(\bar{v} \cdot C_{km} + C_h + \frac{\alpha\lambda_d}{v1}(VoT_d + C_E \cdot E_d + F_c) + C_E \cdot E_d\right)}\right)^{1/2} \quad (3.78)$$

Where the first term is the minimum fleet size for serving all trips and the second term is the extra fleet needed for providing a better LoS to customers, while maintaining a satisfactory profit to taxi drivers and respecting the congestion level created by the taxis. The constant value of this extra fleet is directly proportional to the VoT and inversely proportional to C_h, C_{km}, $\bar{v}$ and α, presenting similar behavior as the one observed for the dispatching market.

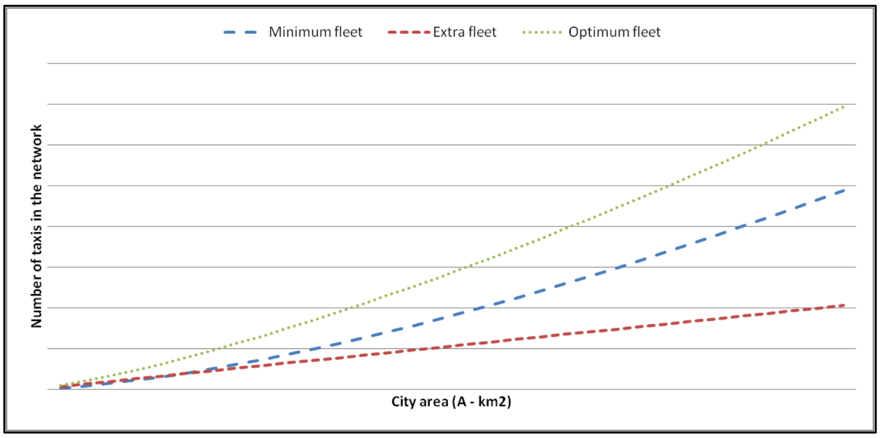

Figure 3-16 Minimum, extra and optimum fleet in relation to the city area

Figure 3-16 shows the minimum and the optimum fleet sizes obtained by the formulation presented above in relation to the area size for a generic case. It has a similar behavior to the other operation modes.

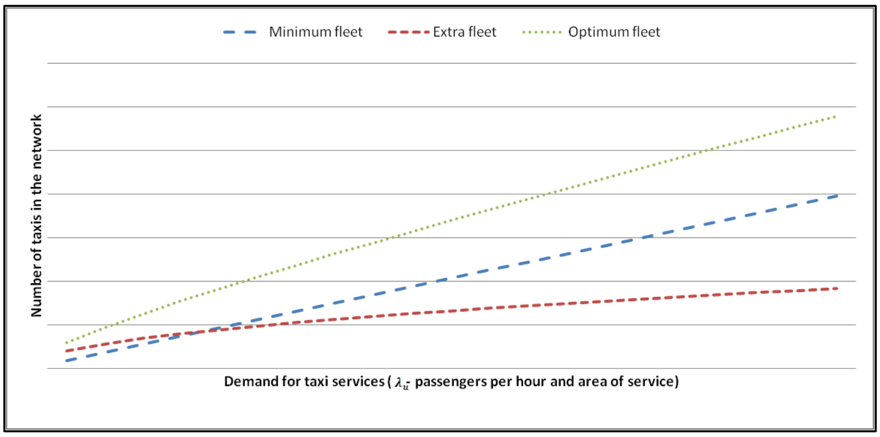

Figure 3-17 Minimum, extra and optimum fleet in relation to the demand for taxi services

Figure 3-17 shows the optimum fleet and its composition (minimum fleet and extra fleet) in relation to the demand level. The relation between the minimum fleet and the extra fleet decreases as the demand level increases.

The associated waiting time to the optimum supply is the one presented in Equation 3.79:

$$T_W = \left(\bar{v}\left(\lambda_u \frac{rA^{1/2}}{2\bar{v}} + \left(\frac{\lambda_u \alpha_W VoT_d}{\bar{v}\left(\bar{v}\cdot C_{km} + C_h + \frac{\alpha\lambda_d}{v1}(VoT_d + C_E\cdot E_d + F_c) + C_E\cdot E_d\right)}\right)^{1/2} - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\right)\right)^{-1}$$
$$= \left(\frac{\bar{v}\cdot C_{km} + C_h + \frac{\alpha\lambda_d}{v1}(VoT_d + C_E\cdot E_d + F_c) + C_E\cdot E_d}{\lambda_u \bar{v}\alpha_W VoT_d}\right)^{1/2}$$
$$= \left(\frac{\alpha_W VoT_d}{\bar{v}\cdot C_{km} + C_h + \frac{\alpha\lambda_d}{v1}(VoT_d + C_E\cdot E_d + F_c) + C_E\cdot E_d}\lambda_u \bar{v}\right)^{-1/2} \quad (3.79)$$

The waiting time is multiplied by the same factor obtained in Equation 3.78 (directly proportional to the VoT and inversely proportional to $C_h, C_{km}, \bar{v}$ and α). Finally, by introducing the Equation 3.78 and Equaiton 3.79 into the generalized cost function, Equation 3.80 is obtained:

$$Z_u + Z_d + Z_v = A\left[\frac{\lambda_d{}^*(C_h + \bar{v}C_{km})}{VoT_d} + \lambda_u \alpha_{IV}\frac{rA^{1/2}}{2} + \lambda_u \alpha_W T_W{}^* + \lambda_v \cdot A\left[\frac{\alpha\lambda_d}{v1}\left(1 + \frac{C_E\cdot E_d}{VoT_u} + \frac{F_c}{VoT_u}\right)\right] + \frac{\lambda_d\cdot A\cdot C_E\cdot E_d}{VoT_d}\right] \quad (3.80)$$

where the first term corresponds to the fixed hourly cost of the taxi fleet, the second term represents the customer variable cost due to the trips, the third term is the waiting time cost of the customers and the final term is the congestion cost of the other drivers. Rearranging the terms and introducing the formulations of the optimum fleet (Equation 3.17) and minimum waiting time (Equation 3.18) the following actors (Equation 3.81 and Equation 3.82) and system unitary costs (Equation 3.83) are obtained:

$$Z_u = \lambda_u\cdot A\cdot\left[\alpha_W\cdot\left(\frac{\alpha_W VoT_d}{\bar{v}\cdot C_{km} + C_h + \frac{\alpha\lambda_v}{v1}}\lambda_u\bar{v}\right)^{-1/2} + \alpha_{IV}\cdot\frac{rA^{1/2}}{2\bar{v}} + \frac{D + \frac{rA^{1/2}}{2}\cdot\tau_{km}}{VoT_u}\right] \quad (3.81)$$

$$Z_d = \left(\lambda_u\frac{rA^{1/2}}{2\bar{v}} + \left(\frac{\lambda_u\alpha_W VoT_d}{\bar{v}\left(\bar{v}\cdot C_{km} + C_h + \frac{\alpha\lambda_v}{v1}\right)}\right)^{1/2}\right)\frac{A}{VoT_d}\left[-\lambda_u\cdot\left(D + \frac{rA^{1/2}}{2}\cdot\tau_{km}\right) + \bar{v}\cdot C_{km} + C_h\right] \quad (3.82)$$

$$\mathrm{Z_v} = \lambda_v\cdot A\left[\frac{\alpha\lambda_d}{v1}\left(1 + \frac{C_E\cdot E_d}{VoT_u} + \frac{F_c}{VoT_u}\right)\right] + \frac{\lambda_d\cdot A\cdot C_E\cdot E_d}{VoT_d} \quad (3.83)$$

$$\frac{Z_u + Z_d + Z_v}{\lambda_u \cdot A} = \left(\frac{rA^{1/2}}{2\bar{v}} + \left(\frac{\alpha_W VoT_d}{\bar{v}\lambda_u \left(\bar{v} \cdot C_{km} + C_h + \frac{\alpha\lambda_v}{v1} \right)} \right)^{1/2} \right) \cdot \frac{\bar{v} \cdot C_{km} + C_h}{VoT_d} + \alpha_W \cdot \left(\frac{\alpha_W VoT_d}{\bar{v} \cdot C_{km} + C_h} \lambda_u \bar{v} \right)^{-1/2} + \alpha_{IV} \cdot \frac{rA^{1/2}}{2\bar{v}} + \lambda_v \cdot \frac{1}{\lambda_u} \left[\frac{\alpha\lambda_d}{v1} \left(1 + \frac{C_E \cdot E_d}{VoT_u} + \frac{F_c}{VoT_u} \right) \right] + \frac{\lambda_d \cdot C_E \cdot E_d}{\lambda_u VoT_d} = 2 \left(\frac{\alpha_W \left(\bar{v} \cdot C_{km} + C_h + \frac{\alpha\lambda_v}{v1} \right)}{VoT_d \bar{v} \lambda_u} \right)^{1/2} + \frac{rA^{1/2}}{2\bar{v}} \cdot \left(\frac{\bar{v} \cdot C_{km} + C_h + \frac{\alpha\lambda_v}{v1}}{VoT_d} + \alpha_{IV} \right) + \lambda_v \cdot \frac{1}{\lambda_u} \left[\frac{\alpha\lambda_d}{v1} \left(1 + \frac{C_E \cdot E_d}{VoT_u} + \frac{F_c}{VoT_u} \right) \right] + \frac{\lambda_d \cdot C_E \cdot E_d}{\lambda_u VoT_d} \quad (3.84)$$

By representing the unitary costs, the results shown in Figure 3-18 are obtained, where the minimum and optimum fleet are represented using the formulations presented in Equation 3.78.

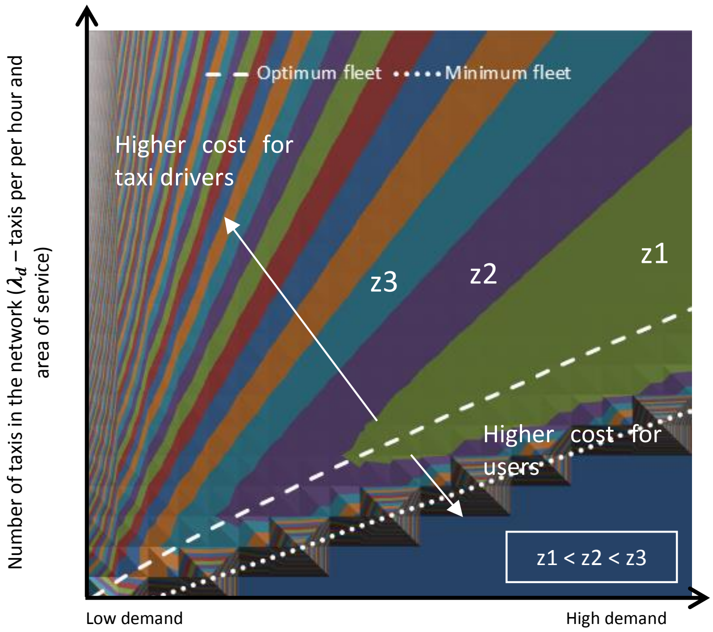

Figure 3-18 System unitary cost (z) of each demand and supply configurations for the hailing market

Similar characteristics as the commented in the dispatching application can be observed in Figure 3-18 and Figure 3-19.

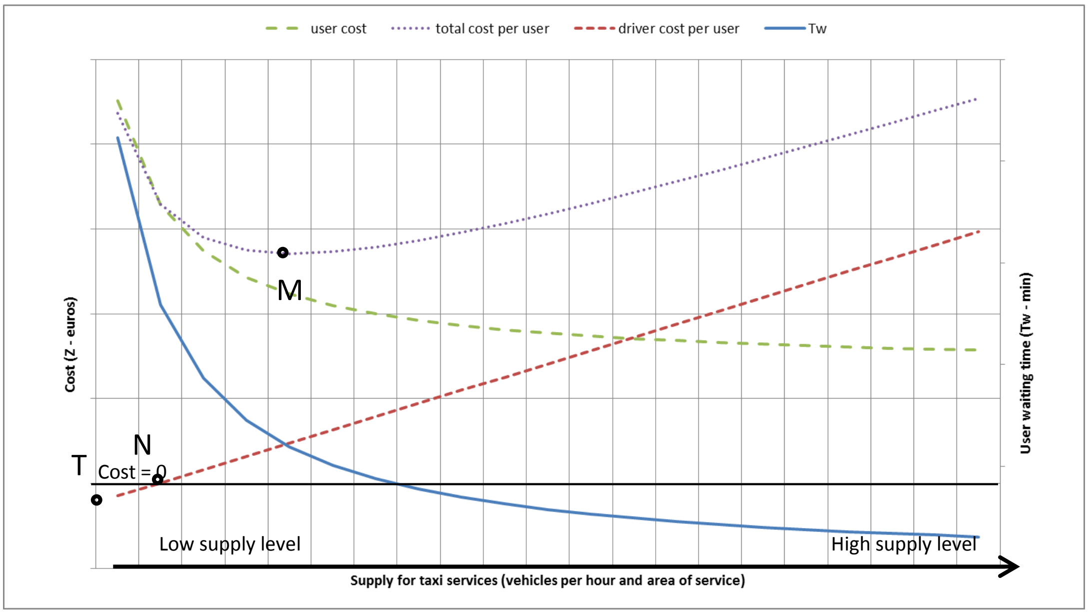

Figure 3-19 Waiting time, customer, driver and system unitary costs of different fleet sizes for the hailing market

The formulation of the second best solution can be obtained by solving Equation 3.85:

$$Z_d = \frac{\lambda_d \cdot A}{VoT_u}\left[-\frac{\lambda_u}{\lambda_d}(D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \bar{v} \cdot C_{km} + C_h\right] = 0 \tag{3.85}$$

$$\lambda_d = \frac{\lambda_u(D + \frac{rA^{1/2}}{2} \cdot \tau_{km})}{\bar{v} \cdot C_{km} + C_h} \tag{3.86}$$

The formulation of the point T (Figure 3-19) is presented in Equation 3.87:

$$Z_d = \frac{A}{VoT_d}\left[-\frac{\lambda_u}{\lambda_d}(D + \frac{rA^{1/2}}{2} \cdot \tau_{km}) + \bar{v} \cdot C_{km} + C_h\right] = B \tag{3.87}$$

$$\lambda_d = \frac{\lambda_u(D + \frac{rA^{1/2}}{2} \cdot \tau_{km})}{\bar{v} \cdot C_{km} + C_h - \frac{B \cdot VoT_d}{A}} \tag{3.88}$$

3.7. Comparison between the hailing, the dispatching and the stand taxi markets

In order to identify the optimum operation mode for each range of demand and to identify the most significant variables and parameters that define these ranges, a comparison of the three developed models is presented below. Reference values for the demand, the supply and the area (which defines the average trip distance) are used for generating the comparative analyses.

- Demand for taxi trips: 25 - 50 customers per hour and km^2
- Area of services: 100 - 400 km^2
- Supply: 15 - 35 taxis per hour and unit of area of service

The analyses done below are based on the variation of one of the three variables within the whole range while the other two values remain constant and equal to the average value of the above ranges.

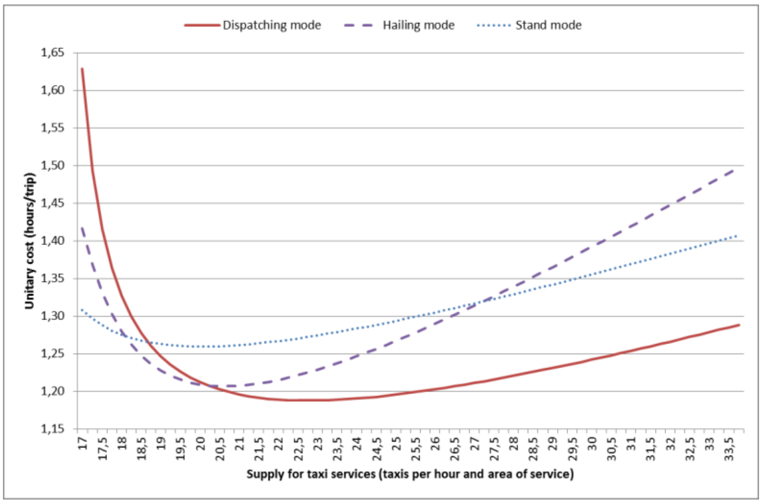

Figure 3-20 Unitary costs for various fleet sizes and operation mode

For fixed demand ($\lambda_u = ct$) and area ($A = ct$) levels the following can be stated from Figure 3-20:

- The stand mode has the lowest unitary cost of operation if the taxi fleet is small.
- The hailing mode has the lowest unitary cost if the taxi fleet size is medium.
- The dispatching mode has the lowest unitary cost if the taxi fleet is large. It should be taken into account that there are technological barriers since a dispatching center and the respective software should be implemented and maintained.

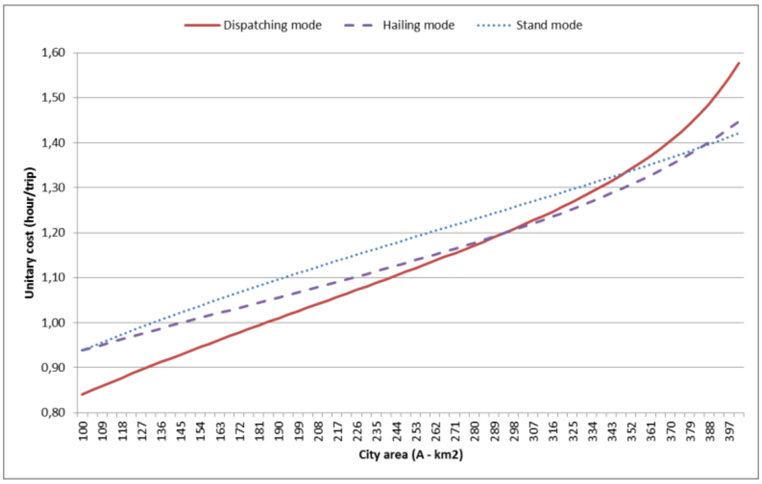

Figure 3-21 Unitary costs for various area levels and operation mode

For fixed demand ($\lambda_u = ct$) and supply ($\lambda_d = ct$) levels the following can be stated from Figure 3-21:

- The dispatching mode has the lowest unitary cost of operation in small to medium regions.
- The hailing mode has the lowest unitary cost in medium to large regions.
- The stand mode has the lowest unitary cost in very large regions.

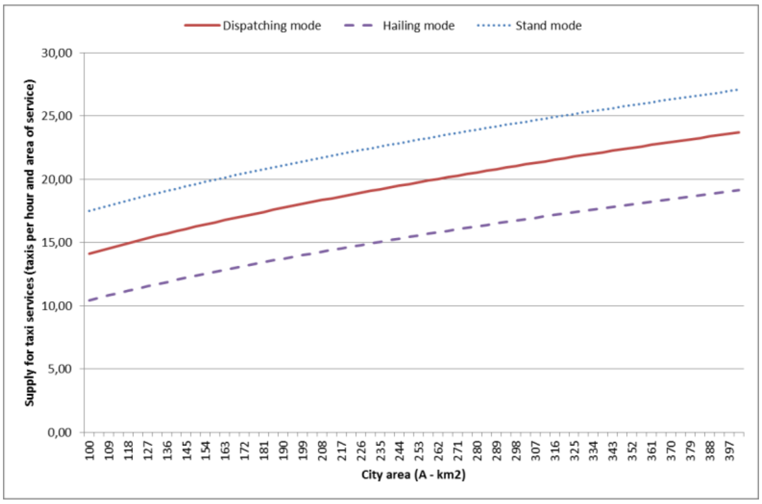

Figure 3-22 Optimum fleet size for various area levels and operation mode

In the case of fixed demand level and independently of the area, the hailing optimum fleet is the smallest one, while the stand optimum fleet is the largest.

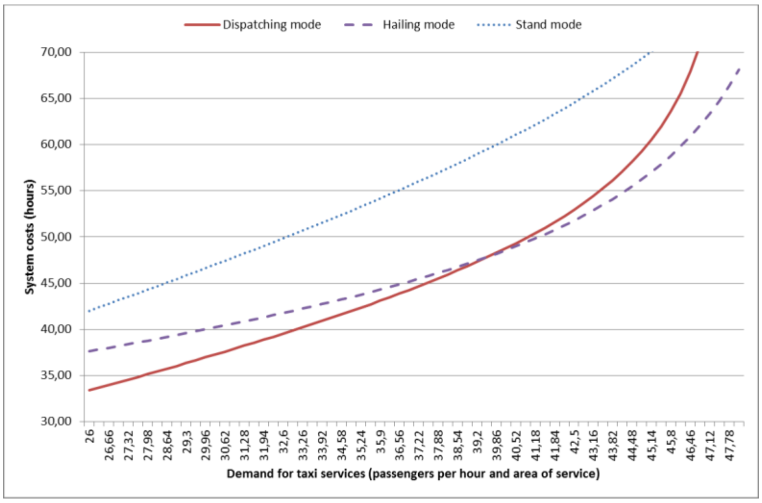

Figure 3-23 Unitary costs for various demand levels and operation mode

For fixed supply ($\lambda_d = ct$) and area ($A = \text{ct}$) levels the following can be stated from Figure 3-23:

- The stand mode has the lowest unitary cost of operation for low demand levels.
- The dispatching mode has the lowest unitary cost for low to medium demand levels.
- The hailing mode has the lowest unitary cost for high demand levels.

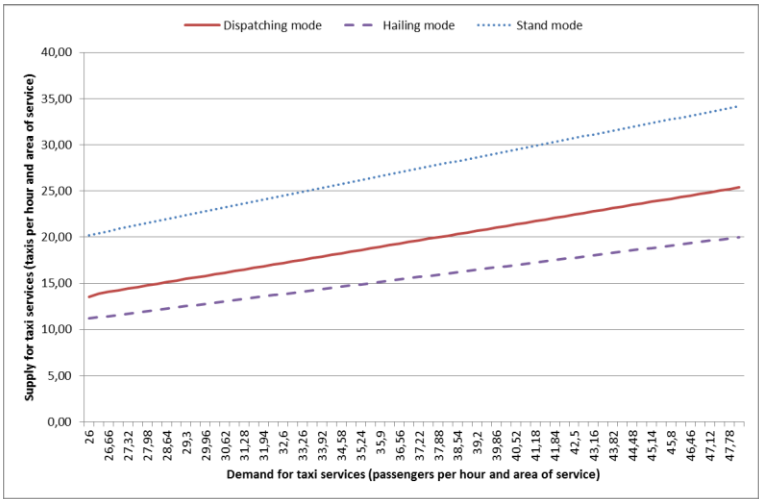

Figure 3-24 Optimum fleet size for various demand levels and operation mode

In the case of fixed area, the dispatching optimal fleet is the smallest one for low demand regions, while the hailing optimal fleet is the smallest one for high demand regions. The stand optimal fleet is the largest independently of the demand level.

It is important to highlight that three parameters have great influence on the above comparisons: the operational cost, the average speed and the VoT. The relation between these three parameters defines the demand ranges where each operation mode has the minimum unitary cost. The analytical formulations showing the above statement are presented in Equation 3.90, Equation 3.94 and Equation 3.96.

3.7.1. Mathematical formulation of the demand levels with equal system costs

The mathematical expressions of the points where the unitary costs are equal for each pair of modes of operation are presented in Figure 3-25:

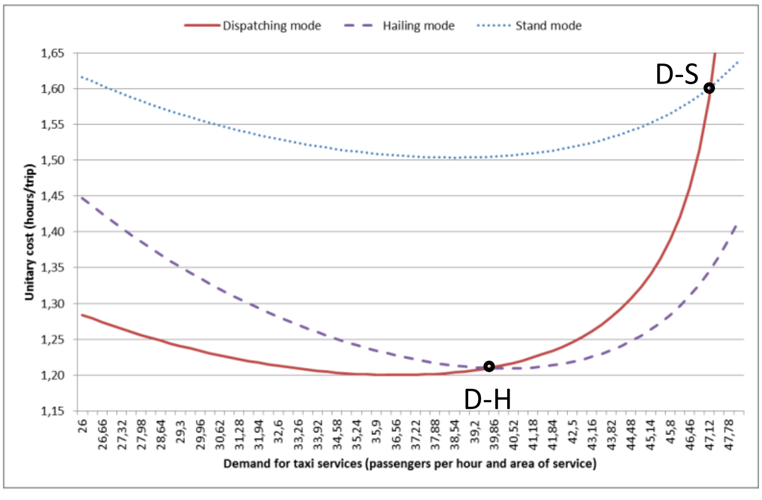

Figure 3-25 Unitary costs for various fleet sizes and operation mode

Figure 3-25 defines the ranges where each operation mode provides the minimum unitary cost. The analytical formulations of the intersection points of Figure 3-25 are provided below.

3.7.1.1. Dispatching versus stand

When comparing the unitary cost of the dispatching and the stand markets, it is seen that their basic difference is the cost of taxis traveling to the origin of the customers in comparison of the costs of customers walking to the taxi stands. This can be mathematically expressed as follows:

$$\frac{rA^{1/2}}{2}\frac{\varepsilon}{2}\frac{\left(C_{km}+{C_h}/{v}\right)}{VoT_d}=\left(1{,}35\frac{VoT_u\overline{v_u}^2\lambda_u}{(C_h+C_s)\alpha_A{}^2}\right)^{-1/3}-\left(0{,}926\frac{VoT\bar{v}^2\lambda_u}{(\alpha_W r)^2C_h}\right)^{-1/3} \qquad (3.89)$$

The extra cost for taxi drivers for picking up the customers at their origin is expressed on the left side of the Equation, while the difference between the waiting time and the access time of customers is expressed on the right side. The demand level for which the two unitary costs are equal can be expressed as follows:

$$\lambda_{u}{}'_{DS}=\left(4\frac{\left(1{,}35\frac{\overline{v_u}^2}{(C_h+C_s)VoT^2\alpha_A{}^2}\right)^{-1/3}-\left(0{,}926\frac{\bar{v}^2}{(\alpha_W r)^2VoT^2C_h}\right)^{-1/3}}{rA^{1/2}\left(C_{km}+{C_h}/{v}\right)\varepsilon}\right)^3 \qquad (3.90)$$

Higher demand levels than this value will produce lower unitary costs when served by the stand market. The difference between the trip speed ($\bar{v}$) and the pedestrian speed ($\overline{v_u}$) weighted by the operational costs has a clear effect on this value. In congested hours it would be cheaper for customers (and for the system) to walk to taxi stands rather than wait for the taxi to pick them up.

3.7.1.2. Dispatching and stand versus hailing

The formulations obtained for both comparisons are very similar due to the fact that the optimum costs for both operation modes have very similar expressions. The point of equal unitary cost for the dispatching and the hailing modes can be expressed as follows:

$$\frac{rA^{1/2}}{2vVoT}\left[(\varepsilon-1)(\bar{v}\cdot C_{km}+C_h)-\frac{\alpha\lambda_d}{v1}\right]-\lambda_v\cdot\frac{1}{\lambda_u}\left[\frac{\alpha\lambda_d}{v1}\left(1+\frac{C_E\cdot E_d}{VoT_u}+\frac{F_c}{VoT_u}\right)\right]+\frac{\lambda_d\cdot C_E\cdot E_d}{\lambda_u VoT_d}=2\left(\frac{\alpha_W\left(\bar{v}\cdot C_{km}+C_h+\frac{\alpha\lambda_v}{v1}\right)}{VoT_d\bar{v}\lambda_u}\right)^{1/2}-\left(0{,}926\frac{VoT\bar{v}^2\lambda_u}{(\alpha_W r)^2C_h}\right)^{-1/3} \qquad (3.91)$$

The difference between the extra distance traveled by the drivers when traveling to the customers origin in the dispatching mode and the extra distance of the taxis circulating constantly in the hailing marked are compared to the external costs of the hailing market in the left side of Equation 3.91, while the difference between the waiting times is expressed on the right side of Equation 3.91. The formulation for the stand mode is presented in Equation 3.92:

$$\frac{rA^{1/2}}{2vVoT}\left[\left(\frac{\varepsilon}{2}-1\right)(\bar{v}\cdot C_{km}+C_h)-\frac{\alpha\lambda_d}{v1}\right]-\lambda_v\cdot\frac{1}{\lambda_u}\left[\frac{\alpha\lambda_d}{v1}\left(1+\frac{C_E\cdot E_d}{VoT_u}+\frac{F_c}{VoT_u}\right)\right]+\frac{\lambda_d\cdot C_E\cdot E_d}{\lambda_u VoT_d}=2\left(\frac{\alpha_W\left(\bar{v}\cdot C_{km}+C_h+\frac{\alpha\lambda_v}{v1}\right)}{VoT_d\bar{v}\lambda_u}\right)^{1/2}-\left(1{,}35\frac{VoT_u\overline{v_u}^2\lambda_u}{(C_h+C_s)\alpha_A{}^2}\right)^{-1/3} \tag{3.92}$$

In order to simplify Equation 3.91 and Equation 3.92, two hypotheses are proposed: The elimination of the externalities of the hailing market and the elimination of the extra distance traveled from/to the stands to the customers' origin and destination ($\varepsilon = 1$). For the dispatching market Equation 3.94 is obtained:

$$2\left(\frac{\alpha_W(\bar{v}\cdot C_{km}+C_h)}{VoT_d\bar{v}\lambda_u}\right)^{1/2}=\left(0{,}926\frac{VoT\bar{v}^2\lambda_u}{(\alpha_W r)^2C_h}\right)^{-1/3} \tag{3.93}$$

$$\lambda'_{u_{DH}}=6.86\frac{\bar{v}(\bar{v}\cdot C_{km}+C_h)^3}{VoT_d(\alpha_W r)^4C_h{}^2} \tag{3.94}$$

Very similar formulations are obtained for the stand market:

$$2\left(\frac{\alpha_W(\bar{v}\cdot C_{km}+C_h)}{VoT_d\bar{v}\lambda_u}\right)^{1/2}=\left(1{,}35\frac{VoT_u\overline{v_u}^2\lambda_u}{(C_h+C_s)\alpha_A{}^2}\right)^{-1/3} \tag{3.95}$$

$$\lambda'_{u_{SH}}=14.6\frac{\frac{\overline{v_u}^4}{\bar{v}^3}\frac{\alpha_W{}^3}{\alpha_A{}^4}\frac{(\bar{v}\cdot C_{km}+C_h)^3}{(C_h+C_s)^2}}{VoT_d} \tag{3.96}$$

3.8. Impact of pricing in the taxi model

The impact of the pricing in the models presented in the chapters above is internal, since the monetary cost of the trip applies to the customers and the drivers with opposite sign, having no impact in the system cost. This internal cost has an impact on the viable number of taxis from the driver point of view. The formulations related ot the second best solution above include the trip cost, which means that the second best solution can also be achieved by increasing the cost for taxi trips. This sub-chapter analyses this possibility by introducing elastic demand in the formulations, since the increase of tariffs will create a reduction on the demand for taxi trips.

An exponential demand function of the customers' utility similar to the one proposed by Wong et al. (2001) is used in order to analyze the impact of the pricinig policy (Equation 3.97). The utility function of the users is composed by the waiting time and the trip monetary cost calculated as a flag-drop and the trip distance multiplied by a kilometric cost in temporal units.

$$\lambda_u = \widetilde{\lambda_u} \cdot e^{-\gamma\left(\alpha_W \cdot T_W + \frac{D + \frac{rA^{1/2}}{2} \cdot \tau_{km}}{VoT_u}\right)} \qquad (3.97)$$

where,
$\widetilde{\lambda_u}$ is the maximum demand for taxi trips
γ is the scaling parameter

The fact that the demand depends on the waiting time and, at the same time the waiting time depends on the demand creates the necessity for an iterative bi-level optimization problem, where the optimum fleet is calculated at the upper level by minimizing the unitary system costs while the waiting time related to this supply and the demand related to this waiting time is calculated at the lower level.

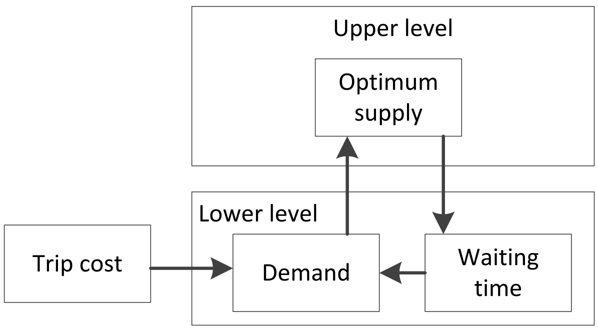

Figure 3-26 Bi-level optimization problem with elastic demand

Applying the formulations of the waiting time and the optimum supply presented in the dispatching market, and calibrating the model with elastic demand and the waiting time related to this elastic demand, Figure 3-27 is obtained, depicting the convergence of the total demand and waiting time as the number of iterations increase.

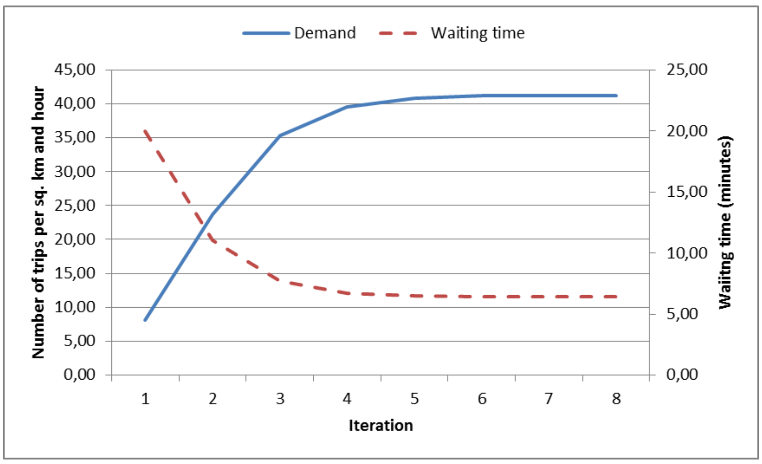

Figure 3-27 Demand and waiting time obtained from the bi-lvele optimization problem

The results of the optimization problem are independent of the initial waiting time value, as shown with the convergence of the values in Figure 3-28.

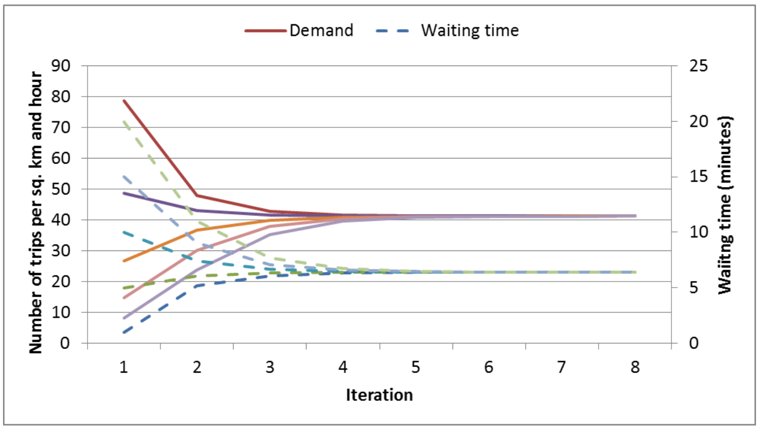

Figure 3-28 Convergence of the demand and the waiting time

The above methodology is applied for various fleet sizes and fees, obtaining the demand for taxi trips for each pair of values (number of taxis and applied fare).

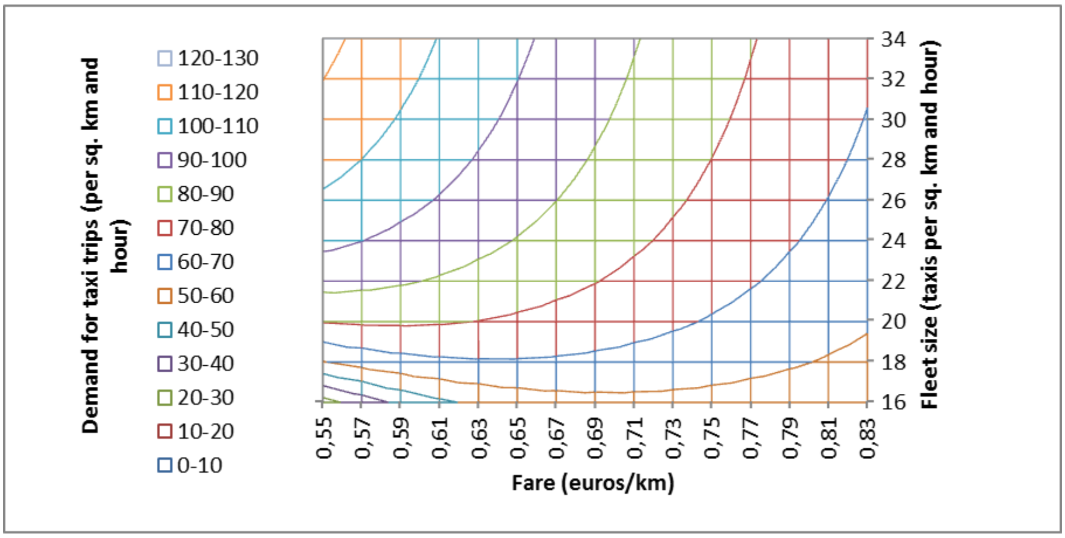

Figure 3-29 Effect of fares and fleet size on demand for taxi trips

Two regions can be observed in Figure 3-29. One region is identified where the number of taxis is very low and the demand does not depend significantly on the fare (between 16 and 22 taxis per km^2 and hour). The second region represents a large taxi density where the demand highly depends on the fare (between 22 and 34 taxis per km^2 and hour). The customers are more sensible to the waiting time for low taxi densities, where the fare has small impact on the demand. Oppositely, for large fleets, the waiting time is not the constraining factor that customers take into account, as opposed to the monetary cost of the trip itself.

The same analysis can be conducted for the drivers' profit, the consumer surplus and the social welfare.

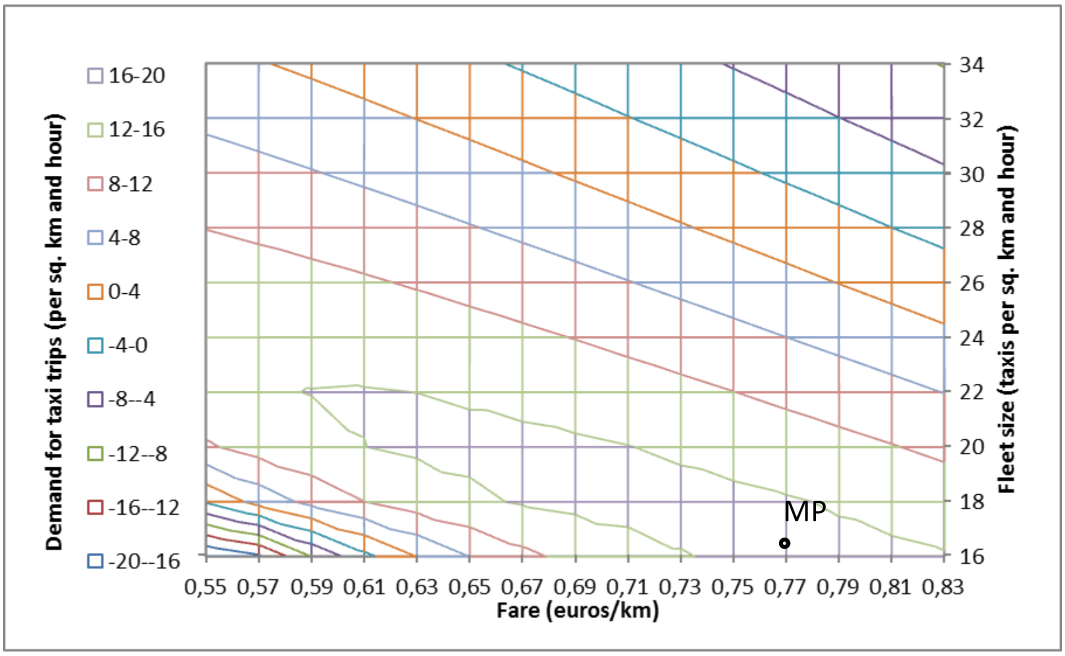

Figure 3-30 Effect of fares and fleet size on drivers' profit

The maximum profit (MP) is obtained with a low number of taxis and high fees, which can be related to a provision of high-quality services to a demand sector with high economic possibilities. Oppositely, the maximum consumer surplus (MCS) is obtained with large fleets and low fares as depicted in Figure 3-30.

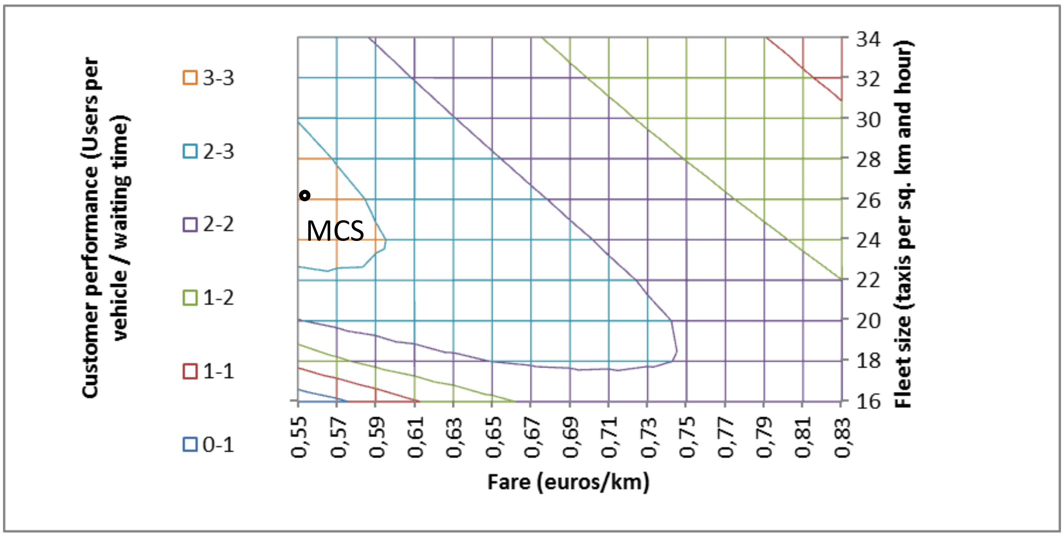

Figure 3-31 Effect of fares and fleet size on consumer surplus

Combining the consumers'surplus and the producers's surplus, the social welfare is obtained where the optimum region (MSW) is defined by both the maximum consumers' and producers' surplus.

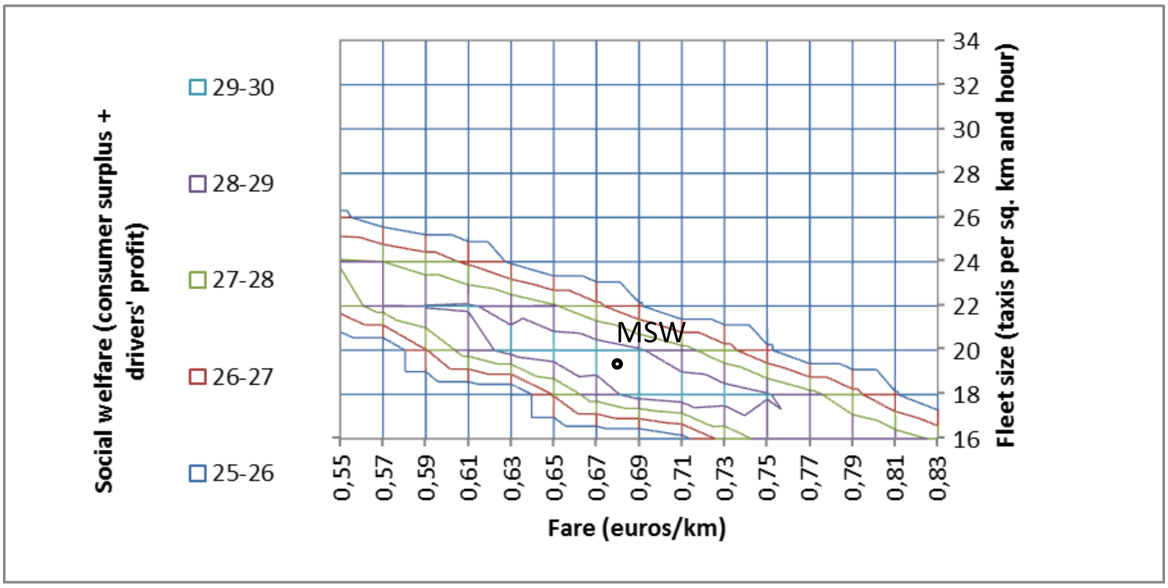

Figure 3-32 Effect of fares and fleet size on social welfare

The results presented in Figure 3-29, Figure 3-30, Figure 3-31 and Figure 3-32 are in the same line with the results presented in Figure 2-15 and Figure 2-16 by Yang et al. (2002).

3.9. Adaptation of the optimal fleet size to the shifts policy

The optimum taxi fleet size of the above models is quantified in vehicle-hours per hour, and should be obtained by real taxi fleets by applying policies and regulations to the taxi sector. Most of the taxi markets around the world have regulated the working days of the taxis in order to regulate the externalities produced, while protecting the drivers themselves. These shifts can have various durations, from 2 hours to 8 hours.

An optimization algorithm has been developed for adapting the shifts policy to the results obtained by the model. The algorithm calculates the shifts policy to be applied in order to provide the optimum fleet size at each time interval. The algorithm solves an optimization problem, which tries to minimize the difference between the optimum and the provided taxi-hours during the day.

$$min \sum_{i=1}^{24} (O_i^* - AS_i)^2 \qquad (3.98)$$

Subject to:

$$AS_i > \overline{O_i} \qquad (3.99)$$

$$\sum_{i=1}^{24} S_i = M * \left({}^{L}/_{l} \right) \qquad (3.100)$$

$$S_i > 0 \ \forall i = 1:24 \qquad (3.101)$$

where,
O_i^* is the optimum number of taxi-hours during hour i
AS_i is the number of taxi-hours during hour i
$\overline{O_i}$ is the minimum number of taxi-hours during hour i
S_i is the number of taxis starting their shift at hour i
M is the maximum daily number of taxis working
L is the maximum daily working hours per driver
l is the duration of the shift
A is the shift matrix, where $A_{ij}=\begin{cases} 1 \; if \; i \leq j \leq i+l \\ 0 \; elsewhere \end{cases}$

The algorithm minimizes the objective function based on Quasi-Newton Methods and Function Splitting (Brayton et al. (1979)).

The difference between the supply level provided by shifts policy and the optimum supply level can be easily measured as the area between the two curves. An example is presented in Figure 3-33.

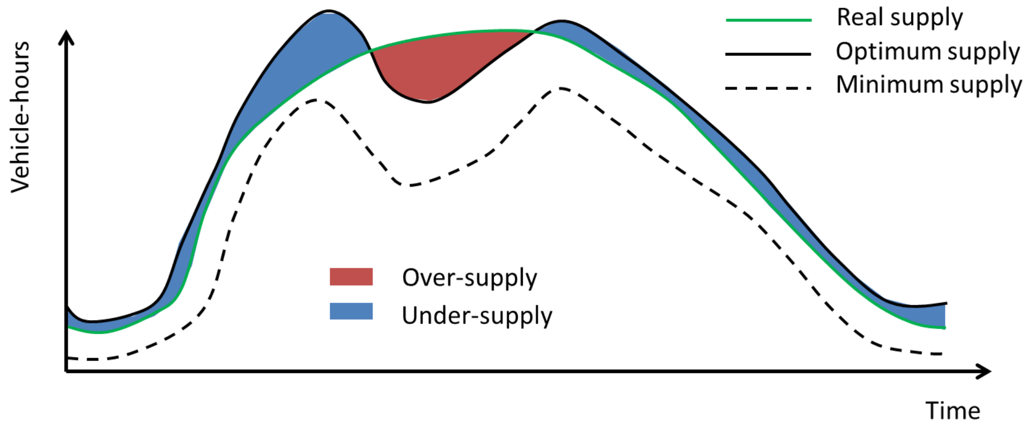

Figure 3-33 Real, optimum and minimum supply during the day

The impacts of the differences between the two curves can be estimated by using the formulations presented in the proposed model. They are basically increase/decrease of waiting time of customers and increase/decrease in the benefits of taxi drivers depending on the sign of this difference. More taxis than the optimum value will reduce the waiting time but will also reduce the benefits of the taxi drivers. In comparison, less taxis than the optimum value will increase the benefits of the taxi drivers, but will at the same time increase the waiting time of customers.

3.10. Elasticities of the three modes of operation

Using the formulations obtained above, the elasticities of the cost function with regards to the demand, the supply and the value of time can be obtained both analytically and empirically.

