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Two-Period Model of Bank Lending Channel: Basel II Regulatory Constraints

(Model Dua Tempoh Saluran Pinjaman Bank: Kekangan Regulatori Basel II)

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ABSTRACT

This paper predicts the dynamic model of the bank lending channel under Basel II regulatory constraints with monopolistic competition. The two-period model is chosen in order to demonstrate the effects of new Basel capital constraints on the risks of banks assets during both periods; and the amount of equity in the second period. The prediction of period one and two are shown to have the same effect and the only difference is the constraint. The regulatory constraint in periods one and two are predicted depending on the regulatory parameters and constraints for both periods. Thus, the effect of optimal rates on a policy rate is felt greater or less during the first period than during the second period, which means tightening capital requirements increases or decreases the risks of assets and banks taking higher or lower risks, respectively,during the first period than during the second period.

Keywords: Basel II; Bellman equation; two period model; bank lending channel.

ABSTRAK

Kajian ini menganggarkan model dinamik saluran pinjaman bank di bawah kekangan regulatori Basel II dengan persaingan monopoli. Model dua tempoh ini dipilih untuk menunjukkan kesan kekangan modal Basel yang baru terhadap aset bank di kedua-dua tempoh tersebut dan juga jumlah ekuiti di tempoh kedua. Penganggaran bagi tempoh pertama dan kedua ditunjukkan mempunyai kesan yang sama dan yang membezakannya adalah kekangan. Kekangan regulatori bagi tempoh pertama dan kedua dianggarkan bergantung kepada parameter regulatori dan kekangan kedua-dua tempoh. Maka, kesan kadar optimum bagi kadar faedah diperolehi kesannya lebih atau kurang semasa tempoh pertama daripada tempoh kedua. Ini bermaksud pengetatan keperluan modal meningkatkan atau mengurangkan risiko asset dan bank menghadapi risiko yang tinggi atau rendah semasa tempoh pertama daripada tempoh kedua.

Kata kunci: Basel II; persamaan Bellman; model dua tempoh; saluran pinjaman bank.

INTRODUCTION

The main objective of the present study is to predict the dynamic model of the bank lending channel under Basel II regulatory constraints with monopolistic competition, which was originally analysed by Kishan and Opiela (2000) and Baglioni (2007) using a static model. The two period model is chosen in order demonstrate the effect of new capital constraints on the risks of bank assets during both periods; and the amount of equity in the second period.

The present study expands upon extant studies in two principal manners. First, the present study explicitly considers the time period of banks by choosing interest rate charged on loans; the interest rate paid on deposits; and determining how much to borrow from the money market. Second, the present study predicts the different impacts of optimal rates on the effect of new regulatory constraints, monetary policies and credit risks for each period.

Malaysian banks implemented the Basel II regulatory constraints in the early part of 2008. The first phase of the Basel II regulatory constraints is the standard Internal Rating Based (IRB) approach. Under the standard IRB approach, exposure[1] to banking institutions shall be accorded risk weights based on their external credit ratings, which can be in the form of either long-term or short-term ratings.[2] Banks have always borrowed from the interbank market because the constraint of capital restricts the amount of loans and securities that can be offered to borrowers. The main role of the Basel II regulatory constraints is to prevent banks from any difficulties, such as bankruptcy and liquidity problems. On the other side, if banks always borrow from the interbank market, such banks are exposed to higher risk if the banksare unable to repay the borrowing.

The present study primarily contributes to literature concerning the dynamic model of bank lending channel under the Basel II regulatory constraints. The present study predicts the theoretical impact of bank rates on the monetary policy and regulatory constraint in two period models with monopolistic competition.

The remainder of the paper is structured as follows. Section 2 contains the literature review; and section 3 shows the theoretical model and the predictions. Finally, section 4 presents the overall findings of the study. The appendix contains all proofs and tables.

LITERATURE REVIEW

The dynamic model of the lending channel has not been discussed in detail. A transmission mechanism of monetary policy is important in determining the behaviour of the bank lending channel. Banks set their own interest rates and behave as though they exist in an environment of monopolistic competition. The two period model is chosen in order demonstrate the effect of new capital constraints on the risks of bank assets during both periods; and the amount of equity during the second period.

It is vital to highlight the role of the new Basel regulatory constraints in the model employed in the present study because the new Basel regulatory constraints may result in different impacts on banks' balance sheets. Why do banks need to be regulated? Banks are exposed to credit and liquidity risks. Banks are also faced with the possibility of borrowers defaulting on loan repayments; and not having enough cash to meet deposit withdrawals. The higher the risk of a bank's assets (e.g., the ratio of loans to total assets), the more vulnerable the banks are likely to be.

Extant studies that are similar to the present study include Kishan and Opiela (2000), Baglioni (2007) and Honda (2004), who use a static model of bank lending channels under the Basel I Capital Accord of 1998 (Basel I). Jaques (2008), Ahmad (2006) and Kashyap and Stein (2004) analyse the adverse macroeconomics effect of Basel I, especially in regards to its procyclicality and the neglect of the endogeneity of financial risk. Jaques (2008) develops a theoretical model to examine how commercial loans of varying credit quality are likely to respond to an adverse capital shock under the Basel II regulatory constraints. The results of the study suggest low credit risk loans may actually increase with the increased differentiation of credit risk introduced by the revised standards under the Basel II regulatory constraints. Ahmad (2006) concludes that the new capital requirements can have both good and bad effects on the targeted financial institutions and markets. The recent study made by Boivin et al. (2010) reviews the empirical evidence concerning the changes in the effect of monetary policy actions on real activity and inflation; and present new evidence using a relatively unrestricted factor-augmented vector auto regression (FAVAR) and a DSGE model. The results indicate notable changes in policy behaviour (with policy more focused on price stability) and in the reduced form correlations of policy interest rates in relation to activity in the United States. Both approaches employed by Boivin et al. (2010) yield similar results. Additionally,due to competition on the assets side, Repullo and Suarez (2004) argue that banks eligible for the IRB approach have a competitive advantage in the provision of low-risk loans because the IRB approach has a lower capital requirement, while the less sophisticated banks have a competitive advantage in the provision of high risk loans because the standardised approach has a lower capital requirement.

The present study makes different findings due to the use of a methodology that differs in several important respects from those used by Jaques (2008), Ahmad (2006), Kashyap and Stein (2004), Repullo and Suarez (2004) and Kishan and Opiela (2000). First, the aforementioned studies analyse the bank lending channel by assuming that banks operate in an imperfect-competitive market. According to the assumption of the aforementioned studies, the correct bank strategic variable is quantity instead of price. In other words, each bank decides its optimal volume of loans, taking, as given, the volumes of loans supplied by the other banks. The equilibrium price is the one that equates to the aggregate supply and demand for loans. However, the present research differs from the aforementioned studies since the present study assumes that each of the banks behave as if in monopolistic competition (an assumption inspired by Baglioni (2007) and Boivin et al. (2010)). This market structure is suitable for describing the market for bank loans, despite the presence of many players in the market, in which each of them retains the power of setting its own price at the desired level. The reason for choosing a monopolistic competition market over an imperfect competition market in the present analysis is that loans are not perfect substitutes to borrowers (a monopolistic competition market can be differentiated). Each bank has some market power in the market for loans (faces downward-sloped demand for loans with finite elasticity) and time deposits. The difference between the present analysis and the analysis of Boivin et al. (2010) is that the disaggregated data of banks are used and the behaviour of banks is analysed by changes in the policy rate. The study of Boivin et al. concentrates more on the changes of monetary policy actions in relation to real activity and inflation without looking into the behaviour of individual banks.

Second, Jaques (2008) models bank competition on the asset side and ignores competition on the liabilities side. However, the present analysis considers competition on both the assets and liabilities of banks' balance sheets. In other words, the present study examines whether small or large banks (bank size) become more or less competitive in engaging a higher or lower risk of loans and securities; and whether high or low risk loans/securities are more competitive under the Basel II regulatory constraints.

Third, a two period model is chosen in order to demonstrate the effect of the bank lending channel by setting bank prices as the optimal decisions in the different time periods. The two period model is chosen over the infinite model in order to provide clear evidence of whether banks are holding more risky assets or less risky assets during the first and second periods when the new regulatory constraints are imposed at the start of period 1. This is essential especially to determine the amount of equity in the second period. If banks hold more risky assets (less risky assets) during the first period, the amount of equity during the second period will increase (decrease) if the investment is success. For example, if banks are assumed to impose new capital requirements at the start of period 1, tightening the capital requirements will either decrease the risk of assets (Blum 1999) or increases the risk of assets (Ahmad 2006) depending upon whether the new requirements motivate banks from taking lower risks or higher risks during the first and second periods. Thus, the two periods chosen are sufficient to demonstrate banks' investment decisions during the first period,while all costs are paid and returns are received during the second period. The bank operation will continue over time if the model is assumed to be in the n-period or infinite horizon. However, this is not pursued since a two period model can sufficiently prove the main objective of the present study. Miyake and Nakamura (2007) conclude that the timing of the introduction of tight regulations is important. If the regulations become tighter when a negative productivity shock occurs, the economy falls into a long and severe slump. This is consistent with the state of the Japanese economy after experiencing the bubble economy in 2007. In addition, Naceur (2009) investigates the effects of capital regulations on the cost of intermediation and profitability. The study finds a higher capital adequacy increase in the interest rate of shareholders in managing banks' portfolios. The reduction in economic activity has opposite effects on banks' profitability.