3.10.1. Elasticity of the dispatching model

The total cost of the system can be expressed as presented in Equation 3.102:

$$Z = A\left[\frac{\lambda_d C_h}{VoT_d} + \lambda_u \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u \alpha_A \frac{0{,}4\mathrm{r}}{\bar{v}\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon}}\right] \quad (3.102)$$

The derivatives of the cost with respect to the supply, the demand and the value of time are presented in Equation 3.103, Equation 3.104 and Equation 3.105:

$$\frac{\partial Z}{\partial \lambda_d} = A\left[\frac{C_h}{VoT_d} - \lambda_u \alpha_A \frac{0{,}4\mathrm{r}}{2\bar{v}\left(\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon}\right)^3}\right] \quad (3.103)$$

$$\frac{\partial Z}{\partial \lambda_u} = A\left[\frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\varepsilon + \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u \alpha_{A}{}_{0{,}4\mathrm{r}} \frac{\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon} - 0.5\lambda_u\left(\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon}\right)^{-1}}{\bar{v}\left(\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon\right)}\right] \quad (3.104)$$

$$\frac{\partial Z}{\partial VoT} = A\left[-\frac{\lambda_d C_h}{{VoT_d}^2} - \lambda_u \frac{rA^{1/2}}{2}\frac{C_{km}}{{VoT_d}^2}\varepsilon\right] \quad (3.105)$$

The elasticities of the cost with respect to the supply (E_{Z,λ_d}), demand (E_{Z,λ_u}) and value of time ($E_{Z,VoT}$) are:

$$E_{Z,\lambda_d} = \frac{\frac{C_h}{VoT_d} - \lambda_u \alpha_A \frac{0{,}4\mathrm{r}}{2\bar{v}\left(\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon}\right)^3}}{\frac{C_h}{VoT_d} + \lambda_u \frac{rA^{1/2}}{2\lambda_d}\left(\frac{C_{km}}{VoT_d}\varepsilon + \frac{\alpha_{IV}}{\bar{v}}\right) + \frac{\lambda_u}{\lambda_d}\alpha_A \frac{0{,}4\mathrm{r}}{\bar{v}\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon}}} \quad (3.106)$$

$$E_{Z,\lambda_u} = \frac{\frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\varepsilon + \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u \alpha_A 0{,}4\mathrm{r} \frac{\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon} - 0.5\lambda_u\left(\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon}\right)^{-1}}{\bar{v}\left(\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon\right)}}{\frac{\lambda_d C_h}{\lambda_u VoT_d} + \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}}\right) + \alpha_A \frac{0{,}4\mathrm{r}}{\bar{v}\sqrt{\lambda_d - \lambda_u \frac{\mathrm{rA}^{1/2}}{2\bar{v}}\varepsilon}}} \quad (3.107)$$

$$E_{Z,VoT} = \frac{-\lambda_d C_h - \lambda_u \frac{rA^{1/2}}{2} C_{km}\varepsilon}{\lambda_d C_h + \lambda_u \frac{rA^{1/2}}{2}\left(C_{km}\varepsilon + VoT_d \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u \alpha_A \frac{0{,}4 r VoT_d}{\bar{v}\sqrt{\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\varepsilon}}} \tag{3.108}$$

Using the analytical formulations, the variation of the system costs in relation to the variation of each variable can be calculated per actor and is presented in Figure 3-34:

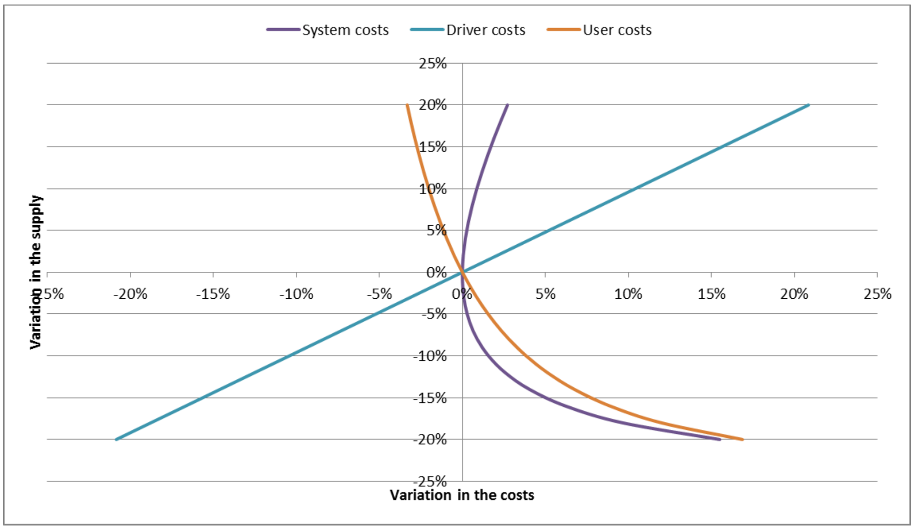

Figure 3-34 Elasticity of the total costs with respect to the supply for the dispatching market

The relation between the variation in the supply and the variation in the driver costs is directly linear, while the relation between the variation in the supply and the variation in the customer costs is inverse and non-linear.

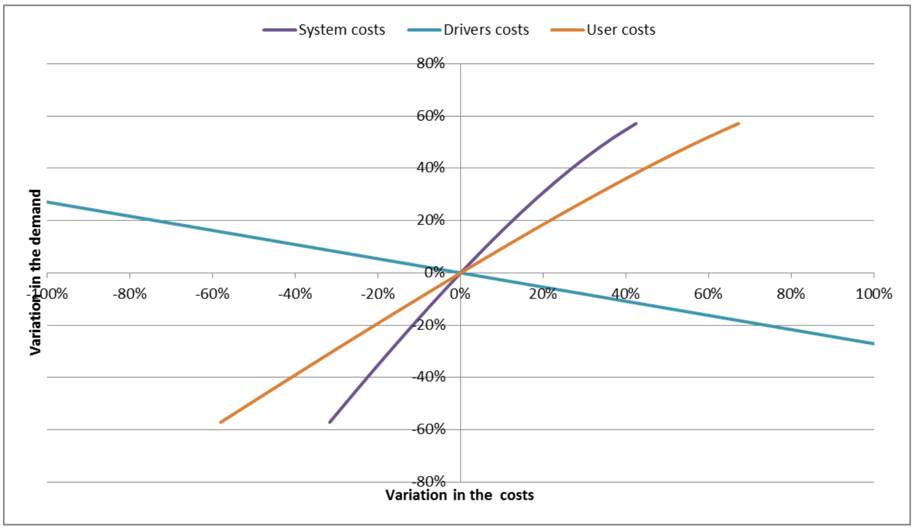

Figure 3-35 Elasticity of the total costs with respect to the demand for the dispatching market

The relation between the variation in the demand and the variation in the driver costs is inverse linear, while the relation between the variation in the supply and the variation in the customer costs is almost directly linear.

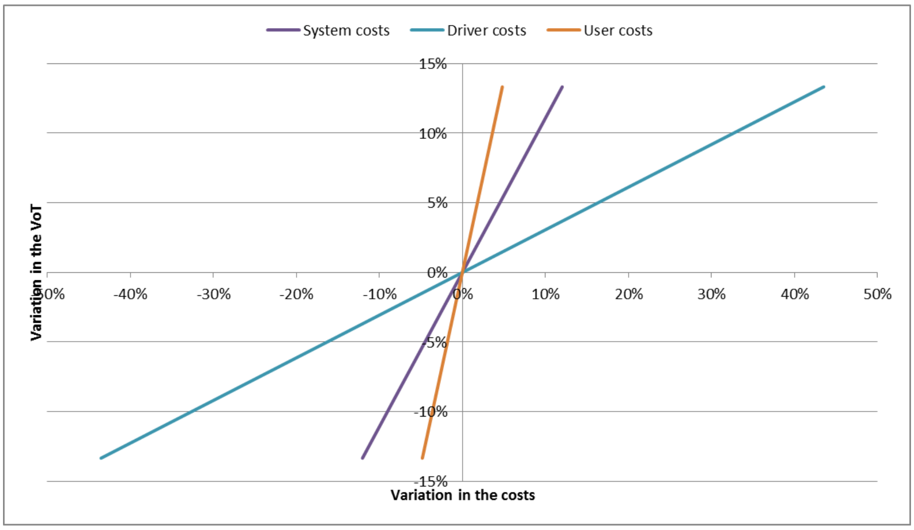

Figure 3-36 Elasticity of the total costs with respect to the value of time for the dispatching market

The relation between the variation in the VoT and the variations in the driver and user costs directly linear.

3.10.2. Elasticity of the stand model

The total cost of the system can be expressed as presented in Equation 3.109:

$$Z = A\left[\frac{\lambda_d C_h}{VoT_d} + \lambda_u \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u \alpha_A \frac{1}{2\overline{v_u}\sqrt{\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}} + \frac{C_s}{VoT_d}\left[\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right]\right] \tag{3.109}$$

The derivative of the cost with respect to the supply, demand and value of time is presented Equation 3.110, Equation 3.111 and Equation 3.112:

$$\frac{\partial Z}{\partial \lambda_d} = A\left[\frac{C_h + C_s}{VoT_d} - \lambda_u \alpha_A \frac{0{,}4\mathrm{r}}{2\overline{v_u}\left(\sqrt{\lambda_d - \lambda_u \frac{\mathrm{r}A^{1/2}}{2\bar{v}}\varepsilon}\right)^3}\right] \tag{3.110}$$

$$\frac{\partial Z}{\partial \lambda_u} = A\left[\frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}} + \frac{C_s}{\bar{v}VoT_d}\frac{\varepsilon}{2}\right) + \lambda_u \alpha_A 0{,}4\text{r}\frac{\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}} - 0.5\lambda_u\left(\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}\right)^{-1}}{2\overline{v_u}\left(\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right)}\right] \quad (3.111)$$

$$\frac{\partial Z}{\partial VoT} = A\left[-\frac{\lambda_d C_h}{{VoT_d}^2} - \lambda_u \frac{rA^{1/2}}{2}\frac{C_{km}}{{VoT_d}^2}\frac{\varepsilon}{2} - \frac{C_s}{{VoT_d}^2}\left[\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right]\right] \quad (3.112)$$

The elasticities of the cost with respect to the supply (E_{Z,λ_d}), demand (E_{Z,λ_u}) and value of time ($E_{Z,VoT}$) are:

$$E_{Z,\lambda_d} = \frac{\frac{C_h + C_s}{VoT_d} - \lambda_u \alpha_A \frac{0{,}4\text{r}}{4\overline{v_u}\left(\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}\right)^3}}{\frac{C_h}{VoT_d} + \frac{\lambda_u}{\lambda_d}\frac{rA^{1/2}}{2\lambda_d}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}}\right) + \frac{\lambda_u}{\lambda_d}\alpha_A \frac{0{,}4\text{r}}{2\overline{v_u}\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}} + \frac{C_s}{VoT_d}\left[1 - \frac{\lambda_u}{\lambda_d}\frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right]} \quad (3.113)$$

$$E_{Z,\lambda_u} = \frac{\frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}} + \frac{C_s}{\bar{v}VoT_d}\frac{\varepsilon}{2}\right) + \lambda_u \alpha_A 0{,}4\text{r}\frac{\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}} - 0.5\lambda_u\left(\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}\right)^{-1}}{2\overline{v_u}\left(\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right)}}{\frac{\lambda_d C_h}{\lambda_u VoT_d} + \frac{rA^{1/2}}{2}\left(\frac{C_{km}}{VoT_d}\frac{\varepsilon}{2} + \frac{\alpha_{IV}}{\bar{v}}\right) + \alpha_A \frac{0{,}4\text{r}}{2\overline{v_u}\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}} + \frac{C_s}{VoT_d}\left[\frac{\lambda_d}{\lambda_u} - 1\frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right]} \quad (3.114)$$

$$E_{Z,VoT} = \frac{-\lambda_d C_h - \lambda_u \frac{rA^{1/2}}{2} C_{km}\frac{\varepsilon}{2} - C_s\left[\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right]}{\lambda_d C_h + \lambda_u \frac{rA^{1/2}}{2}\left(C_{km}\frac{\varepsilon}{2} + VoT_d \frac{\alpha_{IV}}{\bar{v}}\right) + \lambda_u \alpha_A \frac{0{,}4\text{r}VoT_d}{2\overline{v_u}\sqrt{\lambda_d - \lambda_u \frac{\text{rA}^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}}} + C_s\left[\lambda_d - \lambda_u \frac{rA^{1/2}}{2\bar{v}}\frac{\varepsilon}{2}\right]} \quad (3.115)$$

Using the analytical formulations the variation of the system costs in relation to the variation of each variable can be calculated and is presented in Figure 3-37:

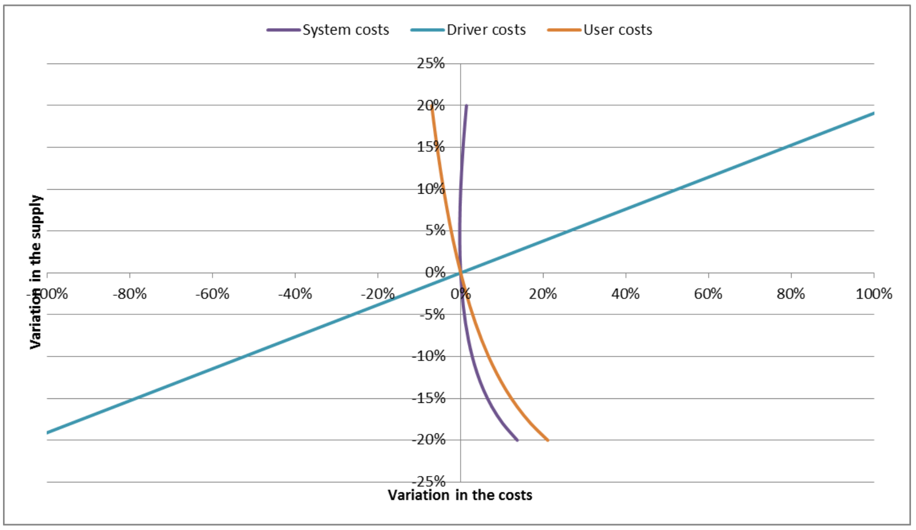

Figure 3-37 Elasticity of the total costs with respect to the supply for the stand market

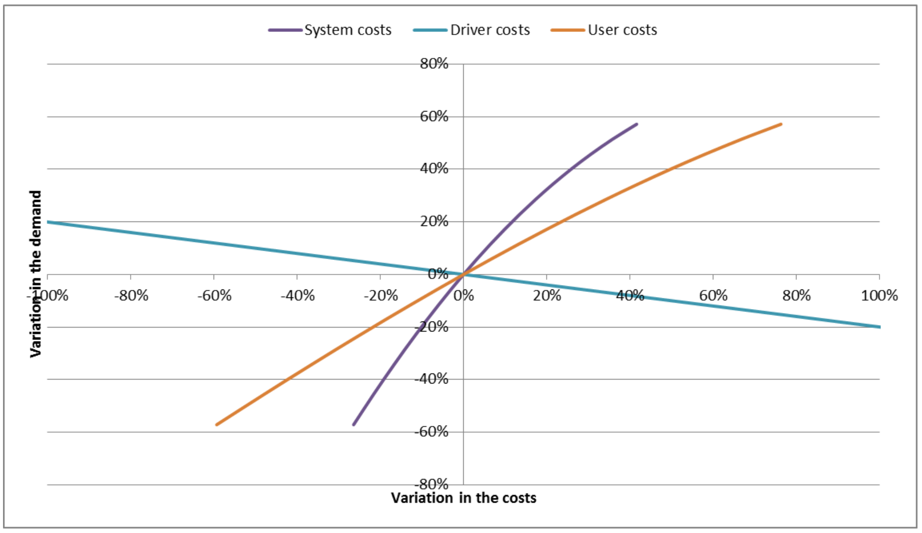

Figure 3-38 Elasticity of the total costs with respect to the demand for the stand market

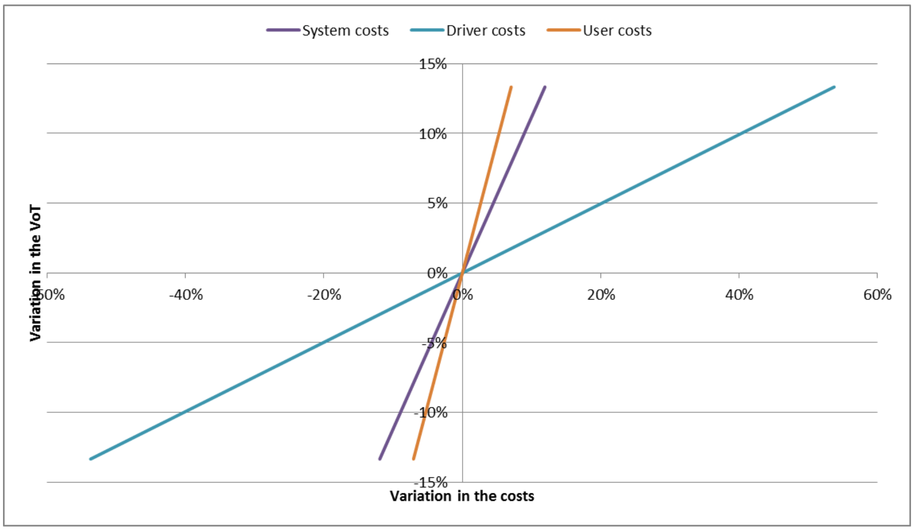

Figure 3-39 Elasticity of the total costs with respect to the value of time for the stand market

3.10.3. Elasticity of the hailing model

The total cost of the system can be expressed in Equation 3.116:

$$Z = A\left[\frac{\lambda_d(C_h+\bar{v}C_{km})}{VoT_d} + \lambda_u\alpha_{IV}\frac{rA^{1/2}}{2} + \lambda_u\alpha_W\frac{1}{\bar{v}\left(\lambda_d-\lambda_u\frac{rA^{1/2}}{2\bar{v}}\right)} + \frac{\lambda_v}{VoT_u}*\frac{\alpha\lambda_d}{v1}\right] \tag{3.116}$$

The derivative of the cost with respect to the supply, demand and value of time is presented Equation 3.117, Equation 3.118 and Equation 3.119:

$$\frac{\partial Z}{\partial\lambda_d} = A\left[\frac{(C_h+\bar{v}C_{km})}{VoT_d} - \lambda_u\alpha_W\frac{1}{\bar{v}\left(\lambda_d-\lambda_u\frac{rA^{1/2}}{2\bar{v}}\right)^2} + \frac{\lambda_v}{VoT_u}*\frac{\alpha}{v1}\right] \tag{3.117}$$

$$\frac{\partial Z}{\partial\lambda_u} = A\left[\alpha_{IV}\frac{rA^{1/2}}{2} + \alpha_W\frac{\left(\bar{v}\lambda_d-\lambda_u\frac{rA^{1/2}}{2}\right)+\lambda_u\frac{rA^{1/2}}{2}}{\left(\bar{v}\lambda_d-\lambda_u\frac{rA^{1/2}}{2}\right)^2}\right] \tag{3.118}$$

$$\frac{\partial Z}{\partial VoT} = A\left[-\frac{\lambda_d(C_h+\bar{v}C_{km})}{{VoT_d}^2} - \frac{\lambda_v}{{VoT_u}^2}*\frac{\alpha\lambda_d}{v1}\right] \tag{3.119}$$

The elasticities of the cost with respect to the supply (E_{Z,λ_d}), demand (E_{Z,λ_u}) and value of time ($E_{Z,VoT}$) are:

$$E_{Z,\lambda_d} = \frac{\frac{(C_h+\bar{v}C_{km})}{VoT_d} - \lambda_u\alpha_W\frac{1}{\bar{v}\left(\lambda_d-\lambda_u\frac{rA^{1/2}}{2\bar{v}}\right)^2} + \frac{\lambda_v}{VoT_u}*\frac{\alpha}{v1}}{\frac{(C_h+\bar{v}C_{km})}{VoT_d} + \frac{\lambda_u}{\lambda_d}\alpha_{IV}\frac{rA^{1/2}}{2\lambda_d} + \frac{\lambda_u}{\lambda_d}\alpha_W\frac{1}{\bar{v}\left(\lambda_d-\lambda_u\frac{rA^{1/2}}{2\bar{v}}\right)} + \frac{\lambda_v}{VoT_u}*\frac{\alpha}{v1}} \tag{3.120}$$

$$E_{Z,\lambda_u} = \frac{\alpha_{IV}\frac{rA^{1/2}}{2} + \alpha_W\frac{\left(\bar{v}\lambda_d-\lambda_u\frac{rA^{1/2}}{2}\right)+\lambda_u\frac{rA^{1/2}}{2}}{\left(\bar{v}\lambda_d-\lambda_u\frac{rA^{1/2}}{2}\right)^2}}{\frac{\lambda_d(C_h+\bar{v}C_{km})}{\lambda_u VoT_d} + \alpha_{IV}\frac{rA^{1/2}}{2} + \alpha_W\frac{1}{\bar{v}\left(\lambda_d-\lambda_u\frac{rA^{1/2}}{2\bar{v}}\right)} + \frac{\lambda_v}{\lambda_u VoT_u}*\frac{\alpha\lambda_d}{v1}} \tag{3.121}$$

$$E_{Z,VoT} = \frac{-\lambda_d(C_h+\bar{v}C_{km}) - \lambda_v*\frac{\alpha\lambda_d}{v1}}{\lambda_d(C_h+\bar{v}C_{km}) + VoT_d\lambda_u\alpha_{IV}\frac{rA^{1/2}}{2} + \lambda_u\alpha_W\frac{VoT_d}{\bar{v}\left(\lambda_d-\lambda_u\frac{rA^{1/2}}{2\bar{v}}\right)} + \lambda_v*\frac{\alpha\lambda_d}{v1}} \tag{3.122}$$

Using the analytical formulations the variation of the system costs in relation to the variation of each variable can be estimated and is presented in Figure 3-40:

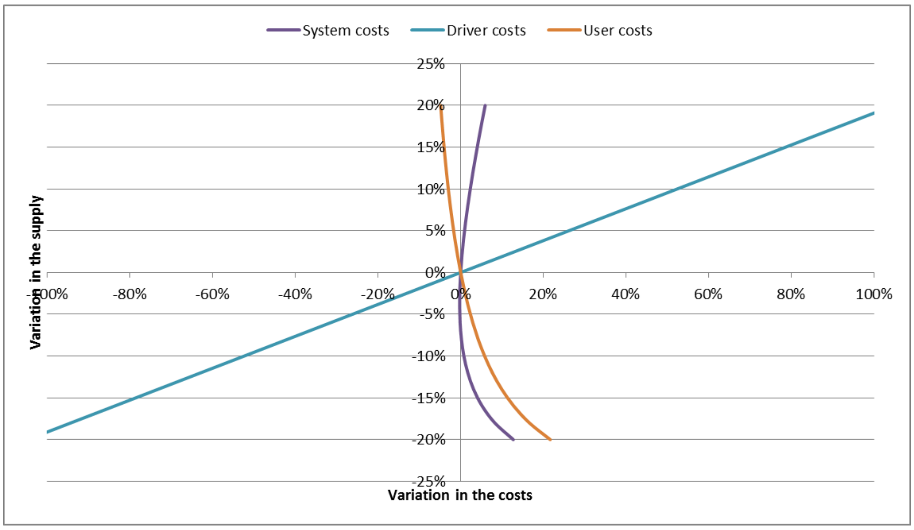

Figure 3-40 Elasticity of the total costs with respect to the supply for the stand market

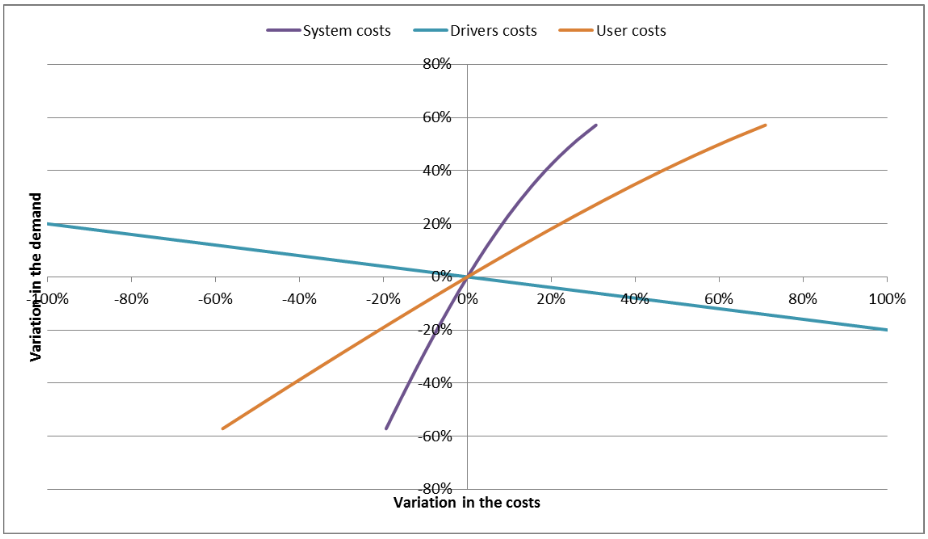

Figure 3-41 Elasticity of the total costs with respect to the demand for the hailing market

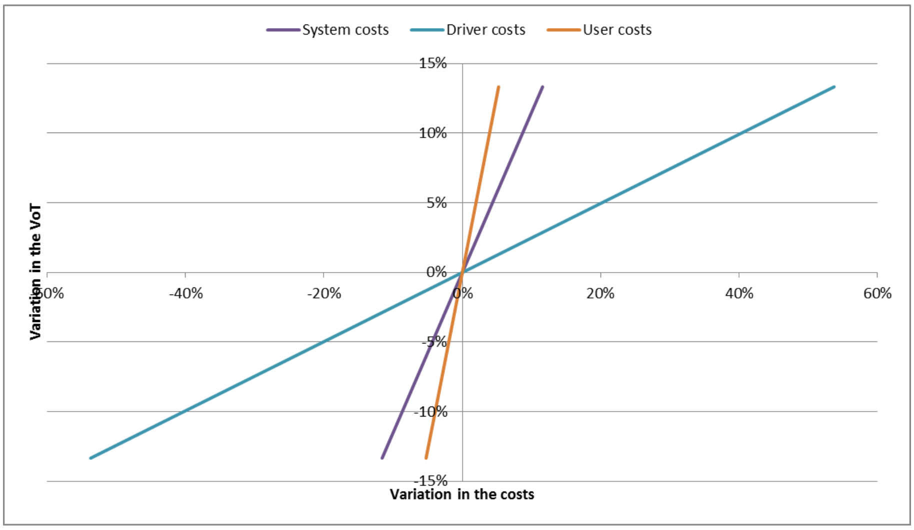

Figure 3-42 Elasticity of the total costs with respect to the value of time for the hailing market

3.10.4. Elasticity comparison between the three operation modes

The elasticitiy curves are very similar for the three operation modes.

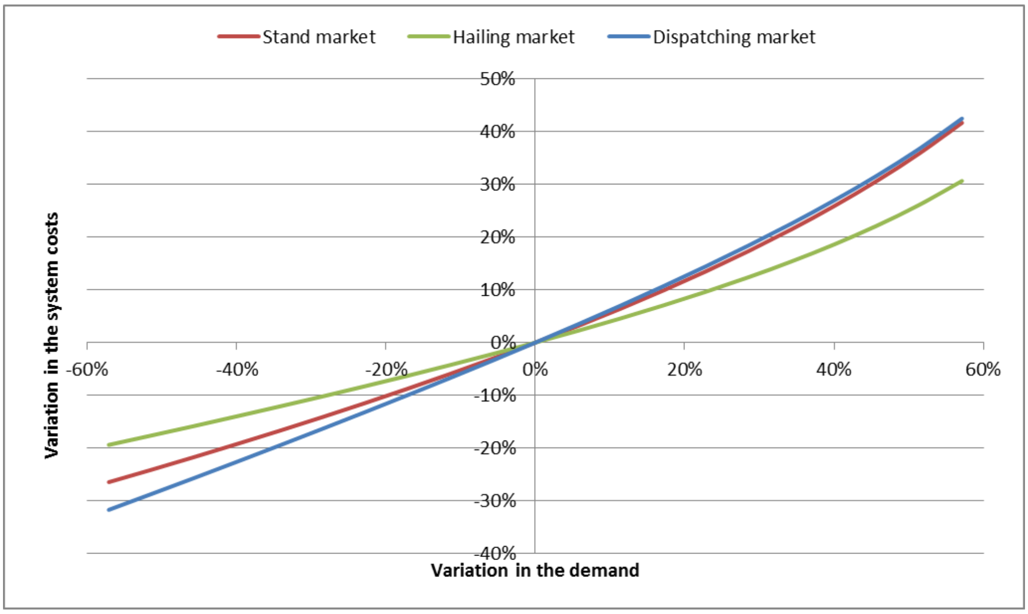

Figure 3-43 Elasticity of the total costs with respect to the demand for the three modes of operation

The elasticity of the system costs with respect to the demand ranges from 0.33 to 0.74 depending on the mode of operation and the absolute values of the demand.

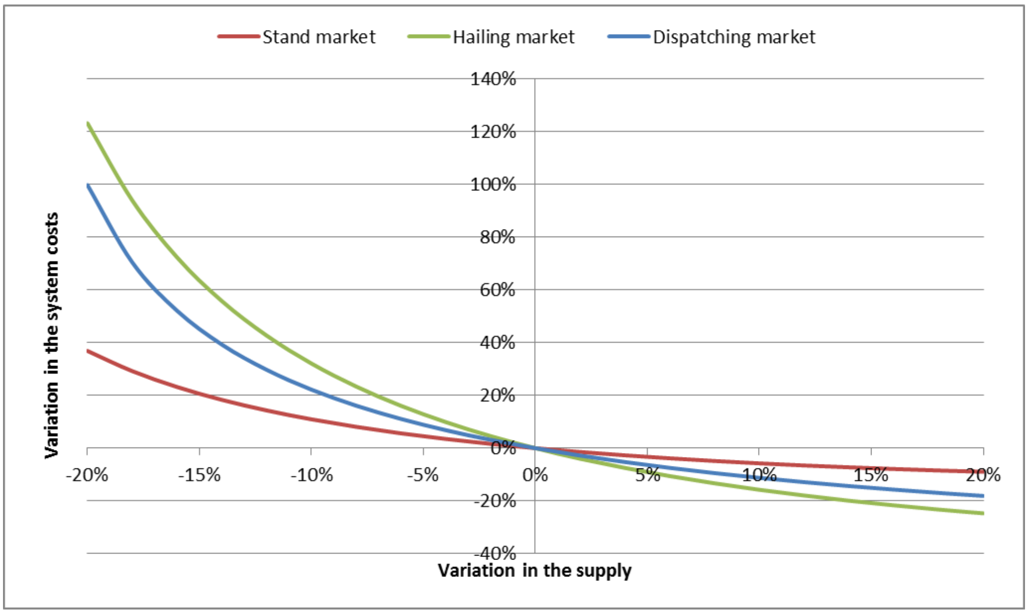

Figure 3-44 Elasticity of the total costs with respect to the supply for the three modes of operation

The elasticity of the system costs with respect to the supply ranges from -0.44 to -6.15 depending on the mode of operation and the absolute values of the demand.

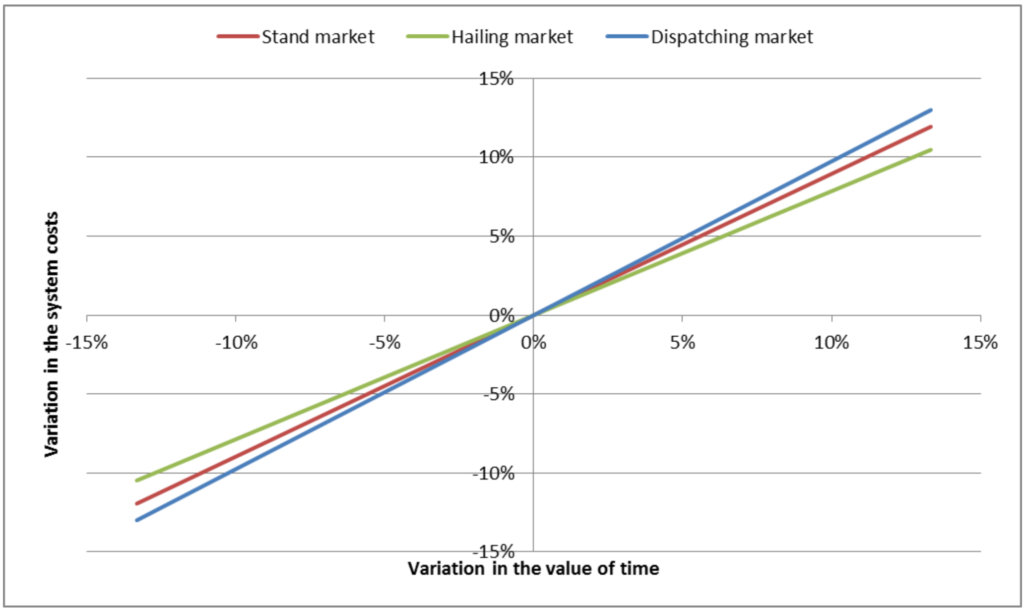

Figure 3-45 Elasticity of the total costs with respect to the value of time for the three modes of operation

The elasticity of the system costs with respect to the value of time ranges from 0.78 to 0.97 depending on the mode of operation and the absolute values of the demand.

3.10.5. Elasticity of the demand to with respect to the fare

Using the formulation of the demand proposed in Equation 3.97, the elasticity of the demand with respect to the fare is the one presented in Equation 3.123:

$$E_{Z,\lambda_u} = \frac{\partial \lambda_u}{\partial \tau_{km}} \frac{\tau_{km}}{\lambda_u} = -\gamma \frac{rA^{1/2}}{2VoT_u} \tau_{km} \qquad (3.123)$$

The elasticity is always negative, which means that an increase in the fare will generate a decrease in the demand and viceversa.

3.11. Conclusions

A formulation for the dispatching, the hailing and the stand taxi markets based on the generalized cost has been presented and studied, concluding in minimum and optimum taxi fleets for different city sizes and demand levels. While the minimum taxi fleet ensures that all trip requests will be served, the optimum fleet ensures a minimum LoS to travellers while providing positive benefits to taxi drivers. This extra fleet size is directly proportional to the value of time of customers and inversely proportional to the speed and hourly operation costs of taxi drivers. The relation between demand and supply has been presented for different city sizes. The optimum fleet has been calculated mathematically and represented graphically, providing in this last case its exact formulation. Analytic formulations of the first and second best solutions have been also obtained for the three operation modes.

The findings prove that small cities or cities with low demand for taxi services must have dispatching taxi market rather than a stand or hailing market. At the same time, if the taxi fleet is large, it is recommended to have a dispatching market rather than a hailing or a stand one, which are optimal for medium to small fleets. The range of the optimum demand level for each operation mode depends significantly on the Value of Time, the network congestion and the operational costs of the taxi fleet. Finally, a methodology for matching the optimum fleet size obtained by the models to the current shifts policies has been proposed.

The impact of pricing on the demand and the Level of Service has been provided by defining an elastic demand function. The optimum combination of fare and fleet size have been identified, providing a small number of taxis and high fares as optimum for the drivers, and a large number of taxis with low fares as optimum for the customers. Elasticities of the demand with regard to the fare and elasticities of the total system costs with regard to the demand, the supply and the VoT have been provided for the different modes of operation in both analytical and graphical format.

All the presented models need to be calibrated with real world or simulation data in order to understand the behavior of the variables and to validate their hypotheses. Agent-based models should be developed for simulating the drivers and customers behavior in the network and support the proposed models with simulation results. Further research is also needed in the demand estimation methods, since the waiting time depends on the demand and at the same time demand depends on the waiting time, forming a bi-level problem where the demand is obtained in the upper level and the waiting time associated to the demand in the second level. Finally, real time data and forecasting procedures should be applied to the taxi models in order to evaluate the impact on the level of service and the income of taxi drivers. This new characteristic of the taxi services will anticipate customer requests and traffic condition in the city (as proposed by Wong and Bell (2006)), optimizing the management of the taxi fleet.

3.12. References

Bautista (1985) Models de distribució del temps d'espera del taxi. *Tesina final de Carrera,* Escola Tècnica Superior d'Enginyeria Industrial de Barcelona.

Chang S. K. and Huang S. M. (2003) Optimal fare and unoccupancy rate for taxi market. *Transportation Planning Journal* **32** (2), 341 – 363.

Chang S. K. and Chu-Hsiao Chu (2009) Taxi vacancy rate, fare and subsidy with maximum social willingness-to-pay under log-linear demand function. *Transportation Research Record: Journal of the Transportation Research Board* **2111**, 90 – 99.

Chang S. K. J., Wu C. H., Wang K. Y. and Lin C. H. (2010) Comparison of Environmental Benefits between Satellite Scheduled Dispatching and Cruising Taxi Services; *Proc. of the 89th Annual Meeting of the Transportation Research Board,* Washington D.C..

CENIT (2013) Observatori del Taxi 2012. *Document Final. Institut Metropolità del Taxi*.

Daganzo C. F. (1984) The length of tours in zones of different shapes. *Transportation Research Part B* **18B** (2), 135-145.

Daganzo C. F. (2010a) Lesson notes (available at http://www.ce.berkeley.edu/~daganzo/index.htm).

Estrada M., F. Robusté, C. Amat, H. Badia and J. Barceló (2011) On the optimal length of the transit network with transit performance simulation. Application to Barcelona. *Proc. of the 90th Annual Meeting of the Transportation Research Board,* Washington D.C.

Fernandez L. J. E., de Cea Ch. J. and Malbran R. H. (2008) Demand responsive urban public transport system design: Methodology and application. *Transportation Research Part A* **42**, 951 – 972.

Geroliminis N. and Daganzo C.F. (2008) Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B* **42** (9), 759-770.

Greenberg H. (1959) An analysis of traffic flow. *Operations Research* **7**, pp 78-85.

Greenshield B. D. (1935) A study of traffic capacity. *Highway Research Board Proceedings* **14**, 448-477.

Kittelson & Associates (2003) Transit Capacity and Quality of Service Manual. *Transit cooperative research program report 100*, 2nd Edition.

Lo H. K., Yip C. W. and Wan Q. K. (2004). Modeling Competitive Multi-modal Transit services: A Nested Logit Apporach. *Transportation Research Part C* **12C** (3-4), 251-272.

Meyer R. F. and Wolfe H. B. (1961) The organization and operation of a taxi fleet. *Naval Research Logistics* Quarterly **8**, 137–150.

Ntzichristos L. and Samaras Z. (2012) Update of the chapter Passenger cars, light-duty trucks, heavy-duty vehicles including buses and motorcycles. European Monitoring and Evaluation Programme / European Environment Agency *air pollutant emission inventory guidebook*.

Raveau S., Muñoz J. C. and de Grange L. (2011) A topological route choice model for metro. *Transportation Research Part A* **45** (2), 138-147.

Underwood R.T. (1961), Speed, Volume, and Density Relationships: Quality and Theory of Traffic Flow. *Yale Bureau of Highway Traffic*, New Haven, CT, USA, 141-188.

Wong K. I., Wong S. C. and Yang H. (2001) Modeling urban taxi services in congested road networks with elastic demand. *Transportation ResearchPart B* **35**, 819 – 842.

Yang H., Wong S. C. and Wong K. I. (2002) Demand-supply equilibrium of taxi services in a network under competition and regulation. *Transportation Research B* **36**, 799 – 819.

Zamora D. (1996) Modelització dels costos unitaris d'una flota de taxis. *Tesina final de carrera. ETSECCPB Barcelona.*

4. AGENT BASED MODEL FOR THE ESTIMATION OF THE TAXI SUPPLY

4.1. Introduction

One of the main limitations of the aggregated model presented in chapter 3 was the assumption of uniform demand distribution over the region of service. Therefore, an agent based simulation model has been developed, which considers the spatial and temporal dimension of both the demand and the supply, providing more detailed results.

Discrete-event simulations model a complex system as a discrete sequence of events which occur at a particular instant. The agent-based model developed in this document, which is based on discrete-event simulation, proposes the use of agents running in a real city road network and taking their own decisions for completing as many trips as possible. There are three basic types of agents, each one of which correspond to the operational taxi modes.

4.2. Actors and variables presentation

The two basic inputs that define the system performance and costs are the supply and the demand. The supply is determined by the number of taxis running in the city network looking for a customer. On the other hand, the demand is composed by the customers looking for a ride. Then, the three basic actors of the model are the city, characterized by a street network and taxi stands, the taxi drivers and the customers.

4.2.1. Input parameters

- Total demand (absolute value and geographical distribution)
- Supply (taxi fleet size and composition)
- Network congestion (vehicle flow in each link)
- Taxi fares (fixed fare and delay-time-based charge rates)

4.2.2. Dependent outputs

- The city
    - Number of kilometers realized by the taxi fleet (veh-km)
    - Number of hours the taxis are running in the streets (veh-hours)
    - Number of trips realized by the taxi fleet
    - Average waiting time of customers
    - Average benefits of the taxi drivers
- The taxi drivers
    - Number of kilometers.
        - Number of occupied kilometers
        - Number of vacant kilometers
    - Number of hours.
        - Occupied journey time
        - Vacant journey time
    - Taxi occupancy
    - Vacant taxi headway

  - Benefits
- The taxi customers
  - Waiting time
  - Travel time
  - Cost of trip

4.3. Input parameters and variables

The model simulates the real taxi market, where taxis are looking for customers. Taxis and customers are modeled as agents, with their own rules, objectives and behavior. In each time interval every agent is taking decisions, taxis are circulating looking for a customer and customers are waiting or looking for a taxi. Three basic agents are designed for the taxis, related to the three operational modes.

4.3.1. The road network

Let $G\ (N, A)$ be a directed graph, where N is the set of nodes and A the set of links. The set of nodes N contains nodes of the following types:

- Intersections between two links
- Taxi stands
- Trip origin and destination zones

The set of links A has the following characteristics:

- Length (meters)
- Free flow travel time (minuts)
- Volume-delay parameters
- Flow of vehicles (veh/hour)
- Capacity (veh/hour)

4.3.2. The demand for taxi trips

Customers appear in each zone following a two-dimensional geographic distribution considered to be uniform. The demand is generated by an origin-destination (OD) matrix, which contains the number of trips between each pair of zones. In order to generate the events, the OD matrix is converted into a vector containing for each trip in the matrix, its origin, its destination and the starting time, which are distributed uniformly within the time of the simulation. The demand level from each origin to each destination is generated as presented in Equation 4.1:

$$D_{ij} = D_0 \cdot f_i \cdot f_j \qquad (4.1)$$

where,

D_{ij} is the demand for trips with origin in zone i and destination in zone j

$\mathrm{f_k}$ is the probability density function of trips of zone k.

Hailing customers wait at the streets until a hailing taxi reaches them. Stand customers wait at the taxi stand, forming queues served by a FIFO system. Dispatching customers call for a taxi service before appearing in the network, and then wait until the assigned taxi reaches them.

4.3.3. Link performance function (Sheffield)

The speed of a taxi is considered to be equal to the speed of the rest of the traffic, which is calculated using the link performance function (Sheffi, 1984), formulated according to the proposal of the Bureau of Public Roads (BPR). It is presented in Equation 4.2:

$$t_i(x_i) = t_{0_i}(1 + \alpha_i({}^{x_i}/_{c_i})^{\beta_i}) \quad (4.2)$$

where,
t_{0_i} is the free flow travel time for the link i
α_i and β_i are the i-link parameters (normally $\alpha = 0.15$ and $\beta = 4$ for all links)
c_i the capacity of the link i
x_i the flow on link i

The rules of the developed model are presented in the following sub-chapters for the different operation modes.

4.3.4. Pricing structures

The trip cost is calculated by using the same logic as the one used in reality. In addition to the flag-drop charge, two counters are used for charging the cost increments based on time or distance depending on the speed of the vehicle at each moment. This is different to the methodology used in the aggregated model, where the trip cost was calculated using only the trip distance and the flag-drop charge.

While moving, the taxi is calculating the trip cost using a distance-based charge or a time-based charge, always charging the first value of both that reaches the roof value (depending on the congestion of each link). Two counters are used, one for the time (c_t^1) and one for the distance (c_t^2).

$$c_{t+1}^1 = c_t^1 + \left\| (x_i, y_i)_{t+1} - (x_i, y_i)_t \right\| \quad (4.3)$$

$$c_{t+1}^2 = c_t^2 + \Delta t \quad (4.4)$$

When one of the two counters reaches the roof value, the related charge is added to the cost of the trip ($\mathrm{c_t}$) and the counters are set to zero.

$$c_{t+1} = c_t + \begin{matrix} c^1 \; if \; c_{t+1}^1 > M_1 \\ c^2 \; if \; c_{t+1}^2 > M_2 \end{matrix} \quad (4.5)$$

where,
c^1 is the distance-based charge (euros per $\mathrm{M_1}$meters)
c^2 is the time-based charge (euros per $\mathrm{M_2}$ seconds)
M_1 is the charging distance of the distance-based charge methodology
M_2 is the charging time of the time-based charge methodology

4.4. Taxi-customer processes

Figure 4-1 shows the time distribution and relation of the customers and the taxis in the three markets.

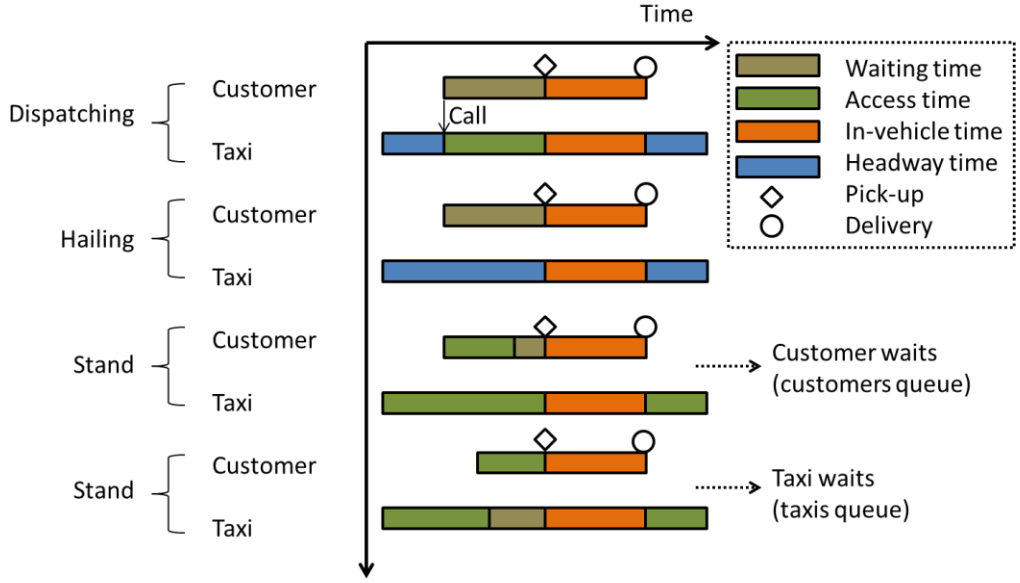

Figure 4-1 Time distribution of the customers and the taxis

4.5. Logic modules

The logic modules developed for the states (waiting, access, in-vehicle) presented in Figure 4-1 are presented below. Figure 4-2 shows the agent-based model of the taxi market.

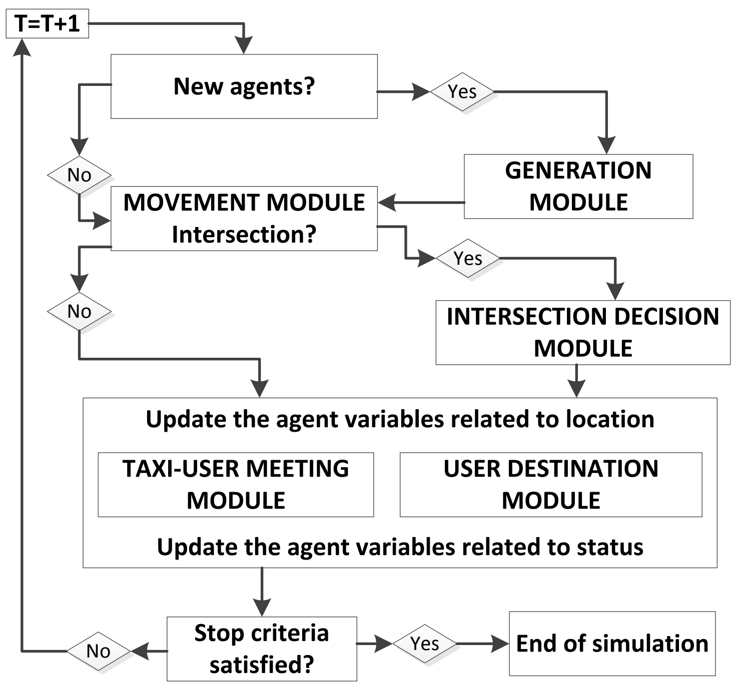

Figure 4-2 Agent-based proposed model

As shown, as the taxis and customers are being generated and moved, the variables are being created and updated. Each module is represented and explained below.