THEORETICAL MODEL

The present study examines a dynamic balance sheet model of bank lending and portfolio decisions. Banks invest in loans and securities while obtaining funds from deposits, own capital and the money market. When making decisions, banks are bound by the regulatory capital requirements and risk weights imposed by the Basel accords. Banks act in a partial equilibrium monopolistic competition environment, where decisions concerning interest rates on time deposits and on loans are based upon their analysis of the equilibrium interest rate in both loan and deposit markets. In this setting the transmission of monetary policy into bank lending and portfolio composition is examined.

The model developed in the present study is a two period extension of the previous work conducted by Kishan and Opiela (2000) and Baglioni (2007). Banks start their first period with an exogenous own capital endowment. The amount of equity in the second period is not fixed, but can actually be influenced by the investment decisions made during the first period. By decreasing (increasing) risk today, the banks have a lower (higher) amount of equity available tomorrow in case of success. Therefore, the introduction of a new capital requirement for today induces lower risks (higher risks) tomorrow depending upon whether banks are taking risks to maximize profits tomorrow.

During the first period, each bank observes the demand for deposits and the demand for loans directed at their own bank. Based upon this information, the banks decide how much to borrow from the money market, how much to reward deposits and how much to charge for loans, effectively choosing their preferred position in these idiosyncratic demand curves.

During the second period, loans are repaid (or defaulted upon) and risky securities yield their return. Meanwhile, banks must pay back their depositors and also what was borrowed from the money market. If any money is left over, that constitutes the own capital of the bank for next period's exercise.[3]

THE BALANCE SHEET CONSTRAINT

The model is built on the definition of the balance sheet of each bank, which equates the following assets and liabilities:

$$R_{jt} + S_{jt} + L_{jt} = D_{jt} + T_{jt} + B_{jt} + K_{jt} \qquad (1)$$

On the asset side, R denotes required reserves, S denotes securities, and L denotes loans. On the liability side, D denotes demand deposits, T denotes time deposits, K denotes the bank's own capital, and B denotes the interbank borrowing. All items have subscripts (jt) because the present study utilizes a panel structure, where the subscript (j) identifies the bank and the subscript (t) identifies the period.

The equality of balance sheets in equation 1 is an *ex-ante* definition: At the first period banks were given the choice of choosing time deposit and loan rates; and the choice of how much to borrow from the money market, the amount of loans issued, the amount of securities bought and reserves set aside must obey this relationship. At the end of the second period, some loans may be defaulted and the amount of equity is not fixed, but can actually be influenced by the investment decisions made in the first period. Therefore, the equality of this equation will no longer hold. But it must hold *ex-ante*.

RESERVES

Banks do not hold excess reserves, only required reserves. As in Kishan and Opiela (2000), the required reserves are assumed to be a constant fraction α of demand deposits at each period t = 1,2 period:

$$R_{jt} = \alpha D_{jt} \quad (2)$$

The reserve requirement fraction α is set by the central bank at 4 percent of demand deposits. Required reserves receive no returns.

SECURITIES

Banks also hold marketable financial assets, such as government and private bonds; and bills. Banks are assumed to hold securities if it is costly to liquidate loans in the short run, as opposed to Kashyap and Stein (1995). Furthermore, banks may hold a buffer stock of securities to insulate themselves, at least partially (Stein 1998). The rate of return on securities, r_{st}, is given by:

$$r_{st} = e_0 + e_1 i_t + v_t \quad (3)$$

The current inter-bank or money market rate i_t has a direct influence on the current rate of return on securities, where e_0 and e_1 are parameters and v_t is a random error term that summarizes all other factors influencing the rate of return. Other factors in the random error can include changes in the total factor productivity of firms.[4] The money market rate, i_t, is observed at the start of period 1 before decisions are made, but the error term is only realised at the end of period 2 after the decisions are made.

LOANS

The loan market is in monopolistic competition, where each bank sets its own loan interest rate, taking as given the 'market' interest rate, r_{Ljt}. The demand for loans faced by bank j in period t is given by

$$L_{jt} = b_0 - b_1(r_{Ljt} - \bar{r}_{Lt}) - b_2\bar{r}_{Lt} + v_{jt} \quad (4)$$

where is an error term. Individuals and firms demand loans based on the loan rate and incur some costs when changing banks, which generates local monopoly power for each bank. The error term is not correlated with other variables and is observed at the start of each period so that the exact location of the demand curve is known and can be explored by the bank. The monopolistic competition assumption follows Baglioni (2007).

Loans are subject to ex-post default. The default rate is a random variable with an expected value (1-q). The bank, therefore, expects to recover qL (performing loans) of the loans made.

DEPOSITS

The banks' sources of funds are deposits, equity and money market borrowing. Demand and time deposits are separated and demand deposits are assumed to be outside of the control of any bank. All deposits are returned to customers at the end of period 2.

The demand for demand deposits faced by bank j during each period is inversely related to the interbank rate and varies over time by error term ε_t, the realization of which is observed at the start of each period.[5]

$$D_{jt} = c_0 - c_1 i_t + \varepsilon_t \quad (5)$$

The interest rate paid on demand deposits, r_{Dt}, is determined as given by interbank rates or mean market interest rates; and is exogenous to commercial banks. Every bank has the same interest rate on demand deposits.

The demand for time deposits directed at bank j is a function of the spread between banks j's rate, r_{Tjt}, and the average rate in the market. This demand varies over time by error term ω_{jt}, the realization of which is known at the start of each period before decisions are made. If banks want to attract more time deposits, they must raise interest rates to increase their market share. The demand for time deposits faced by a bank is given by

$$T_{jt} = d_0 + d_1(r_{Tjt} - \bar{r}_{Tt}) + d_2\bar{r}_{Tt} + \omega_{jt} \quad (6)$$

BORROWING

Banks' borrowing is understood as borrowing from the interbank market or money market. The cost of borrowing is assumed to depend upon the policy rate, which originates in the interbank market and risk free market repurchase agreements (repos).

CAPITAL

The initial level of equity K_{jt} is exogenously determined, either being derived from retained earnings or capital injections and profits of the previous period. The use of equity capital must conform with capital requirement regulations imposed by Basel I, which limits the bank's exposure to non performing loans and securities. The reserves requirement is the amount that should be possessed by banks before loans and securities. This constraint always binds, as demonstrated below. The capital constraint is given by

$$K_{jt} \geq \mu(R_{jt} + \delta_S S_{jt} + \delta_L L_{jt}) \quad (7)$$

This equation states that banks are subject to risk-based capital requirements, where μ measures the minimum capital requirements for reserves, securities and loans. In accordance with Basel I, all loans and securities in the private sector are given the average capital requirement $\mu = 0.08$. However, under the Basel II regulatory constraints, a different calculation of risk-weighted assets across loans and securities is utilized, which depends on the borrowers' ratings or quality of portfolio held by the bank. Therefore, δ_S and δ_L are assumed to be the risk weights on securities and loans.[6] These risk factors are essential to banks exposed to high levels of risk for loans and securities. The crucial property here is that:

$$\delta_S, \delta_L \geq 1 \quad (8)$$

PROFIT MAXIMIZATION IN PERIOD 2

At the start of each period, banks choose the interest rate they offer on time deposits, r_{Tj2}, and charge on loans, r_{Lj2}, as well as how much is to be borrowed from the money market, B_{j2}. The model is solved backwards and the decision at the start of period 2 is examined first. Given positive capital and positive demand for deposits (no bank run), the bank chooses (r_{Tj2}, r_{Lj2}, B_{j2}) to maximize profits subject to the constraints and relationships described above. Expected profits are given by

$$E\pi_{j2} = E_q r_{Lj2} q L_{j2} + E_{rS} r_{S2} S_{j2} - r_{D2} D_{j2} - r_{Tj2} T_{j2} - i_2 B_{j2} - \mathrm{E}_{1-q}(1-q)L_{j2} \quad (9)$$