4.5.1. The developed modules

4.5.1.1. Generation module

The generation module creates the demand and the supply at each time interval assigning the zone and the operational mode that will be simulated.

When a taxi is created, it is assigned to an origin zone (as for the demand) and operation mode. If the operation mode is hailing or dispatching, random coordinates within the zone are defined for the first position of the taxi; if the taxi operation mode is the stand mode, a random stand on the zone is assigned to it. If there is a customers' queue in the stand, the first customer in the queue is assigned to the taxi, otherwise the taxi joins the taxi queue in the last place.

When a customer is created, origin, destination and operational mode are set. If the operation mode is hailing or dispatching, random coordinates within the origin zone are defined for the waiting position of the customer. In the dispatching case, the nearest (in real network travel time) taxi is assigned to the customer. If the operation mode is stand, the customer is assigned to a random stand within the origin zone, if there is a taxi queue in the stand, the customer is assigned to the first taxi in the queue, if there is no taxi queue in the stand, the customer joins the customers' queue in the last place.

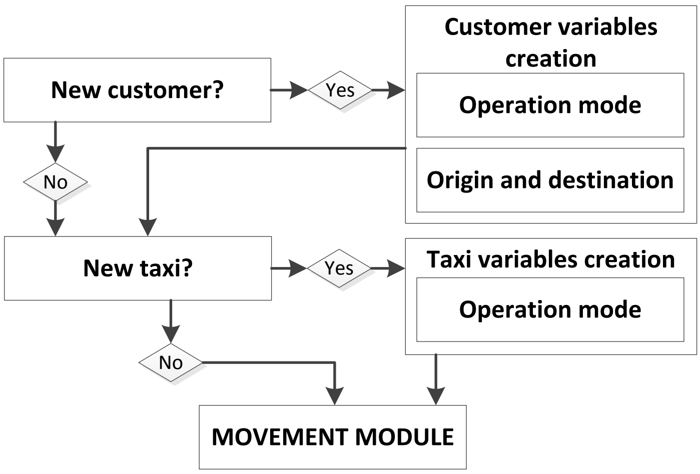

Figure 4-3 Generation module

If neither taxis, nor customers are created, the model moves directly to the movement module.

4.5.1.2. Movement module

The movement module moves each taxi depending on the congestion of the correspondent link. If the taxi arrives to an intersection, the intersection decision module decides the route that the taxi will follow (the next link). If the taxi continues in the same link, the position of the taxi is updated.

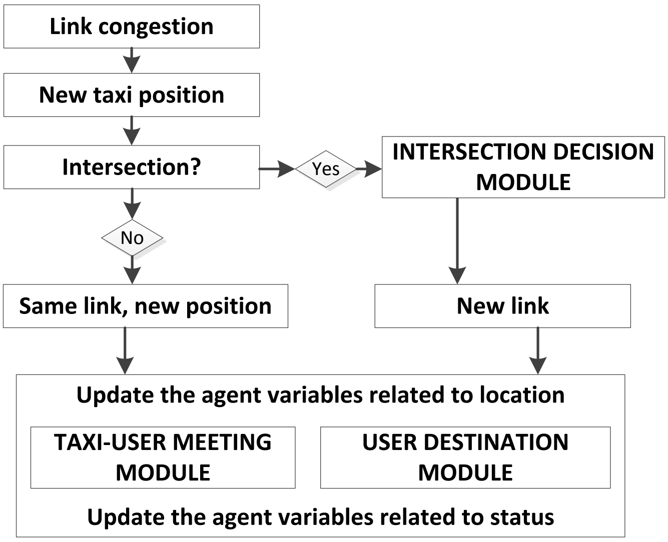

Figure 4-4 Movement module

4.5.1.3. Intersection decision module

When a vacant, not assigned taxi reaches an intersection, it chooses where to go based on experience. When an occupied taxi, stand taxi or assigned taxi reaches an intersection, it follows the defined path in the correspondent shortest path (between the origin and the destination of the customer, looking for the nearest stand or looking for the assigned customer respectively).

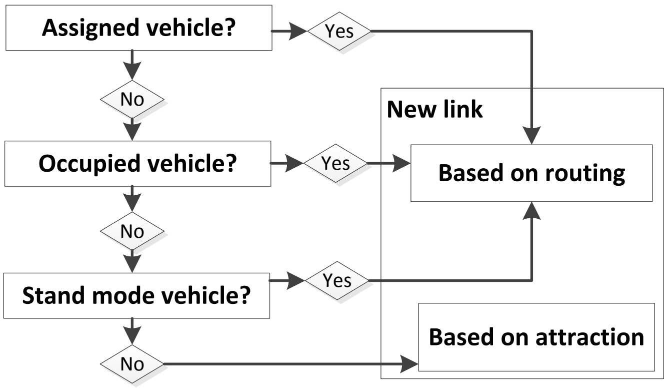

Figure 4-5 Intersection decision module

4.5.1.4. Taxi/user meeting module and user destination module

If a vacant or assigned taxi meets a customer during the last movement, the taxi/user meeting module will decide if the taxi will pick up the customer or not. If an assigned taxi meets its assigned customer, the taxi will pick up the customer. If a vacant hailing taxi meets a hailing customer, the taxi will pick up the customer. Dispatching taxis pick up only their assigned customers. Stand taxis pick up only customers at taxi stands. Hailing taxis pick up only hailing customers. When a taxi picks up a customer, the shortest route between the origin and the destination is calculated and established in the intersection decision module.

If an occupied taxi passes by the destination point of its customer, the trip ends and the taxi will be free for the next time interval. Depending on the operation mode the taxi will find then the nearest taxi stand and define the correspondent path for reaching the stand or will circulate randomly waiting for a new order.

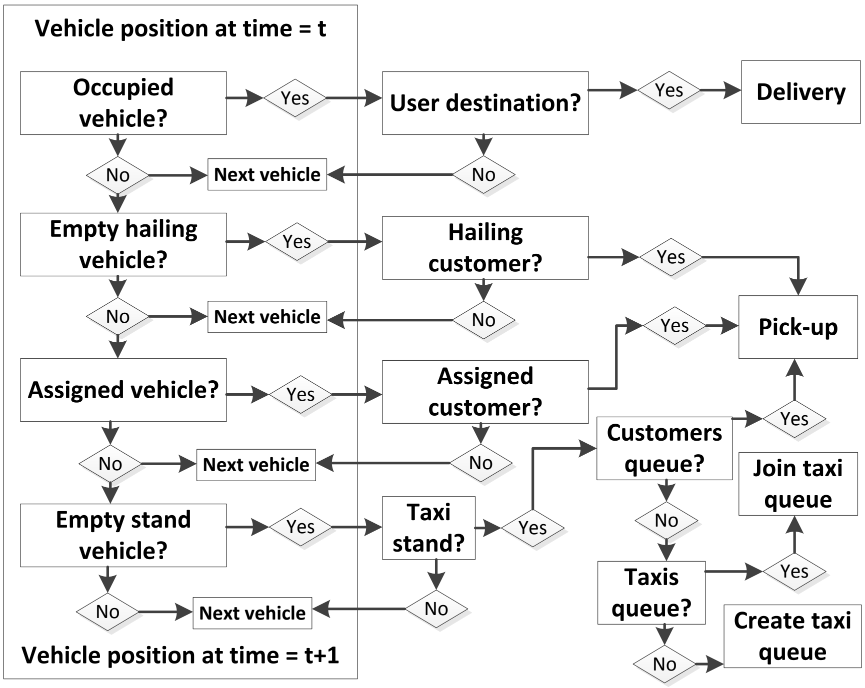

Figure 4-6 Taxi/user meeting and user destination modules

4.5.2. Formulation used in the modules

4.5.2.1. <u>Vacant movement</u>

4.5.2.1.1. Dispatching and hailing markets

If the taxi is assigned, it follows the route to the assigned customer. If not, it moves along a network link waiting for a new order. The distance traveled within the time period is related to the duration of the time period and the congestion of the link.

$$(x_i, y_i)_{t+1} = (x_i, y_i)_t + \frac{\Delta t}{t_j}\left[(x_E, y_E)_j - (x_S, y_S)_j\right] \quad (4.6)$$

where,
$(x_i, y_i)_t$ are the coordinates of the vehicle i at the time interval t
$(x_S, y_S)_j$ are the coordinates of the starting node of the link j
$(x_E, y_E)_j$ are the coordinates of the ending node of the link j
t_j is the travel time of link j
Δt is the duration of the time interval

When the taxi arrives to an intersection, it chooses the next link (i) based on the selection probability ($P(A_i)$). This is detected by checking at each time step that the coordinates of the vehicle are inside the square defined by the coordinated of the start and end of the link. If they are outside the square, the driver chooses the next link as follows:

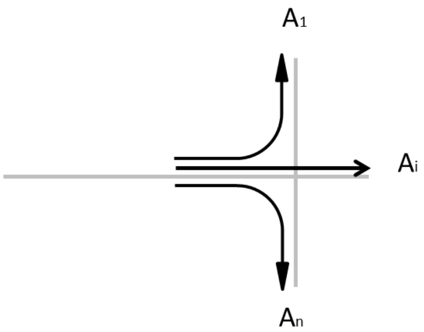

Figure 4-7 Intersection sample in the agent-based model

All the alternatives (links with the origin in the intersection) are listed clockwise from A_1 to A_n, where $n+1$ is the number of links having the intersection as their origin (u-turns are prohibited, all other turn movements are permitted). The probability of each link at intersection depends on its attractiveness and is calculated as follows:

$$P(A_i) = \frac{w(A_i)}{\sum_{j=1}^{n} w(A_j)} \qquad (4.7)$$

where $w(A_i)$ is the attractiveness of link i calculated as the total number of trip origins in link i during the last Ns simulations, which provides a quite simple learning procedure to the drivers that can simulate their experience. Results demonstrate that N_s=10 is enough for providing a good network knowledge.

Once the probabilities have been defined, when a taxi arrives at each ending node (intersection) a random number (Rn) is generated. The decision procedure has the logic presented in Figure 4-8:

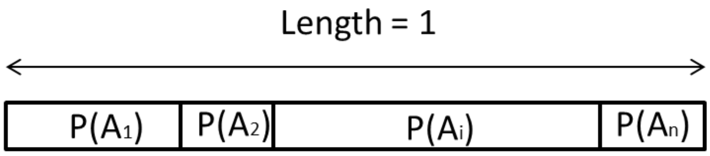

Figure 4-8 Roulette for the intersection decision procedure

$$next\ link = A_i if \sum_{j=1}^{i-1} P(A_j) < Rn < \sum_{j=1}^{i-1} P(A_j) + P(A_i) \qquad (4.8)$$

4.5.2.1.2. Stand market

When a taxi moves along a network link, the distance traveled within the time period is related to the duration of the time period and the congestion of the link, using a constant free flow speed for taxis. Vacant taxis running in stand mode are always looking for a taxi stand near their current location (following the shortest path between their current situation and the nearest taxi stand). When the taxi arrives to an intersection, it chooses the next link based on the calculated shortest path to the nearest taxi stand.

4.5.2.2. Assignment of a customer

This module is applied to the dispatching market. When a customer asks for a taxi, the nearest free dispatching taxi (distance is calculated within the network using the Dijkstra algorithm (1959) based on the travel time of each link) is assigned to him/her. This is made by a proxy variable (x_{ij}).

$$x_{ij} = \begin{cases} 1 \; if \; d_{ij} = min_k\{d_{ik}\} \\ 0 \; otherwise \end{cases} \tag{4.9}$$

Where,

d_{ij} is the network distance betweencustomer i and vehicle j

The taxi then finds the shortest path between its current location and the location of the customer using the Dijkstra algorithm (1959) and follows it until the customer location is reached. In order to avoid u-turns, the Dijkstra algorithm is link-based instead of node based, using the current link and the destination link instead of origin and destination nodes.

Various assignment procedures can be simulated with this tool, such as the assignment to the driver which was waiting for a longer time or dynamic bidding by the drivers, but these options are not developed in this methodology.

4.5.2.3. Picking up and delivery of a customer

The customer pick-up and delivery as well as the arrival to a taxi stand are detected by using a local coordinates system for all agents. Instead of using the geographic coordinates, the agents are located in the network by knowing the link and the distance from the origin of the link. It is important to highlight that all links present a one-way direction, and if a street has two directions, two links are created. The detection of a customer origin, destination or a taxi stand is executed then by comparing the link where each agent is and the last two relative positions within the link.

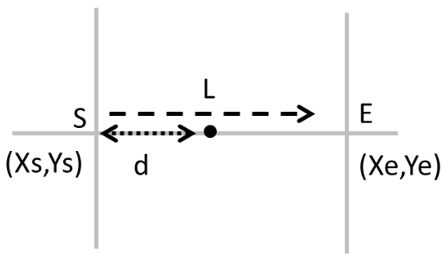

Figure 4-9 Location of an agent

The relation between the geographical location and the one used in the agent-based model (link identity - L, distance from the origin - d) is the one presented in Equation 4.10 and Equation 4.11.

$$x = x_S + \frac{d}{\sqrt{(x_E - x_S)^2 + (y_E - y_S)^2}}(x_E - x_S) \tag{4.10}$$

$$y = y_s + \frac{d}{\sqrt{(x_E - x_S)^2 + (y_E - y_S)^2}}(y_E - y_S) \quad (4.11)$$

4.5.2.3.1. Dispatching market

If the taxi finds the assigned customer, it picks him/her up. The taxi situation then changes to occupied, and the shortest path between the customer origin and destination is then calculated, depending on the traffic congestion of each link at that moment, setting the path to be followed by the taxi.

4.5.2.3.2. Hailing market

If the taxi finds a free customer (not waiting for an assigned taxi or at a stand), it picks him/her up. The taxi then changes to occupied, and the shortest path between the customer origin and destination is then calculated, depending on the traffic congestion of each link at that moment, setting the path to be followed by the taxi.

4.5.2.3.3. Stand market

Taxis and customers are assigned in each taxi stand based on a FIFO system.

4.5.2.4. Occupied movement

The taxi and the customer move along the links, as in the vacant movement, but when arriving to an intersection, the route correspondent to the shortest path is followed until the destination is reached.

4.5.2.5. Delivering a customer

4.5.2.5.1. Dispatching and hailing markets

When the destination of the customer is reached, the taxi calculates the cost of the last interval. The customer then disappears from the network, and the trip cost is charged to both agents (income for the taxi and cost for the customer). The taxi becomes vacant and continues looking for the next customer.

4.5.2.5.2. Stand market

When the destination of the customer is reached, the taxi calculates the cost of the last interval. The customer then disappears from the network, and the trip cost is charged to both agents (income for the taxi and cost for the customer). The taxi becomes free and calculates the shortest path to the nearest taxi stand.

4.5.2.6. Arriving to a taxi stand

This module is applied to the stand market. When a taxi arrives to a taxi stand, it joins the taxi queue (if it exists). If there are no taxis on the stand, the taxi picks up the first customer in the queue (if it exists). If there are neither taxis, nor customers, the arriving taxi forms the taxi queue.

4.5.3. Space-time diagrams of the three operation modes

Figure 4-10 shows the space-time diagram of the activities realized by the agents in the dispatching market.

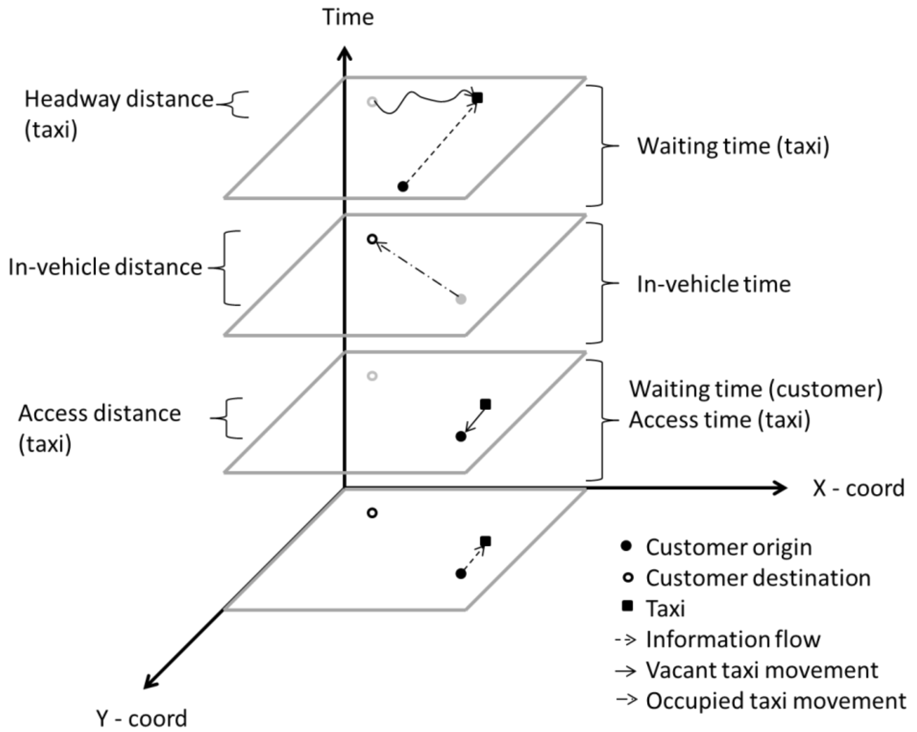

Figure 4-10 Space-time diagram of the dispatching market activities

Figure 4-11 shows the space-time diagram of the activities realized by the agents in the hailing market.

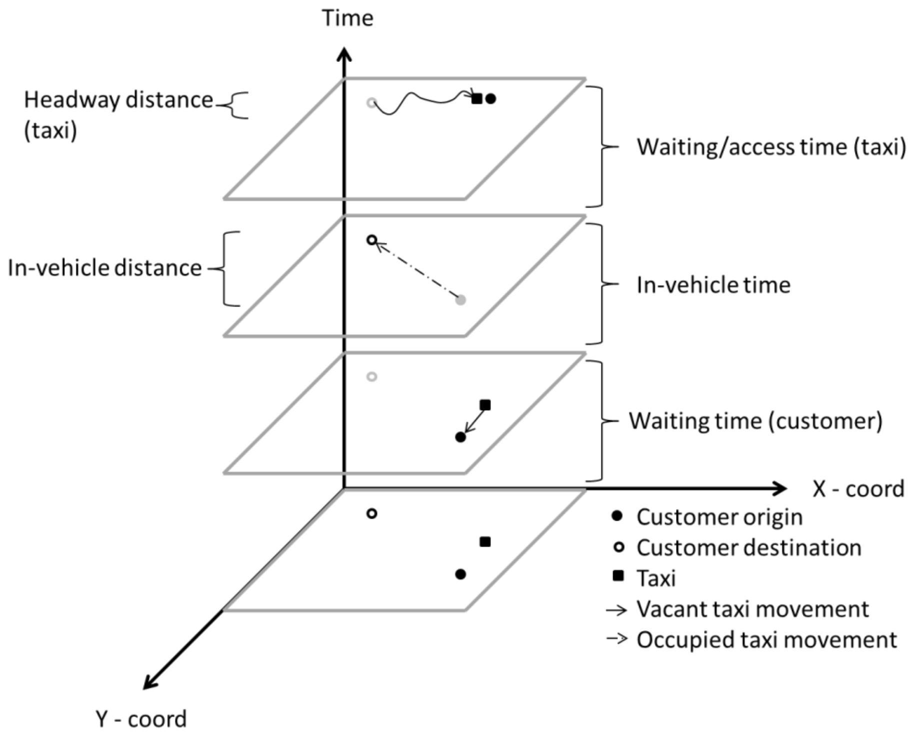

Figure 4-11 Space-time diagram of the hailing market activities

Figure 4-12 shows the space-time diagram of the activities realized by the agents in the stand market.

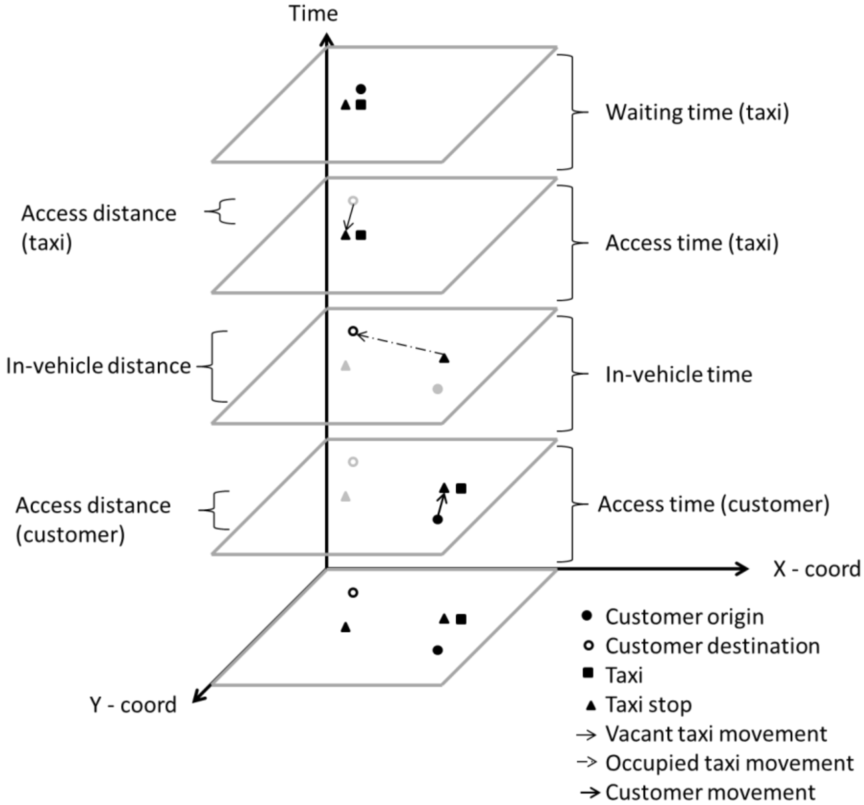

Figure 4-12 Space-time diagram of the stand market activities

4.6. Theoretical use case: the Sioux Falls network

The current model has been applied in the Sioux Falls network, a small-scale test network proposed by Leblanc et al. (1975) and adapted by Bar-Gera (2012). It is widely used in the literature for testing different algorithms, from simple routing algorithms to complex Transport Assignment Problem solvers. Five figures depicting the graph of study are presented in Annex II.

The agent-based model has been tested for a fixed origin-destination demand matrix and 25 different fleet sizes, generating a total of 2.500 customer trips served by 650 vehicles. In order to calculate average results of the performance indicators, 50 iterations have been run for each supply, creating a total of 125,000 trips satisfied by 32.500 vehicles. The results obtained are presented for both the drivers and the customers in the graphs below. Performance indicators have been obtained for each operation mode and agent. The driver indicators are total distance traveled, occupied and vacant time, occupied and vacant distance, income and benefits. The customer indicators are waiting time and travel cost. Finally, the system costs and the optimum fleet are also presented.

4.6.1. Dispatching Model

Figure 4-13 shows the relation between total benefits of drivers, the total system costs and customer costs obtained from the agent-based model.

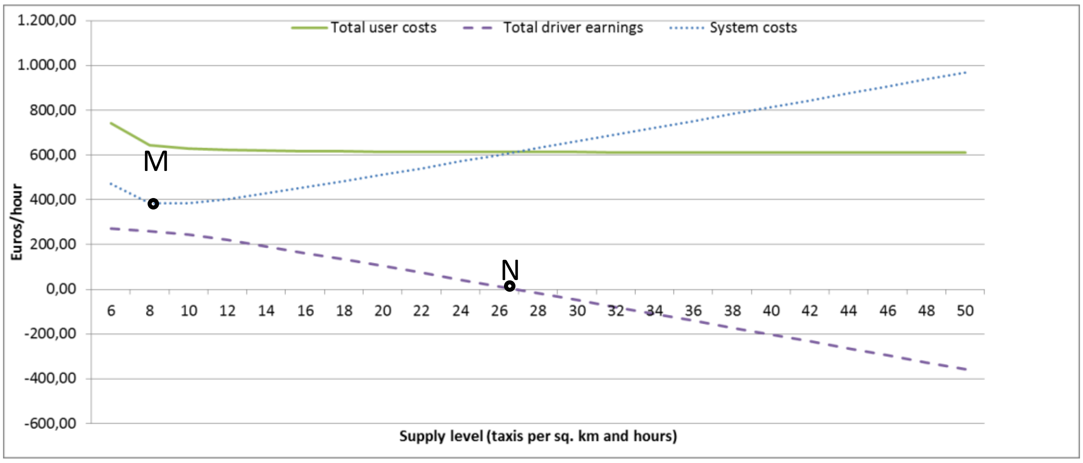

Figure 4-13 Customer costs, system costs and driver benefits related to different fleet sizes and to a fix demand for the dispatching mode model.

It can be observed that the system's optimum fleet is where the system cost is minimum. Smaller fleet than this optimum fleet produces more benefit to the taxi drivers due to the higher number of trips, but the customers' costs due to the waiting time are higher. Higher fleet than the optimum fleet has no significant effects on reducing the waiting time of customers, but the benefits of the drivers are dramatically reduced due to the lower number of trips. The first best solution (M) and the second best solution (N) can be also identified.

It is interesting to compare the results obtained from both the aggregated and the simulation models when applied to the dispatching market for the Sioux Falls theoretical network, whih are shown in Figure 4-14.

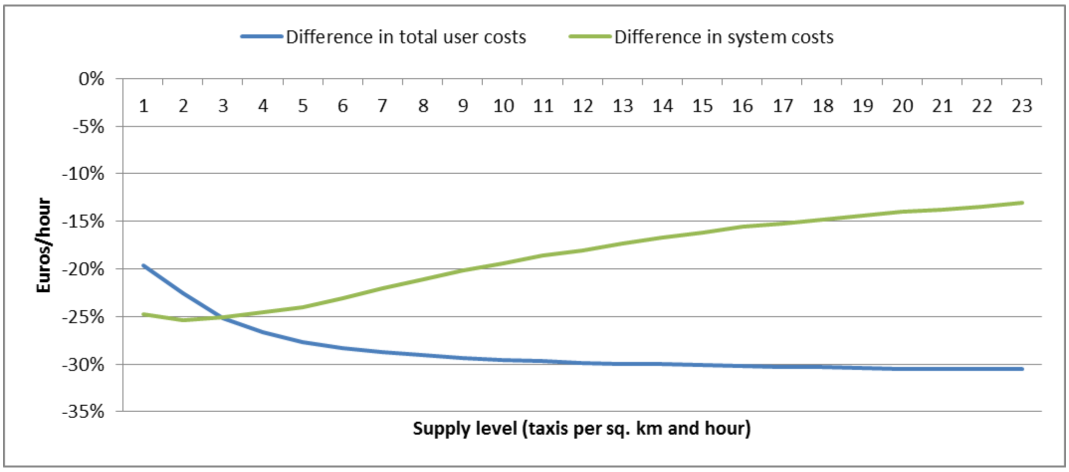

Figure 4-14 Difference in the customer costs, system costs and driver benefits between the aggregated and the simulation models.

The difference between the two models in the estimation of the user costs ranges from -20% to -30%, depending on the number of taxis (the aggregate model underestimates the user costs). At the same time, the difference in the system costs ranges from -25% to -15% depending on the number of taxis, which means that when increasing the number of taxis, the error between both models is reduced.

Figure 4-15 shows the relation between vacant and occupied kilometers and their ratio, which is defined as the rate between the occupied and vacant distances for each fleet size.

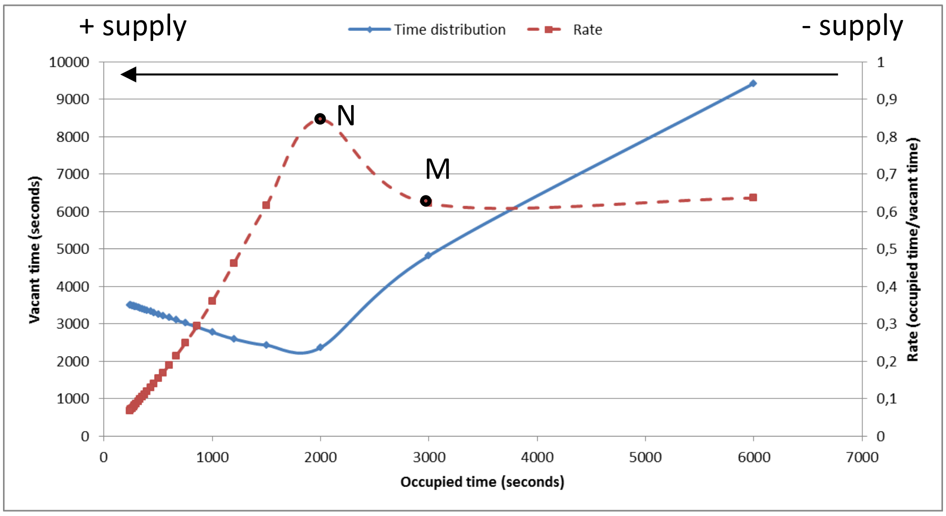

Figure 4-15 Relation between the vacant and the occupied times for the dispatching mode model.

When the fleet size is small, the occupied time is high, but the vacant time is also high due to inefficiencies of the system; at the same time the waiting time of the customers is high. When the supply is high, the number of occupied kilometers is low, while the number of vacant kilometers is high. The first best solution (M) and the second best solution (N) can also be identified. The driver optimum fleet corresponds to the maximum rate between occupied and vacant times, as shown in Figure 4-15, where the time composition (vacant and occupied) and the occupancy rate are presented. This driver optimum fleet is smaller than the system optimum fleet. The rate value of the system optimum fleet is 0.65, which means that the occupied time is 35-40% of the total time. The rate value of the drivers' optimum is 0.82, which means that the occupied time is 45%.

4.6.2. Stand and hailing Models

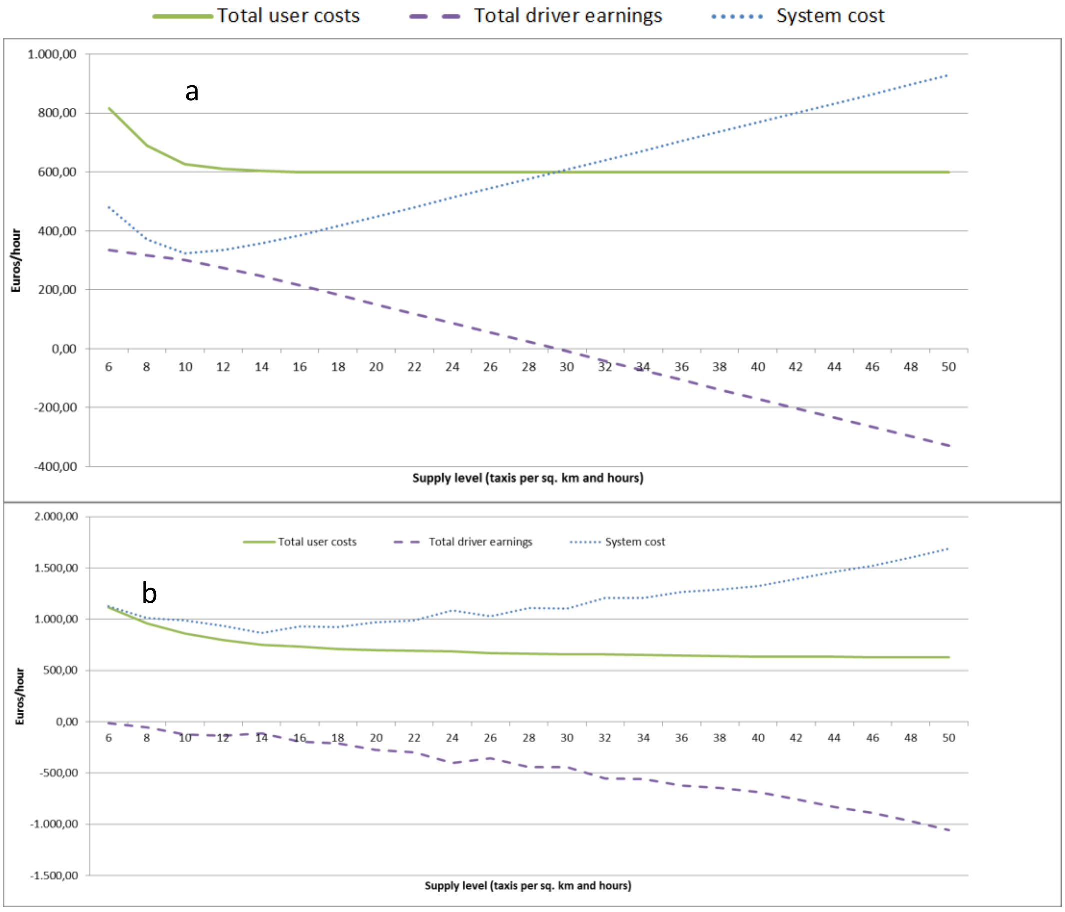

Figure 4-16 Customer costs, system costs and driver benefits related to different fleet sizes and to a fix demand for the stand (a) and hailing (b) models.

Figure 4-16 shows the driver benefits and system's and customers' costs for the stand and the hailing modes. In the hailing mode (Figure 4-16b), the system optimum fleet and the respective system costs are larger than the respective values in the other transport modes. The benefits of the drivers are always negative, which means that in this case, subsidization for maintaining a minimum threshold of waiting time is needed. The results obtained in the stand market (Figure 4-16a) are very similar to the results obtained for the dispatching market.

Figure 4-17 shows the relation between vacant and occupied time. In the stand mode, (Figure 4-17a) this relation is linear, and the rate of the system's optimum fleet is 1.3, which means that the occupied time is 55-60% of the total time. In the hailing mode, (Figure 4-17b) the relation between vacant time and occupied time is also linear, and the rate of the system optimum fleet is 0.16, which means that the occupied time is 10-15% of the total time.

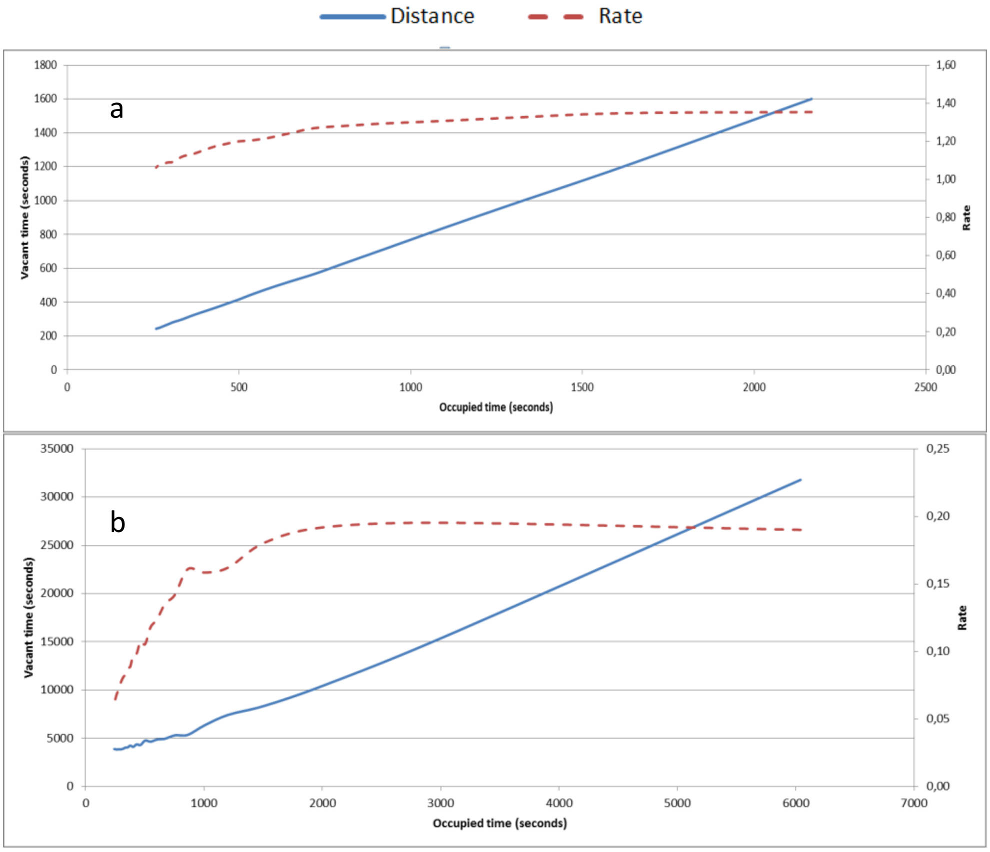

Figure 4-17 Relation between the vacant and the occupied times for the stand (a) and hailing (b) mode model.

4.6.3. Comparison of the three operation modes

Average results from the simulations are presented in Table 4-1.

Table 4-1 Simulation results of the three operation modes.

	Dispatching	Stand	Hailing*
Optimum fleet for system cost (vehicles)	8	10	14
Average occupied distance (m)	6.100	4.900	3.500
Average vacant distance (m)	4.200	330	14,000
Average occupied time (min)	25	20	14,5
Average vacant moving time (min)	35	16	90
Average vacant standped time (min)	0	24	0
Income (euros/h)	67	54	38
Driver unitary benefits (euros/hour)	32,5	30	-8
Rate occupied/vacant moving time	0.65 (35-40%)	1.3 (55-60%)	0.16 (10-15%)
Rate occupied/vacant distance	1.5 (60%)	15 (94%)	0.25 (20%)
Average customer waiting time (sec)	136	65	392
Customer unitary cost (euros/hour)	6,45	6,25	7,5
System cost (euros/hour)	385	325	865

*In the hailing case, more than one hour was needed in the majority of the simulations for completing all trips since the taxis are randomly looking for customers.

The system's optimum fleet of the dispatching mode for the same demand is the smallest one; the hailing optimum fleet is the highest one, which is due to the reason that taxis circulate randomly looking for customers while in the other markets there is a mechanism for matching customers with vehicles (taxi stands or dispatching centers). The relation between occupied and vacant time is 10-15% for the hailing market since the taxis are constantly circulating and randomly looking for customers; 35-40% for the dispatching market, where customers call taxis, reducing the vacant distance; 55-60% for the stand market where taxis wait at taxi stands, having to return there before picking up a new customer.

4.7. Discussion on the hypothesis of uniform demand

A uniform demand has been assumed in both aggregated and agent-based simulation models. In order to study the impacts of the spatial distribution of the demand in the waiting time of customers, different spatial demand distribution profiles have been proposed. Each demand profile has been run with different fleet sizes and operation modes and tested using the simulation model in the Sioux Falls network. In order to define intermediate distribution configurations, the Gauss distribution has been used with different coefficients.

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$

where,
a and c have been set to 1 and 12.5 respectively
b has different values between 0.1 and 100 in order to generate intermediate distributions
x is the zone identity

The demand profiles are presented in Annex III.

4.7.1. Comparison of the results obtained for each demand distribution

A total of nine demand distributions have been developed, ranging from uniform to linear or Gaussian distributions. The nine distributions can be observed in Figure 4-18 in a cummulative trip diagram.

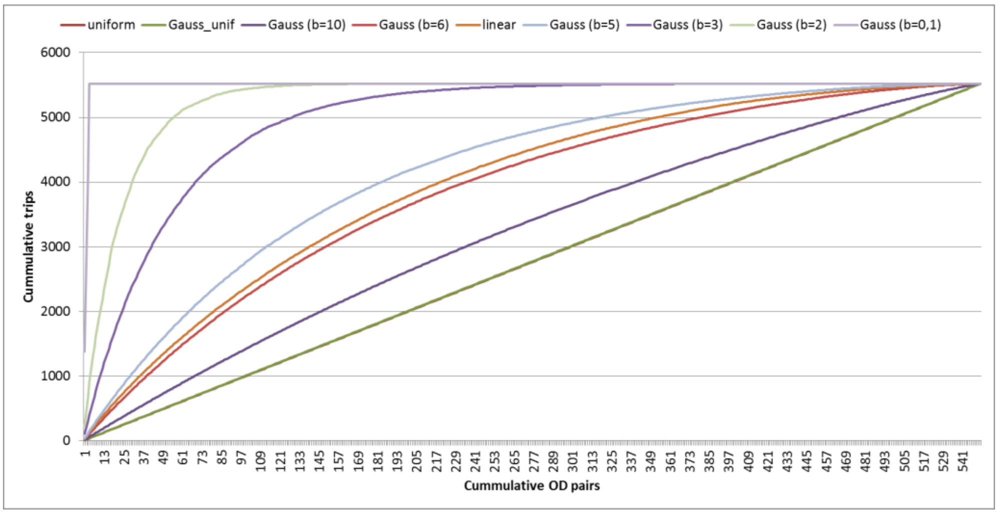

Figure 4-18 Lorenz curves for the different demand distributions

For each one of the demand distributions presented above, the GINI coefficient has been calculated and presented below. The GINI coefficient proposed by Gini (1912) estimates the degree of uniformity of the distribution using the Lorenz curve, and it is calculated as shown in Figure 4-19:

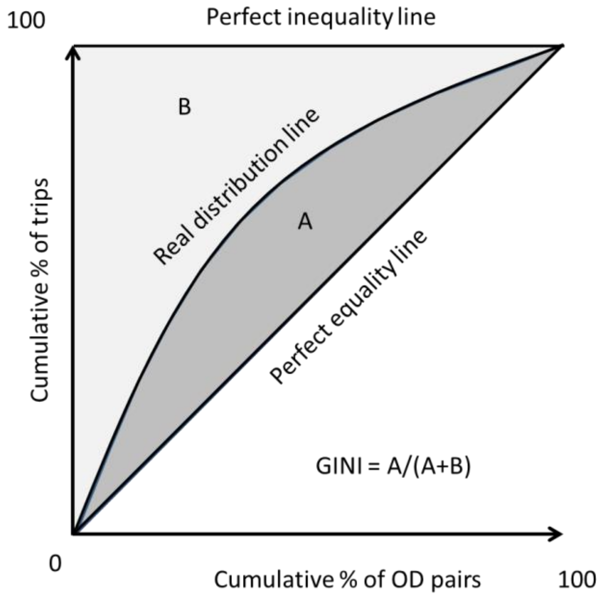

Figure 4-19 Methodology for the calculation of the GINI coefficient.

The perfect equality line, which represents a uniform distribution, has a GINI coefficient equal to 0. The perfect inequality line, which represents that all trips have the same origin and destination, has a GINI value of 1.

The GINI coefficient values of the above distributions are presented in Figure 4-20.

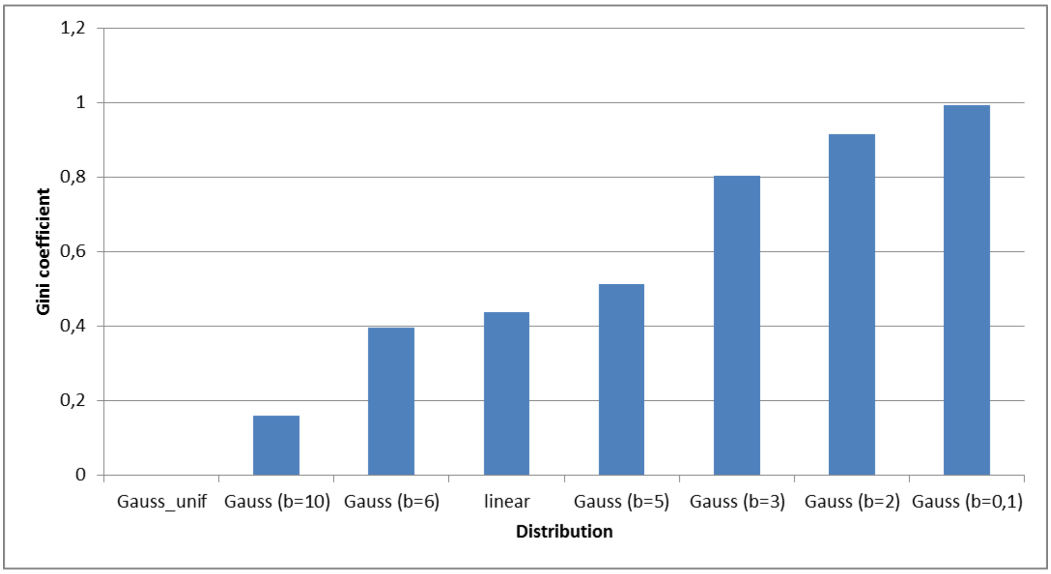

Figure 4-20 GINI coefficient values of the nine demand distributions

The 9 demand distributions have been used for the creation of 9 OD matrices, with a total of 1.100 trips. A total of 5 fleet sizes (100, 200, 300, 400 and 500 vehicles) and 3 operation modes (hailing, dispatching and stand) have been tested for the nine demand profiles, generating a total of 135 configurations. Each configuration has been tested 10 times in order to obtain average values of the waiting time of customers, which means that the total satisfied demand is approximately 1.500,000 trips. The results obtained are presented in Figure 4-21.

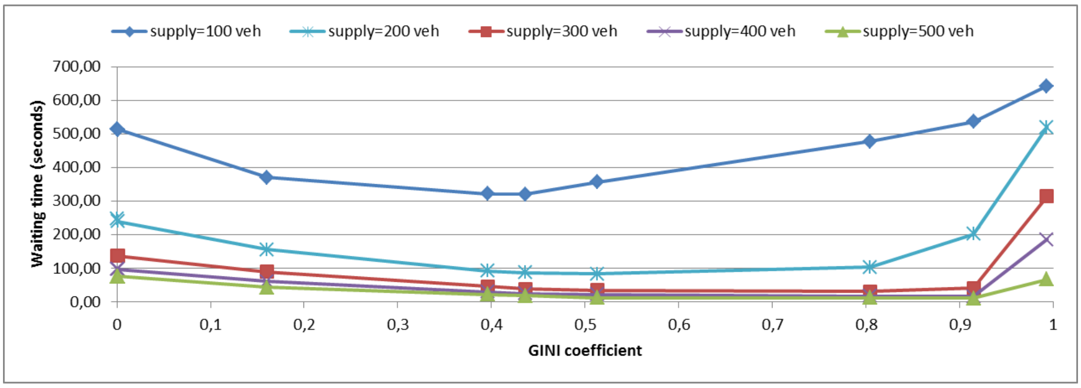

Figure 4-21 Waiting time for the different demand and supply configurations for the hailing operation mode

In the case of the hailing mode, there is an important reduction of 65% - 75% when the GINI coefficient is near to 0,5 depending on the fleet size. For GINI values between 0,5 and 0,9 the waiting time values remain stable for high fleet sizes, while for low fleet sizes it increases to the uniform demand values. Finally, for GINI values near to 1 (highly concentrated demand), most of the waiting times are worse than the waiting times of the uniform distributions.

The waiting times obtained by the aggregated model are slightly underestimated, as it observed in Figure 4-22.

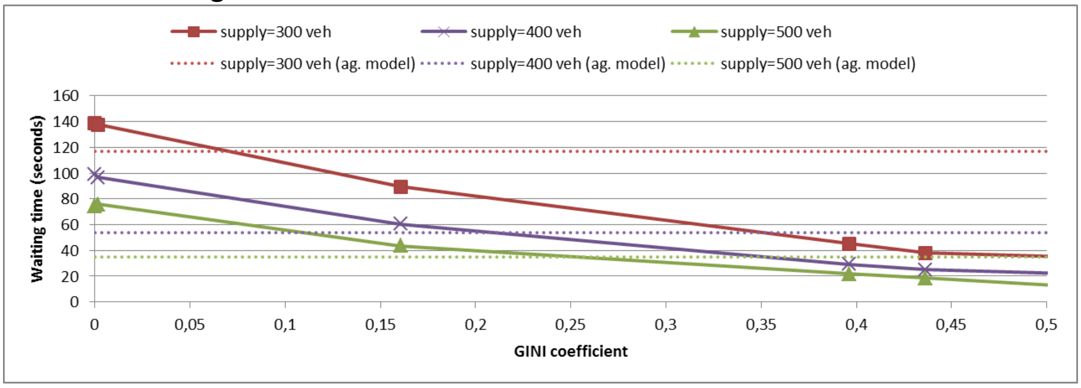

Figure 4-22 Waiting time underestimation by the aggregated model for the hailing operation mode

The underestimation increases with the size of the fleet. This underestimation is counter-balanced with a certain grade of non-uniformity, increasing with the fleet size. For uniform demand (GINI coefficient equal to 0) the aggregated model underestimated the waiting time of customers from 15% to 50% depending on the number of taxis. For demand profiles with a 0.5 GINI coefficient, the aggregated model overestimates the waiting time from 100% (large fleets) to 300% (small fleets).

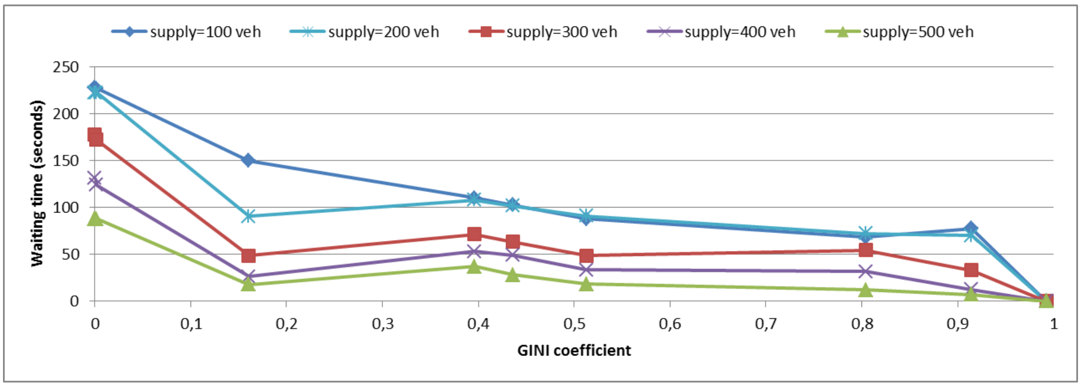

Figure 4-23 Waiting time for the different demand and supply configurations for the stand operation mode

In the case of the stand mode, there is an initial reduction of 40% to 80% depending on the fleet size, and the values remains stable between GINI coefficient values of 0,1 – 0,8. For the lowest fleet size the reduction is almost linear.

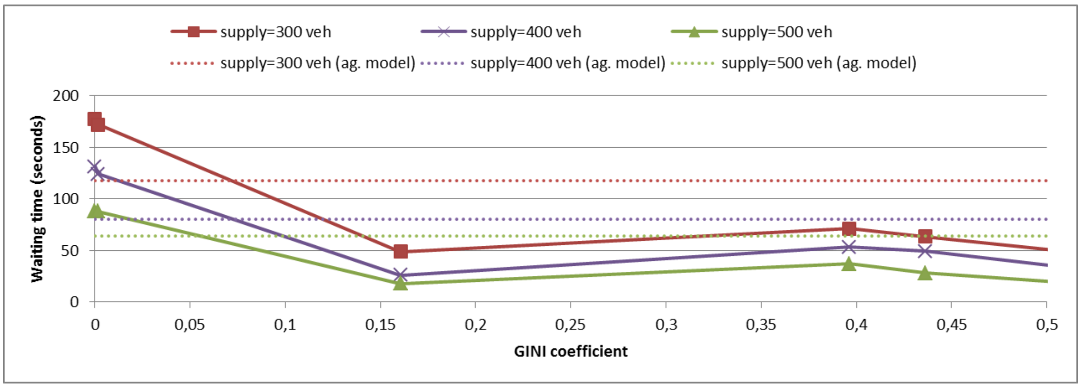

Figure 4-24 Waiting time underestimation by the aggregated model for the stand operation mode

Similar results are obtained when comparing the results obtained by the aggregated model for a uniform demand. For uniform demand (GINI coefficient equal to 0) the aggregated model underestimated the waiting time of customers from 30% to 40% depending on the number of taxis. In this case, the waiting time is equal to the waiting time of a demand related to a GINI coefficient of 0.075, while the overestimation for higher GINI coefficients presents values from 100% to 300%.

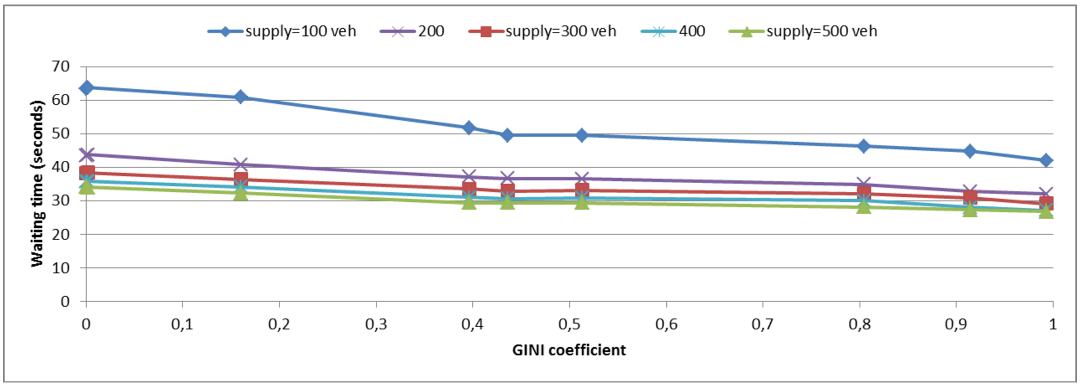

Figure 4-25 Waiting time for the different demand and supply configurations for the dispatching operation mode

In the case of the dispatching mode, the reduction is lower, between 20% and 30%, with a clear linear tendency as the GINI coefficient value increases.

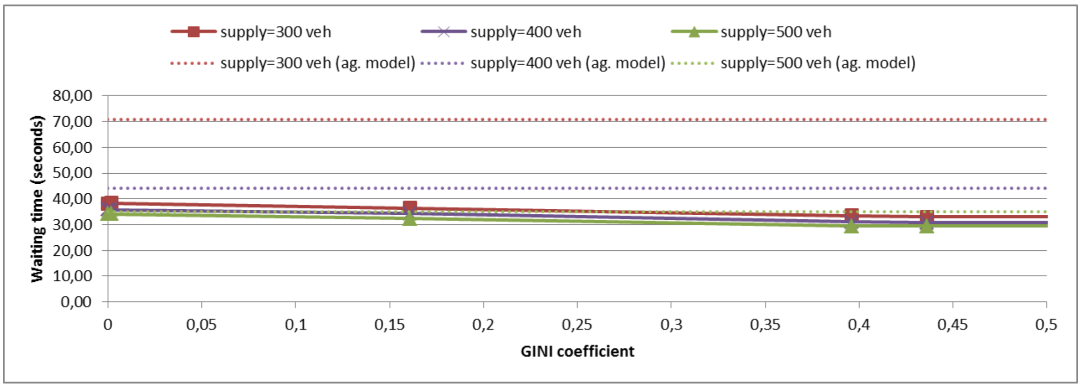

Figure 4-26 Waiting time underestimation by the aggregated model for the dispatching operation mode

For the dispatching market, there is always an overestimation of the waiting time by the aggregated model, ranging from 100% for small fleets to 0% for large fleets.

The overall conclusion is that the same waiting time can be related to different fleet sizes for the same demand, depending on the spatial profile of the demand. The spatial distribution of the demand for taxi services is therefore a significant factor when defining the optimum taxi fleet size. The aggregated model underestimates the waiting time of the uniform demand distribution while overestimates it for non-uniform demand distributions.

4.8. Conclusions

The developed agent-based model is capable of simulating the three operation modes (hailing, stand and dispatching). Results for the dispatching model have been compared to a mathematical aggregated model, presenting similar tendency and results, therefore validating the formulatios of the aggregated model proposed in chapter 3. The model outputs in terms of system costs and total time composition (vacant and occupied time) have been presented, concluding in the system´s and the drivers´ optimum fleet. As concluded by most authors, the drivers' optimum fleet is smaller than the system optimum fleet.

The aggregated model underestimates the costs in comparison to the agent-based model, ranging from -20% to -30% for the customer costs and from -25% to -15% for the system costs. The spatial distribution of the demand has a significant effect in the waiting time (when it is more concentrated, waiting time is lower). This reduction depends on the mode of operation and the taxi fleet size, presenting higher variations for smaller fleets. The hypothesis of non-uniform demand has been tested, obtaining overestimations of the waiting time up to 300% in comparison to the uniform demand scenario. Aggregate formulations should be provided for the non-uniform demand case.

Future research should focus in the customers' behavior, giving them the possibility of changing operation mode if the waiting time exceeds a threshold value, or presenting customer classes with different willingness to pay for taxi services. In order to obtain more realistic and representative results, there is a necessity for testing different demand profiles and levels in different network geometries, calibrating the parameters of the model with real world data. Finally, new technologies should be tested using simulation models in order to prove their benefits and performance levels and create the necessary rules for providing information to drivers about the hot spots of the city or demand forecasts in the different city zones.

4.9. References

Bar-Gera H. (2012) Transportation Network Test Problems, (available online at http://www.bgu.ac.il/~bargera/tntp/, accessed on July, 30 2012).

Dijkstra E. (1959) A note on two problems in connexion with graphs. *Numerische Mathematik* **1**,. 269-271.

Gini C. (1912) Italian: Variabilità e mutabilità, contributo allo studio delle distribuzioni e delle relazioni statistiche.

LeBlanc, L.J., Morlok, E.K., Pierskalla, W.P. (1975) An efficient approach to solving the road network equilibrium traffic assignment problem. *Transportation Research A* Vol. **9**, pp. 309-318.

Rodrigue J-P, Comtois C. and Slack B. (2012) The Geography of Transport Systems. Hofstra University, Department of Global Studies & Geography.

5. CASE STUDY: BARCELONA

In this section, the two developed models are applied to the taxi sector of the city of Barcelona, providing significant results in terms of minimum and optimum fleet sizes.

5.1. Introduction to the Barcelona taxi sector

The taxi sector has a high importance in the Metropolitan Area of Barcelona (AMB), being an important transport mode in a continuous urban region of more than 3 milions inhabitants and roughly 500 km^2 area extension. The public body responsible for the management of the taxi sector is the Institut Metropolità del Taxi (IMT), which defines the regulation related to the number of licenses, the fares, the requirements asked from the drivers and the characteristics of the vehicles.

The current number of taxi licenses is 10.523, from which 5% belong to private companies and the 95% to individuals. The number of licenses has been frozen since 2005 (1 license per 275 inhabitants). However, the number of driver licenses has increased significantly during the last years (13.136 in 2010, with rate drivers/vehicles of 1.26 while this rate was 1.14 in 2007). The evolution of the number of taxi licenses and driver licenses is presented in Figure 5-1.

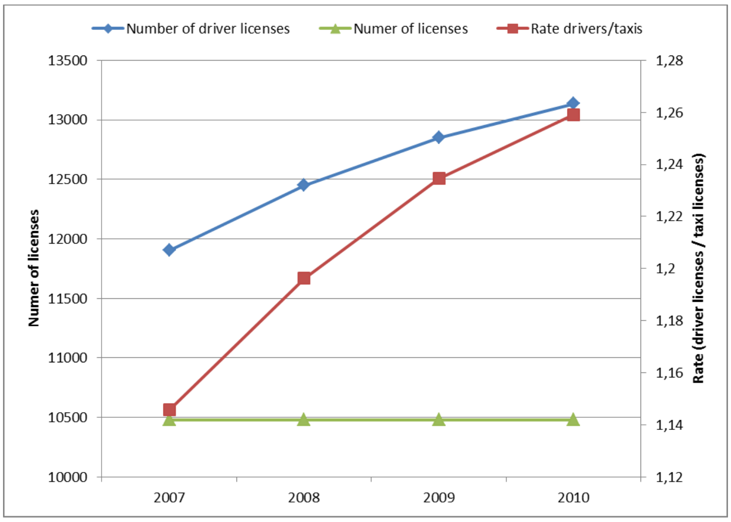

Figure 5-1 Evolution of the number of licenses and the rate drivers versus taxis

The number of companies has been significantly increased during the last 4 years, with an increase of roughly 100%. The number of licenses per company has decreased significantly, while the number of drivers per company remains stable. The combination of these two facts results in an increase in the rate of drivers/taxis, as observed in Figure 5-2.

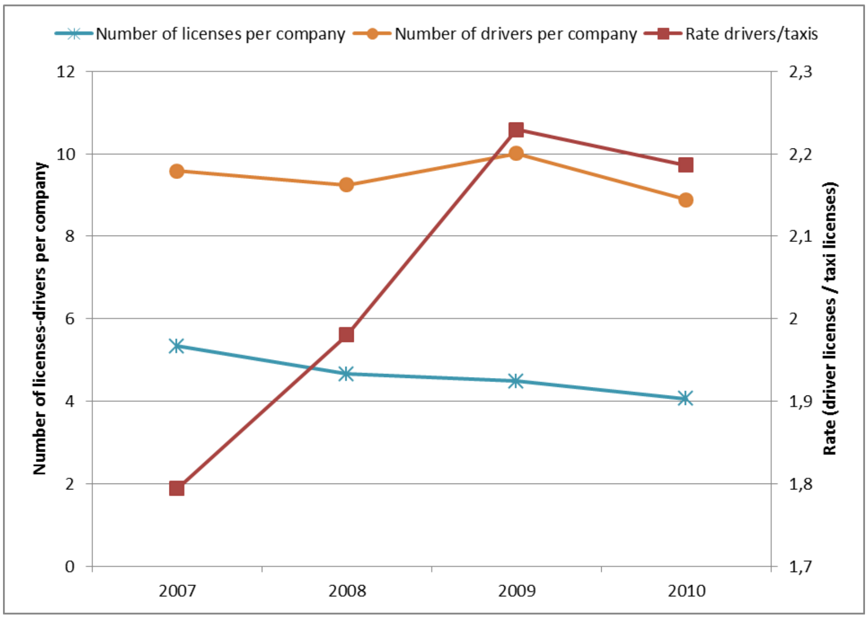

Figure 5-2 Evaluation of the number of licenses per company and the rate drivers versus taxis

The number of dispatching centers is lower than the number of companies (roughly 25%) but the number of license holders collaborating with them is considerably higher (roughly 800%).

This increase in the number of drivers per license, and therefore in the taxi-hours, along with the fact that the demand has remained stable during the last years (SACon & IMT 2003) caused a significant reduction of the benefits for the taxi drivers of Barcelona. Anaylsis of the data collected by CENIT (Amat, 2010) show that the trip cost remains stable, which means that the trip distance is being reduced. This situation is reflected in the price of a taxi license, which is decreasing since 2007 (Amat, 2010).

The taxi sector in Barcelona has been traditionally regulated by the prohibition of 2 working days per week, only one of them within the weekend. There is a new regulation in 2013 in order to reduce the number of circualting taxis in Barcelona. Three working shifts have been created, depending on the type of license holder (individual or company).

The supply of taxi services has been calculated in (CENIT, 2013) for the year 2007 and is presented in Table 5-1.

Table 5-1 Hourly taxi offer during working days and weekends

		2007
Working day	*Day shift*	5.461
	Night shift	1.278
Weekend	*Day shift*	3.158
	Night shift	1.521

The distribution of the offer within the day has also been calculated in (CENIT,2013) for the year 2007 and is presented in Figure 5-3. The total number of daily taxi-hours is roughly 100,000.

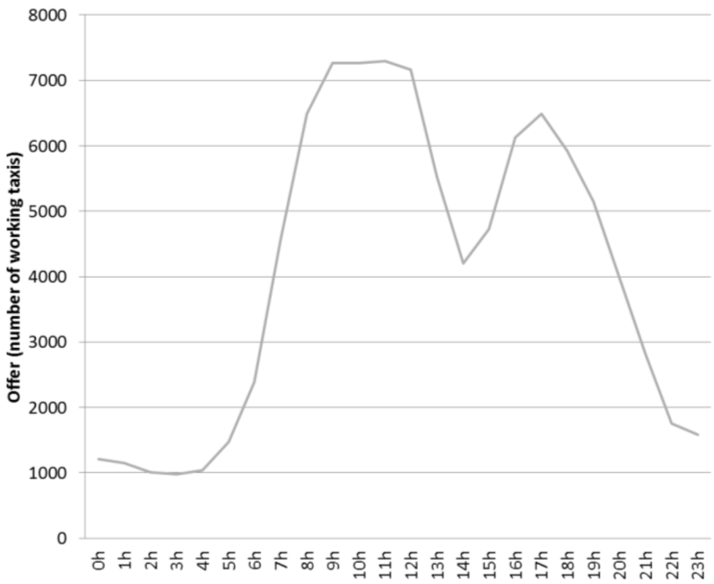

Figure 5-3 Taxi supply during a working day in Barcelona

The number of taxi stands is roughly 250, where 3 of each 4 stands are frequently used (Amat, 2010).

The demand for taxi services has been estimated by IMT in more than 200,000 trips per day, which corresponds to 1.5 – 2 % of the total trips of the AMB (roughly 13,000,000 daily trips). CENIT (2013) estimates the demand for taxi trips in 60,000,000 trips per year. The waiting time of the taxi customers has been estimated in (Amat 2010) in 3 - 4 minutes during the day and 6 - 7 minutes during the night.

The fares applied to the taxi sector depend on the time of the day and are presented in Table 5-2 (Amat 2010 and IMT).

Table 5-2 Urban taxi fares between 2004 and 2013

	Shift	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Shifts	T-2	6h a 22h		7h a 21h		8h a 20h					
	T-1	22h a 6h		21h a 7h		20h a 8h					
Flag-Down	T-2	1,2	1,3	1,45	1,75	1,8	2			2,05	
	T-1	1,35	1,4	1,55	1,85	1,9					
kilometric charge	T-2	0,71	0,74	0,78	0,78	0,82	0,86	-	0,9	0,93	0,98
	T-1	0,91	0,96	1	1	1,04	1,1	-	1,15	1,18	1,24
hourly charge	T-2	15,33	16	16,95	16,95	17,8	18,6	-	19,49	20	21,23
	T-1	15,54	16,33	17,42	17,42	17,9	18,8	-	19,59	20,44	21,7

5.2. Data of the Barcelona taxi sector

5.2.1. The network

Barcelona road network consists of more than 1.600 kilometers of streets and more than 8,000 intersections. The network used for the modeling of the city consists of more than 13,000 links, 62% of which are bidirectional, which means that the total number of one-way links is near to 20,000.

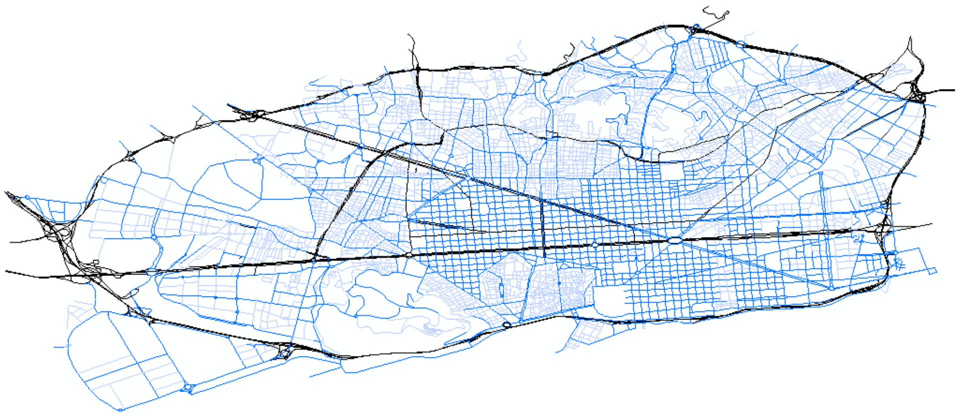

Figure 5-4 Barcelona network used in the agent-based taxi services model

5.2.2. The OD matrix

The sample used for the estimation of the OD matrix for all transport modes is composed by 40,000 trip descriptions for the day before with a confidence level of 95% obtained from 33,249 phone and household surveys to people older than 15 years in 11 municipalities during the period 01/02/2007 - 31/05 /2007. A total of 7.35 millions of motorized trips were obtained, a 68.3% of which had origin or destination in the city of Barcelona.

5.2.3. The taxi trips database

A database of more than 1,200,000 taxi trips during the last nine years (2004-2012) is analyzed and significant results of the evolution of the most important performance indicators are presented. The database is used for the calibration of the presented models. Both models are applied to the “real data” of the taxi sector of Barcelona and compared to each other. The results are presented in terms of waiting time of customers, and minimum and optimum density of taxis. The first best solution and second best solutions are identified and commented in the results.

The databases have been provided by Center for Innovation in Transport (CENIT). CENIT collaborates with the Metropolitan Institute of Transport (IMT) in the city for the creation of a taxi Observatory. The taxi Observatory is a tool for the evaluation of the actual situation and the implementation of a new structure in the taxi market for a more efficient relation between taxi drivers and the demand. A large amount of information is needed for the Observatory, and technologies such as ICT, GIS and GPS can assure flow of information between taxis and the system regulators, achieving a better mobility.

Table 5-3 and Table 5-4 present the most important indicators of the database.

Table 5-3 Average cost, travel and idle time, distance of the database of taxi recorded trips

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Average cost (euros)	6,69	7,18	7,83	7,94	8,47	8,58	9,22	9,51	9,84
Average travel time (min)	13,12	12,85	13,38	12,73	12,58	12,23	12,64	12,71	12,70
Average distance (km)	4,97	5,40	5,45	5,23	5,05	5,09	5,69	5,76	5,83
Average idle time (min)	15,62	17,16	15,98	16,13	17,89	20,07	23,42	26,50	27,31
Average idle distance (km)	4,64	4,62	4,50	3,64	3,82	3,13	2,52	5,21	6,46

Table 5-4 Total number of trips, vehicles, drivers, costs, occupied ad vacant time and distance of the database of taxi recorded trips

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Taxi trips	17.760	114.988	90.250	64.779	54.820	256.140	313.127	147.996	119.858
Taxi vehicles	15	25	29	18	12	39	69	40	26
Taxi drivers	15	27	29	18	12	108	130	77	68
Daily trips per driver	20,6	22,2	21,7	20,7	19,5	17,5	18,0	15,3	14,2
Total trip costs (euros)	119.253	855.299	738.266	556.369	485.987	2.421.596	1.651.637	1.554.178	1.295.433
Total occupied time (min)	233.030	1.477.992	1.207.482	824.763	689.612	3.131.664	2.024.899	1.880.590	1.522.234
Total occupied distance (km)	88.323	621.348	491.830	338.782	276.581	1.302.949	911.998	851.843	698.963
Total vacant time (min)	277.460	1.973.712	1.441.912	1.045.087	980.459	5.141.504	3.751.581	3.922.250	3.273.474
Total vacant distance (km)	82.480	531.118	406.054	235.461	209.286	802.126	403.835	771.542	774.303

The data contained in the database is analyzed in order to identify the evolution of the most important indicators of the taxi services in Barcelona. The number of daily trips per driver has decreased significantly, from 22 in 2005 to 14 in 2012, which means a decrease of 36% due to the crisis. All the data contained in Table 5-4 are plotted together in Figure 5-5 in order to relate their variation and understand the significant changes in the taxi market in Barcelona.

The cost of the trip has increased significantly since 2004, but at the same time the number of daily trips per driver has been reduced. The characteristics of the trips (duration and length) have remained more or less the same, with a small increase in the trip length during the last years. Five periods can be recognized in Figure 5-5:

- 2004-2005: increase in the number of daily trips per driver and the trip cost, which means more income for the drivers, increase of the daily working time and distance of the drivers.
- 2005-2008: decrease of the number of daily trips and their length, increase in the trip cost. Drivers reduce the variable costs but reduce the idle distance while maintaining the working hours.
- 2008-2009: decrease of the number of daily trips but small increase in their length, increase in the trip cost.
- 2009-2010: stable number of daily trips and significant increase in their length, increase in the trip cost. Increase in the total distance, significant increase of the working hours together with a reduction of the idle distance, the drivers wait at taxi stands. Small increase in the working hours.
- 2010-2012: decrease of the number of daily trips but small increase in their length, increase in the trip cost. Redution of the working hours and significant increase in the idle distance, which means significant reduction of the income per kilometer. The total distance remains stable.

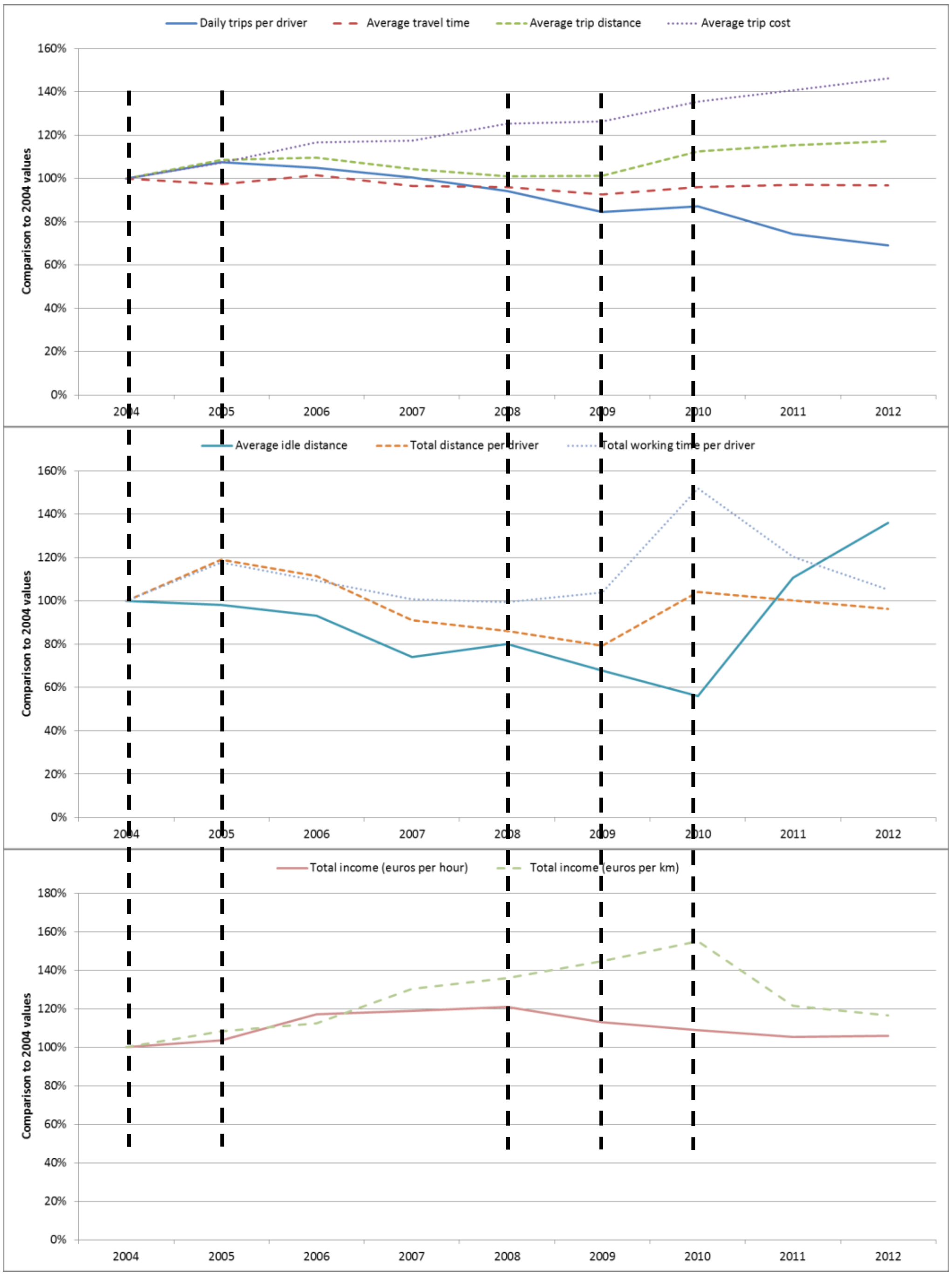

Figure 5-5 Evaluation of the most significant performance indicators of the taxi market in Barcelona

5.2.4. The taxi trips spatial database

The 33% of the trips recorded during the years 2009 and 2012 include the GPS coordinates of the trip origin and destination. Figure 5-6 shows the matching of the origin coordinates of the 270,000 trips with the network. An important concentration of origins can be observed in the two airport terminals of the city on the left side of the picture (these trips will not be taken into account in the application of the model).

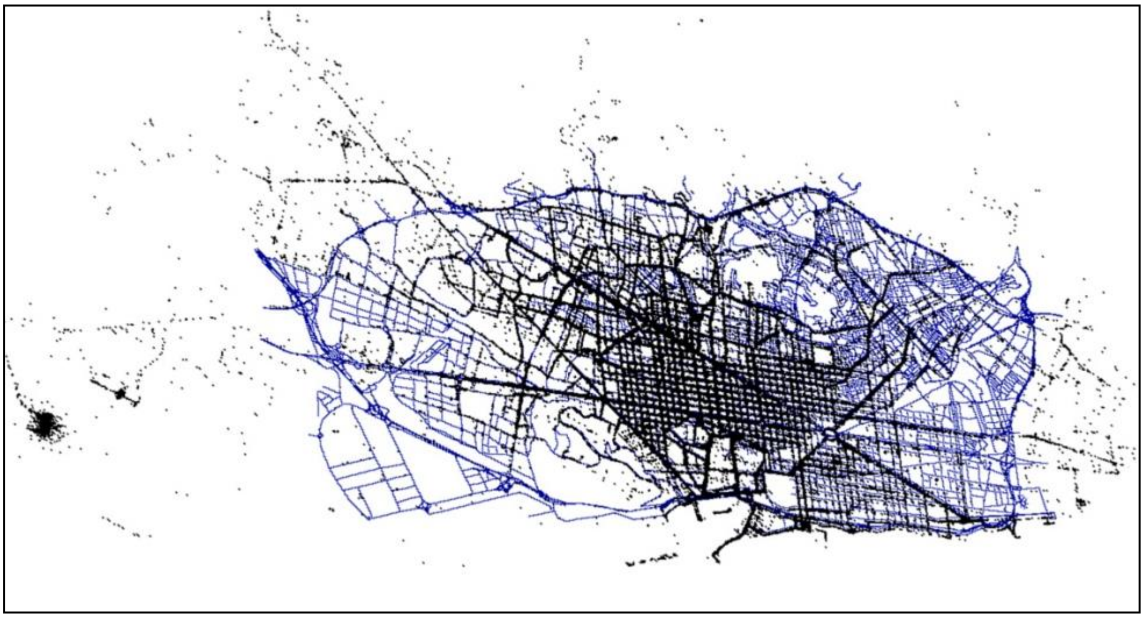

Figure 5-6 Matching of taxi origins recorded by the GPS system and the real Barcelona network

This data can be used for estimating the origin-destination matrices of the taxi trips. The sample seems to be sigfincantly representative in spatial terms since it covers the entire city. Figure 5-7 shows the trips originated and finished in each link, where the links with a high concentration of trip ends and trips starts are related to hot spots of the city.

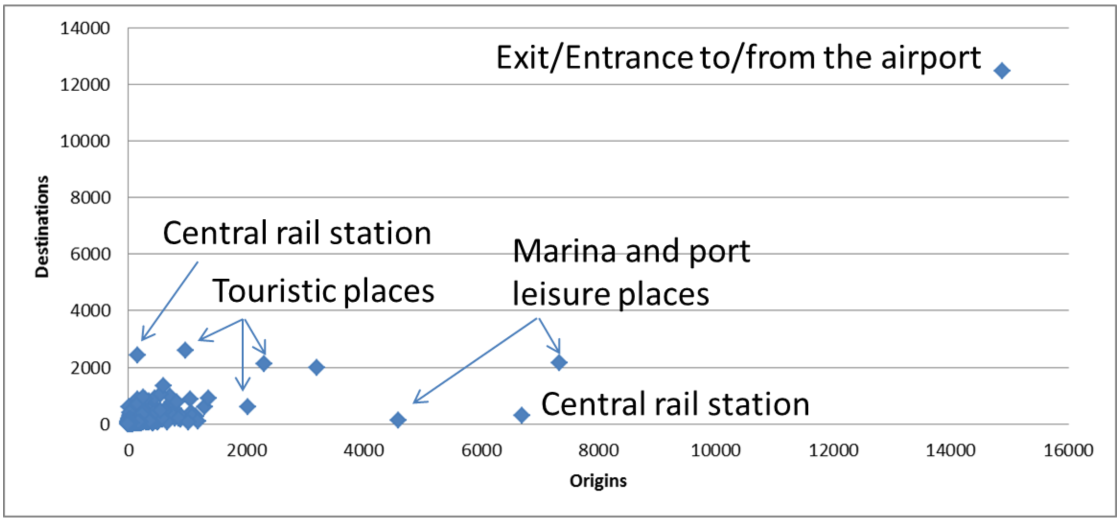

Figure 5-7 Hot spots of the taxi OD (concentration of trip origins and destinations from the taxi trips database)

Figure 5-8 shows the concentration of origins and destinations within the city. The hot spots of the city indicated in Figure 5-8 can be observed in red and orange colors for both the origins and the destinations.

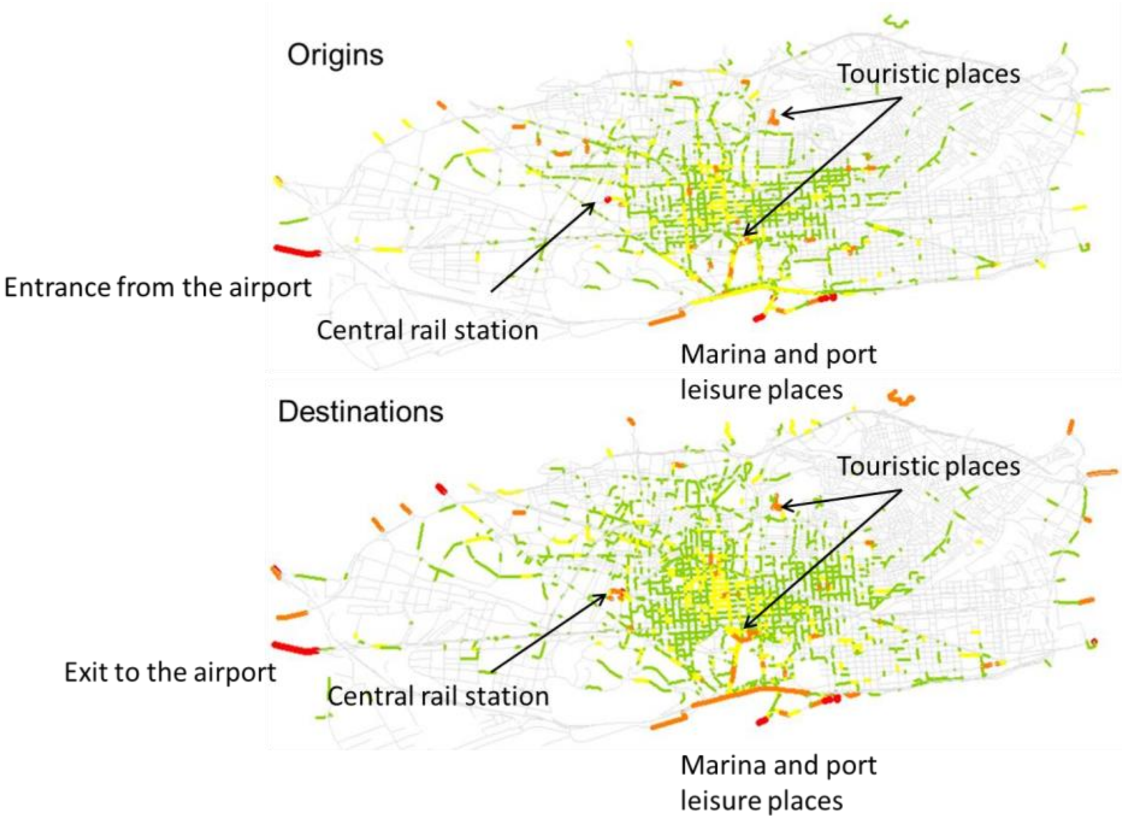

Figure 5-8 Attractions and generations of the links of Barcelona

5.3. Model implementation

The database is used for the calibration and implementation of the two developed models. The obtained results are presented and discussed below.

5.3.1. Results of the aggregated model

The reference values have been obtained from a database of more than 1,200,000 taxi trips recorded during 9 years (2004-2012) by more than 100 vehicles and 200 drivers and a total transaction of more than 10,000,000 euros. The total number of kilometers driven by the drivers is roughly 10,000,000, half of them occupied. The total number of hours driven by the drivers is roughly 600,000, a third of them occupied.

Table 5-5 Variables definition

Variable		Reference values (Barcelona)
Model outputs	Z is the cost of the system (€) - z is the unitary system cost (€/trip)	-
	Z_d is the cost of the drivers (min) - z_d is the unitary cost of the drivers (min/trip)	-
	Z_u is the cost of the customers (min) - z_u is the unitary cost of the customers (min/trip)	-
	Z_c is the additional cost for the city (min) - z_c is the unitary cost for the city (min/trip)	-
	G is the cost of the infrastructure (min) - g is the unitary infrastructure cost (min/trip)	-
	T_W is the waiting time of customers (min)	2 - 6
	T_{IV} is the in-vehicle time of customers (min)	12 - 14
	$\bar{c}$ is the average trip fare (€)	7 - 10
	$\bar{d}$ is the average distance of the trip (km)	5 - 6
	$\bar{n}$ is the average number of trips per hour and driver (trips)	1 - 3

Decision variables	λ_d is the taxi hourly supply (vehicles per hour and area of service)	20 - 60
	D is the flag-drop charge (€)	2
	τ_{km} is the taxi fee per unit of distance (€/km)	0.93
	τ_{sec} is the taxi fee per unit of time (€/min)	0.30
Model inputs (variables)	λ_u is the hourly demand for taxi trips (trips per hour and area of service)	50 - 250
	A is the area of the region (km^2)	100
	λ_v is the hourly circulating vehicles (vehicles per hour and area of service)	8,000
	$\bar{v}$ is the average speed of the trip (km/h)	30
Model inputs (parameters)	VoT_u is the value of time of the taxi customers (€/min)	20
	VoT_d is the value of time of the taxi drivers (€/min)	20
	VoT_v is the value of time of the other drivers (€/min)	20
	α_A is the customer perception factor of the access time	3
	α_W is the customer perception factor of the waiting time	3
	α_{IV} is the customer perception factor of the in-vehicle time	1
	C_{km} is the operational cost per unit of distance of taxis (€/km)	0.35
	C_h is the hourly operational cost of the moving taxis (€/min)	0.33
	r is the area and network parameter	1.27

The data presented in Table 5-6 has been obtained from both the IMT and Amat (2010) and are introduced in the model. Table 5-6 presents and compares the obtained results with the measured values from the database.

Table 5-6 Results of the validation of the aggregated model to the city of Barcelona

	Model prediction	Measured value (2012)	Deviation
Average cost (euros)	7.96	9.84	19%
Average travel time (min)	11.2	12.70	12%
Average distance (km)	6.35	5.83	-9%

The values obtained from the model are independent of the mode of operation since they refer to the taxi demand. The predicted average distance is higher due to the non-homogeneity of the demand since most of the trips are concentrated in the city center. The average travel time is lower due to the lower congestion levels considered in the model, which applies the same congestion level for all the trips during the day, while in the database each trip has different speed depending on the congestion level and the streets included in the path. The difference on the cost is a consequence of these differences since cost in the model is calculated using the distance, while cost in the reality depends on the congestion levels. The results obtained by the model in terms of waiting time and costs for the different actors are presented in Figure 5-9, Figure 5-10 and Figure 5-11.

5.3.1.1. Application of the hailing model

It can be observed that the number of taxis per hour and km^2 should be higher than 31 since the waiting time for smaller fleets is very high due to the low number of free taxis. The real minimum fleet size is 29 taxis/hour*km^2, which means that with this taxi fleet size, the number of demanded customer hours are equal to the offered vehicle hours. It can be also observed that the maximum number of taxis with positive benefits is 36 taxis/hour*km^2 since more taxis than this value will generate losses to the drivers. Therefore, 36 is the second best solution. The optimum number of taxis taking into account the system costs is 34 –38 taxis/hour*km^2 (first best solution).

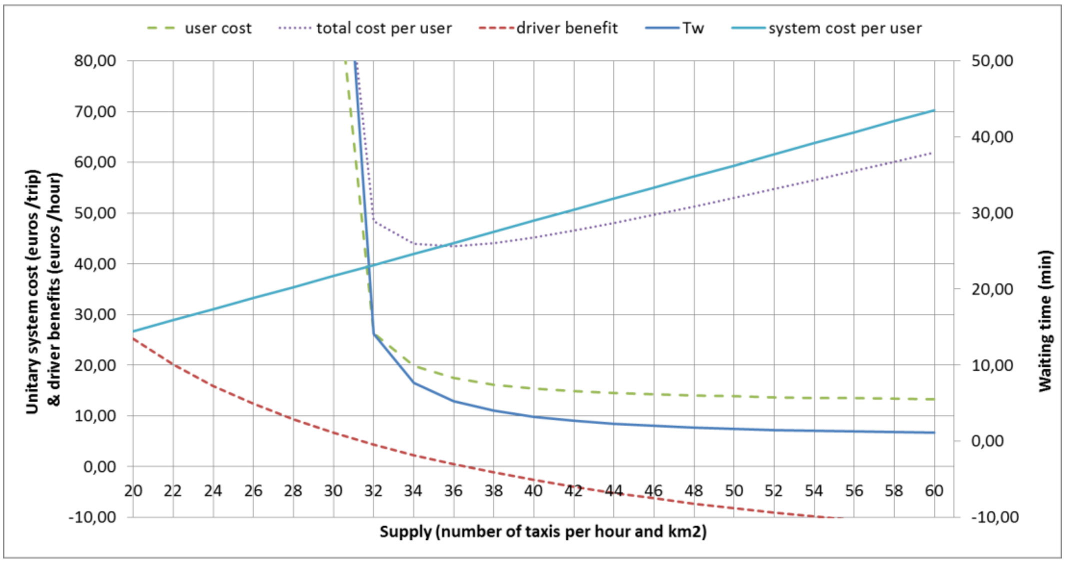

Figure 5-9 Waiting time, driver benefits and unitary costs for each fleet size obtained by the aggregated model (hailing operation mode operation mode)

In the hailing market the first and the second best solutions are equal, which is due to the presence of externalities such as congestion and pollution, which increase linearly with the number of taxis. These externalities reduce the optimum number of taxis of the first best solution to levels where the taxi drivers have benefits.

Table 5-7 Results of the application of the aggregated hailing model to the city of Barcelona

Minimum number of taxis	29 taxis/hour*km^2
First best solution	34 – 38 taxis/hour*km^2
Second best solution	36 taxis/hour*km^2
Subsidy	-

5.3.1.2. *Application of the dispatching model*

Similar results can be obtained from Figure 5-10 for the dispatching mode, but without the presence of externalities, the first best solution is larger than the second best solution, which generates losses to the taxi drivers. These losses can be subsidized by the state in order to provide a better level of service to the taxi customers by an amount of 3-4 euros per trip.

Table 5-8 Results of the application of the aggregated dispatching model to the city of Barcelona

Minimum number of taxis	29 taxis/hour*km^2
First best solution	44 – 46 taxis/hour*km^2
Second best solution	38 taxis/hour*km^2
Subsidy	3 – 4 euros per trip

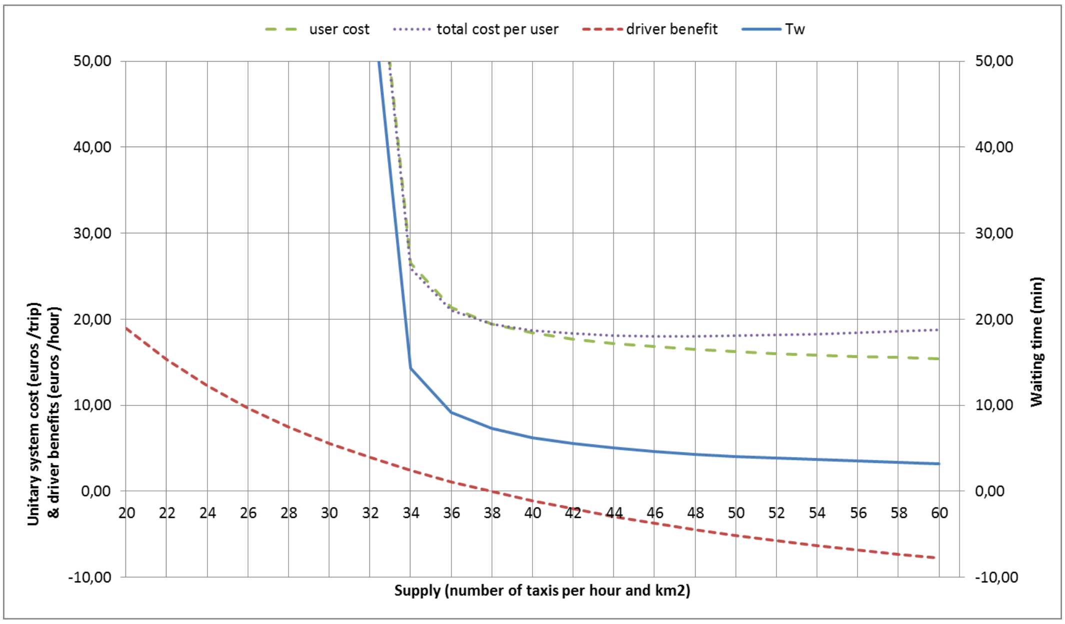

Figure 5-10 Waiting time, driver benefits and unitary costs for each fleet size obtained by the aggregated model (dispatching operation mode operation mode)

5.3.1.3. Application of the stand model

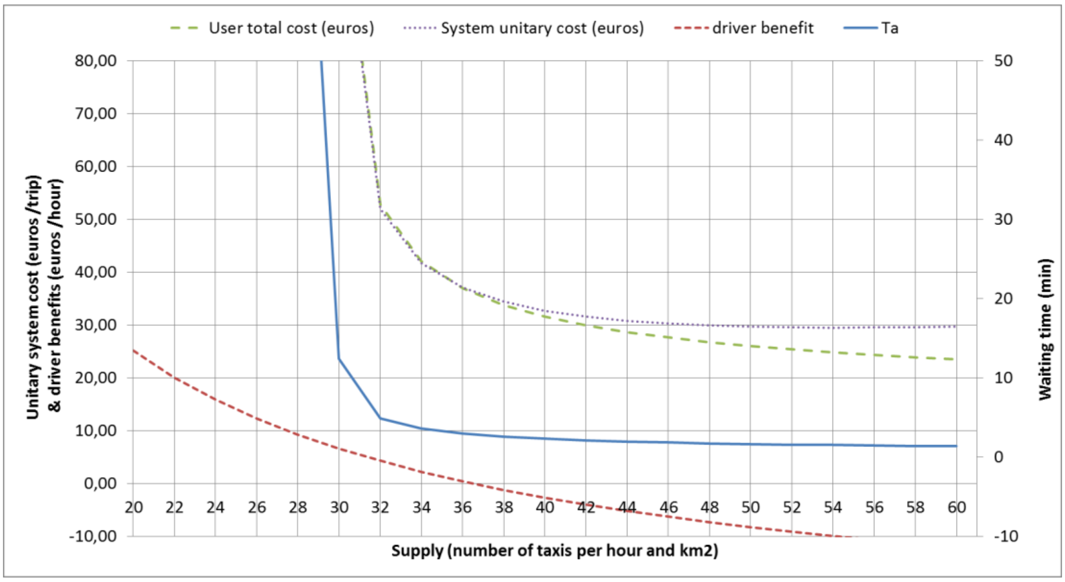

Figure 5-11 Waiting time, driver benefits and unitary costs for each fleet size obtained by the aggregated model (stand operation mode operation mode)

In this case, the system cost is almost horizontal for large fleet sizes, which makes it more difficult to detect the minimum value.

Table 5-9 Results of the application of the aggregated stand model to the city of Barcelona

Minimum number of taxis	29 taxis/hour*km^2
First best solution	52 – 54 taxis/hour*km^2
Second best solution	37 taxis/hour*km^2
Subsidy	10 euros per trip

5.3.1.4. *Comparison of the three operation modes*

The operation mode with the minimum optimum number of taxis is the hailing mode, due to the impact of the externalities of the circulating taxis, while the operation mode with the maximum optimum number of taxis is the stand mode. The maximum number of taxis with non-negative profits (second best) is very similar for the three operation modes. Finally, the subsidization of the stand mode is the highest one, since the first best and the second best have the largest difference, while in the dispatching mode the difference is lower. There is no need for subsidization in the hailing market, since the inclusion of the externalities has reduced the optimum number of taxis below the second best value. This subsidization is the amount of money that taxi drivers lose if the number of taxis is equal to the social optimum. It can also be seen as the amount of money that should be provided to drivers per trip in order to provide the waiting times to customers of the first best solution.

All the above is related to the peak hour, which means that, for different hours, the results can vary significantly. The three models have been applied to 24 hourly demand levels in order to acquire a better understanding of the provision of services during the day. The demand level is presented in figure 5-21 together with the optimum fleets obtained for each operation mode.

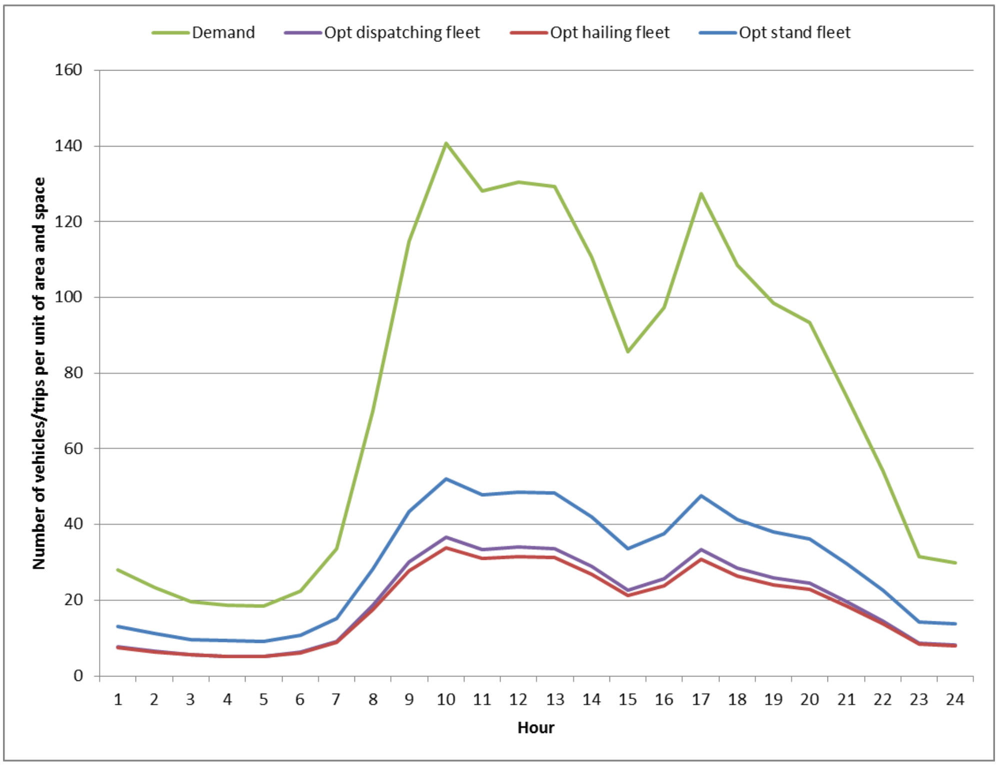

Figure 5-12 Demand and optimum fleets for each operation mode along the day

The waiting/access time, the unitary costs and the benefit of the drivers are presented in Figure 5-13, Figure 5-14 and Figure 5-15.

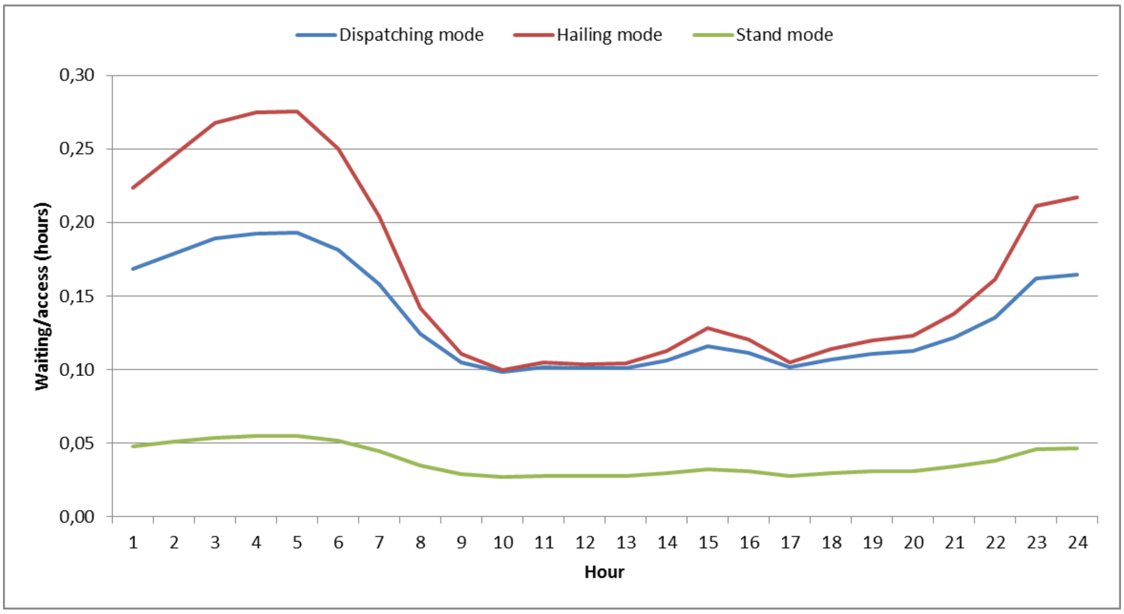

Figure 5-13 Waiting/access time for each operation mode along the day

The access times of the stand mode are significantly lower than waiting times of the dispatching and the hailing modes, but this is because the number of taxis in the stand mode is higher. At the same time, the waiting/access time at night are higher than during the day, but the increase is different for each operation mode, being the hailing mode the one which presents the highest impact of the low demand levels at night.

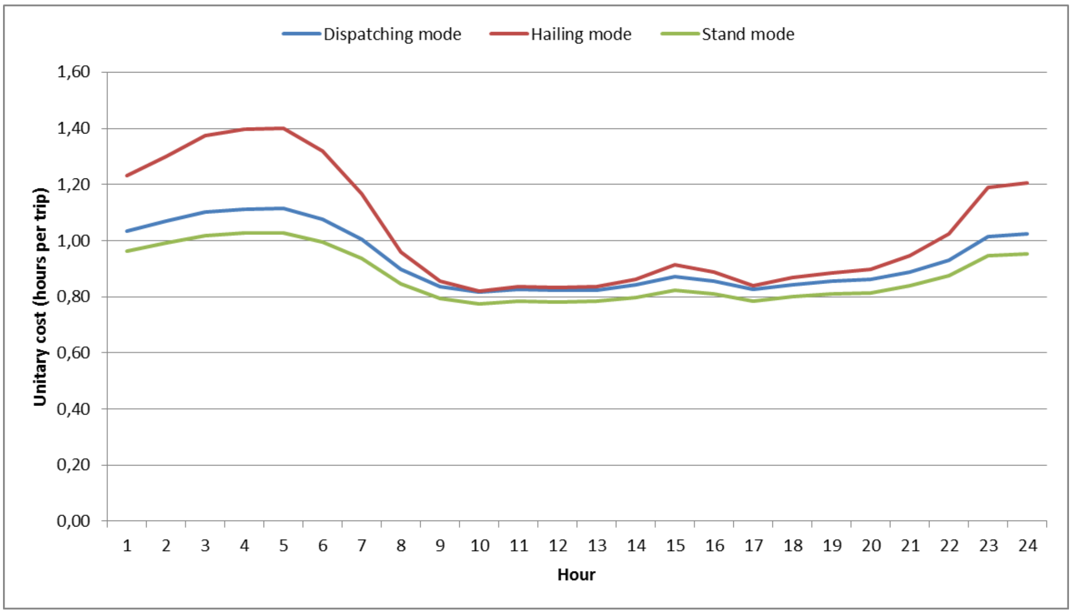

Figure 5-14 Unitary cost for each operation mode along the day

It is interesting to highlight that the unitary costs are very similar during the day, but at night the hailing mode presents a higher unitary costs due to the low demand and unproductive vehicle-kilometers.

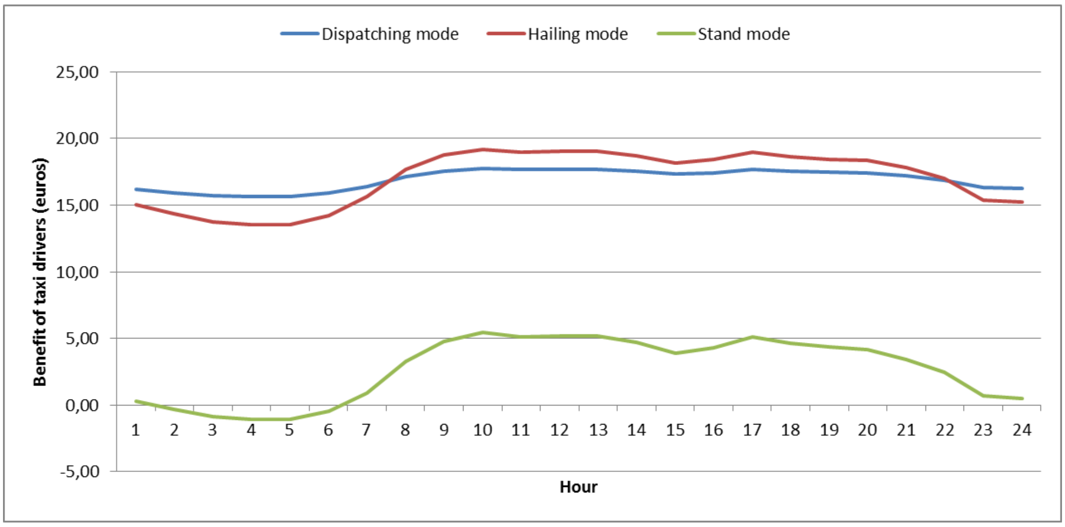

Figure 5-15 Benefit of taxi drivers for each operation mode along the day

The benefit of the drivers is similar for both the hailing and the dispatching modes, while the stand mode presents the lowest benefits, especially at night.

5.3.2. Results of the simulation model

The agent-based dispatching model has been applied to the Barcelona network. The supply side is composed by more than 20,000 links and 8,000 nodes, representing most of the streets of the city. The lower road categories have not been taken into account in order to reduce the processing time. The demand side is composed by more than 270,000 recorded taxi trips during 4 years (2009-2012) and the OD matrices of 2007 for all modes.

Table 5-10 Variables definition

Variable		Reference value (Barcelona)
Model outputs	Waiting time of customers (min)	2 - 6
	In-vehicle time of customers (min)	11
	Trip cost of customers (€)	6
	Distance of the trip of customers (km)	5
	Number of trips of drivers (trips/hour)	1 - 6
	Vacant distance (km/h)	8 - 18
	Occupied distance (km/h)	8 - 18
	Vacant time (min)	25 - 44
	Occupied time (min)	16 - 35
	Driver income (€/h)	14 - 30
	Driver benefits (€/h)	(-6) - 5
Model inputs	Network geometry	-
	Taxi hourly supply (total number of vehicles per hour and km^2)	20 - 60
	Hourly demand for taxi trips (OD matrix)	14000
	Average speed of each link (km/h)	4 - 90
	D is the flag-drop charge (€)	2.05
	τ_{km} is the taxi fee per unit of distance (€/km)	0.86
	τ_{sec} is the taxi fee per unit of time (€/min)	0.3

The data presented in Table 5-10 has been provided by IMT and Amat (2010). The data of the taxi database has been filtered in order to use the trips with origin and destination within the city network, generating a total of 235,000 valid trips. A small sample of 1.200 trips between 8 and 9 of all Tuesdays has been used for validating the model in terms of travel distance, time and cost.

Table 5-6 presents and compares the obtained results with the measured values from the database.

Table 5-11 Results of the validation of the aggregated model to the city of Barcelona

	Model prediction	Measured value (2012)	Deviation
Average cost (euros)	5,53	7,1	22%
Average travel time (min)	8,16	12,38	34%
Average distance (km)	3,56	3,7	4%

In this case the measured distance in the agent model and in the reality are almost equal, while the cost and duration are underestimated by the agent model. The agent model has a unique congestion level in each link, which is lower than the congestion levels of the measured trips. The difference in the trip cost is due to the difference on the two other parameters (travel time and trip cost). The dispersion of the above calibration values is presented in Figure 5-16:

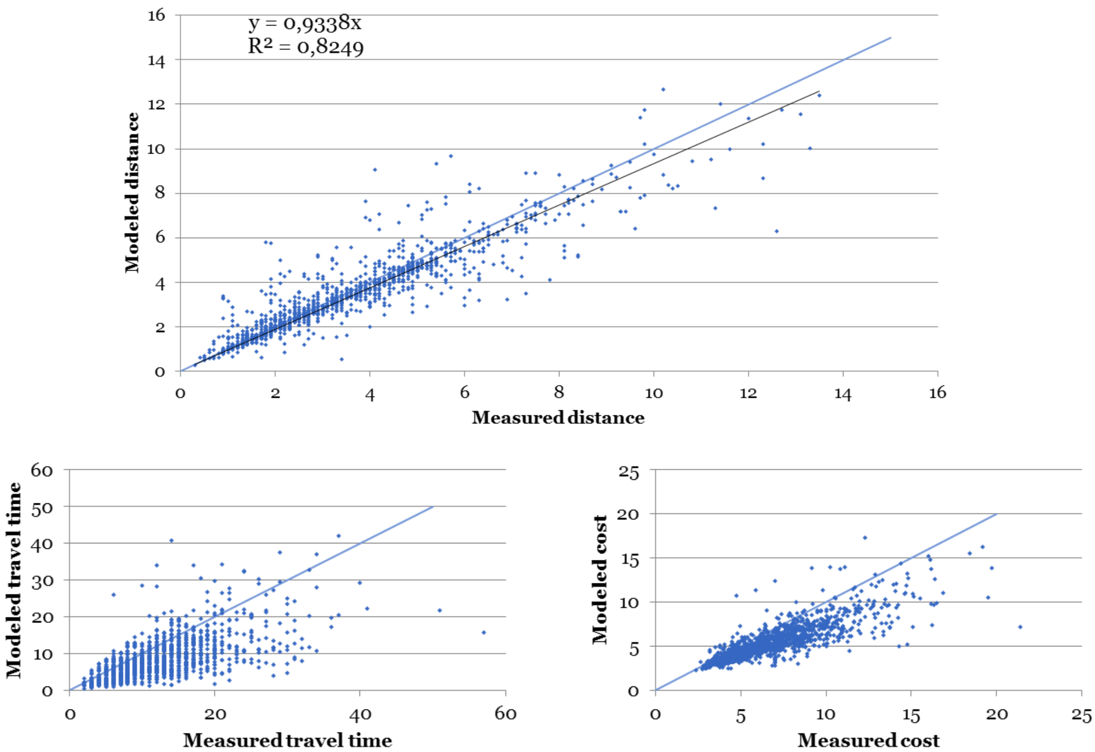

Figure 5-16. Validation of the agent-based model in terms of travel time, distance and cost

The relation between the measured travel distance and the modeled one shows the validity of the model. There are two issues that affect this relation, one is the detail of the network used in the model, where the lowest street categories have been eliminated; the second issue is the route chosen by the taxi driver that cannot be the shortest one in some cases. The relation between the measured travel times and the modeled ones present more scattered results. The reason is that the model uses an average congestion factor for each link, but since this value is changing within a day and within days, it will not always be representative of the real congestion levels. This fact influences the relation between the measured costs and the modeled ones, which presents a small deviation off the 45 degree line. The trip cost presents a good fit, but the fitted curve presents a constant term, which can be related to a difference in the value of the flag-drop.

The demand between 8 and 9 of all Mondays, Wednesdays and all Thursdays (4.800 trips) has been introduced in the model and different fleet sizes have been tested with the dispatching model, obtaining the waiting time and the system costs presented in Figure 5-17.

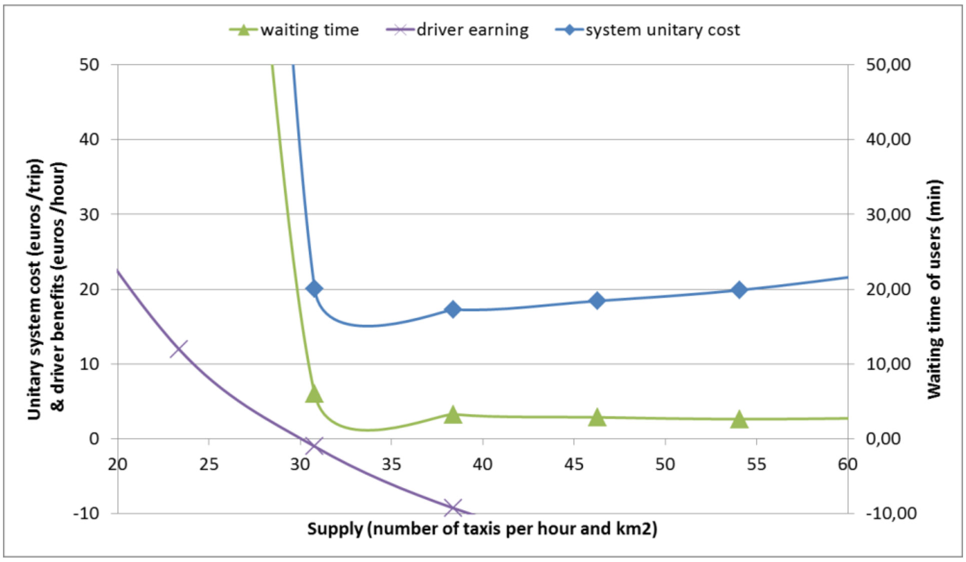

Figure 5-17 Waiting time, driver benefits and unitary costs for each fleet size obtained by the agent-based model

It can be observed that the satisfactory number of taxis per hour and km^2 is higher than 28 since the waiting time for smaller fleets is very high due to the low number of free taxis. It can also be observed that the maximum number of taxis with positive benefits is 30 taxis/hour*km^2 since more taxis than this value will generate losses to the drivers (second best solution). The optimum number of taxis taking into account the system costs is 33 – 34 taxis/hour*km^2 (first best solution).

Table 5-12 Results of the application of the agent-based model to the city of Barcelona

Minimum number of taxis	28 taxis/hour*km^2	29 taxis/hour*km^2
First best solution	33 – 34 taxis/hour*km^2	44 – 46 taxis/hour*km^2
Second best solution	30 taxis/hour*km^2	38 taxis/hour*km^2
Subsidy	4 – 5 euros per trip	3 – 4 euros per trip

The minimum number of taxis is equal in the two models, but the number of taxis related to the first and second best solutions are larger in the aggregated model in comparison to the agent-based model.

5.4. Shifts policy in Barcelona

The above models provide the optimum number of vehicle-hours to be offered during each hour of the day at strategic level. This value should be obtained at operational level by applying the correct shifts policy to the taxi market. The city of Barcelona has recently applied a new policy for regulating the working hours during the day, and not only the working days during the week. These shifts can have various durations, from 2 to 8 hours.

Three scenarios have been tested in order to define the best policy. The supply level used is the one obtained by the dispatching model, but the methodology is valid for the other operation modes by changing the optimum supply levels:

- One shift of eight hours
- Two shifts of four hours
- Four shifts of two hours

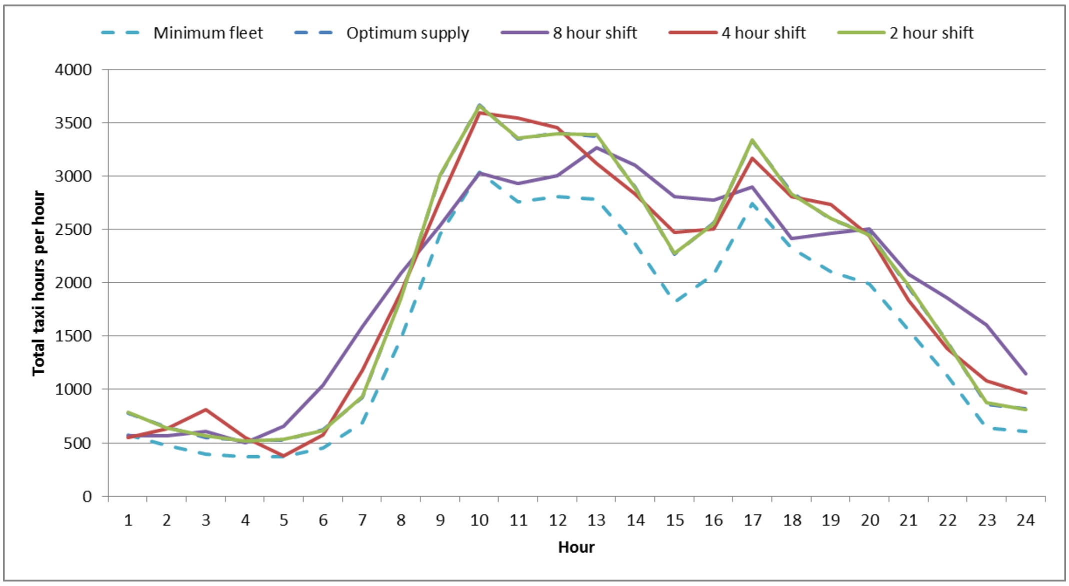

Figure 5-18 Taxi-hours provided by the three policy scenarios in comparison to the optimum fleet size

The 2 hour shift and the optimum supply curves coincide, which means that the optimum supply level can be achieved by 2 hour shifts. Smaller shifts allow a better fit of the provided taxi-hours to the optimum values (as expected). The areas between each line and the optimum value represent the over or under supply, which is related to lower income for taxi drivers and higher waiting time for taxi customers respectively. This area goes from 7,000 hours in the 8-hour shift to 275 hours in the 2-hour shift.

The impact of each policy on the most significant indicators of the taxi market is presented in Figure 5-19, Figure 5-20 and Figure 5-21.

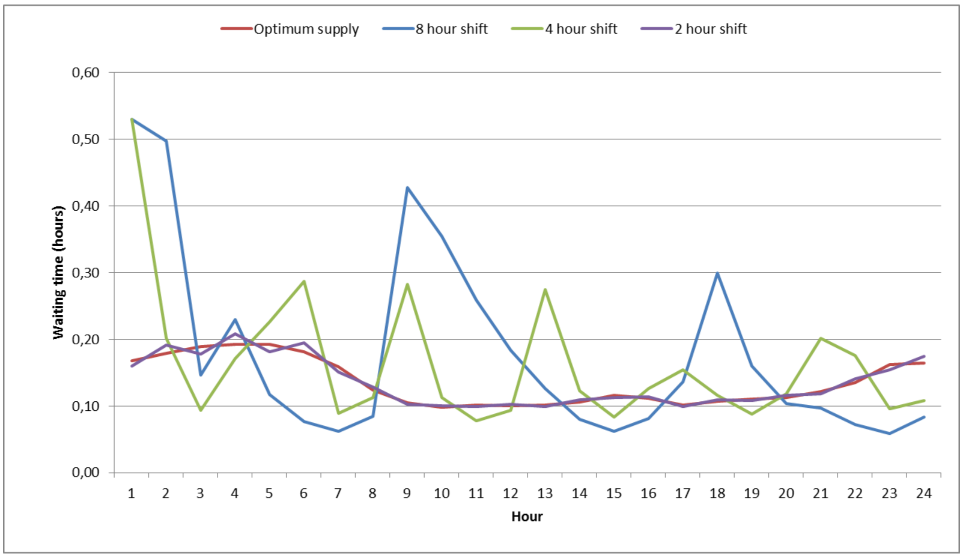

Figure 5-19 Customer waiting time for the various proposed policies

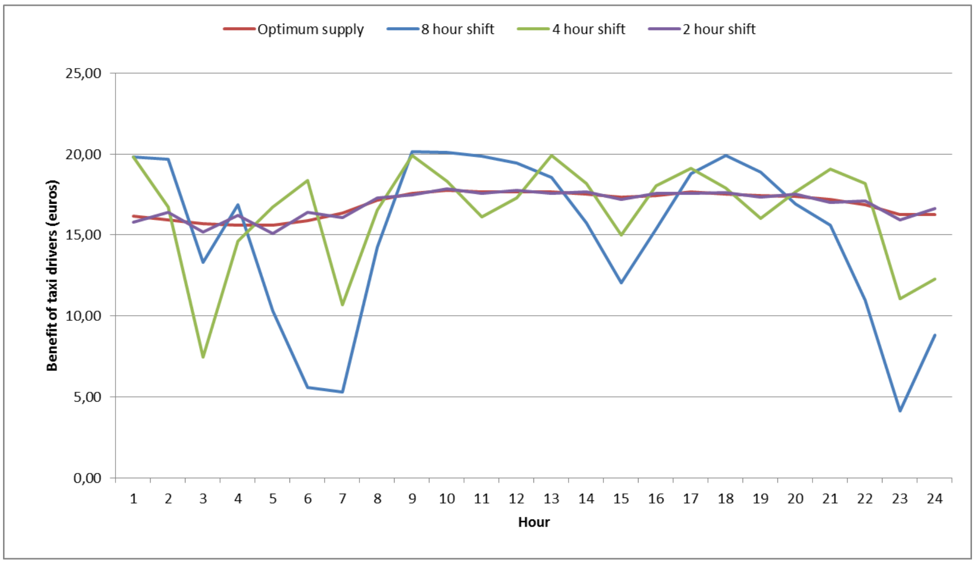

Figure 5-20 Benefit of the taxi drivers for the various proposed policies

The waiting time variation in relation to the optimum waiting time depends on the difference between the optimum and the real number of taxi-hours offered. When the benefits of the taxi drivers are higher than the ones of the optimum supply levels, the waiting time of customers is higher, and oppositely, when the benefits of taxi drivers are lower than the expected, the waiting time of customers are lower. It is interesting to highlight that the benefit of the optimum taxi-hour supplied are equal during the day, which means that the benefit of each working hour are similar, providing equity to the taxi drivers independently of the interval of the shifts assigned. Oppositely, the benefits obtained by the 4-hour or the 8-hour shift have important variations along the day, which means that the shifts should be assigned taking into account equity issues related to the benefit of each working hour.

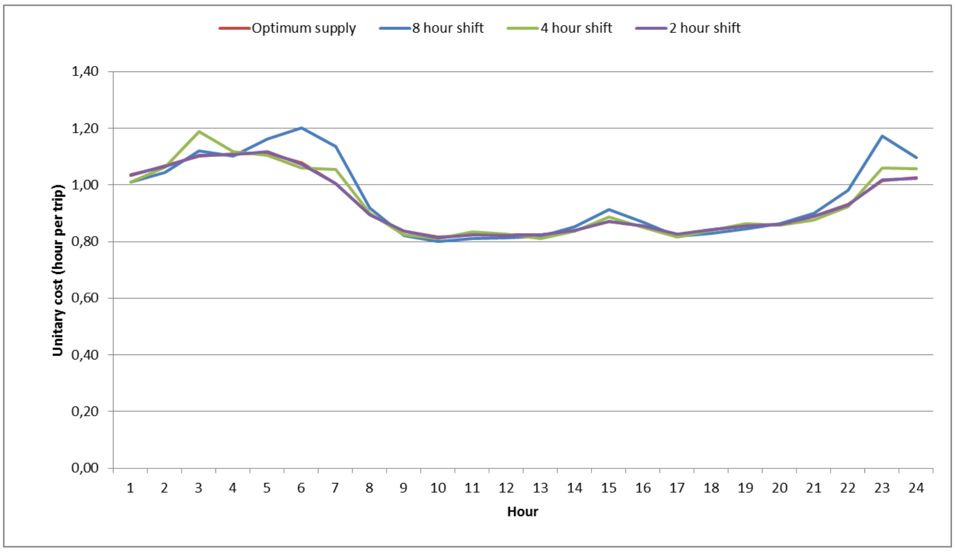

Figure 5-21 Unitary cost of the various proposed policies

Tha variation of the unitary costs is smoother due to the compensation effect between the increase/decrease of the waiting time of customers and the respective decrease/increase benefits of drivers. It is interesting to analyze the accumulated over/under-offered hours during the day.

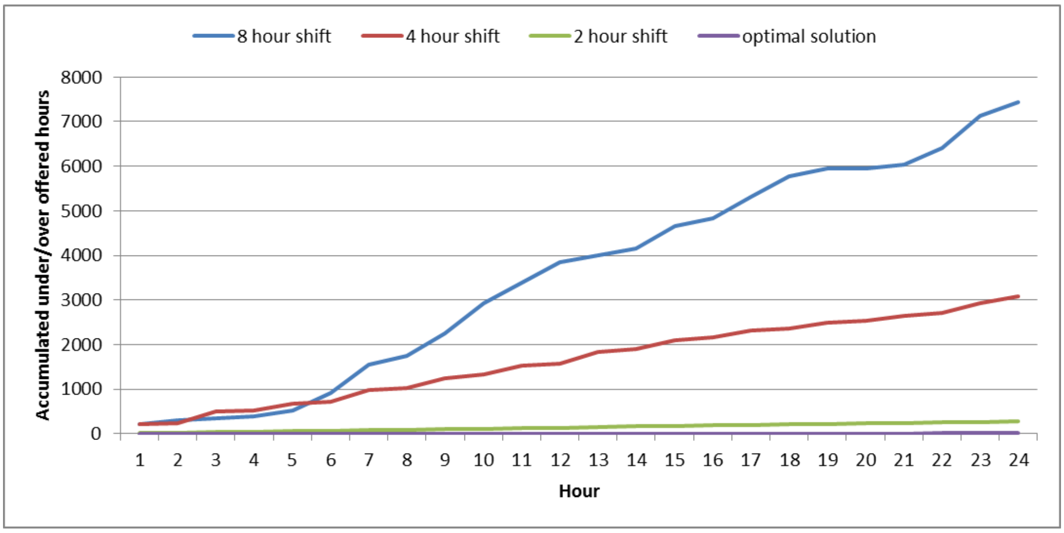

Figure 5-22 Accumulated under/over-offered hours during the day

The total number of hours over/under-offered during the day is more than 7,000 when applying 8-hour shifts, while in the case of 4-hour shifts this value is less than the half.

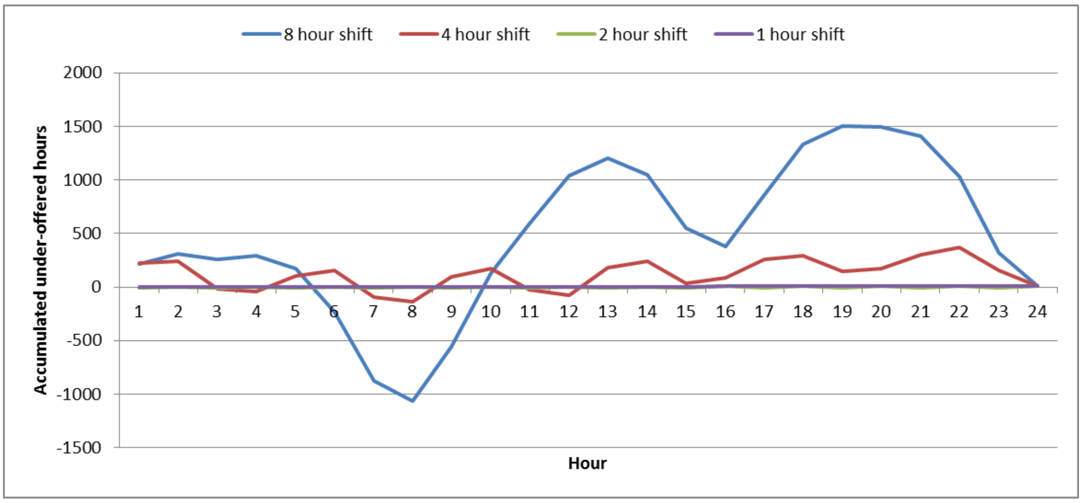

Figure 5-23 Accumulated under-offered hours during the day

The maximum accumulated deficit of taxi hours is always below 500 in the 4-hour shift, while in the 8-hour shift this value reaches an over-supply of 1,000 hours during the morning peak and an under-supply of 1.500 hours during the afternoon peak.

5.4.1. Proposed shifts policy strategy for the city of Barcelona

From all the above, the best solution is a 2-hour shift policy, but this is not feasible at all. A 4-hour shift is more feasible and the results are much better than the results of an 8-hour shift policy. In order to make the 4-hour shifts more attractive to the taxi drivers, they can be grouped in pairs of shifts where the second one will start when the first one finishes, generating 8-hour shifts. This cannot be applied to all the shifts, but only to some of them. The percentage of shifts that can be grouped is calculated by measuring the area of the peaks that cannot be covered by the 4-hour horizontal line, equal to 95% in the case of Barcelona as it can be observed in Figure 5-24.

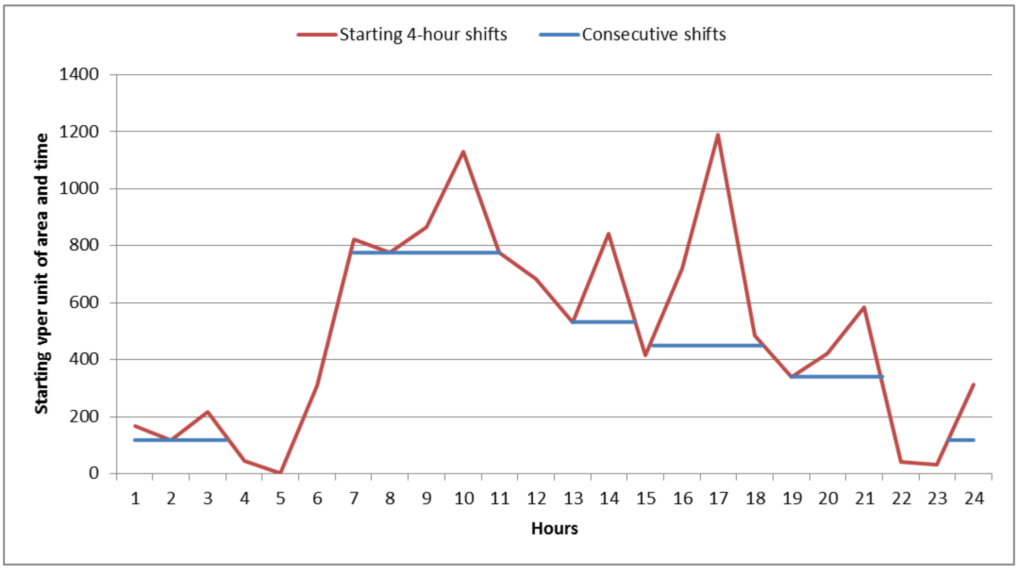

Figure 5-24 Grouping of pairs of 4-hour shifts

5.5. Results discussion

The two proposed models have been applied to the city of Barcelona using real data from the network and the conducted taxi trips. The models have been validated with data collected from taximeters of more than 100 taxis, which have conducted more than 1.200,000 trips during the last 9 years. The results obtained in terms of number of vehicles per hour and area, are slightly lower in the simulation model than the ones obtained by the aggregated model. The hailing and dispatching models have concluded that the system´s optimum number of taxis in Barcelona ranges from 34 to 46 vehicles per hour and km^2 depending on the operation mode, while the drivers optimum size is slightly lower (30 – 38 vehicles per hour and km^2, depending on the operation mode). Subsidization values for each operation mode have been provided. With this subsidization, the optimum number of vehicle-hours of the first best solution can be offered to the customers retaining drivers´benefits, which means that the waiting time of the customers will be reduced to the one of the first best solutions.

The aggregated model has been applied to the 24-hour demand curve, presenting the optimum number of vehicle-hours for each time of day. A 4-hour shift policy has been proposed and a methodology has been developed for adapting its distribution to the optimum supply level obtained by the model. The impacts of the over or under supply provided in relation to the optimum levels have been identified and quantified. Finally, a methodology for matching two consecutive 4-hour shifts has been developed in order to make the 4-hour shifts more attractive to the taxi drivers by creating 8-hour shifts. The total percentage of 4-hour matched shifts in the case of Barcelona is around 95%, which means that only 5% of the shifts will have a duration of 4 hours, while all other shifts will have a duration of 8 hours.

5.6. References

Amat i Bertran C. (2010) Anàlisi de l’eficiència del servei de taxi a Barcelona. Propostes de millora. *Tesina final de Carrera. ETSECCPB Barcelona*.

Brayton R.K., Director S.W., Hachtel G.D. and Vidigal L. (1979) A New Algorithm for Statistical Circuit Design Based on Quasi-Newton Methods and Function Splitting. *IEEE Trans. Circuits and Systems* CAS **26**, 784-794.

CENIT (2013) Observatori del Taxi 2012. Document Final. Institut Metropolità del Taxi.

Institut Metropolità del Taxi (IMT), www.taxibarcelona.cat

SACon & IMT 2003. Enquesta Òmnibus municipal. Presentació de resultats, publicació del mes de setembre.

SHEFFI, Y. (1985) *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*, Prentice-Hall, Englewood Cliffs, New Jersey.

U. S. Bureau of Public Roads (1964). *Traffic assignment manual, 11*. S. Bureau of Public Roads, U. S. Government Printing Office, Washington, D.C.

6. EPILOGUE

6.1. Introduction

This chapter reviews the conclusions and the highlights of the research conducted in this thesis in relation to the modeling of taxi markets, both individually for each chapter and overall for the whole research, presenting an overview of the taxi modeling and the outputs of the research, as well as future research directions.

6.2. Valuation of the thesis

An extensive review of the modeling of taxi markets has been presented in the second chapter, classifying them into aggregated, equilibrium and simulation models and highlighting the advantages as well as the disadvantages of each type of model. The approach proposed by the author is a combination of the use of all models, using the aggregated one for the overview of the variables of the taxi market and the simulation one for the detailed study of the operational issues of the adopted solution and the fine-tunning of the selected values. The most significant variables of each model have been identified and the formulations have been presented, giving emphasis in the most operational ones.

A new formulation has been presented in chapter three, taking into account the taxi customers, the drivers and the city and has been applied to the three operation modes: hailing, stand and dispatching. This formulation has been used for analyzing the cost elasticity and for obtaining the optimum fleet size and the most significant variables related to this fleet, such as the customers´ waiting time and the drivers´ benefit. The three operation modes have been compared in terms of unitary costs, presenting the values of demand and area where each operation mode has the minimum unitary costs. Formulations for the first best solution, the second best solution and the waiting and access times have been presented. The pricing policy of taxi services has been analyzed by using an elastic demand function. The system performance for various combinations of fare and fleet size has been presented.

A new simulation model has been developed for analyzing the operational issues of the taxi markets, being able to collect more detailed results, yet by using more detailed data. The results of the simulation model are richer because distributions of values are obtained rather than just average values. The results of both models have been compared, presenting similar tendencies and values, which provides validity to the models. The aggregated models present underestimations between 15% and 30% of the costs for uniform demand distributions. The impact of the non-uniformity of the demand has been obtained by the simulation model, ranging from 20% to 300% overestimation of costs.

The two models have been applied to a case study, using real data from the city of Barcelona.

6.3. Valuation of the research output

The formulations proposed for the modeling of the taxi market by various authors have been presented and analyzed. New formulations have been presented in detail for the estimation of various variables related to the taxi market. Cost formulations have been developed for each operation mode taking into account the externalities generated by the taxi services provision. Analytic formulations for the optimum fleet size and the waiting and access time related to this fleet have been provided. The formulation for the minimum unitary costs related to the optimum fleet size has been also provided. Comparison of these unitary costs have been presented both analytically and graphically, presenting the area sizes and demand levels for which each operation mode is optimum. Analytic fomulations for the calculation of the cost elasticities with relation to the supply, the demand and the value of time have been provided.

The performance of the three operation modes has been compared by using simulation tools, presenting the ratios of the most significant variables of the taxi market for each operation mode. The simulation model has been used for validating the results of the aggregated model, concluding in that the aggregated model underestimates the costs for uniform demand, but overestimates them for non-uniform demand distributions. The impact of the demand distribution on the waiting time has been presented and quantified by applying various spatial demand distributions. Variations in the demand for taxi services depending on the fare and the number of taxis have been presented by using an elastic demand. A methodology for defining the shifts duration and distribution has been proposed.

A detailed overview of the taxi sector in the city of Barcelona has been presented, applying the developed models and presenting the optimum fleet sizes for both the customers and the drivers and the most significant performance indicators. The optimum shifts policy for the city of Barcelona has been presented, which is composed by a 95% of 8-hour shifts and a 5% of 4-hour shifts. The impacts of the difference between the optimum supply and the one proposed by the shifts policy have been quantified.

6.4. Future directions of research

Future research should develop analytic taxi demand estimation methods, which will be taken into account in order to analyze the long-term impacts of taxi policies on the performance of the taxi market. Formulations for the non-uniform demand should be provided for the three operation modes, especially when the presence of taxi hot spots or aiports is significant in the study zone. Forecast and real-time estimations should be also included in the taxi modeling, in order to analyze the impact of new technologies to the level of service provided to the taxi customers.

Simulation-based models should be further developed and used for analyzing the performance of the taxi market. These kinds of models are able to provide detailed operational data, but also to test different configurations and scenarios in order to compare or estimate the impacts of new policy measures.

There is a strong need for data collection related to the taxi market in order to develop and calibrate more realistic models. The technology for providing this data already exists and is widely used by most of the taxi fleets. Collaboration between research and taxi operators should be established in order to share this data, creating win-win situations, where the taxi industry will receive benefits from the research conducted with the provided data. New business models should be developed based on this data, not only for the provison of taxi services, but also for the provision of traffic-related services by using taxis as Floating Car Data collectors.

7. ANNEXES

ANNEX I: Source code of the Matlab programs (agent-based model)

tot.m
```
programa_enunciat
programa_moviment
%resultats
T(:,1:6)=taxi(:,5:10);
C(:,1)=client(:,1);
C(:,2:4)=client(:,6:8);
C(:,5)=client(:,10);
C(:,6:11)=client(:,13:18);
%dibuixar oferta i demanda resultant
figure(4)
subplot(4,1,1);bar(vec_dem(1:24,1),'DisplayName','demm(1:24,2)');figure(gcf)
subplot(4,1,2);bar(vec_of(1:24,1),'DisplayName','off(1:24,1)');figure(gcf)
subplot(4,1,3);bar(demm(1:24,1),'DisplayName','demm(1:24,2)');figure(gcf)
subplot(4,1,4);bar(off(1:24,1),'DisplayName','off(1:24,1)');figure(gcf)
```

programa_enunciat.m
```
%carrego la xarxa i la dibuixo
dataSF
%%
%carrego les parades
parades
%%
%carrego la oferta i la dibuixo
ofe
%%
%carrego la demanda i la dibuixo
dem
%%
%carrego la demanda i la dibuixo
cong
%%
%tanco els clients i els taxis
client(:,1)=-999999999999;
taxi(:,4)=-999999999999;
hold off
```

dataSF.m
```
% ho esborro tot
clc
clear all
%carreguem les dades de la xarxa (FFv=30km/h)
%FR TO CAP FFTT A B VOL CURT XFR YFR XTO YTO XCE YCE LEN PES
L=(1   2   25900   32  0   4   4495    32  50  510 320 510 185 510 270 1
…
24  23  5079    10  0   4   7862    18  130 50  130 130 130 90  80  1);
%%
%dibuixo la xarxa
figure(1)
for i=1:length(L)
   a=(L(i,9),L(i,11));
   b=(L(i,10),L(i,12));
   plot (a,b,'k')
   hold on
end
axis((0 450 0 550))
```

parades.m
```
%id link dist x y taxis clients
parad=(1 27 15
…
   4 8 15);
numparad=length(parad);
for h=1:numparad
   aaa=L(parad(h,2),9)+parad(h,3)*(L(parad(h,2),11)-L(parad(h,2),9))/L(parad(h,2),15);
```

```
   bbb=L(parad(h,2),10)+parad(h,3)*(L(parad(h,2),12)-L(parad(h,2),10))/L(parad(h,2),15);
   parad(h,4)=aaa;
   parad(h,5)=bbb;
end
parad(:,6)=0;
parad(:,7)=0;
plot(parad(:,4),parad(:,5),'ks')
fifota(numparad,2)=0;
fifocl(numparad,2)=0;
```

ofe.m

```
%%genero la oferta
%defineixo el cost de la baixada de bandera i la tarificacio
bd=3;
tar_time=12;
tar_dist=50;
tar=0.2;
%carrego la oferta per a cada hora (flota dispatching stop)
oferta=(0   500 500 500
…
23  500 500 500);
%defineixo la jornada de treball
jornada=28800;
%inicio el contador de taxis
post=1;
%inicio el vector real doferta
vec_of(24,1)=0;
%genero lo oferta acumulada en fucnio de la jornada laboral
off=oferta(:,2)-500;
for i=24:-1:1
   if i<=8
      off(i)=sum(off(1:i));
   end
   if i>8
      off(i)=sum(off(i-8:i));
   end
end
%dibuixo el perfil de la oferta
figure(2)
subplot(3,1,1);bar(off(1:24,1),'DisplayName','off(1:24,1)');figure(gcf)
taxi(1,26)=0;
```

dem.m

```
%perfil de demanda per a cada hora (demanda absoluta i dispatching)
%TIME DEMANDA DISPATCHING
demanda=(0  500 500 500
1   500 500 500
…
23  500 500 500);
%dibuixo la demanda al llarg del dia
demm=demanda(:,2)-500;
figure(2)
subplot(3,1,2);bar(demm(1:24,1),'DisplayName','demm(1:24,2)');figure(gcf)
%creo el vector real de demanda
vec_dem(24,1)=0;
%inicio el contador de clients i el vector de clients
posc=1;
client(1,26)=0;
```

cong.m

```
%perfil de congestio que multiplica el temps de viatge en funcio de lhora
%TIME valor
congestio=(0    1
1   1
2   1
3   1
4   1
5   1
6   1
7   1
8   1.1
```

```
9   1.2
10  1.1
11  1
12  1
13  1
14  1.1
15  1
16  1
17  1.1
18  1.2
19  1.1
20  1
21  1
22  1
23  1);
%dibuixo el graf de la congestio al llarg del dia
figure(2)
subplot(3,1,3);bar(congestio(1:24,2),'DisplayName','demm(1:24,2)');figure(gcf)
hold off
```

programa_moviment.m

```
%defineixo velocitat d'animacio, linici, el final i dibuixo la xarxa base
speed=100000000000000000;
pas=1;
inici=21600;
final=86400;
temps=final-inici;
figure(3)
for i=1:length(L)
   a=(L(i,9),L(i,11));
   b=(L(i,10),L(i,12));
   plot (a,b,'k')
   hold on
end
plot(parad(:,4),parad(:,5),'bs')
axis((1 450 1 550))
pause(1/speed)
%%
%input de dades
taxnor=input('hailing??');
taxdis=input('dispatching??');
taxst=input('stops??');
%%
%corre el temps
for t=inici:pas:final-1
   %contador de temps
   t/final
   %carrego la oferta
   vehicle=round(oferta(round(24*t/final-0.5)+1,2)*rand()/1000);
   %vehicle=1;
   %if t==21600
   %%%%molts vehicles
   %vehicle=1;
   %end
   vehi(t,1)=vehicle;
   vec_of(round(24*t/final-0.5)+1,1)=vec_of(round(24*t/final-0.5)+1,1)+vehicle;
   %actualitzo el vector de taxis (% link distlink dispatching situacio dbuit dple tbuit tple ingres ttotal
   % client xbuit ybuit tempstorn tempsrestant xple yple distanciatemporal xdisp ydisp assignat stop paradalink parada dist
xstop ystop)
   if vehicle==1
      taxi(post,1)=round((length(L)-1)*rand())+1;
      %taxi(post,30)=taxi(post,1);
      taxi(post,2)=L(taxi(post,1),15)*rand();
      taxi(post,4)=0;
      taxi(post,10)=0;
      taxi(post,15)=jornada;
      taxi(post,21)=0;
      if taxnor==0
         moneda=round(rand);
         taxi(post,3)=taxdis*abs(taxdis-taxst)+taxdis*taxst*moneda;
         if taxi(post,3)==0
```

```
            taxi(post,22)=1;
         end
      end
      %dispatching?
      if taxdis==1 & taxnor==1
         taxi(post,3)=round(oferta(round(24*t/final-0.5)+1,3)*rand()/1000);
      end
      if taxi(post,3)==0
         if taxst==1 & taxnor==1
            %stop?
            taxi(post,22)=round(oferta(round(24*t/final-0.5)+1,4)*rand()/1000);
         end
         fet=0;
         if taxi(post,22)==1
            (a b)=min(abs(taxi(post,1)-parad(:,2)));
            if a==0 & taxi(post,2)<parad(b,3)
               fet=1;
               parada_as=b;
               %pathstaxis(post,:)=pathsparades(parada_as,:);
               taxi(post,23)=parad(parada_as,2);
               taxi(post,22)=parada_as;
               taxi(post,24)=parad(parada_as,3);
            end
         end
         if fet==0 & taxi(post,22)==1
            %lassigno a una parada
            stops
         end
      end
      post=post+1;
   end
   %carrego la demanda
   trip=round(demanda(round(24*t/final-0.5)+1,2)*rand()/1000);
   if taxdis==1 & taxnor+taxst==0 & max((taxi(:,3)-taxi(:,21)).*(1-taxi(:,11)))<1
      trip=0;
   end
   %trip=1;
   tri(t,1)=trip;
   vec_dem(round(24*t/final-0.5)+1,1)=vec_dem(round(24*t/final-0.5)+1,1)+trip;
   %actualitzo el vector clients (sit frL frD toL toD wtime ttime dist dispatching cost x y
   %taxi frL frD toL toD temps_inici contador_temps contador_distancia xdispmovblau ydispmovblau xdispmovgroc
ydispmovgroc xdispstop ydispstop)
   if trip==1
      client(posc,1)=1;
      client(posc,2)=round((length(L)-1)*rand())+1;
      client(posc,14)=client(posc,2);
      client(posc,3)=0.25*L(client(posc,2),15)+0.5*L(client(posc,2),15)*rand();
      client(posc,15)=client(posc,3);
      client(posc,18)=t;
      ts=0;
      while ts==0
         client(posc,4)=round((length(L)-1)*rand())+1;
         client(posc,16)=client(posc,4);
         client(posc,5)=0.25*L(client(posc,4),15)+0.50*L(client(posc,4),15)*rand();
         client(posc,17)=client(posc,5);
         %comprobacio que el viatge te sentit
         sentit1=(L(client(posc,2),1) L(client(posc,2),1) L(client(posc,2),2) L(client(posc,2),2));
         sentit2=(L(client(posc,4),1) L(client(posc,4),2) L(client(posc,4),1) L(client(posc,4),2));
         ts=min(abs(sentit1-sentit2));
      end
      dij
      while dijkstra(Destination,4)==0
         ts=0;
         while ts==0
            client(posc,4)=round((length(L)-1)*rand())+1;
            client(posc,16)=client(posc,4);
            client(posc,5)=0.25*L(client(posc,4),15)+0.50*L(client(posc,4),15)*rand();
            client(posc,17)=client(posc,5);
            %comprobacio que el viatge te sentit
            sentit1=(L(client(posc,2),1) L(client(posc,2),1) L(client(posc,2),2) L(client(posc,2),2));
            sentit2=(L(client(posc,4),1) L(client(posc,4),2) L(client(posc,4),1) L(client(posc,4),2));
```

```
            ts=min(abs(sentit1-sentit2));
        end
        dij
    end
    client(posc,10)=0;
    client(posc,11)=L(client(posc,2),9)+client(posc,3)*(L(client(posc,2),11)-L(client(posc,2),9))/L(client(posc,2),15);
    client(posc,12)=L(client(posc,2),10)+client(posc,3)*(L(client(posc,2),12)-L(client(posc,2),10))/L(client(posc,2),15);
    client(posc,13)=0;
    if taxnor==0
        moneda=round(rand);
        client(posc,9)=taxdis*abs(taxdis-taxst)+taxdis*taxst*moneda;
        if client(posc,9)==0
            client(posc,9)=-1;
        end
    end
    if taxdis==1 & taxnor==1
        %dispatching?
        client(posc,9)=round(demanda(round(24*t/final-0.5)+1,3)*rand()/1000);
    end
    if max((taxi(:,3)-taxi(:,21)).*(1-taxi(:,11)))<1 & client(posc,9)==1
        client(posc,9)=0;
    end
    if client(posc,9)==1
        client(posc,21)=client(posc,9)*client(posc,11);
        client(posc,22)=client(posc,9)*client(posc,12);
    end
    %assigno els clients dispatching
    if client(posc,9)==1
        %hi ha algun taxi al mateix link??
        fet=0;
        (a b)=min(1000000*(abs(1-taxi(:,3)-taxi(:,21)))+1000000*abs(taxi(:,1)-client(posc,2))+abs(taxi(:,2)-client(posc,3)));
            if a<1000000 & taxi(b,2)<client(posc,3)
                fet=1;
                %ja lagafara quan passi per davant
                taxi(b,4)=client(posc,2);
                taxi(b,5)=client(posc,3);
                taxi(b,11)=posc;
                taxi(b,16)=taxi(b,12);
                taxi(b,17)=taxi(b,13);
                taxi(b,12)=0;
                taxi(b,13)=0;
                taxi(b,19)=0;
                taxi(b,20)=0;
                taxi(b,21)=1;
                client(posc,2)=0;
                client(posc,23)=client(posc,11);
                client(posc,24)=client(posc,12);
                client(posc,13)=b;
                client(posc,10)=bd;
            end
            if fet==0
                %trobo el taxi mes proper (no directe, sino per dikjstra, funciona amb els links al reves amb origen el client)
                dij2
            end
    end
    if taxst==1 & taxnor==1
        %stop?
        if client(posc,9)==0
          client(posc,9)=-round(demanda(round(24*t/final-0.5)+1,4)*rand()/1000);
        end
    end
    if client(posc,9)==-1
        %lassigno a una parada
        parada=1+round((numparad-1)*rand);
        client(posc,2)=parad(parada,2);
        client(posc,3)=parad(parada,3);
        %comprobacio si el viatge te sentit
        ts=0;
        while ts==0
            client(posc,4)=round((length(L)-1)*rand())+1;
            client(posc,16)=client(posc,4);
```

```
            client(posc,5)=0.25*L(client(posc,4),15)+0.50*L(client(posc,4),15)*rand();
            client(posc,17)=client(posc,5);
            %comprobacio que el viatge te sentit
            sentit1=(L(client(posc,2),1) L(client(posc,2),1) L(client(posc,2),2) L(client(posc,2),2));
            sentit2=(L(client(posc,4),1) L(client(posc,4),2) L(client(posc,4),1) L(client(posc,4),2));
            ts=min(abs(sentit1-sentit2));
          end
          dij
          %si a la parada no hi ha taxis es posa a la cua
          if parad(parada,6)==0
            parad(parada,7)=parad(parada,7)+1;
            fifocl(parada,parad(parada,7))=posc;
            client(posc,11)=parad(parada,4);
            client(posc,12)=parad(parada,5);
          end
          %si a la parada hi ha taxis agafa el primer
          if parad(parada,6)>0
            parad(parada,6)=parad(parada,6)-1;
            veh_cua=fifota(parada,1);
            client(posc,1)=0;
            client(posc,2)=0;
            client(posc,11)=0;
            client(posc,12)=0;
            client(posc,21)=0;
            client(posc,22)=0;
            client(posc,25)=taxi(veh_cua,12);
            client(posc,26)=taxi(veh_cua,13);
            client(posc,13)=veh_cua;
            client(posc,10)=bd;
            client(posc,9)=0;
            taxi(veh_cua,4)=client(posc,4);
            taxi(veh_cua,5)=client(posc,5);
            taxi(veh_cua,11)=posc;
            taxi(veh_cua,19)=0;
            taxi(veh_cua,20)=0;
            taxi(veh_cua,16)=taxi(veh_cua,12);
            taxi(veh_cua,17)=taxi(veh_cua,13);
            taxi(veh_cua,12)=0;
            taxi(veh_cua,13)=0;
            fifota(parada,1:end-1)=fifota(parada,2:end);
            fifota(parada,end)=0;
            pathstaxis(veh_cua,:)=paths(posc,:);
          end
        end
        posc=posc+1;
      end
      %moc els taxis
      o=size(taxi);
      for i=1:o(1,1)
        if taxi(i,4)==0
          if taxi(i,15)>0 %si el taxi esta a la xarxa, buit i te temps disponible
            temp=taxi(i,2)+L(taxi(i,1),15)/L(taxi(i,1),8);
            taxi(i,18)=L(taxi(i,1),15)/L(taxi(i,1),8);
            if temp<L(taxi(i,1),15)
              taxi(i,2)=temp;
            end
            if temp>=L(taxi(i,1),15) & taxi(i,22)==0
              %canvi link
              taxi(i,2)=temp-L(taxi(i,1),15);
              canvi_link
              taxi(i,1)=sol;
            end
            if temp>=L(taxi(i,1),15) & taxi(i,22)>0 & abs(taxi(i,1)-taxi(i,23))>0%taxi va a parada i no hi ha arribat
              taxi(i,2)=temp-L(taxi(i,1),15);
              canvi_link3
              taxi(i,1)=sol;
            end
            taxi(i,10)=taxi(i,10)+1;
            %calculo les coordenades del taxi
            aa=L(taxi(i,1),9)+taxi(i,2)*(L(taxi(i,1),11)-L(taxi(i,1),9))/L(taxi(i,1),15);
            bb=L(taxi(i,1),10)+taxi(i,2)*(L(taxi(i,1),12)-L(taxi(i,1),10))/L(taxi(i,1),15);
```

```
        end
        if taxi(i,15)==1
            taxi(i,4)=-1;
            taxi(i,12)=0;
            taxi(i,13)=0;
        end
    end
    if taxi(i,4)>0
        if taxi(i,15)>0 %si el taxi esta a la xarxa, ple i te temps disponible
            temp=taxi(i,2)+L(taxi(i,1),15)/L(taxi(i,1),8);
            taxi(i,18)=L(taxi(i,1),15)/L(taxi(i,1),8);
            if temp<L(taxi(i,1),15)
                taxi(i,2)=temp;
            end
            if temp>=L(taxi(i,1),15)
                %canvi link
                taxi(i,2)=temp-L(taxi(i,1),15);
                if taxi(i,21)==1%taxi assignat
                    canvi_link3
                end
                if taxi(i,21)==0%taxi no assignat
                    canvi_link2
                end
                taxi(i,1)=sol;
            end
            taxi(i,10)=taxi(i,10)+1;
            %calculo les coordenades del taxi
            aa=L(taxi(i,1),9)+taxi(i,2)*(L(taxi(i,1),11)-L(taxi(i,1),9))/L(taxi(i,1),15);
            bb=L(taxi(i,1),10)+taxi(i,2)*(L(taxi(i,1),12)-L(taxi(i,1),10))/L(taxi(i,1),15);
        end
        if taxi(i,15)==1
            taxi(i,15)=taxi(i,15)+1;
        end
    end
    %taxi buit
    if taxi(i,4)==0
        taxi(i,7)=taxi(i,7)+1;
        taxi(i,15)=taxi(i,15)-1;
        taxi(i,12)=(1-taxi(i,3))*aa;
        taxi(i,13)=(1-taxi(i,3))*bb;
        taxi(i,19)=taxi(i,3)*aa;
        taxi(i,20)=taxi(i,3)*bb;
        if taxi(i,22)>0
            taxi(i,25)=aa;
            taxi(i,26)=bb;
        end
        taxi(i,5)=taxi(i,5)+taxi(i,18);
        %recollida client?
        (a b)=min(1000000*abs(taxi(i,1)-client(:,2))+abs(taxi(i,2)-client(:,3)));
        if a<1000000 & taxi(i,2)>=client(b,3) & taxi(i,2)-taxi(i,18)<=client(b,3) & client(b,9)==taxi(i,3) & taxi(i,22)==0
            client(b,1)=0;
            client(b,2)=0;
            client(b,11)=0;
            client(b,12)=0;
            client(b,21)=0;
            client(b,22)=0;
            client(b,13)=i;
            client(b,10)=bd;
            taxi(i,4)=client(b,4);
            taxi(i,5)=client(b,5);
            taxi(i,11)=b;
            taxi(i,12)=0;
            taxi(i,13)=0;
            taxi(i,19)=0;
            taxi(i,20)=0;
            taxi(i,16)=aa;
            taxi(i,17)=bb;
        end
        %arriba a parada i esta buida?
        if taxi(i,1)==taxi(i,23) & taxi(i,2)>=taxi(i,24) & parad(taxi(i,22),7)==0
        taxi(i,12)=parad(taxi(i,22),4);
```

```
        taxi(i,13)=parad(taxi(i,22),5);
        parad(taxi(i,22),6)=parad(taxi(i,22),6)+1;
        fifota(taxi(i,22),parad(taxi(i,22),6))=i;
        taxi(i,4)=-1;
        taxi(i,25)=parad(taxi(i,22),4);
        taxi(i,26)=parad(taxi(i,22),5);
        end
        %arriba a parada i hi ha cua?
        if taxi(i,1)==taxi(i,23) & taxi(i,2)>=taxi(i,24) & parad(taxi(i,22),7)>0
          parad(taxi(i,22),7)=parad(taxi(i,22),7)-1;
          cli_cua=fifocl(taxi(i,22),1);
          client(cli_cua,1)=0;
          client(cli_cua,2)=0;
          client(cli_cua,11)=0;
          client(cli_cua,12)=0;
          client(cli_cua,21)=0;
          client(cli_cua,22)=0;
          client(cli_cua,25)=taxi(i,12);
          client(cli_cua,26)=taxi(i,13);
          client(cli_cua,13)=i;
          client(cli_cua,10)=bd;
          client(cli_cua,9)=0;
          taxi(i,4)=client(cli_cua,4);
          taxi(i,5)=client(cli_cua,5);
          taxi(i,11)=cli_cua;
          taxi(i,19)=0;
          taxi(i,20)=0;
          taxi(i,16)=taxi(i,12);
          taxi(i,17)=taxi(i,13);
          taxi(i,12)=0;
          taxi(i,13)=0;
          taxi(i,25)=0;
          taxi(i,26)=0;
          fifocl(taxi(i,22),1:end-1)=fifocl(taxi(i,22),2:end);
          fifocl(taxi(i,22),end)=0;
          pathstaxis(i,:)=paths(cli_cua,:);
        end
      end
      %taxi ocupat
      if taxi(i,4)>0 & taxi(i,21)==0 %sense assignacio
        taxi(i,8)=taxi(i,8)+1;
        taxi(i,15)=taxi(i,15)-1;
        taxi(i,16)=aa;
        taxi(i,17)=bb;
        taxi(i,6)=taxi(i,6)+taxi(i,18);
        %lliurament de client?
        (a b)=min(abs(taxi(i,1)-taxi(i,4)));
        if a==0 & taxi(i,2)>=client(taxi(i,11),5) & taxi(i,11)>0
          taxi(i,8)=taxi(i,8)-1;
          taxi(i,15)=taxi(i,15)+1;
          client(taxi(i,11),1)=-9999999999;
          taxi(i,4)=0;
          client(taxi(i,11),2)=0;
          client(taxi(i,11),4)=0;
          client(taxi(i,11),8)=client(taxi(i,11),8)+taxi(client(taxi(i,11),13),18);
          client(taxi(i,11),21)=0;
          client(taxi(i,11),22)=0;
          client(taxi(i,11),23)=0;
          client(taxi(i,11),24)=0;
          client(taxi(i,11),25)=0;
          client(taxi(i,11),26)=0;
          taxi(i,25)=0;
          taxi(i,26)=0;
          %taxi(i,8)=taxi(i,8)+client(taxi(i,11),5);
          taxi(i,16)=0;
          taxi(i,17)=0;
          taxi(i,12)=(1-taxi(i,3))*aa;
          taxi(i,13)=(1-taxi(i,3))*bb;
          taxi(i,19)=taxi(i,3)*aa;
          taxi(i,20)=taxi(i,3)*bb;
          fare2
```

```
          taxi(i,9)=taxi(i,9)+client(taxi(i,11),10);
          taxi(i,11)=0;
          %va a parada??
          fet=0;
          if taxi(i,22)>0
             taxi(i,25)=aa;
             taxi(i,26)=bb;
             (a b)=min(abs(taxi(i,1)-parad(:,2)));
             if a==0 & taxi(i,2)<parad(b,3)
                fet=1;
                parada_as=b;
                %pathstaxis(i,:)=pathsparades(parada_as,:);
                taxi(i,23)=parad(parada_as,2);
                taxi(i,22)=parada_as;
                taxi(i,24)=parad(parada_as,3);
             end
          end
          if fet==0 & taxi(i,22)>0
             %lassigno a una parada
             stops2
          end
       end
    end
    %taxi ocupat
    if taxi(i,4)>0 & taxi(i,21)==1 %assignat
       taxi(i,8)=taxi(i,8)+1;
       taxi(i,15)=taxi(i,15)-1;
       taxi(i,6)=taxi(i,6)+taxi(i,18);
       taxi(i,16)=aa;
       taxi(i,17)=bb;
       %recollida de client?
       a=taxi(i,1)-taxi(i,4);
       if a==0 & taxi(i,2)>=client(taxi(i,11),3)
          client(taxi(i,11),1)=0;
          client(taxi(i,11),2)=0;
          client(taxi(i,11),11)=0;
          client(taxi(i,11),12)=0;
          client(taxi(i,11),21)=0;
          client(taxi(i,11),22)=0;
          client(taxi(i,11),23)=0;
          client(taxi(i,11),24)=0;
          %client(b,21)=0;
          %client(b,13)=i;
          %client(b,10)=bd;
          %client(taxi(i,11),9)=0;
          taxi(i,4)=client(taxi(i,11),4);
          taxi(i,5)=client(taxi(i,11),5);
          %taxi(i,11)=b;
          taxi(i,12)=0;
          taxi(i,13)=0;
          taxi(i,16)=aa;
          taxi(i,17)=bb;
          taxi(i,19)=0;
          taxi(i,20)=0;
          taxi(i,21)=0;
       end
    end
 end
 %"moc" els clients
 figure(3)
 o=size(client);
 for u=1:o(1,1)
    if client(u,1)>0
      client(u,6)=client(u,6)+1;
      x=1.01*client(u,11);
      y=1.01*client(u,12);
      no=client(u,6);
      client(u,23)=0;
      client(u,24)=0;
      client(u,25)=0;
      client(u,26)=0;
```

```
        if client(u,9)==-1
            client(u,21)=0;
            client(u,22)=0;
            client(u,25)=client(u,11);
            client(u,26)=client(u,12);
        end
        %eval(('text(x,y,' 'num2str(no)' ')'))
      end
      if client(u,1)==0 & abs(client(u,9))==0
        client(u,7)=client(u,7)+1;
        client(u,19)=client(u,19)+1;
        client(u,8)=client(u,8)+taxi(client(u,13),18);
        client(u,20)=client(u,20)+taxi(client(u,13),18);
        xx=taxi(client(u,13),16);
        yy=taxi(client(u,13),17);
        client(u,21)=xx;
        client(u,22)=yy;
        client(u,23)=0;
        client(u,24)=0;
        client(u,25)=0;
        client(u,26)=0;
        if taxi(client(u,13),22)>0
            client(u,21)=0;
            client(u,22)=0;
            client(u,25)=xx;
            client(u,26)=yy;
        end
        %no=client(taxi(client(u,13),11),7);
        %eval(('text(xx,yy,' 'num2str(no)' ')'))
        %es mou el taximetre?
        fare
      end
      if abs(client(u,9))>0 & client(u,1)==0
        client(u,7)=client(u,7)+1;
        client(u,19)=client(u,19)+1;
        client(u,8)=client(u,8)+taxi(client(u,13),18);
        client(u,20)=client(u,20)+taxi(client(u,13),18);
        xx=taxi(client(u,13),16);
        yy=taxi(client(u,13),17);
        client(u,23)=0;
        client(u,24)=0;
        client(u,25)=0;
        client(u,26)=0;
        if client(u,9)==1
            client(u,23)=xx;
            client(u,24)=yy;
        end
        if taxi(client(u,13),22)>0
            client(u,21)=0;
            client(u,22)=0;
            client(u,25)=xx;
            client(u,26)=yy;
        end
        no=client(taxi(client(u,13),11),7);
        %eval(('text(xx,yy,' 'num2str(no)' ')'))
        %es mou el taximetre?
        fare
      end
      %pause(1/speed)
   end
   pause(1/speed)
   %dibuixo els taxis i els clients
   figure(3)
   for h=1:length(L)
   a=(L(h,9),L(h,11));
   b=(L(h,10),L(h,12));
   plot (a,b,'k')
   hold on
   end
   plot(parad(:,4),parad(:,5),'bs')
   for u=1:numparad
```

```
      xp=parad(u,4)+5;
      yp=parad(u,5)+10;
      nop=parad(u,6);
      eval(('text(xp,yp,' 'num2str(nop)' ')'))
      xp=parad(u,4)-10;
      yp=parad(u,5)+10;
      nop=parad(u,7);
      eval(('text(xp,yp,' 'num2str(nop)' ')'))
   end
   plot(client(:,11),client(:,12),'r*')
   plot(client(:,21),client(:,22),'g*')
   plot(client(:,23),client(:,24),'m*')
   plot(client(:,25),client(:,26),'b*')
   axis((1 450 1 550))
   plot(taxi(:,12),taxi(:,13),'ro')
   plot(taxi(:,19),taxi(:,20),'ko')
   plot(taxi(:,16),taxi(:,17),'go')
   plot(taxi(:,25),taxi(:,26),'bo')
   %plot(taxi(:,16),taxi(:,17),'g*')
   %dades
   x=400;
   y=450;
   no=t;
   %eval(('text(x,y,' 'num2str(no)' ')'))
   hold off
end
```

dij.m

```
%trobo el shortest path pels clients
o=size(client);
NoN=24;
NoL=76;
%genero un vector temporal de links
LL=L;
%esborro els antics vectors de path i dijkstra
clear dijkstra
clear path
%elimino el path on sha originat el client i es path on vol anar per evitar
%girs de 180 graus
w=LL(client(posc,2),1);
ww=LL(client(posc,2),2);
q=LL(client(posc,4),1);
qq=LL(client(posc,4),2);
LL(client(posc,2),:)=();
esb=0;
for n=1:NoL-1
   if LL(n,1)==ww
      if LL(n,2)==w
         nn=n;
         esb=1;
      end
   end
end
if esb==1
   LL(nn,:)=();
end
esb=0;
for m=1:NoL-2
   if LL(m,1)==qq
      if LL(m,2)==q
         mm=m;
         esb=1;
      end
   end
end
if esb==1
   LL(mm,:)=();
end
%genero el vector dikjstra
dijkstra(:,1) = 1:NoN;
dijkstra(:,2) = ones(NoN,1);
```

```
dijkstra(:,3) = 9999999999*ones(NoN,1);
dijkstra(:,4) = zeros(NoN,1);
dijkstra(:,5) = 9999999999*ones(NoN,1);
%dono origen i destinacio
Origin=L(client(posc,2),2);
Destination=L(client(posc,4),1);
%resolem dijkstra
dijkstra(Origin,2)=0;%kleinei to prwto node (poy einai iso me to origin)
dijkstra(Origin,3)=0; % midenizo to kostos tou prwto node
for i=1:NoL-3
   if LL(i,1)==Origin
      temp=dijkstra(Origin,3)+ LL(i,15).*congestio(round(24*t/final-0.5)+1,2);
      if temp < dijkstra(LL(i,2),3)
       dijkstra(LL(i,2),3)= temp;
       dijkstra(LL(i,2),4) = LL(i,1);
       dijkstra(LL(i,2),5)= temp;
      end
   end
    dijkstra(Origin,5)= 9999999999;
end
contador=1;
while dijkstra(Destination,2)>0 & contador<NoL+1
   contador=contador+1;
   (value position)=min(dijkstra(:,5));
   position;
   for i=1:NoL-3
      if LL(i,1)==position
         temp=dijkstra(position,3)+ LL(i,15).*congestio(round(24*t/final-0.5)+1,2);
         if temp < dijkstra(LL(i,2),3)
          dijkstra(LL(i,2),3)= temp;
          dijkstra(LL(i,2),4) = LL(i,1);
          dijkstra(LL(i,2),5)= temp;
         end
      end
   end
    dijkstra(position,5)= 9999999999;
    dijkstra(position,2)= 0;
    %plot(NodeID(position,2),NodeID(position,3),'yo')
    %hold on
end
%genero el path a seguir i lescric a la matriu de paths
path = Destination;
count=2;
if dijkstra(Destination,4)>0
   while abs(path(count-1)- Origin) > 0
   path(count)=dijkstra(path(count-1),4);
   count=count+1;
   end
path(1:length(path))=path(length(path):-1:1);
path(1,length(path)+1)=L(client(posc,4),2);
path(NoL+1)=0;
paths(posc,NoL+1)=0;
paths(posc,:)=path();
paths(posc,NoL+1)=1;
end
```

<u>dij2.m</u>

```
%trobo el shortest path pels taxis
o=size(client);
NoN=24;
NoL=76;
%genero un vector temporal de links
LLL=L;
LLL(:,1)=L(:,2);
LLL(:,2)=L(:,1);
%esborro els antics vectors de path i dijkstra
clear dijkstra
clear path
clear distancies
%elimino el path oposat al que esta esperant el client
w=LLL(client(posc,2),1);
```

```
ww=LLL(client(posc,2),2);
LLL(client(posc,2),:)=();
for n=1:NoL-1
   if LLL(n,1)==ww
      if LLL(n,2)==w
         nn=n;
      end
   end
end
LLL(nn,:)=();
LL=LLL;
o=size(taxi);
distancies(1:o(1,1))=99999999999;
% per cada taxi trobo la distancia minima
for u=1:o(1,1)
   if taxi(u,3)==1 & taxi(u,4)==0
      LL=LLL;
      clear dijkstra;
      clear path
      w=L(taxi(u,1),1);
      ww=L(taxi(u,1),2);
      esb=0;
      for n=1:NoL-2
         if LL(n,2)==ww
            if LL(n,1)==w
               nn=n;
               esb=1;
            end
         end
      end
      if esb==1
         LL(nn,:)=();
      end
      esb=0;
      for n=1:NoL-3
         if LL(n,1)==ww
            if LL(n,2)==w
               nn=n;
               esb=1;
            end
         end
      end
      if esb==1
         LL(nn,:)=();
      end
      dijkstra(:,1) = 1:NoN;
      dijkstra(:,2) = ones(NoN,1);
      dijkstra(:,3) = 9999999999*ones(NoN,1);
      dijkstra(:,4) = zeros(NoN,1);
      dijkstra(:,5) = 9999999999*ones(NoN,1);
      Origin=L(client(posc,2),1);
      Destination=L(taxi(u,1),2);
      o=size(taxi);
      dijkstra(Origin,2)=0;%kleinei to prwto node (poy einai iso me to origin)
      dijkstra(Origin,3)=0; % midenizo to kostos tou prwto node
      for ii=1:NoL-4
         if LL(ii,1)==Origin
            temp=dijkstra(Origin,3)+ LL(ii,15).*congestio(round(24*t/final-0.5)+1,2);
            if temp < dijkstra(LL(ii,2),3)
             dijkstra(LL(ii,2),3)= temp;
             dijkstra(LL(ii,2),4) = LL(ii,1);
             dijkstra(LL(ii,2),5)= temp;
            end
         end
          dijkstra(Origin,5)= 9999999999;
      end
      contador=1;
      while dijkstra(Destination,2)>0 & contador<NoL+1
         contador=contador+1;
         (value position)=min(dijkstra(:,5));
         position;
```

```
            for ii=1:NoL-4
                if LL(ii,1)==position
                    temp=dijkstra(position,3)+ LL(ii,15).*congestio(round(24*t/final-0.5)+1,2);
                    if temp < dijkstra(LL(ii,2),3)
                     dijkstra(LL(ii,2),3)= temp;
                     dijkstra(LL(ii,2),4) = LL(ii,1);
                     dijkstra(LL(ii,2),5)= temp;
                    end
                end
            end
             dijkstra(position,5)= 9999999999;
             dijkstra(position,2)= 0;
             %plot(NodeID(position,2),NodeID(position,3),'yo')
             %hold on
        end
        path = Destination;
        count=2;
        if dijkstra(Destination,4)>0
            while abs(path(count-1)- Origin) > 0
                path(count)=dijkstra(path(count-1),4);
                count=count+1;
            end
            path(1,length(path)+1)=L(client(posc,2),2);
            path(NoL+1)=0;
            pathstaxis(u,NoL+1)=0;
            pathstaxis(u,:)=path();
            pathstaxis(u,NoL+1)=1;
            distancies(u)=dijkstra(Destination,3);
        end
    end
end
%escullo el taxi mes proper
(a b)=min(distancies);
taxi_as=b;
if a<99999999999
    taxi(taxi_as,4)=client(posc,14);
    taxi(taxi_as,5)=client(posc,15);
    taxi(taxi_as,11)=posc;
    taxi(taxi_as,12)=0;
    taxi(taxi_as,13)=0;
    taxi(taxi_as,16)=aa;
    taxi(taxi_as,17)=bb;
    taxi(taxi_as,19)=0;
    taxi(taxi_as,20)=0;
    taxi(taxi_as,21)=1;
    client(posc,2)=0;
    client(posc,23)=client(posc,11);
    client(posc,24)=client(posc,12);
    client(posc,13)=taxi_as;
    client(posc,10)=bd;
end
```

<u>fare.m</u>

```
%cost a cada bhma en funcio de la distancia o del temps (vmax 30km/h)
%3 euros baixada de bandera
%0.2 euro per 50m
%0.2 euro per 12 seg
%quan va a 15km/h es el mateix cobrar per distancia o per temps, per sota
%de 15km/h guanya mes per temps, per sobre de 15km/h per distancia
if client(u,19)>=tar_time
    client(u,10)=client(u,10)+tar;
    client(u,19)=0;
    client(u,20)=0;
end
if client(u,20)>=tar_dist
    client(u,10)=client(u,10)+tar;
    client(u,19)=1;
    client(u,20)=client(u,20)-tar_dist;
end
```

fare2.m

```
%carrego la ultima part de la tarifa
```

```
cost_dist=tar*client(taxi(i,11),19)/tar_time;
cost_time=tar*client(taxi(i,11),20)/tar_dist;
if cost_dist>=cost_time
   client(taxi(i,11),10)=client(taxi(i,11),10)+cost_dist;
   client(taxi(i,11),19)=0;
   client(taxi(i,11),20)=0;
end
if cost_dist<cost_time
   client(taxi(i,11),10)=client(taxi(i,11),10)+cost_time;
   client(taxi(i,11),19)=0;
   client(taxi(i,11),20)=0;
end
```

stops.m

```
%trobo la parada mes propera
o=size(client);
NoN=24;
NoL=76;
pathstaxis(post,1:NoL+1)=0;
%genero un vector temporal de links
LLL=L;
%esborro els antics vectors de path i dijkstra
clear dijkstra
clear path
clear distancies
clear pathsparades
%elimino el path oposat al que esta el taxi
w=LLL(taxi(post,1),1);
ww=LLL(taxi(post,1),2);
LLL(taxi(post,1),:)=();
for n=1:NoL-1
   if LLL(n,1)==ww
      if LLL(n,2)==w
         nn=n;
      end
   end
end
LLL(nn,:)=();
LL=LLL;
o=size(parad);
distancies(1:o(1,1))=99999999999;
% per cada taxi parada la distancia minima
for u=1:o(1,1)
   if abs(parad(u,2)-taxi(post,1))>0
      LL=LLL;
      clear dijkstra;
      clear path
      w=L(parad(u,2),1);
      ww=L(parad(u,2),2);
      esb=0;
      for n=1:NoL-2
         if LL(n,1)==ww
            if LL(n,2)==w
               nn=n;
               esb=1;
            end
         end
      end
      if esb==1
         LL(nn,:)=();
      end
      esb=0;
      w=L(parad(u,2),1);
      ww=L(parad(u,2),2);
      for n=1:NoL-3
         if LL(n,1)==w
            if LL(n,2)==ww
               nn=n;
               esb=1;
            end
         end
```

```
    end
    if esb==1
      LL(nn,:)=();
    end
    dijkstra(:,1) = 1:NoN;
    dijkstra(:,2) = ones(NoN,1);
    dijkstra(:,3) = 9999999999*ones(NoN,1);
    dijkstra(:,4) = zeros(NoN,1);
    dijkstra(:,5) = 9999999999*ones(NoN,1);
    Origin=L(taxi(post,1),2);
    Destination=L(parad(u,2),1);
    if length(LL)<73
      o=size(taxi);
      dijkstra(Origin,2)=0;%kleinei to prwto node (poy einai iso me to origin)
      dijkstra(Origin,3)=0; % midenizo to kostos tou prwto node
      for ii=1:NoL-4
        if LL(ii,1)==Origin
          temp=dijkstra(Origin,3)+ LL(ii,15).*congestio(round(24*t/final-0.5)+1,2);
          if temp < dijkstra(LL(ii,2),3)
           dijkstra(LL(ii,2),3)= temp;
           dijkstra(LL(ii,2),4) = LL(ii,1);
           dijkstra(LL(ii,2),5)= temp;
          end
        end
         dijkstra(Origin,5)= 9999999999;
      end
      contador=1;
      while dijkstra(Destination,2)>0 & contador<NoL+1
        contador=contador+1;
        (value position)=min(dijkstra(:,5));
        position;
        for ii=1:NoL-4
          if LL(ii,1)==position
            temp=dijkstra(position,3)+ LL(ii,15).*congestio(round(24*t/final-0.5)+1,2);
            if temp < dijkstra(LL(ii,2),3)
             dijkstra(LL(ii,2),3)= temp;
             dijkstra(LL(ii,2),4) = LL(ii,1);
             dijkstra(LL(ii,2),5)= temp;
            end
          end
        end
         dijkstra(position,5)= 9999999999;
         dijkstra(position,2)= 0;
         %plot(NodeID(position,2),NodeID(position,3),'yo')
         %hold on
      end
      path = L(parad(u,2),2);
      path (2) = Destination;
      count=3;
      if dijkstra(Destination,4)>0
        while abs(path(count-1)- Origin) > 0
          path(count)=dijkstra(path(count-1),4);
          count=count+1;
        end
        path(1:end)=path(end:-1:1);
        path(NoL+1)=0;
        pathsparades(u,NoL+1)=0;
        pathsparades(u,:)=path(:);
        pathsparades(u,NoL+1)=1;
        distancies(u)=dijkstra(Destination,3);
      end
    end
  end
end
%escullo el taxi mes proper
(a b)=min(distancies);
parada_as=b;
pathstaxis(post,:)=pathsparades(parada_as,:);
taxi(post,23)=parad(parada_as,2);
taxi(post,22)=parada_as;
taxi(post,24)=parad(parada_as,3);
```

stops2.m

```
%trobo la parada mes propera
o=size(client);
NoN=24;
NoL=76;
pathstaxis(post,1:NoL+1)=0;
%genero un vector temporal de links
LLL=L;
%esborro els antics vectors de path i dijkstra
clear dijkstra
clear path
clear distancies
clear pathsparades
%elimino el path oposat al que esta el taxi
w=LLL(taxi(i,1),1);
ww=LLL(taxi(i,1),2);
LLL(taxi(i,1),:)=();
for n=1:NoL-1
   if LLL(n,1)==ww
      if LLL(n,2)==w
         nn=n;
      end
   end
end
LLL(nn,:)=();
LL=LLL;
o=size(parad);
distancies(1:o(1,1))=99999999999;
% per cada taxi parada la distancia minima
for u=1:o(1,1)
   if abs(parad(u,2)-taxi(i,1))>0
      LL=LLL;
      clear dijkstra;
      clear path
      w=L(parad(u,2),1);
      ww=L(parad(u,2),2);
      esb=0;
      for n=1:NoL-2
         if LL(n,1)==ww
            if LL(n,2)==w
               nn=n;
               esb=1;
            end
         end
      end
      if esb==1
         LL(nn,:)=();
      end
      esb=0;
      w=L(parad(u,2),1);
      ww=L(parad(u,2),2);
      for n=1:NoL-3
         if LL(n,1)==w
            if LL(n,2)==ww
               nn=n;
               esb=1;
            end
         end
      end
      if esb==1
         LL(nn,:)=();
      end
      dijkstra(:,1) = 1:NoN;
      dijkstra(:,2) = ones(NoN,1);
      dijkstra(:,3) = 9999999999*ones(NoN,1);
      dijkstra(:,4) = zeros(NoN,1);
      dijkstra(:,5) = 9999999999*ones(NoN,1);
      Origin=L(taxi(i,1),2);
      Destination=L(parad(u,2),1);
      if length(LL)<73
```

```
            o=size(taxi);
            dijkstra(Origin,2)=0;%kleinei to prwto node (poy einai iso me to origin)
            dijkstra(Origin,3)=0; % midenizo to kostos tou prwto node
            for ii=1:NoL-4
               if LL(ii,1)==Origin
                  temp=dijkstra(Origin,3)+ LL(ii,15).*congestio(round(24*t/final-0.5)+1,2);
                  if temp < dijkstra(LL(ii,2),3)
                   dijkstra(LL(ii,2),3)= temp;
                   dijkstra(LL(ii,2),4) = LL(ii,1);
                   dijkstra(LL(ii,2),5)= temp;
                  end
               end
                dijkstra(Origin,5)= 9999999999;
            end
            contador=1;
            while dijkstra(Destination,2)>0 & contador<NoL+1
               contador=contador+1;
               (value position)=min(dijkstra(:,5));
               position;
               for ii=1:NoL-4
                  if LL(ii,1)==position
                     temp=dijkstra(position,3)+ LL(ii,15).*congestio(round(24*t/final-0.5)+1,2);
                     if temp < dijkstra(LL(ii,2),3)
                      dijkstra(LL(ii,2),3)= temp;
                      dijkstra(LL(ii,2),4) = LL(ii,1);
                      dijkstra(LL(ii,2),5)= temp;
                     end
                  end
               end
                dijkstra(position,5)= 9999999999;
                dijkstra(position,2)= 0;
                %plot(NodeID(position,2),NodeID(position,3),'yo')
                %hold on
            end
            path = L(parad(u,2),2);
            path (2) = Destination;
            count=3;
            if dijkstra(Destination,4)>0
               while abs(path(count-1)- Origin) > 0
                  path(count)=dijkstra(path(count-1),4);
                  count=count+1;
               end
               path(1:end)=path(end:-1:1);
               path(NoL+1)=0;
               pathsparades(u,NoL+1)=0;
               pathsparades(u,:)=path(:);
               pathsparades(u,NoL+1)=1;
               distancies(u)=dijkstra(Destination,3);
            end
         end
   end
end
%escullo el taxi mes proper
(a b)=min(distancies);
parada_as=b;
pathstaxis(i,:)=pathsparades(parada_as,:);
taxi(i,23)=parad(parada_as,2);
taxi(i,22)=parada_as;
taxi(i,24)=parad(parada_as,3);
```

<u>canvi_link.m</u>

```
%%%canvi link taxi quan esta buit
%%contatge d'opcions
%inicio un contador
op=0;
%esborro els antics vectors de posibilitat i alternatives
clear pos
clear alt
%creo la llista dalternatives llegint la llista de links
for j=1:length(L)
   %comprobo si el node on ha arribat el taxi es el node inicial del link
```

```
  if L(j,1)==L(taxi(i,1),2)
    %comprobo que no es el node don venia el taxi
    if abs(L(j,2)-L(taxi(i,1),1))>0
      %actualitzo el contador dopcions
      op=op+1;
      %a les alternatives tinc el codic del link
      alt(op)=j;
      %a les posibilitats tinc el pes del link
      pos(op)=L(j,16);
    end
  end
end
%%genero el vector de possibilitats
%inicio un contador
p=0;
%esborro lantic vector de probabilitats
clear prob
%genero el vector de probabilitat en funcio del pes de cada link
for j=1:length(alt)
  prob(2*j-1)=p+1;
  prob(2*j)=p+pos(j);
  p=prob(2*j);
end
%escullo de forma aleatoria el node (sol)
sort=p*rand;
(asort bsort)=min(abs(prob-sort));
sol=alt(round(bsort/2));
```

canvi_link2.m

```
%%%canvi link taxi quan esta ocupat (ruta shortest path)
%%contatge d'opcions
%inicio un contador
op=0;
%esborro els antics vectors de node i alternatives
clear alt
clear nod
%creo la llista dalternatives llegint la llista de links
for j=1:length(L)
  %comprobo si el node on ha arribat el taxi es el node inicial del link
  if L(j,1)==L(taxi(i,1),2)
    %comprobo que no es el node don venia el taxi
    if abs(L(j,2)-L(taxi(i,1),1))>0
      %actualitzo el contador dopcions
      op=op+1;
      %a les alternatives tinc el codic del link
      alt(op)=j;
      %als nodes tinc el seguent node on aniria seguint el link
      %corresponent
      nod(op)=L(j,2);
    end
  end
end
%llegeixo del vector de paths dels clients el seguent node a visitar(sol)
paths(taxi(i,11),NoL+1)=paths(taxi(i,11),NoL+1)+1;
seg=paths(taxi(i,11),paths(taxi(i,11),NoL+1));
(val pos)=min(abs(nod-seg));
sol=alt(pos);
```

canvi_link3.m

```
%%%canvi link taxi quan el taxi va a buscar al client (ruta shortest path)
%%contatge d'opcions
%inicio un contador
op=0;
%esborro els antics vectors de node i alternatives
clear alt
clear nod
%creo la llista dalternatives llegint la llista de links
for j=1:length(L)
  %comprobo si el node on ha arribat el taxi es el node inicial del link
  if L(j,1)==L(taxi(i,1),2)
    %comprobo que no es el node don venia el taxi
```

```
    if abs(L(j,2)-L(taxi(i,1),1))>0
      %actualitzo el contador dopcions
      op=op+1;
      %a les alternatives tinc el codic del link
      alt(op)=j;
      %als nodes tinc el seguent node on aniria seguint el link
      %corresponent
      nod(op)=L(j,2);
    end
  end
end
%llegeixo del vector de paths dels taxis el seguent node a visitar(sol)
pathstaxis(i,NoL+1)=pathstaxis(i,NoL+1)+1;
seg=pathstaxis(i,pathstaxis(i,NoL+1));
(val pos)=min(abs(nod-seg));
sol=alt(pos);
```

Source code of the Matlab programs (methodology for minimizing the difference between the optimal supply and the one provided by the shifts system).

data.m
```
clear all
clc
l=1;
M=5912*(8/l);
A=zeros(24);

for i=1:24
    A(i,i:i+l-1)=1;
end

for i=1:l-1
    A(:,i)=A(:,i)+A(:,i+24);
end

AA=A(1:24,1:24);
AA=AA';
Oo=[776
651
553
526
522
626
923
1863
3005
3667
3348
3406
3374
2900
2265
2564
3331
2848
2592
2462
1961
1453
864
823];
B=[16
16
16
16
16
16
16
17
18
18
18
18
18
18
17
17
18
18
17
17
17
17
16
16];
ub=[M
    M
M
M
M
M
M
```

```
M
M
M
M
M
M
M
M
M
M
M
M
M
M
M
M
M];
x0=sum(Oo)*ones(24,1)/(8*24);
C=ones(1,24);
lb=[0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0
    0];

mh=[573
473
395
374
371
453
692
1477
2460
3038
2759
2810
2782
2369
1820
2078
2744
2323
2102
1990
1560
1131
644
611];
```

eval_x.m

```
function f = eval_x(x)

l=1;
A=zeros(24);

for i=1:24
```

```
    A(i,i:i+l-1)=1;
end

for i=1:l-1
    A(:,i)=A(:,i)+A(:,i+24);
end

AA=A(1:24,1:24);
AA=AA';

Oo=[776
651
553
526
522
626
923
1863
3005
3667
3348
3406
3374
2900
2265
2564
3331
2848
2592
2462
1961
1453
864
823];

B=[16
16
16
16
16
16
16
17
18
18
18
18
18
18
17
17
18
18
17
17
17
17
16
16];

O=AA*x;

Be=AA*B/l;



for i=1:24
    m(i)=(Oo(i)-O(i))^2;
end


f=sum(m);
```

solver.m

```
function [x,fval,maxfval,exitflag,output,lambda] =
solver2(x0,Aineq,bineq,Aeq,beq,lb,ub)
```

```
%% This is an auto generated MATLAB file from Optimization Tool.

%% Start with the default options
options = optimset;
%% Modify options setting
options = optimset(options,'Display', 'off');
options = optimset(options,'PlotFcns', {  @optimplotx @optimplotfval });
[x,fval,maxfval,exitflag,output,lambda] = ...
fminimax(@eval_x,x0,Aineq,bineq,Aeq,beq,lb,ub,[],options);
```

ANNEX II: Screenshots of the Sioux Falls Network and the various configurations of the agent based simulation model

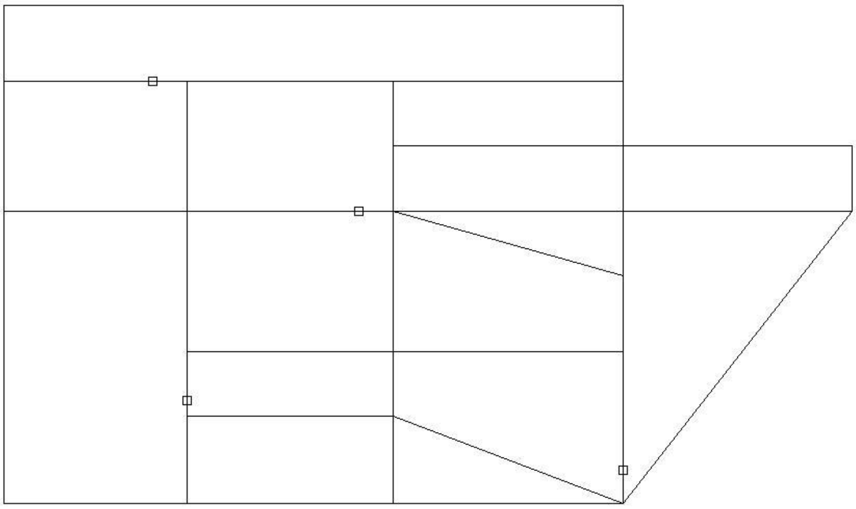

Figure 7-1 Sioux Falls Network and stands location

Figure 7-1 shows the links and nodes of the Sioux Falls network, composed by 24 nodes and 76 links (38 bidirectional links). Four taxi stands have been arbitrary added to the network for the graphical representation, represented by a square.

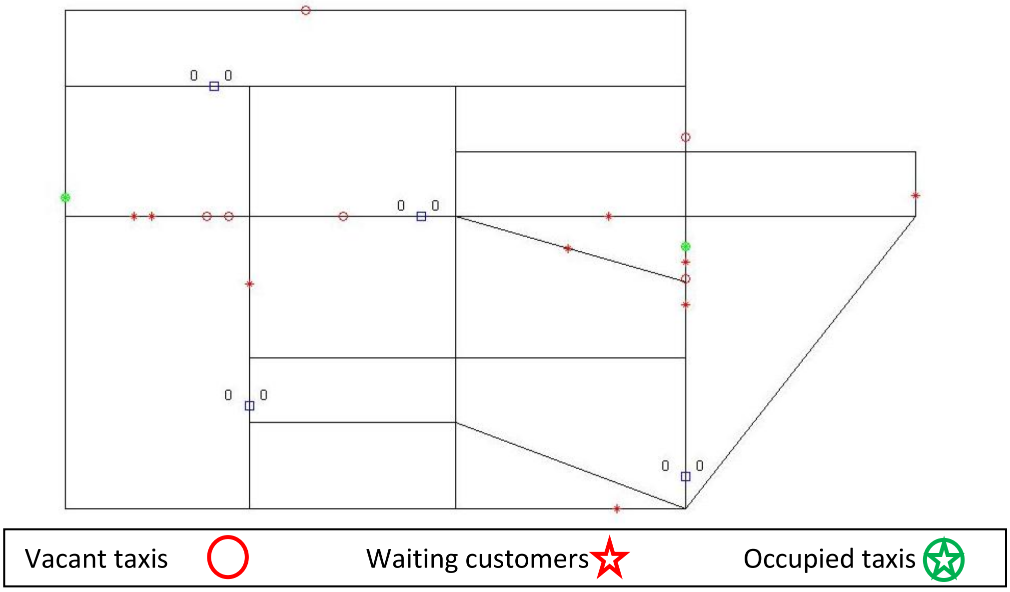

Figure 7-2 Hailing model

Figure 7-2 shows a screenshot of the hailing model, where red circles are vacant taxis looking for a ride and red stars are customers waiting for a taxi. Green circles and stars are occupied taxis with their respective customers traveling to the customer´s destination.

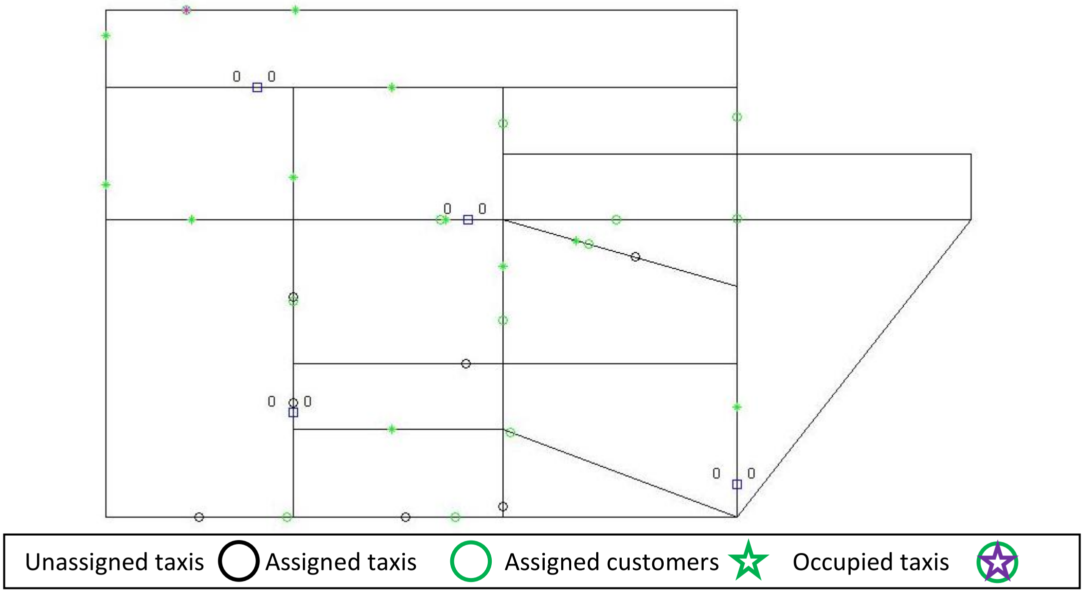

Figure 7-3 Dispatching model

Figure 7-3 shows a screenshot of the dispatching model, where black circles are unassigned circulating taxis waiting for an assignment, green circles are assigned taxis travelling to the assigned customer´s origin and green stars are assigned customers waiting for their assigned taxi. Green circles and rose stars are occupied taxis with their respective customers traveling to the customer´s destination.

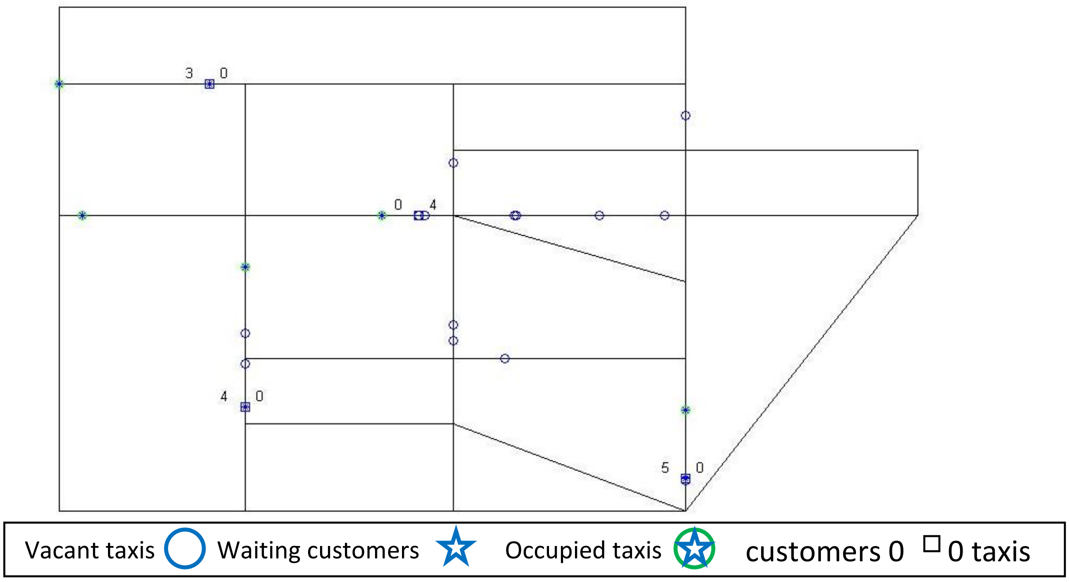

Figure 7-4 Stand model

Figure 7-4 shows a screenshot of the stand model, where the right number in each taxi stand is the number of taxis in the queue, while the left number is the number of customers in the queue. Blue circles are vacant taxis traveling to the nearest taxi stand, while blue stars are customer waiting at taxi stands. Green taxis with blue stars are occupied taxis with their respective customers traveling to the customer´s destination.

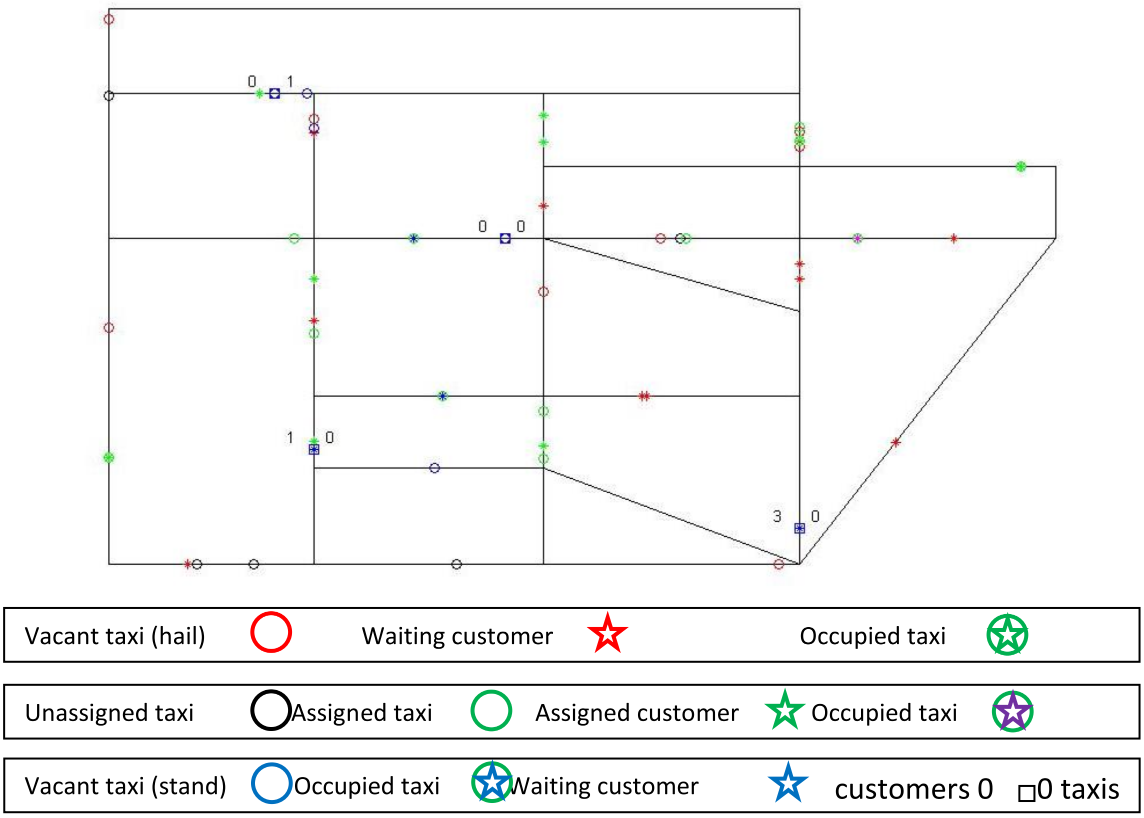

Figure 7-5 Hailing, dispatching and stand models running together

Figure 7-5 shows the three models running together, using the symbols described in each individual model.

ANNEX III: Demand distributions used in the agent based simulation model

7.1.1. Uniform distribution

The weight in this case is the same for all zones, meaning that the same amount of trips is generated between all zones. The weight of each zone and OD pair are presented in the figures below.

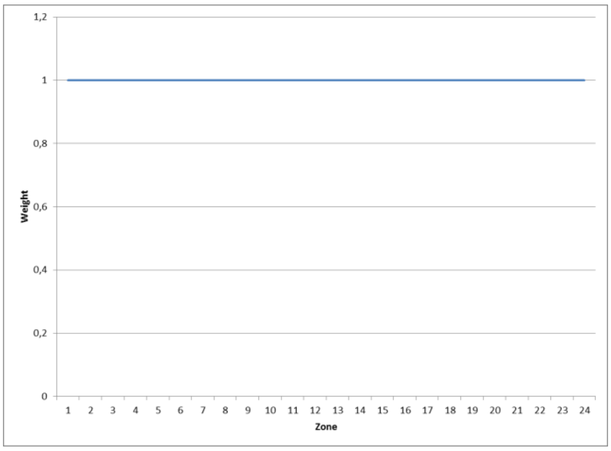

Figure 7-6 Demand weight of each node for the uniform distribution for the Sioux Falls Network

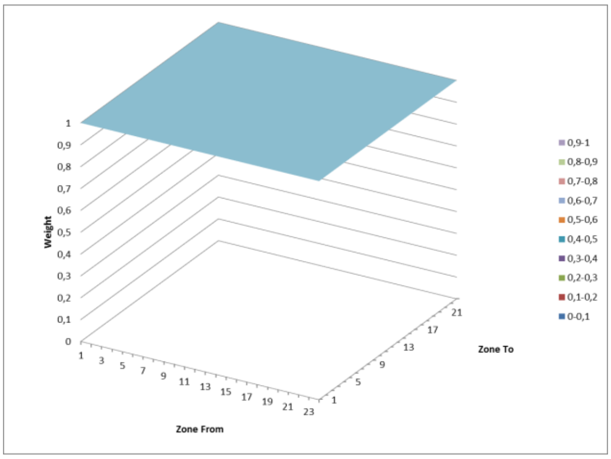

Figure 7-7 Demand weight of each OD pair for the uniform distribution for the Sioux Falls Network

7.1.2. Linear distribution

A linear distribution has been used in order to define the weight of the zones, which is higher for the central zones of the OD and lower for the zones in the extremes of the OD matrix.

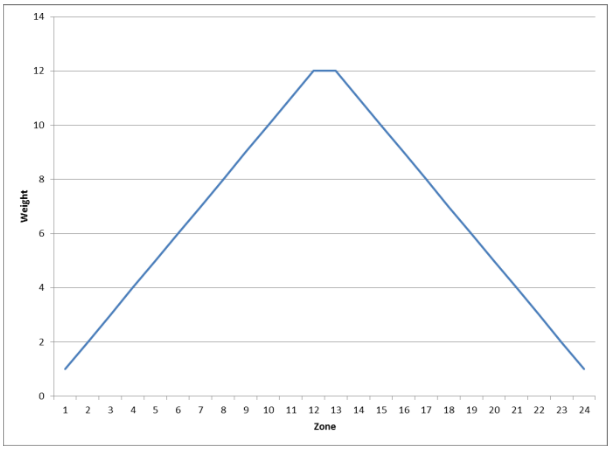

Figure 7-8 Demand weight of each node for the linear distribution for the Sioux Falls Network

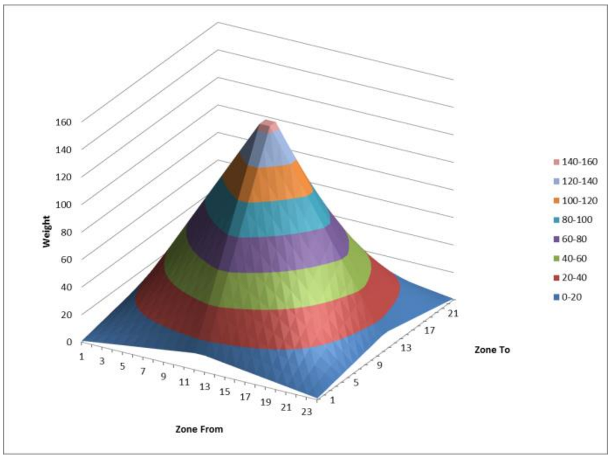

Figure 7-9 Demand weight of each OD pair for the linear distribution for the Sioux Falls Network

7.1.3. Gauss distribution with coefficient b=100

This distribution is equivalent to the uniform distribution.

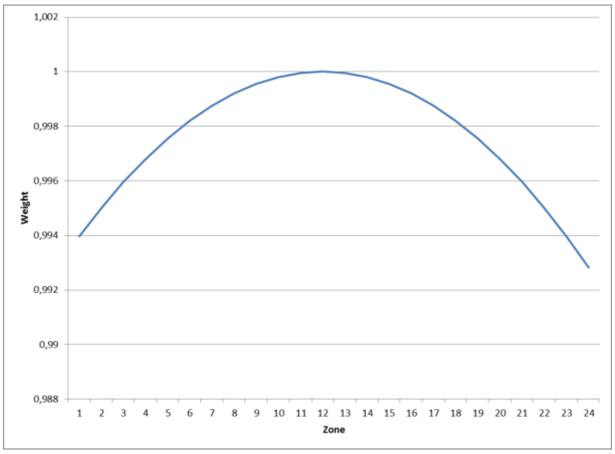

Figure 7-10 Demand weight of each node for the Gauss distribution (b=100) for the Sioux Falls Network

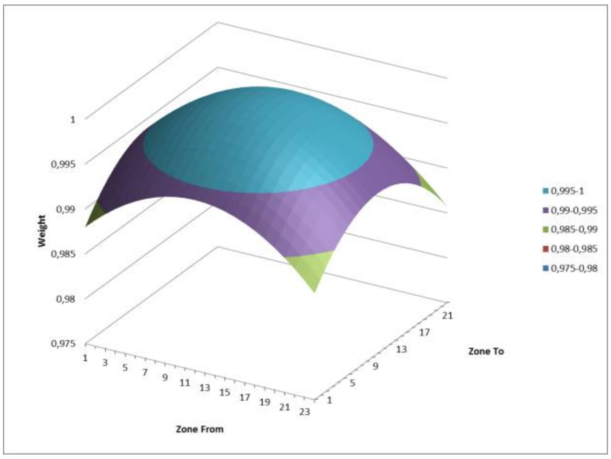

Figure 7-11 Demand weight of each OD pair for the Gauss distribution (b=100) for the Sioux Falls Network

7.1.4. Gauss distribution with coefficient 10

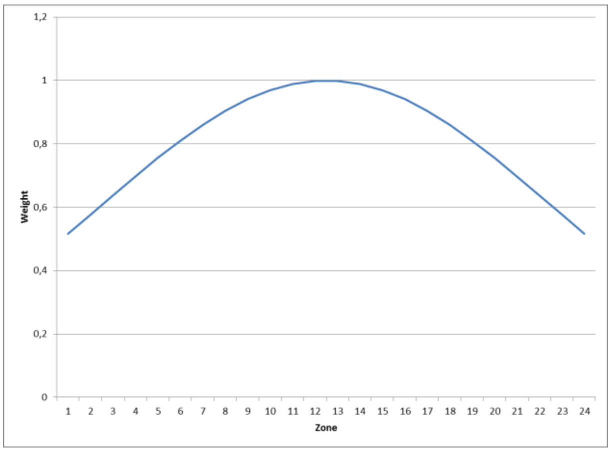

Figure 7-12 Demand weight of each node for the Gauss distribution (b=10) for the Sioux Falls Network

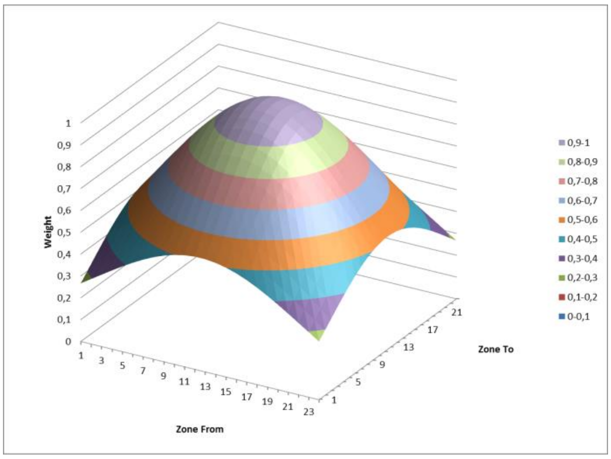

Figure 7-13 Demand weight of each OD pair for the Gauss distribution (b=10) for the Sioux Falls Network

7.1.5. Gauss distribution with coefficient 6

This distribution is very similar to thelinear presented in 7.1.2.

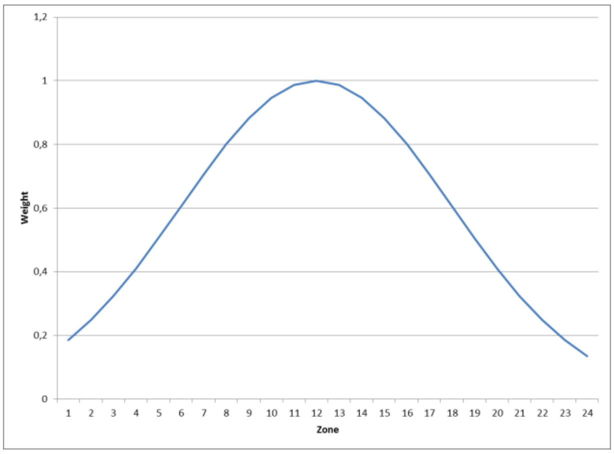

Figure 7-14 Demand weight of each node for the Gauss distribution (b=6) for the Sioux Falls Network

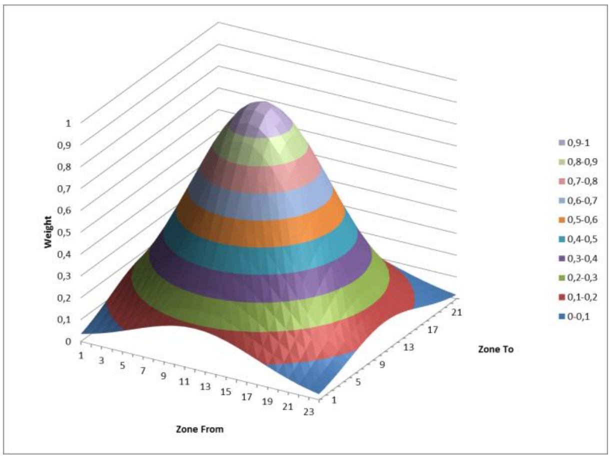

Figure 7-15 Demand weight of each OD pair for the Gauss distribution (b=6) for the Sioux Falls Network

7.1.6. Gauss distribution with coefficient 5

This distribution is very similar to thelinear presented in 7.1.2.

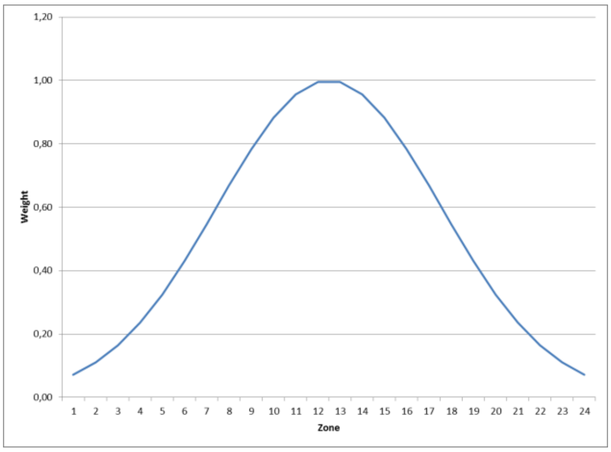

Figure 7-16 Demand weight of each node for the Gauss distribution (b=5) for the Sioux Falls Network

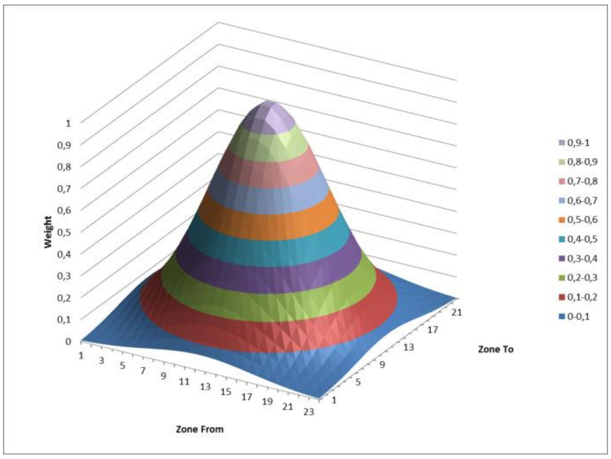

Figure 7-17 Demand weight of each OD pair for the Gauss distribution (b=5) for the Sioux Falls Network

7.1.7. Gauss distribution with coefficient 3

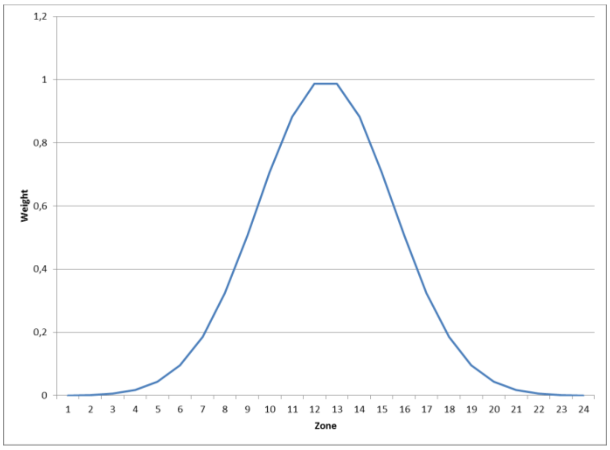

Figure 7-18 Demand weight of each node for the Gauss distribution (b=3) for the Sioux Falls Network

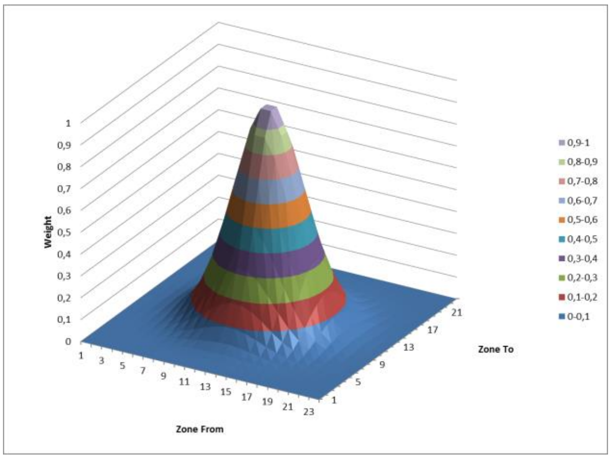

Figure 7-19 Demand weight of each OD pair for the Gauss distribution (b=3) for the Sioux Falls Network

7.1.8. Gauss distribution with coefficient 2

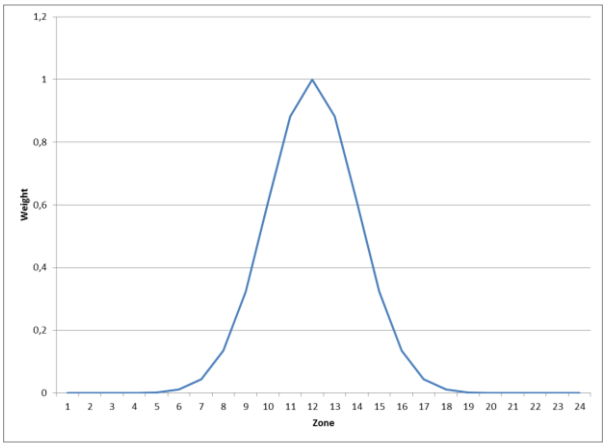

Figure 7-20 Demand weight of each node for the Gauss distribution (b=2) for the Sioux Falls Network

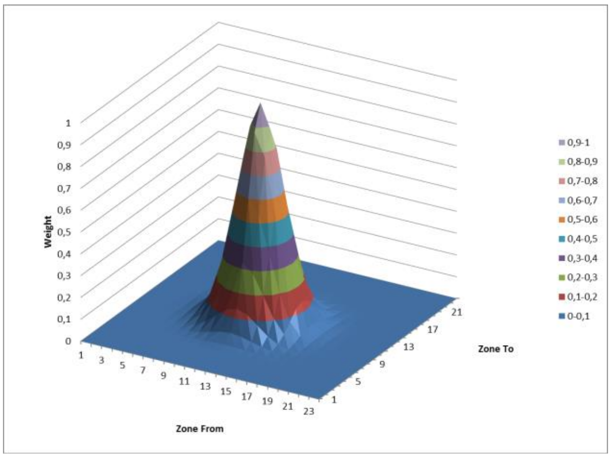

Figure 7-21 Demand weight of each OD pair for the Gauss distribution (b=2) for the Sioux Falls Network

7.1.9. Gauss distribution with coefficient 0.1

This distribution is equivalent to the concentration of all trips in a very small number of OD pairs.

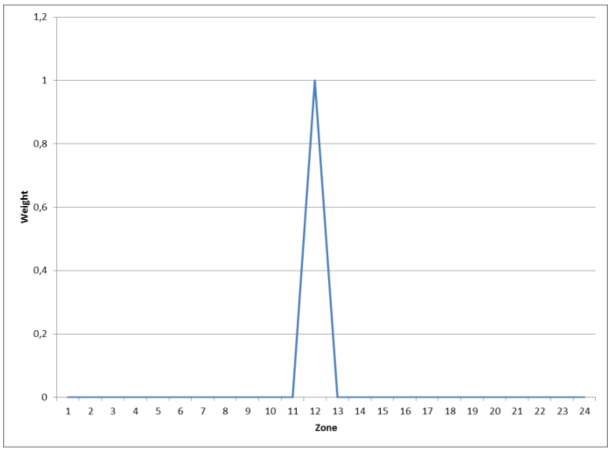

Figure 7-22 Demand weight of each node for the Gauss distribution (b=0.1) for the Sioux Falls Network

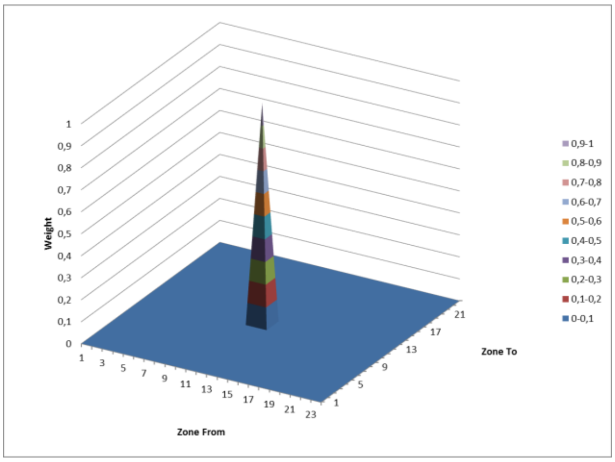

Figure 7-23 Demand weight of each OD pair for the Gauss distribution (b=0.1) for the Sioux Falls Network

ANNEX IV: Publications related to the research done in the thesis

Available online at www.sciencedirect.com

ScienceDirect

Procedia Social and Behavioral Sciences 20 (2011) 150–161

Procedia
Social and Behavioral Sciences

14th EWGT & 26th MEC & 1st RH

A review of the modeling of taxi services

Josep Maria Salanova[a,*], Miquel Estrada[b], Georgia Aifadopoulou[a], Evangelos Mitsakis[a]

[a]*Centre for Research and Technology Hellas (CERTH) – Hellenic Institute of Transport (HIT), Thessaloniki 57001, Greece*
[b]*Center for Innovation in Transport (CENIT), Barcelona Tech, Technical University of Catalonia (UPC), Barcelona 08034, Spain*

Abstract

This paper presents a review of the different models developed for the taxicab problem. The presented models are grouped in two categories, aggregated and equilibrium models. Each model is analyzed from different points of view, such as market organization, operational organization and regulation issues. Conclusions extracted by authors are presented, listed and compared, analyzing each affirmation in terms of market regulation and organization. Finally, a state of the practice is presented, analyzing the configuration of the taxi market regulations along the world, linking the conclusions obtained by the authors with the real market situations.

Keywords: The taxicab problem; Modeling taxi services; Taxi regulation; Transportation; Taxi

1. Introduction

Actual cities are oversaturated, on one hand most of the population is concentrated in large cities (in 2030 more than 80% (UNFPA 2007) of the population will live in urban areas), on the other hand mobility needs of the modern population are growing continuously. While urban demand for trips is growing constantly, supply (capacity of city streets) is limited, and must be optimized, not increased (most of the times not possible inside the city). Well-planned, efficiently operated, and cost-effective transportation system management (TSM) strategies can improve mobility of existing systems for transportation users, especially in urban environments, where a good optimization of the infrastructure is needed (considering the high cost of building new facilities and the continuously increasing demand resulting from economical and population growth). Last years tendencies are shifting person trips from private vehicles to public vehicles, increasing the Public Transport share importantly. The most used Public Transports are the "Mass Transports" such as metro, tram or bus. This kind of transport usually has a centralized management which uses ITS technologies developed in the last decade for an optimal operation of the service. Unfortunately, inflexibility, long total travel time and insufficient service coverage of Mass Transport systems cause a lower usage of them in most metropolitan areas. Oppositely, the taxi-cab sector is a more convenient mode due to its speediness, door-to-door attribute, privacy, comfort, long-time operation and lack of parking fees. The great

* Corresponding author. Tel.: 00302310498433; Fax: 00302310498269.
E-mail address: jose@certh.gr

doi:10.1016/j.sbspro.2011.08.020

inconvenience is the lack of central management; each taxi is operated by an independent driver, taking his own decisions continuously, with a weak intent of control by the policy issues of each city such as license control or distributing the working days of the taxi vehicles (normally the control is imposed on vehicles, not on drivers, generating double shift and increasing the use of taxis). An important percentage of the cars (e. g. 60% in Hong Kong (Yang et al. 2000)) in the daily flow are taxis, most of them empty taxies. This situation is creating two problems, an internal problem to the taxi drivers (higher empty kilometers means lower benefits) and an external problem to the citizens (congestion and pollution). The first problem is being aggravated with the actual economic crisis, which is breaking the market equilibrium: demand is decreasing due to the lower incomes of the population and offer is increasing due to the increasing number of taxi drivers (not taxi licenses). Market equilibrium cannot be achieved in this concrete market because of the regulations (price is not established freely), and cannot go to the next equilibrium point due to the price policies imposed in each city. This is a vicious cycle, where empty hours are increasing, and taxi drivers need to work more time in order to have the same income, which means lower income per hour (Daniel (2006)). In this situation, taxi drivers prefer to stop at taxi stands and wait for a client, without expending fuel in empty trips and consequently saturating the taxi stands. If taxi stops network is not well designed, this situation will create a decrease in the Level of Service of the passengers, decreasing the demand and congesting the streets near the taxi stops.

The taxi sector has been traditionally a regulated market in terms of fares and entry control. The objective of this regulation is to correct the defects of the taxi sector, such as externalities (congestion and contamination), low level of service offered and anticompetitive behavior of the market. A fundamental distinction in types of taxi regulations is between quantity regulation, quality regulation and market conduct regulation. Quality regulation embraces the standard of vehicles, driver and operator; this type of regulation is more a safety regulation than a competitiveness one. Market conduct regulation includes rules regarding pick up of passengers, or affiliation to a radio network. Quantity regulations include price regulation and entry restriction. From now and on, the term regulation will refer to quantity regulation. Restrictions on entry to the taxi market have been applied by many cities around the world, but actually many cities are deregulating their markets. The most common justifications used for controlling the entrance to the taxi market are the protection of the taxi drivers incomes and the externalities (pollution and congestion) caused by the circulating taxis, but when decisions are taken without a good justification or implementation plan, entry restrictions and fare regulations are distorting economically the taxi sector, leading to important welfare losses. As a result of entry control, the price of the licenses in markets where taxi licenses are tradeable are higher (Paris 125.000 €, Sydney 300.000 $, Melbourne 500.000$, New York 600.000$ [OECD 2007]), and they are rising up constantly due to the exploitation of their owners. Reforms have often been opposed to reduce the incomes of drivers, which are normally low, and restrictive conditions have been applied in this direction, but there is no evidence that taxi incomes are higher in markets with regulated entry conditions. Oppositely, license owners is the group who is being beneficiated by these measures, and not the drivers (Melbourne, as commented above has taxi licenses valuated in 500.000$, but driver incomes are estimated at 8 – 14$ per hour [OECD 2007]). Deregulation has most of the times positive impacts, resulting in lower waiting times, increased consumer satisfaction and price falling (OECD 2007). Market liberalization is an interesting challenge for many cities, but in cities where strong supply restrictions have been applied, there will be a strong opposition to reform proposals from the license-owners. Arguments support that license-owners must be compensated in that case: one approach (first used in Ireland) is to give the additional licenses to each license-owner, ensuring that the new monopoly will remain in their hands; alternatively the new license can be given to taxi drivers without taxi license (OECD 2007). In Melbourne, a 12 year program is adding to the stock of licenses a number of licenses equal to the yearly demand growth. Other concepts are important in relation to deregulation, most of the times quantity deregulation means quality regulation, ensuring safety and minimum service standards.

The paper is structured as follows: the second chapter presents the taxi market, describing the operational modes. The third chapter resumes the different models presented in the literature, from the aggregated models until the equilibrium models. The next chapter highlights the most important ideas and results from the literature review, analyzing the operational modes, the market equilibrium and the regulation of the taxicab markets. The fifth chapter presents an overview of the taxi markets in different cities around the world, resuming the deregulation consequences observed in the deregulated markets. Finally, the last chapter contains the conclusions obtained from the literature and state of the practice review and proposes the development of a new model for the study of the taxicab market.

2. The taxi market

Taxis are private vehicles used for public transport services providing door to door personal transport. Taxi services can be divided into three broad categories: rank market, hail market and prebooked market.

- Rank places are designated places where taxi can wait for passengers and vice versa. Taxis and customers are forming queues regulated by a FIFO system. Disadvantages are that due to the FIFO policy established price has no effects on customer choice, and that customers must walk until the nearest taxi stop.
- In the hail market clients hail a cruising taxi on the street. There is uncertainty about the waiting time and the quality/fare of the service customers will find. Advantage here is that customer mustn't walk until the taxi stop. In this case a monopolistic market is possible.
- In the pre-booked market consumers telephone a dispatching center asking for an immediate taxi service or for a later taxi service. Only in this kind of market consumers can choose between different service providers or companies. At the same time, companies can fidelize clients with a good door to door service. The market here is a competitive market where larger companies can offer smaller waiting times.

3. Taxi models review

From the early 70's many studies have been published in relation to the taxi sector. While first studies (1970-1990) were related to the profitability of the sector and the necessity for regulation using aggregated models, later studies (1990-2010) implemented more realistic models in the taxi sector: from the most simple model of Wong developed in 1997 for a little taxi fleet until the most sophisticated model of Wong (2009) being able to simulate congestion, elasticity of demand, different user classes, external congestion and non linear costs, taking into account different market configurations. Douglas (1972) developed the first taxi model in an aggregated way, using economic relationships from other sectors (goods and services). Many authors (de Vany (1975), Beesley (1973), Beesley and Glaster (1983) and Schroeter (1983)) used the model proposed by Douglas for developing their models and tested them in the different market configurations. Manski and Wright (1976), Arnott (1996) and Cairns and Liston-Heyes (1996) developed structural models, obtaining more realistic results. Yang and Wong (1997-2010c) developed accurate models, taking into account the spatial distribution of demand and supply in the city using traffic assignment models. Last models proposed by Wong et al. (2005) and Yang et al. (2010b) assume a bidirectional function taking account the willingness to pay of customers, making it much more realistic. New technologies applied to the taxi market such as GPS, GIS and GPRS were also simulated in the different models, proving their benefits and justifying their use. Many of the models developed have been tested in different cities around the world using data from different sources. Beesley (1973) and Beesley and Gaister (1983) studied the data obtained from questionnaires in different cities in the UK, especially from London. Schroeter (1983) is the first to use data from taximeters in his model, using the data from a taxi company in Minneapolis (EEUU). Schaller (2007) uses interviews and questionnaires from taxi agents and customers in different cities of the EEUU.

3.1. Aggregated models

Douglas (1972) was the precursor of the first studies related to the taxi sector. He considered a taxicab market where taxicabs can be engaged anywhere along the city streets, with scheduled (by a regulatory authority) fares, and free entry. He concluded that the maximum revenue to the industry occurs at the point where demand is less than maximum, characterizing social welfare as an efficient but unfeasible (deficit) equilibrium. He also proved that taking into account the social welfare, the points where the number of taxi hours in service is maximized and where demand is max are the same. The formulation proposed by Douglas (1975) has been used as reference formulation by all the later authors. De Vany (1975) proposed solutions for different type of markets: the Monopoly market (with entry and fares regulated), the Competitive market (with free entry and regulated fares) and the Medallion market. In the monopoly solution, the firm's program proposed by De Vany (1975) is to maximize total benefits, while in the competitive solution the owners' objective is to maximize their own benefits. He proved that demand is maximized subject to a zero-profit constraint. He agrees with Douglas (1972) in that the efficient price minimizes output and observes that a comparable increase in the regulated price will be more likely to expand capacity under competition than under monopoly. Beesley (1973) and Beesley and Glaister (1983) also investigated the different markets and their characteristics, trying to establish guidelines for decision makers using a model for simulating relevant inferences in the taxicab market. He identifies and analyzes the important elements and the defects of regulation (monopoly rights, entry conditions and fare control), introducing the external cost (congestion produced by taxi cabs) and testing his ideas in the taxicab data obtained from London, Liverpool, Manchester and Birmingham. He concludes that bigger elasticity than 1 is only possible in a regulated market, and in consequence free markets have lower elasticity than 1 (as postuled by De Vany (1975)). Manski and Wright (1976) concluded that over a certain range, increasing the number of licenses will decrease expected waiting time and increase expected utilization rate. Schroeter (1983) developed a theoretical model in a regulated market where radio dispatch and airport cabstand are the primary modes of operation and applied his methodology to the Minneapolis taxi sector. Daganzo (1978) was the first that studied the travel and waiting time as physical variables. He studied the optimal size of the taxi fleet using the queue theory proposed by De Little. This minimum fleet ensures a minimum level of service at the end of the desired region (bigger waiting times are unacceptable). Foerster and Gilbert (1979) studied the effects of regulation within a framework of eight regulatory scenarios involving different prices, entry policies and type of industry concentration factors. They pointed out the following: in an unorganized industry, price will not be regulated by the market, it will tend to rise without any countervailing down pressure, decreasing the utilization rate; if prices are fixed, monopoly will produce a lower level of output in relation to the level produced by the competitive industry (as concluded by De Vany (1975)); entry control has the same effects, increasing price in both types of industry. They propose different guidelines for Public Policy in relation to their work and suggest that empirical data is necessary to document and prove regulatory impacts. Cairns and Liston-Heyes (1996) analyzed the monopoly market, the social optimum (maximizing the sum of the social and industrial benefits) and the second best (non-negative profits). They observed that profits are zero when taxis are used at their optimal intensity. They showed that price regulation is necessary for producing equilibrium in a simple model of taxi services, but second best can be only achieved if fares and intensity of use of taxi-cabs are controlled, concluding that regulation is needed for achieving second best. Arnott (1996) analyzed the shadow cost of taxis in the first best, proposing subsidization for covering these costs in the vacant trips. He developed a structural model considering a uniform customer demand distribution over a spatially homogenous two-dimensional city, and a dispatching center supply. He concluded that subsidization is necessary, justifying it with the decentralization of the social optimum, observing that the shadow cost is covered only when taxis are busy. Chang and Huang (2003) expanded the research of Douglas (1972) optimizing the vacancy rate and fares. Chang and Chu (2009) continued the work of Chang and Huang (2003) using a more generalized model with the welfare maximization objective for avoiding the elasticity constraint. Their model can analyze and optimize the vacancy rate and fares subsiding in a first-best environment. Daniel (2003) models a taxi-cab market in which fare and entry are regulated, testing it using the data obtained by Schaller (2007). He finds an inelastic relationship between vacant taxicabs and demand. He uses a demand function depending on the price of the service and the number of vacant taxi cabs. Fernandez et al. (2006) studied the characteristics of the cruising taxi market, proving that a unique equilibrium exists for a deregulated market and it corresponds to a monopolistic equilibrium. They conclude that entry regulations are redundant with fare regulations, producing worse industry conditions. They observed that, for an atomized supply of services where many small operators exist, the returns to scale make impossible to obtain the social optimum without subsidy, as postulated by Cairns and Liston-Heyes (1996) and Arnott (1996). They conclude that the need for regulation should be carefully considered case by case, due to the fact that the difference between second best and unregulated free market equilibrium depends on the specified case studied. Massow and Canbolat (2010) develop a model for simulating the taxicab behaviour in a dispatching market where taxis are assigned to virtual queues generated in each zone, and also in high demand points. They conclude that taxis will wait in the borders between zones and propose the creation of super zones for increasing the level of service to customers.

3.2. Equilibrium models

The above studies examined extensively both price and entry controls in the taxicab market, basing their models in aggregate demand and supply and testing them in different markets (monopolistic and competitive). The principal assumptions are the relation between the waiting time and the total number of vacant taxi hours, constant operating cost per hour and demand estimation based in fares and waiting time of passengers. Some of the authors presented above used structural models, going further in the taxi market simulation. These structural models include the work of Manski and Wright (1976), who provided a specified structural model of a single taxi stand, and Arnott (1996), who investigated the first best solution considering a spatial uniform customer demand distribution. Yang and Wong presented a series of models during the years 1997 – 2010 studying the taxicab market in the network of Hong Kong. Their spatial models are more realistic than the aggregated. Yang and Wong (1998) presented a network model describing how vacant and occupied taxis will cruise in an urban network searching customers and providing transportation services to them. They assume stationary taxi movements and customer demand, no demand elasticity, no congestion, "all-or-nothing" routing behaviour and that each taxi tries to minimize its travel time when searching for a new client. They supposed that the expected searching time in each zone is identically distributed following a Gumbel density function and that the probability of a vacant taxi in a zone to meet a customer in another zone follows a logit model, using a parameter of information for taking into account the taxi driver experience (older drivers will find a ride faster), proving that with better knowledge of the supply smaller fleets can have better results for both, taxi drivers and customers. They conclude that taxi fleet and information of taxicabs must be regulated in order to achieve better taxi utilization while maintaining a certain level of service. Wong and Yang (1998) improved the algorithm for guarantying convergence in large-scale applications. Yang et al. (2000) analyzed the demand (taxi availability)-supply (taxi utilization) relationship in the taxi market, developing a nonlinear simultaneous equations system of passenger demand, taxi utilization and level of service. The proposed model is based on the concept of queuing theory and demand-supply equilibrium, using the number of licenses, fare, income and occupied taxi time as exogenous variables, while demand, waiting time, taxi availability, utilization and waiting time of drivers are the endogenous variables. They estimated the parameters of their model using survey data, presenting the value of the endogenous variables listed above in relation to the number of taxis and the fares applied. Wong et al. (2001) added congestion to the network and elasticity to the demand. Evaluating their results they agree with Manski and Wright (1976), Schroeter (1983) and Arnott (1996) in the fact that an increase in the number of taxis will be beneficial for both, customer and drivers, but only in a small taxi fleet since this is an unstable situation, and seldom emerges in a realistic taxi market. Wong and Wong (2002) developed a more efficient solution algorithm and analyzed the social surplus of the taxi market. Wong et al. (2004) simulated the real mode choice with different types of users and mode classes. Yang et al. (2005) investigated the consequences of externalities in the different markets. They postulated that a profitable first-best social optimal emerges in a severely congested taxi market, where the entry of additional taxis into the market has a large marginal congestion effect (and thus the entry should be highly controlled at the social optimum). They conclude that in the competitive market the second-best solution leads to a more efficient use of taxis, with a higher demand served with a smaller fleet and higher fare. All the models commented above use a linear taxi fare structure, making long-distance (from/to the airport) trips more profitable and creating over-supply in airports, wasting many taxi service hours in the airport queue. Schaller (2007) proved that a free entrance to the market in the USA and Canada had as consequence the reduction of the level of service, because taxi drivers will only realize the most profitable trips. In order to diverge excess taxi supply from the airport to other areas, increasing the utilization of the taxi capacity and increasing the quality of the service, Yang et al. (2010a) included a nonlinear taxi pricing of taxi services in their model. They identified the win-win situation (surplus for both producer and consumer) created by a Pareto-improving situation, allocating more efficiently the taxi services in the whole territory. Hyunmyung et al. (2005) added the stochastic behaviour of the demand developing a stochastic modelling approach in a dynamic transportation network. They simulated taxi drivers' learning process implementing the day-to-day evolution approach introduced by Horowitz (1984), Vythoulaks (1990) and Cascetta and Cantarella (1991). They tested their model in a test network, generating demand at each node based on the demand rate at each peak period and the trip distribution pattern, proving drivers capacity in predicting passenger queues at nodes. They also investigate the effectiveness of taxi information systems in reducing unnecessary travels, proving that using information systems is equivalent to an increase in the number of taxis by 20% in regard to the quality of the service (as pointed out by Yang and Wong).

4. Critical review

The extended literature overview presented above is resumed below, highlighting the important factors presented and discussed in the above models, unifying conclusions and identifying debilities and gaps. All authors developed models for analyzing the effects of regulations in the taxi market. They proposed mathematical formulas for calculating demand and supply, simulating different types of markets and obtaining different results for each regulation scheme. Aggregated models calculated total demand and supply using different parameters: Douglas (1972) used the price of the trip and the expected waiting time for calculating the demand, and a flat cost rate for the supply, he stated that if different users have different willingness to pay, the regulator must find a price p for all, maximizing global benefits; De Vany (1975) added an index of the full prices to the calculation of the demand; Cairns and Liston-Heyes (1996) supposed uniform demand within the day decreasing with the increase of the waiting time; Chang and Huang (2003) and Chang and Chu (2009) used log-nonlinear and log-linear functions respectively for simulating demand; Daniel (1978) used a demand function depending on the number of vacant taxis and the price; Fernandez et al. (2006) used the generalized price for obtaining the demand; Manski and Wright (1976) assumed a Poisson process of customer arrivals in a FIFO queue discipline for the rank market. Massow and Canbolat (2010) develop a double queue model simulating a dispatching market, where drivers are assigned to queues in zones and high demand points. Equilibrium models calculated spatial demand and supply: Arnott (1996) considered a uniform demand distribution over a spatially homogenous two-dimensional city; Yang and Wong (1998) used the model of Douglas (1972) in an origin-destination matrix, where demand is fixed for each pair OD; Wong et al. (2001) considered separate demand exponential functions for each pair O-D, depending on waiting time, travel time and trip price, adding elasticity to the demand function; Wong et al. (2004) included multiple user classes and taxi models; Yang et al. (2010a) used a non-linear taxi pricing for treating long-distance trips; Hyunmyung et al. (2005) used a stochastic demand. Figure 1 below shows the evolution of the taxicab models in relation to the added value of each model.

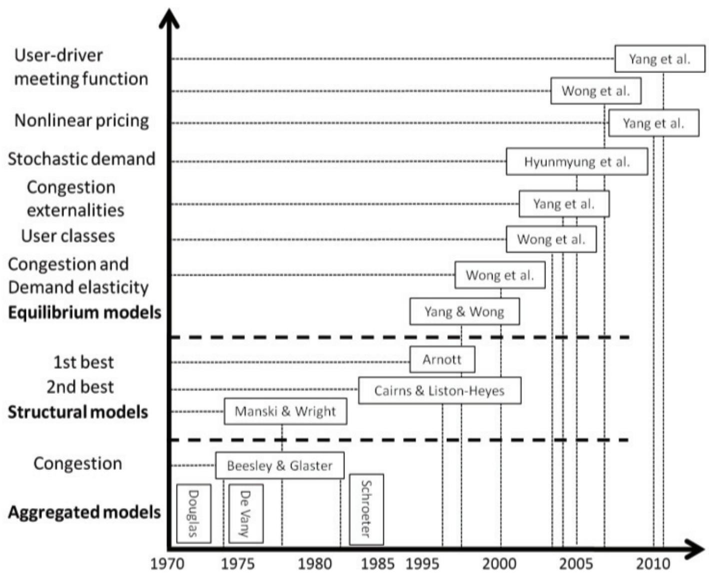

Figure 1 Evolution of the taxicab models

Different fundamental matters of the taxicab market have been investigated by the authors, such as elasticity of demand, external cost, returns to scale and the relation between supply and demand. Elasticity of demand has been an important issue: De Vany (1975) proved that unit elasticity represents zero profit, and higher elasticity than one a negative profit, concluding that elasticity must be less than one. Daniel (2003) obtained an inelastic relationship between vacant taxis and demand. Yang et al. (2005) concluded that the unitary elasticity achieves the maximum competitive taxi fleet size. Congestion was not present in the first models, but later models take it into account, becoming an important factor in the discussions about first best and second best solutions: Beesley (1973) introduced the external cost produced by the congestion generated by the taxis in the network; Fernandez et al. (2006) showed that externalities will reduce waiting time and operational cost. Wong et al. (2001) affirm that demand decreases with congestion, but at the same time trip income increases; Yang et al. (2005) prove that first best can be obtained with congestion. Returns to scale are a matter of discussion in many models: Manski and Wright (1976) assumed increasing returns to scale concluding that increasing the number of licenses, waiting time will decrease while utilization rate will increase; Schroeter (1983) was opposed to the scale economies announced by Manski and Wright (1976), affirming that an increase in the number of taxis will reduce waiting time, increasing demand, but reducing earning for each taxi, in opposition to Daganzo (1978), who stated that taxis do not have significant economies of scale; Fernandez et al. (2006) showed that economies of scale are produced by externalities, and affirms that these returns to scale make impossible the social optimum without subsidy (as observed by Cairns and Liston-Heyes (1996) and Arnott (1996)). All authors agree with the different market equilibriums commented above (first best/second best). Many models studied two equilibrium points for the competitive market, the first best and the second best. Cairns and Liston-Heyes (1996) identified the first best as the social optimum (maximizing the sum of the social and industrial benefits), finding its zero profit character (only covers busy trips) and concluding that regulation is needed for obtaining the second best (non-negative profits); Arnott (1996) proposed subsidization for achieving the first best (basically the subsidy will cover empty trips), oppositely, Yang et al. (2002) showed that in the point at which total surplus is maximized, industry profits are negative (first best). They propose the second best solution instead of subsidization; Chang and Chu (2009) optimized vacancy rate and fare subsidization for obtaining first best, obtaining analytical formulas for the vacant and occupied distance, vacancy rate and fare; Fernandez et al. (2006) agree with the idea that first best does not cover costs, while second best covers operation costs and maximizes social welfare; Yang et al. (2005) postulated that congestion can make profitable the first best, and that second best solution leads to a more efficient use of taxis (higher demand served with smaller fleet and fares). Many authors presented the equilibrium points (first and second best) graphically, representing demand and fare in the axes, and using different mathematical functions for obtaining optimum fleet and fares. Aggregated and equilibrium models have focused exclusively on the taxi availability for calculating the customer waiting time, and therefore the demand resulted. Schroeter (1983) presented a matching function between the taxi availability and the taxi demand; Cairns and Liston-Heyes (1996) used a model of search for drivers and customers; Wong et al. (2005) used stochastic searching behavior with a bilateral searching and meeting function between taxi drivers and customers; Matsushima and Kobayashi (2006) implemented a double-queue system simulating waiting and meeting between taxis and customers in a simple taxi stand; Yang et al. (2010b) modeled a network bilateral searching and meeting between taxis and customers. Yang et al. (2010c) investigated the properties of an aggregate taxi service model using bilateral searching and meeting functions (considering a specific form of the Cobb-Douglas type production function) for characterizing the meeting frictions between vacant taxis and customers. They examined the market profitability at social optimum, finding that taxi services should be subsidized only when there are returns to scale in the meeting function (same conclusion obtained by Fernandez et al. (2006)).

4.1. Market conditions

Most of the models were tested in different market conditions, such as competitive industry or monopoly. De Vany (1975) stated that in the monopoly market, industry tries to obtain the maximum benefit, while in the competitive industry each driver tries to maximize its benefits. General conclusions are that the monopoly industry will obtain the maximum total benefit with a small fleet and high prices, covering only the high-income demand sector, with a poor level of service. Douglas (1972) observed that in the point of maximum benefit for the industry, the total number of taxi hours is maximum, but the demand is not. Foerster and Gilbert (1979) affirmed that if price is fixed, the monopoly market will produce lower level of output. Fernandez et al. (2006) proved that the unique feasible equilibrium in a deregulated market is the monopoly solution. Yang et al. (2005) postulated that the monopolist would charge a price in excess of marginal cost per ride by an amount equal to the consumer´s marginal net willingness-to-pay for a ride. Different market configurations were proposed: De Vany (1975) studied a market with limited entry, but unrestrained price concluding that maximum demand is subjected to zero profit; Foerster and Gilbert (1979) proposed and studied eight different market configurations (monopoly-competitive/regulated-unregulated price/regulated-unregulated entry) concluding that price will rise without control in an unorganized industry while utilization rate decreases; entry control will have the same effects on both types of industry. Each market configuration has different optimal prices and capacities: De Vany (1975) affirms that price and capacity in the monopoly market are lower than price and capacity in the competitive market. From their findings:

- In the monopolistic market, without fare regulation higher fares will satisfy lower demand with a smaller fleet, maximizing the benefit of the operator (Douglas (1972)). With regulated fares, the same market will operate with the fleet in that the marginal benefit is equal to the marginal cost (De Vany (1975)).
- In the competitive market, the operator will try to achieve the first best (social optimum), maximizing the benefits of the society (taking into account externalities commented above). Douglas (1972) and Arnott (1996) proved that in the first best, the driver income will only cover the occupied time, creating the necessity of subsidizing the empty time and therefore achieving the second best.
- Fleet size is one of the most important factors for decision-makers. In a small market, fluctuations in fares will not affect demand because the waiting time has higher importance, but in a big market, fares are important for the demand generation as pointed out by Wong et al. (2001).

4.2. Operational modes

The three operational modes (rank, hail and dispatching center) have been modeled by many authors, some of them differencing the airport market from the rank market due to the special circumstances of the airport taxi stands. Arnott (1996) states that dispatching centers are used in small cities, while in big cities cruising markets are more frequent. The normal situation in many big cities is a mix of the three operation models, but no model has investigated them at the same time. Schaller (2007) proposes a very interesting representation of the situation of each city in relation to the operational modes, using a triangle with Dispacth, Hank and Rail operational mode in each vertex. He represents each city as a point inside the triangle in relation to the situation of the taxi market (only dispatching centers, only rank points, only hail or a mix of them). Farrell (2010) explored patterns of taxi engagement and relationships between generated trips and taxi rank locations for optimizing the taxi rank distribution in relation to the demand patterns in a 3 level (county, town, stand) model. She applies her findings to the Ireland taxicab market, and realized a comparative cost benefit analysis, identifying benefits and disbenefits resulting from developing new taxi stands. He obtained a cost-benefit ratio of 1-11 for the construction of a new rank, and 1-3 for the relocation of an existing rank. Massow and Canbolat (2010) propose the creation of super zones for reducing the waiting time of clients in a dispatching center environment.

4.3. Regulation

Historically most of the taxi markets were regulated (basically controlling entry and fares). Fares are easy to regulate, fixing a maximum price and regulating the way fares are applied to customers (per time, per distance, supplies, etc). Most of the entry regulations were done simply freezing the number of taxi licenses, without supporting in any way why the actual/current number of taxis was optimal, or simply good. Most of the cities maintained the number of taxis at 1980 levels, only some cities increased timidly their number of licenses following the GDP value or other economic indexes. Indeed, Daniel (2006) highlighted that in many regulated markets there is overcapacity. This mistake created in many cities a suboptimal taxi market, or an inefficient taxi market, with more taxicabs than needed or less vehicles than needed. Many authors support that the situation of the market has enormous influence in the results of the regulation, and this situation must be studied in the moment of the regulation for justifying each measure adopted, from the number of taxis until the value of fares, concluding that the starting point of the market is crucial in the success of the regulation policies. Loo et al. (2007) conclude that due to the economic nature of the market, the price of the taxi licenses depends more on economical factors than on the demand for taxi services. Fernandez et al. (2006) affirm that both regulations must not act simultaneously, entry regulations are redundant with fare regulation, and the effect of entry regulation is negative in a market where fares are regulated (and vice versa). A small number of authors tried to develop models for obtaining the optimum number of taxis, Schaller (2007) conducted a regression analysis on seven variables concluding that the taxi demand is generated by households without private cars or trips to the airport. There are two different arguments in favor of entry control in taxicab markets, economic and non-economic. The economic argument is the social welfare achievable with entry control, avoiding market failures. Non-economic arguments are potential cross-modal competition, congestion and pollution issues. Moore and Balaker (2006) stated recently that most of the economic opinions favor open entry to the taxi industry. OECD (2007) identifies arguments against free entry and arguments against controlled entry, (resumed in Table 1).

Table 1 Arguments against free entry and entry control. Source: Own elaboration from OECD (2007) and CENIT (2004).

Arguments	Against free entry	Against entry control
Productivity arguments	Excess of capacity	Augment of demand
	"Diversion" of demand from PT	Most efficient use of resources
Impact on congestion/pollution	More taxis than the optimum (more congestion)	Less Private Vehicles (less congestion)
Distributional arguments an competitiveness	Preserve the income position from incumbent laborers	Reduction of development of new products (rivalry)
Impacts on service quality and information	Reduced standards of taxi services	Absence of information, tools and rules for regulators

5. State of practice

Each country/city has its own regulation for the taxi market. Table 2 shows the regulation characteristics of some countries/cities along the world. As shown in Table 2, a few countries have deregulated the taxicab market; from their experience some deregulation effects can be exposed:

- Sweden (1990): greater taxi fleet, greater accessibility for customers, reduction of waiting time, different type of available vehicles.
- Ireland (2000): quadriplication of the number of license and fare and quality regulation needed for avoiding overcharging and uncompetitive operation of the market (uncertainity of waiting for another taxi and price competition unlikely to work at ranks).
- Japan (2002): 8.4% and 9.7% increase in the number of companies and taxis respectively. Introduction of a large variety of fares, discounts and flat rates.
- United States (Seattle 1979): 5% reduction in fares (taxi-stand rose while radio-dispatch fell); increase in service at the airport, generating queues, but without price reduction due to the FIFO queuing system applied.
- United States (Indianapolis 1994): Increase in the number of cabs and companies, fare reductions, service improvements and reduction in customer complaints.
- United States: fare control is needed for controlling the appropriate level of entry; use of contracts between firms and hotels/airport authorities for avoiding queues at those locations where waiting times are always low.
- Taiwan: over-supply and high vacancy rate, resulting in poor service, unhealthy competition and law-breaking behaviours.
- Number of taxis in Dublin increased by 216% in the two years after deregulation. In New Zealand, the number of taxies increased also by almost 200% following deregulation. In Sweden, the number of taxies was doubled in the first two years after deregulation, but simultaneously, significant innovations had occurred for encouraging taxi use in off-peak periods.

Most of the authors agree with the above, a liberalization of the market will increase the taxi fleet and level of service to the customers, but a fare regulation is needed. As exposed by Fernandez et al. (2010), a fare regulation is enough for controlling the taxi market as concluded in the USA example. The example of deregulation in the United States confirmed the exposed by Schaller (2007); taxi drivers will create over-supply in airports due to the lower waiting time and higher income if there are no regulations. It is important to highlight that the effects of deregulation will depend on the initial pre-deregulation situation. In markets where regulation kept supply close to free entry equilibrium levels and low license values there will be no changes, but in markets where the number of taxis is very low due to the strict applied regulation, supply will increase importantly after deregulation, as occurred in the examples listed above. This entry of new supply will lead to low incomes, high fares and business failure (short terms results), while the adaptation of consumers will be in a long term horizon.

For analyzing the relation between the number of taxis, the population and other economic values, analytical data from 19 European cities is presented below.

Table 2 Regulation issues in different cities around the world. Source: Own elaboration from OECD (2007).

Country - Zone/city	Fare regulation	Entry regulation	Period	Reestrictions / Characteristics
Belgium	yes	yes	5-10 years	1 vehicle per 1.000 hab / personal and intransferable
Czech Republic	yes	no		intransferable
Denmark	yes	yes	10 years	intransferable
France - Paris	yes	yes		100 new licenses per year
Germany	yes	no	5 years	license subjected to a quota
Hungary	yes			
Ireland	yes	no (2000)		license subjected to a fee and quota
Italy	yes	yes		4,5 per 10.000 hab / 1 lic per person
Japan	yes	no (2002)		
Korea	yes	yes		
Netherlands	yes (2004)	no (2002)		
Norway	depending on the city	yes		not tradable, not transferable
Sweden		no (1990)		
Switzerland	depending on the city	yes	3 years	not tradable, not transferable
United States - Seattle	no (1979)	no (1979)		
Romania	yes	yes		4 vehicles per 1.000 hab

Table 3 General data related to the taxi market of different European cities. Source: CENIT (2004)

City	Average trip fare*	Taxi vs Bus**	Taxi vs oil***	Monthly earnings ****	Monthly trips	Population	Urban population density	GDP per inhabitant	Number of taxis	Taxis per thousand inhabitants
Amsterdam	14.8	14.1	2.2	1404	95	850,000	57.3	34100	1504.5	1.77
Athens	7.4	23.1	1.6	256	35	3,900,000	65.7	11600	15249	3.91
Barcelona	8.5	17.7	1.8	594	70	4,390,000	74.7	17100	11765.2	2.68
Berlin	11.3	9.8	1.8	1199	106	3,390,000	54.7	20300	6949.5	2.05
Brussels	14.9	19	2.5	1495	100	964,000	73.6	23900	1243.56	1.29
Budapest	6.2	22.5	1.1	201	32	1,760,000	46.3	9840	5596.8	3.18
Copenhagen	14.9	13	2.3	2661	179	1,810,000	23.5	34100	2805.5	1.55
Dublin	6.2	8.5	1.2	919	148	1,120,000	25.9	35600	1993.6	1.78
Lisbon	5.3	5.6	0.8	441	83	2,680,000	27.9	17100	4529.2	1.69
London	12.5	11.4	1.8	1286	103	7,170,000	54.9	36400	55997.7	7.81
Madrid	9.8	15.8	2	594	61	5,420,000	55.7	20000	14471.4	2.67
Milan	9.9	17.6	1.6	997	101	2,420,000	71.7	30200	4573.8	1.89
Oslo	18.8	9.7	2.5	1570	84	981,000	26.1	42900	2148.39	2.19
Paris	8.7	12	1.3	1095	126	11,100,000	40.5	37200	17538	1.58
Prague	6.8	15.2	1.5	314	46	1,160,000	44	15100	3978.8	3.43
Rome	8.3	19	1.3	997	120	2,810,000	62.6	26600	5816.7	2.07
Stockholm	11.9	7.8	3.5	1879	158	1,840,000	18.1	32700	5207.2	2.83
Vienna	15.7	17.6	3	914	58	1,550,000	66.9	34300	4433	2.86
Warsaw	4.2	12.8	0.8	241	57	1,690,000	51.5	13200	5999.5	3.55

*2002 prices, 5 km trip, day fares, inside the city ** Taxi cost per km/bus cost per km *** Cost per km/cost of 1 liter of oil ****National average

Conclusions obtained from the table 3 are:

- A logic result is that average trip fare is higher in cities with higher GDP. A relation with monthly earnings and cost of taxi in relation to cost of oil exists also.
- The relation taxi cost versus bus cost grows with the density of the city, due to the economies of scale of the Mass Public Transport.
- The number of taxis has a very strong relation with the population of the city (excluding London and Paris). This relation is between 1.3 and 4 taxis per thousand inhabitants (in London this relation is 8 taxis per 1000 inhabitants).

6. Conclusions

As time goes by models are getting more and more realistic. First models used aggregated values, without taking into account that the taxicab market is working in an urban network, sharing the streets with daily traffic and other public transport modes. Later models introduced this spatial characteristic, and many other rules for simulating the real taxi market, as the network knowledge or the learning process of the taxi drivers, with a good effort in the calculation of the passenger trip generation-distribution and assignment. Lastest developed models concentrate their effort in the customer-driver search function, increasing the reality of the simulation of the finding process between a taxi and a customer. Many interesting ideas have been developed in parallel with the evolution of the models, such as the day to day learning process or the use of the logit model for the probability of finding a client in each zone. Main general conclusions obtained from the models presented are:

- Although many models tried to be a tool for decision makers, developed models cannot prove the performance level of the taxi markets. There are no optimum models of taxi supply to guide decision makers.
- Models proposed in the literature are characterized by significant data requirements due to the high number of determinants in the taxi demand and supply. Actually, with the use of GPS and GIS, data recollection is technically easy, but the reticence of the taxi sector to share this data is an important barrier.
- All the models have investigated the taxi market from the point of view of the taxi driver (income) and the customer (waiting time, level of service, total cost), but no model has studied the consequences of the market regulations on the city (contamination, congestion). It is important to add environmental considerations as a determinant factor in the future models since in most of the cities, taxi flows have not only negative consequences on the rest of the traffic, but also in the citizen's health.
- Regulation of entry and fares must not act together; deregulation of access to the taxi market must be achieved in most of the cities, increasing the supply and the level of service of customers. Entry deregulation must be accompanied with new regulations, such as fare regulation (almost with a maximum fare control) and special regulations on high-demand generation points, such as airports, train stations or hotels.

In the opinion of the authors, both approaches are useful, each one used in its respective scale. Aggregated models can explain major variations in the taxi market using fewer variables, simulating fare and entry regulations easily and obtaining clear results. More detailed models can better simulate the taxicab market, taking into account the spatial characteristics of demand and supply, the different types of operational modes working together in the same city, the external and internal factors that are generating the demand, and the congestion (because when the streets are congested is when the demand for trips is higher). They can also work with spatial variables that aggregated models cannot take into account, adapting each model to the reality of each city. Data availability is an important matter for modeling the taxi market, as the more detailed the model is the more accurate the results will be, but the data will be more difficult to recollect; on the contrary, aggregate models need less quantity and not such as high quality of data, but results are not as analytical as they can be in a more detailed model. With new developed ITS and other technologies, a lot of data can be recorded, and more detailed and complex models can be developed.

7. References

Arnott R. (1996). Taxi Travel Should Be Subsidized. *Journal of Urban Economics 40*, 316 – 333.

Beesley M. E. (1973). Regulation of taxis. Royal economic society. *The economic journal*, Vol. 83, No. 329 (Mar., 1973), pp. 150 - 172.

Beesley M. E. And Glaister S. (1983). Information for regulating: the case of taxis. *Royal economic society, The economic journal*, Vol. 93, No. 371, pp. 594 – 615.

Cairns R. D., Liston-Heyes C. (1996). Competition and regulation in the taxi industry. *Journal of Public Economics* 59, 1 – 15.

Cascetta E. and Cantarella G. E. (1991). A Day-to-day and Within-Day Dynamic Stochastic Assignment Model. *Transportation Research A*, vol 25, pp 277 – 291.

CENIT. Metodologia per a l'establiment de les tarifes del taxi a l'AMB i la seva revisió, 2004. Informe final per a l'Institut Metropolità del Taxi.

Chang S. K. And Huang S. M. (2003). Optimal fare and unoccupancy rate for taxi market. *Transportation Planning Journal* Vol 32, No 2, pp 341 – 363.

Chang S. K. And Chu-Hsiao Chu (2009). Taxi vacancy rate, fare and subsidy with maximum social willingness-to-pay under log-linear demand function. *Transportation Research Record: Journal of the Transportation Research Board*, No 2111, pp 90 – 99.

Daganzo (1978). An approximate analytic model of many-to-many demand responsive transportation systems.

Daniel Flores-Guri (2003). An Economic Analysis of Regulated Taxicab Markets. *Review of Industrial Organization* No. 23 pp. 255 – 266.

Daniel S. (2006). No fare. *Toronto life, 40* pp 72.

De Vany A. (1975). Capacity Utilization under Alternative Regulatory Restraints: An Analysis of Taxi Markets. *Chicago Journals, The journal of Political Economy*, Vol. 83, No. 1, pp. 83 – 94.

Douglas G. (1972). Price Regulation and optimal service standards. The taxicab Industry.

Farrell S. (2010). Identifying demand and optimal location for taxi ranks in a liberalized market. *TRB Annual Meeting*.

Fernandez L. J. E., Joaquin de Cea Ch. and Julio Briones M. (2006). A diagrammatic analysis of the market for cruising taxis. *Transportation Research Part E* No. 42 pp. 498 – 526.

Foerster J. F. And Gilbert G. (1979). Taxicab deregulation: economic consequences and regulatory choices. *Elsevier, Transportation 8*, pp 371 – 378.

Horowitz J. L. (1984). The Stability of Stochastic Equilibrium in a Two-Link Transportation Network. *Transportation Research B*, pp 13 – 28.

Hyunmyung Kim, Jun-Seok Oh and R. Jayakrishnan (2005). Effect of Taxi Information System on Efficiency and Quality of Taxi Services. *Transportation Research Record: Journal of the Transportation Research Board*, No 1903, pp 96 – 104.

Loo, Becky P.Y., Leung, Suet Yi, Wong, S. C. and Yang, Hai, (2007) Taxi License Premiums in Hong Kong: Can Their Fluctuations Be Explained by Taxi as a Mode of Public Transport? International Journal of Sustainable Transportation, Vol. 1, Issue 4, pp. 249-266

Manski C. F. and Wright J. D. (1976). Nature of equilibrium in the market for taxi services. *Transportation Research Record* 619, pp 296 – 306.

Massow and Canbolat (2010). Fareplay: An examination of taxicab drivers' response to dispatch policy. *Expert systems with applications 37 (2010)* pp. 2451-2458.

Matsushima K. And Kobayashi K. (2006). Endogenous market formation with matching externality: an implication for taxi spot markets. *Structural Change in Transportation and Communications in the Knowledge Economy* pp. 313 – 336.

Moore & Balaker (2006) Do economists reach a conclusion on taxi deregulation?, *Economical Journal Watch 3 (1) (2006)*, pp. 109–132.

OECD (2007). Taxi Services: Competition and Regulation. *Policy roundtables, Competition law and policy*.

Orr D. (1969). The taxicab problem: A proposed solution. J. P. E. 77, no 1 (January/February).

Schaller B. (2007). Entry controls in taxi regulation: Implications of US and Canadian experience for taxi regulation and deregulation. *Transport Policy 14* (2007) 490 – 506.

Schroeter J. R. (1983). A model of taxi service under fare structure and fleet size regulation. *The Rand Company, The Bell Journal of Economics*, Vol 14, No. 1, pp. 81 – 96.

UNFPA (2007). State of World Population 2007, Unleashing the Potential of Urban Growth.

Vythoulkas P. C. (1990). A Dynamic Stochastic Assignment Model for the Analysis of General Networks. *Transportation Research B*, vol 24, pp 453 - 469.

Wong S. C. and Yang H. (1998). Network Model of Urban Taxi Services. Improved Algorithm. *Transportation Research Record* 1623, 27 – 30.

Wong K. I., Wong S. C. and Yang H. (2001). Modeling urban taxi services in congested road networks with elastic demand. *Transport Research B*, Vol. 35, pp 819 – 842.

Wong K. I. and Wong S. C. (2002). A sensitivity-based solution algorithm for the network model of urban taxi services. *Transportation and Traffic Theory in the 21st Century*.

Wong K. I., Wong S. C., Wu J.H., Yang H. and Lam W.H.K. (2004). A combined distribution, hierarchical mode choice, and assignment network model with multiple user and mode classes. *Urban and regional transportation modeling*.

Wong K. I., Wong S. C. Bell M. G. H. And Yang H. (2005). Modeling the bilateral micro-searching behavior for urban taxi services using the absorbing Markov chain approach. *Journal of Advanced Transportation*, Vol. 39 No. 1, pp 81 - 104.

Yang H. and Wong S. C. (1998). A network model of urban taxi services. *Transport Research B*, Vol. 32, No. 4, pp 235 – 246.

Hai Yang, Yan Wing Lau, Sze Chun Wong and Hong Kam Lo (2000). A macroscopic taxi model for passenger demand, taxi utilization and level of services. Transportation, 27, 317-340.

Yang H., Wong K. I. and Wong S. C. (2001). Modelling Urban Taxi Services in Road Networks: Progress, Problem and Prospect. *Journal of Advanced Transportation*, Vol. 35 No. 3, pp 237 – 258.

Yang H., Wong S. C. and Wong K. I. (2002). Demand-supply equilibrium of taxi services in a network under competition and regulation. *Transport Research B*, Vol. 36, pp 799 – 819.

Yang h., Ye M., Tang W. H. And Wong S. C. (2005). Regulating taxi services in the presence of congestion externality. *Transport Research A*, Vol. 39, pp 17 – 40.

Yang H., Fung C. S., Wong K. I. and Wong S. C. (2010a). Nonlinear pricing of taxi services. *Transport Research A*, Vol. 44, pp 337 – 348.

Yang H., Cowina W. Y. L., Wong S. C. And Michael G. H. Bell (2010b). Equilibria of bilateral taxi-customer searching and meeting on networks. *Transport Research B*, Vol. 44, pp 1067 – 1083.

Yang T., Yang H. And Wong S. C. (2010c). Modeling Taxi Services with a Bilateral Taxi-Customer Searching and Meeting Function. TRB Annual Meeting.

Agent Based Modeling for Simulation of Taxi Services

Josep Maria Salanova Grau
Centre for Research and Technology Hellas/Hellenic Institute of Transport, Thessaloniki, Greece
Email: jose@certh.gr

Miquel Angel Estrada Romeu, Evangelos Mitsakis, and Iraklis Stamos
Technical University of Catalonia, Barcelona, Spain
Centre for Research and Technology Hellas/Hellenic Institute of Transport, Thessaloniki, Greece
Email: miquel.estrada@upc.edu, {emit, stamos}@certh.gr

***Abstract*—This paper presents an agent based model for simulating taxi services in urban areas. The three operation modes (hailing, stand and dispatching) are modeled and tested in the Sioux Falls network. Taxi models presented in the literature are divided into aggregated and equilibrium models, with a very small presence of simulation models. The different programmed modules are presented together with the behavior rules of the agents. Performance indicators are calculated for each operation mode and compared in terms of driver earnings, user cost and vacant versus occupied time.**

***Index Terms*—Agent based modeling, taxi modeling, taxi services.**

I. Introduction

Taxis are present in most of the cities around the world. They combine the comfort of door-to-door transportation of private vehicles with the advantages offered by public transport services. Most of the taxi markets in urban areas combine three operational modes: stand, hailing and dispatching. In the stand market, taxis and users meet each other at predetermined meeting points, called taxi stands or ranks, where a first-in-first-out (FIFO) system applies for both the drivers' and the users' queue. In the hailing mode taxis circulate continuously searching for a user, and users wait for taxis at their origin, while in the dispatching market taxis circulate or just wait for a call from a dispatching center. In the dispatching market, a user contacts the operator asking for taxi services and the nearest taxi in the zone (respecting the queue) is assigned to him/her. Taxi markets in small cities are composed by one or two operational modes, usually the dispatching and stand markets, reserving the hail market only for large cities with high densities of population and a Business District concentrating a high percentage of daily trips.

Since most of the taxi markets are regulated, there is a need for developing models for understanding the behavior of these markets in regard to policy regulations and deregulations. Most of the taxi models developed up to now have studied only one operational mode, using aggregated values or finding equilibrium assignment conditions for calculating system performance indicators, such as level of service of users, earnings of taxi drivers or vacant versus occupied distance. On the contrary, only a few simulation based models have been developed.

The model presented in this paper, proposes the use of agents circulating in an urban road network, taking their own decisions for completing as many trips as possible. There are different types of agents, including the three basic types of agents corresponding to the operational modes listed before, as well as the users of the system.

The paper is structured as follows: A literature review is presented in section two, reviewing the aggregated and equilibrium models and presenting the simulation models developed by various authors. The proposed agent based model is analytically described in section three, while the obtained results are presented in section four. Finally, conclusions and future research guidelines are presented in the last section.

II. Literature Review

A. Aggregated and Equilibrium Models

The initial studies related to the taxi sector (1970-1990) focused on the profitability and the necessity for regulation using aggregated models. Following these, more realistic taxi sector models were implemented in the studies of Yang et al. developed in 1998 for a small taxi fleet [1] until the most sophisticated models of Yang and Wong ([2] and [3]) that are able to simulate congestion, different user classes, elasticity of demand, external congestion and non-linear costs. The first taxi model was developed by Douglas [6] in an aggregated way, using economic relationships from other sectors (goods and services). Other authors based their models on the work presented by Douglas and tested their own models in different market configurations, e. g. de Vany [7], Beesley [8], Beesley and Glaster [9] and Schroeter [10]. An intermediate type of models was developed by Manski and Wright [11], Arnott [12] and Cairns and

Manuscript received December 28, 2012; revised February 27, 2013.

Liston-Heyes [13], obtaining more realistic results with their structural models. Yang and Wong presented accurate and detailed models, where the spatial distribution of demand and supply in the city was taken into account by including traffic assignment procedures in their models ([2] and [3]). The models of Wong et al. [4] and Yang et al. [5] analyzed the user-driver meeting through a bidirectional function and accounted for the willingness of users to pay. Various models have been tested and validated by using real world data. Beesley [8] and Beesley and Gaister [9] used data obtained from questionnaires in various cities in the UK; Schroeter [10] used data from taximeters in his model; Schaller [14] used questionnaires and interviews from taxi drivers and users in various cities of the USA. Recently, Kattan et al. [15] investigated the regression models obtained from work trips conducted by taxi in 25 Canadian cities. A detailed review of the aggregated and equilibrium models can be found in Salanova et al. [16].

B. Simulation based Models

The concept of agent based models was developed in 1940, but it was in 1990 when the advances in computation procedures allowed them to widespread. The first use of the word agent, as it is used today, was initiated by Miller in 1991 [17]. Bailey and Clark [18] investigated changes of performance in the dispatching market related to the number of vehicles, concluding that the waiting time is relatively insensitive to changes in demand but highly sensitive to changes in the number of taxi cabs. Bailey and Clark [19] used a discrete-event method to simulate dispatching taxi services, obtaining a linear relation between total distance and fleet size. Kim et al. [20] developed a simulation based stand taxi services which includes a knowledge building process, proving that the use of information technologies could improve the quality of the service by 20%. Song and Tong [21] and later Tong [22] presented dynamic taxi demand models using the simulation model approach of the taxi stand market. They highlighted the limitations of traditional aggregated models (time-dependent patterns, imperfect information, learning convergence and non-equilibrium in taxi market due to regulation) and tested the effects of Advanced Transport Information Systems (ATIS) in this specific market. Recently, Lioris et al. [23] developed a discrete-event simulation model for reproducing the real taxi on demand market conditions since the mathematical models are out of reach for such a complex multi-agent system (network, stochasticity).

III. The Proposed Agent-based Model

A. Variables Presentation

The two basic variables are supply and demand related. The supply variable is composed by the city's road network and the total number of taxis, while the demand variable is composed by the users in need for taxi trips. The three basic actors of the model are the city, the taxi drivers and the users.

All variables are listed below:

1) Exogenous variables (demand and supply, network geometry and links congestion and taxi fares)

2) Endogenous variables (obtained from the simulation)

- City related variables (number of vehicle-kilometers and vehicle/hours Total system cost (drivers' earnings, city and users' costs)
- Taxi drivers' related variables (circulating time and distance (total, occupied and vacant, taxi occupancy and vacant taxi headway, earnings)
- Taxi users related variables (waiting time and travel time, cost of trip)

B. Vehicle-Agent Rules

Three basic agents are designed for the taxis, related to the three operational modes.

1) Hailing market vehicles

Vacant Movement: While the taxi moves along a link, the distance traveled within the time period is related to the duration of the time period and the congestion of the link, using a constant free flow speed for taxis. When the taxi arrives to an intersection, it randomly chooses the next link based on its weight (attractiveness).

Picking up a User: If the taxi finds a free user (not waiting for an assigned taxi or at a stand), he picks him/her up. The taxi state changes to occupied, and the shortest route to the user's destination is calculated, depending on the traffic congestion of each link at that moment, setting the route to be followed by the taxi.

Occupied Movement: Taxi and user move along the links, as in the vacant movement, but when arriving to an intersection, the link correspondent to the shortest route is followed until the destination is reached. While moving, the taxi is calculating the trip cost using either a distance-based or a time-based charge, charging always according to the first value that reaches a threshold value (either by distance covered or minutes traveled).

Delivering a User: When the destination of a user is reached, the taxi calculates the cost of the last interval and the user disappears. The trip cost is then charged to both agents (income for the taxi and cost for the user). The taxi becomes free and continues looking for a user.

2) Stand market vehicles

Arriving to a Taxi Stand: When a taxi arrives to a taxi stand, it joins the taxi queue (if exists ant). If there are no taxis on the stand, the taxi picks up the first user in the queue. If there are neither taxis, nor users, the arriving taxi forms a taxi queue.

Vacant Movement: Vacant taxis running in stand mode are always looking for a taxi stand near their current location (following the shortest route between their current location and the nearest and most attractive in terms of waiting time taxi stand). When the taxi arrives to an intersection, it chooses the next link based on the calculated shortest route to the nearest taxi stand.

Picking up a User: Taxis and users are assigned in each taxi stand based on a FIFO system.

Occupied Movement and Delivering a user: As in the hailing mode.

3) Dispatching market vehicles

Assignation of a User: When a user asks for a taxi, the nearest free dispatching taxi is assigned to him, according to the distance calculated within the network (not Euclidian). The taxi then finds the shortest route between its actual location and the user, and follows it until the user is reached.

Picking up a User: As in the hailing mode, with the exception that only the assigned user can be picked up.

Vacant Movement, Occupied Movement and Delivering a User: As in the hailing mode.

4) The users

Three basic agents are designed for the users, related to the three operational modes. Users appear in each zone following a two-dimensional geographic distribution. Once the user is assigned to a zone, random coordinates are defined for the location of the user within the zone (if it is a stand user, random taxi stop is chosen within the zone). Hailing users wait until a hailing taxi reaches them. Stand users wait at taxi stands, forming queues served by a FIFO system. Dispatching users call for a taxi service and wait until the assigned taxi reaches them. If there are no free taxis, the user is added to a virtual queue for later assignment as soon as a taxi becomes available.

C. Developed Modules

The diagram below shows the agent-based model of the taxi market. As the taxis and users are being generated, the variables are being created and updated. Each module is represented and explained below.

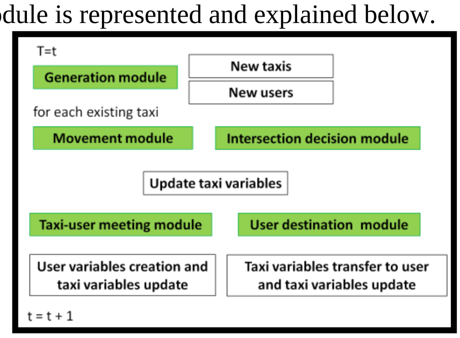

Figure 1. Flowchart of the agent-based model

1) Generation module

The generation module generates the demand and the supply. Users and taxis are statistically generated, depending on the time of the day. Characteristics of both agents, such as origin zone, destination zone or operational mode are also statistically generated.

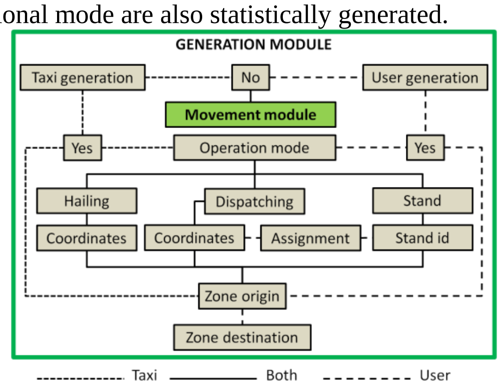

Figure 2. Generation module

When a taxi is created, it is assigned to an origin zone and an operation mode. If the operation mode is hailing or dispatching, random coordinates within the zone are defined for the starting position of the taxi. If the taxi operation mode is the stand mode, a random stand within the zone is assigned to it. If there is a user queue in the stand, the first user in the queue is assigned to the taxi. If there is not a user queue in the stand, the taxi joins the taxi queue.

When a user is created, origin, destination and operational mode are defined. If the operation mode is hailing or dispatching, random coordinates within the zone are defined for the waiting position of the user. In the dispatching case, the nearest (in real network travel time) available taxi is assigned to the user. If there are no available taxis, the user is added to a virtual queue for later assignment as soon as a taxi becomes available. If the operation mode is the stand mode, the user is assigned to a random stand within the zone, while if there is a taxi queue in the stand, the user is assigned to the first taxi in the queue. If there is no taxi queue in the stand, the user joins the user queue.

If neither taxis, nor users are created, the model goes directly to the movement module.

2) Movement module

The movement module moves each taxi depending on the congestion of the correspondent link. If the taxi arrives to an intersection, the intersection decision module decides the route that the taxi will follow (the next link). If the taxi continues in the same link, the position of the taxi is updated.

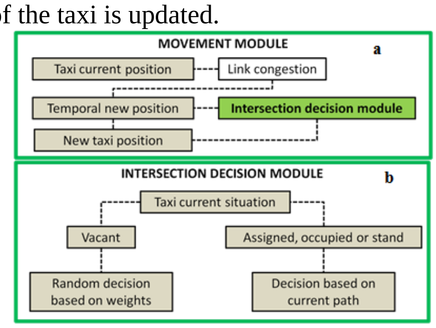

Figure 3. Movement (a) and Intersection decision (b) modules

3) Intersection decision module

When a hailing or not assigned taxi reaches an intersection, it randomly chooses the next link. When an occupied taxi, stand taxi or assigned taxi reaches an intersection, it follows the defined shortest route (between the origin and the destination of the user, looking for the nearest stand or looking for the assigned user respectively). All taxis are informed about the conditions on the network. This is in fact a realistic assumption since nowadays most of the taxis are equipped with real time traffic information technologies.

4) Taxi/user meeting module and user destination module

If a vacant or assigned taxi meets a user while circulating, the taxi/user meeting module decides if the taxi picks up the user or not. If an assigned taxi meets its assigned user, the taxi picks him/her up. The same happens in the case of a vacant hailing taxi and a hailing user. Dispatching taxis pick up only their assigned users. Stand taxis pick up only users at taxi stands. Hailing taxis pick up only hailing users. When a taxi picks up a user, the shortest route between the user's origin and destination is calculated and stored in the intersection decision module.

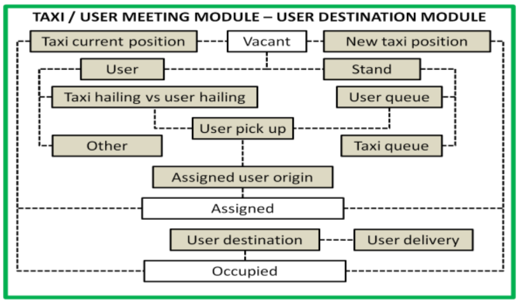

Figure 4. Taxi/user meeting and user destination modules

If an occupied taxi passes by the destination point of its user, the user is delivered, and the taxi is free for the next time interval. A stand taxi finds the nearest taxi stand and the correspondent route for reaching it. A hailing taxi circulates randomly until a new user is found. A dispatching taxi circulates randomly until a new user is assigned, finding the shortest route for reaching the user.

IV. Use Case: Sioux Falls Network

The agent-based model is tested in the Sioux Falls network [24]. The agent-based model has been applied with a fixed origin-destination demand matrix and 25 different supply levels. In order to calculate average results of the performance indicators each supply level has been run 50 times, creating a total of 125.000 trips satisfied by 32.500 vehicles. The results obtained are presented for both the drivers and the users in the graphs below. Performance indicators have been obtained for each operation mode and agent. The driver indicators are total distance, occupied and vacant time, occupied and vacant distance, income and earnings. The user indicator is waiting time. Finally, the system costs and the optimum fleet size are also presented.

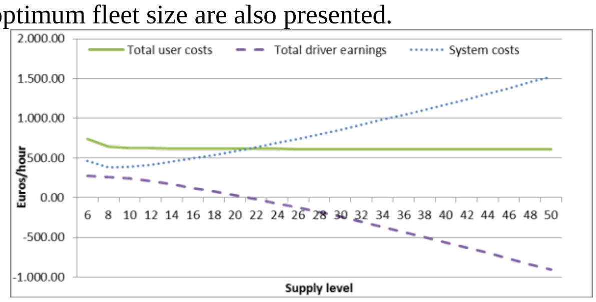

Figure 5. User costs, system costs and driver earnings related to different supply levels and to a fix demand for the dispatching mode model

Fig. 5 shows the relation between total earnings of drivers and total system and user costs in the dispatching market. The system optimum fleet can be observed where the system cost is minimum. Smaller fleet size than this optimum produces more benefit to the taxi drivers due to the higher number of trips, but the users' costs due to the increased waiting time are higher. Higher fleet size than the optimum has no significant effects on reducing the waiting time of users, but the earnings of drivers are reduced dramatically.

The graph below shows the relation between vacant and occupied kilometers and the rate between both terms for each supply level. When the supply level is low, the occupied time is high, but the vacant time is also high; at the same time the waiting time of users is high. When the supply is high, the number of occupied kilometers is low, while the number of vacant kilometers is high.

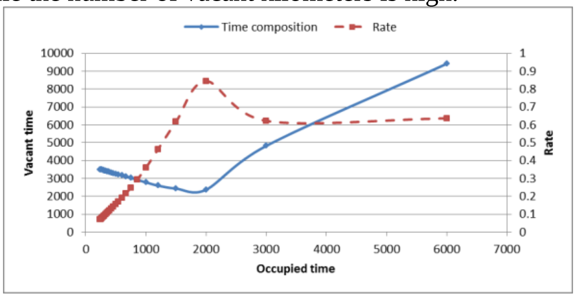

Figure 6. Relation between the vacant and occupied times for the dispatching mode model

The driver optimum fleet corresponds to the maximum rate between occupied and vacant times, as shown in Fig. 7. This driver optimum fleet is smaller than the system optimum fleet. The rate value of the system optimum fleet is 0.65, which means that the occupied time is 35-40% of the total time. Average results from the simulations are presented in the table below.

TABLE I: Simulation Results of the Three Operation Models

	Dispatching	Stand	Hailing[1]
Optimum fleet for system cost (vehicles)	8	10	14
Average occupied distance (m)	6.100	4.900	3.500
Average vacant distance (m)	4.200	330	14.000
Average occupied time (min)	25	20	14,5
Average vacant moving time (min)	35	16	90
Average vacant stopped time (min)	0	24	0
Income (euros/h)	67	54	38
Driver unitary earnings (euros/hour)	32,5	30	-8
Ratio occupied/vacant moving time	0.65 (35-40%)	1.3 (55-60%)	0.16 (10-15%)
Rate occupied/vacant distance	1.5 (60%)	15 (94%)	0.25 (20%)
Average user waiting time (sec)	136	65	392
User unitary cost (euros/hour)	6,45	6,25	7,5
System cost (euros/hour)	385	325	865

V. Conclusions

An agent-based model for simulating taxi services in urban areas has been presented. The model is capable of simulating the three operation modes (hailing, stand and dispatching). Results for the dispatching model have been

[1] In the hailing case more than one hour was needed most of the times for completing all trips since the taxis are randomly looking for users.

compared to a mathematical aggregated model, presenting similar tendency and results. The model outputs in terms of system costs and total time composition (vacant and occupied time) have been presented, concluding in the system's optimum fleet size and the drivers' optimum fleet size, obtained from the system costs and from the relation between occupied and vacant times respectively. The drivers' optimum fleet is smaller than the system's optimum fleet.

The system optimum fleet of the dispatching mode for the same demand is the smallest one; the hailing optimum fleet is the highest one. The ratio of occupied versus vacant time is 10-15% for the hailing market, since the taxis are constantly circulating and randomly looking for users; the respective ratio for the dispatching market is 35-40%, where users call taxis, reducing the vacant distance. Finally, the ratio for the stand market is 55-60%, since taxis wait at taxi stands, having to return there before picking up a new user.

Future research should focus in the users' behavior, giving them the possibility of changing operation mode if the waiting time exceeds a threshold value, or presenting user classes with different willingness to pay for taxi services. In order to obtain more realistic and representative results there is a necessity for testing different demand profiles and levels in different network geometries, calibrating the parameters of the model with real world data. Finally, new technologies should be tested using simulation models, creating the necessary rules for providing information to drivers about the hot spots of the city or demand forecasts in the city.

REFERENCES

[1] H. Yang and S. C. Wong, "A network model of urban taxi services," *Transport Research B*, vol. 32, no. 4, pp 235-246, 1998.

[2] K. I. Wong, S. C. Wong, and H. Yang, "Modeling urban taxi services in congested road networks with elastic demand," *Transport Research B*, vol. 35, pp. 819-842, 2000.

[3] H. Yang, M. Ye, W. H. Tang, and S. C. Wong, "Regulating taxi services in the presence of congestion externality," *Transport Research A*, vol. 39, pp 17-40, 2005.

[4] K. I. Wong, Wong S. C. Bell M. G. H. and H. Yang "Modeling the bilateral micro-searching behavior for urban taxi services using the absorbing Markov chain approach," *Journal of Advanced Transportation*, vol. 39, no. 1, pp. 81-104, 2005.

[5] H. Yang, W. Y. L. Cowina, S. C. Wong and M. G. H. Bell, "Equilibria of bilateral taxi-customer searching and meeting on networks," *Transport Research B*, vol. 44, pp. 1067-1083, 2010.

[6] G. Douglas, "Price Regulation and optimal service standards," *The Taxicab Industry*, 1972

[7] A. De Vany, "Capacity Utilization under Alternative Regulatory Restraints: An Analysis of Taxi Markets," *Chicago Journals, The Journal of Political Economy*, vol. 83, no. 1, pp. 83-94, 1975.

[8] M. E. Beesley, "Regulation of taxis: Royal economic society," *The economic journal*, vol.83, no.329, pp.150-172, 1973.

[9] M. E. Beesley and S. Glaister, "Information for regulating: The case of taxis. Royal economic society," *The Economic Journal*, vol. 93, no. 371, pp. 594-615, 1983.

[10] J. R. Schroeter, "A model of taxi service under fare structure and fleet size regulation," *The Bell Journal of Economics*, vol. 14, no. 1, pp. 81-96, 1983.

[11] C. F. Manski and J. D. Wright, "Nature of equilibrium in the market for taxi services," *Transportation Research Record* 619, pp 296-306, 1976.

[12] R. Arnott, "Taxi travel should be subsidized," *Journal of Urban Economics* 40, 31-333, 1996.

[13] R. D. Cairns, C. Liston-Heyes, "Competition and regulation in the taxi industry," *Journal of Public Economics* 59, pp. 1-15, 1996.

[14] B. Schaller, "Entry controls in taxi regulation: Implications of US and Canadian experience for taxi regulation and deregulation," *Transport Policy* vol. 14, pp. 490-506, 2007.

[15] L. Kattan, A. De Barros and S. C. Wirasinghe, "Analysis of work trips made by taxi in canadian cities," *Journal of Advanced Transportation*, nol. 44, issue 1, pp 11-18, 2010.

[16] J. M. Salanova, M. Estrada, G. Aifadopoulou and E. Mitsakis, "A review of the modeling of taxi services," *Procedia and Social Behavioral Sciences* 20, pp. 150-161, 2011.

[17] J. H. Holland and J. H. Miller, "Artificial adaptive agents in economic theory," *American Economic Review*, vol. 81, pp. 365-71, 1991.

[18] W. A. Bailey and T. D. Clark, "A simulation analysis of demand and fleet size effects on taxicab service rates," in *Proceedings 19th Conference on Winter Simulation*, pp. 838-844, 1987.

[19] W. A. Bailey and T. D. Clark, "Taxi management and route control: a systems study and simulation experiment," in *Proceedings 24th Conference on Winter Simulation*, pp. 1217-1222, 1992.

[20] H. Kim, J. D. Oh, and R. Jayakrishnan, "Effect of taxi information system on efficiency and quality of taxi services," *Transportation Research Record*, vol. 1903, pp. 96-104, 2005.

[21] Z. Q. Song and C. O. Tong, "A simulation based dynamic model of taxi service," in *Proceedings First International Symposium on Dynamic Traffic Assignment*, 2006.

[22] Z. Q. Song, "A simulation based dynamic taxi model," M.S. thesis, University of Hong Kong, 2006.

[23] J. E. Lioris, G. Cohen, and A. La Fortelle, "Evaluation of collecvite taxi systems by discrete-event simulation," in *Proceedings ITE Western Distric*, San Francisco, 2010.

[24] Bar-Gera. "Transportation network test problems." (2012). [Online] Available: http://www.bgu.ac.il/~bargera/tntp/

J.M. Salanova Grau holds a Civil engineering diploma from the Polytechnic University of Catalonia in Barcelona and a MSc in civil and transportation engineering from the Aristotle University of Thessaloniki, Greece. He is a PhD candidate at the Polytechnic University of Catalonia in Barcelona. His PhD thesis is related to the modelling of taxi services. Since 2008 he works as researcher for the Hellenic Institute of Transport of the Centre for Research and Technology Hellas in Thessaloniki, Greece. During the last two years he has been working on the transport model of the city of Thessaloniki and has published several related scientific papers as well as book chapters.

M. A. Estrada holds a PhD in civil and transportation engineering from the Polytechnic University of Catalonia in Barcelona. Since 2003 he works as a researcher for the Center for Innovation in Transport in Barcelona. Since 2004 he is full professor in the Department of Transport Infrastructure and Planning at Polytechnic University of Catalonia in Barcelona. During the last years he has been working on the assessment of Public Transport Systems and has authored several publications in international journals of Science Citation Index (SCI).

E. Mitsakis holds a PhD in civil and transportation engineering from the Aristotle University of Thessaloniki, Greece. Since 2005 he works as a post-doctoral researcher for the Hellenic Institute of Transport of the Centre for Research and Technology Hellas in Thessaloniki, Greece. During the last three years he has been working on analysis and optimization of transport networks and has published several related scientific papers as well as book chapters.

Dr. Mitsakis is a member of several scientific and professional societies in Greece and abroad, as well as an active member of working groups in European transport research organizations.

I. Stamos is a PhD candidate in transportation engineering at the Aristotle University of Thessaloniki, Greece. Since 2011 he works as an associate researcher at the Hellenic Institute of Transport, Centre for Research and Technology Hellas, in Thessaloniki, Greece. During the last two years, he has been working on transportation modelling issues and in the assessment of transportation networks through various modelling techniques. During this time, he co-authored several studies and published scientific papers.