The balance sheet relationship is an identity and is replaced in the objective function to eliminate S. Regulatory constraints under both Basel I and Basel II have the associated Lagrange multiplier lambda, λ_{j2}, defined as $\mathcal{L} = (\pi_{j2} + \lambda_{j2} C)$. The first order conditions for this problem, derived in the appendix, are as follows:

$$\frac{\partial \mathcal{L}_{j2}}{\partial r_{Lj2}} = qL_{j2} - qb_1 r_{Lj2} + r^e_{S2} b_1 + (1-q)b_1 - \lambda_{j2} b_1(\delta_L - \delta_S) = 0 \quad (10)$$

$$\frac{\partial \mathcal{L}_{j2}}{\partial r_{Tj2}} = d_1 r^e_{S2} - T_{j2} - d_1 r_{Tj2} + \lambda_{j2} d_1 \delta_S = 0 \quad (11)$$

$$\frac{\partial \mathcal{L}_{j2}}{\partial B_{j2}} = r^e_{S2} - i_2 + \lambda_{j2}\delta_S = 0 \quad (12)$$

and

$$\frac{\partial \mathcal{L}_{j2}}{\partial \lambda_{j2}} = (\alpha + (1-\alpha)\delta_S)D_{j2} + (\delta_L - \delta_S)L_{j2} + \delta_S T_{j2} + \delta_S B_{j2} - \left(\frac{1}{\mu} - \delta_S\right)K_{j2} = 0 \quad (13)$$

The first condition determines the value of the Lagrange multiplier. It is given by

$$\lambda_{j2} = \frac{i_2 - r^e_{S2}}{\delta_S} < 0 \quad (14)$$

and is negative because the expected return on securities is assumed to be higher than the policy rate, i_2. This shows that only borrowing depends on capital, since banks always borrow from the money market.

The first condition for the constraint is given by:

$$B_{j2} = \left(\frac{1}{\mu} - \delta_S\right)\frac{K_{j2}}{\delta_S} - (\alpha + (1-\alpha)\delta_S)\frac{D_{j2}}{\delta_S} - (\delta_L - \delta_S)\frac{L_{j2}}{\delta_S} - \delta_S \frac{T_{j2}}{\delta_S} \quad (15)$$

The Lagrange multiplier of constraint implies that interbank borrowing at period two depends linearly on the amount of capital during the same period. This shows that banks will decide whether to borrow from the interbank market after observing the amount of capital available. The constraint of capital will determine the amount of borrowing in the money market. This strongly implies that the regulatory constraint always binds. If the bank has enough own capital to invest, it will still borrow from the interbank market until it has purchased enough securities and issued enough loans that it becomes constrained under the Basel II regulatory framework. This is not an unreasonable description of the risk taking behaviour observed recently throughout the international banking system: cheap money and toxic assets.[7]

The second implication of the solution is that the interbank rate affects the model only via the Lagrange multiplier. This is because the policy rate will always be less than the expected rate of return on securities or debt; and the banks always borrow since they assume the rate of returns are profitable.

After a substitution of terms, the following formulas are obtained for the optimal interest rates:

$$r^*_{Lj2} = \frac{[b_0 + (b_1 - b_2)\bar{r}_{L2} + v_{j2}]q - \lambda_{j2} b_1(\delta_L - \delta_S) + b_1 r^e_{S2} + (1-q)b_1}{2qb_1} \quad (16)$$

$$r^*_{Tj2} = \frac{[r^e_{S2} + \lambda_{j2}\delta_S]d_1 - d_0 + (d_1 - d_2)\bar{r}_{T2} - \omega_{j2}}{2d_1} \quad (17)$$

and note that

$$\frac{\partial r^*_{Lj2}}{\partial i_2} = \frac{e_1}{2_q} - \frac{(\delta_L - \delta_S)}{2q}\frac{\partial \lambda_{j2}}{\partial i_2} \quad (18)$$

$$\frac{\partial r^*_{Tj2}}{\partial i_2} = \frac{e_1}{2} + \frac{\delta_S}{2}\frac{\partial \lambda_{j2}}{\partial i_2} \quad (19)$$

The Lagrange multiplier shows a negative derivative $\left(\frac{\partial \lambda_{j2}}{\partial i_2} = \frac{1 - e_1}{\delta_S}\right)$ since the policy rate is always less than the expected return on securities. The Lagrange multiplier in period 2 demonstrates that the effect of the regulatory constraint is always negative. Therefore, the reaction of the optimal rate on loans from the change in policy rate is positive if $\delta_L > \delta_S$, or otherwise if $\delta_S > \delta_L$. This demonstrates that when the credit risk of loans is more than the credit risk of securities, the rate on loans positively reacts to the changes in policy rate since investment in loans are more risky than securities. Thus, an increasing rate of loans will reduce the amount of loans since investments in securities will provide greater returns. Otherwise, if credit risks relating to securities are greater than loans, the rate on loans will increase as the policy rate decreases. This is due to the fact that increasing the credit risk of securities gives more exposures to risk on investment in securities.Therefore, it is more rational for banks to decrease the rate on loans as the policy rate increases since the level of risk on loans is less than the level of risk on securities; and they will prefer to reduce the cost of loans in order to increase the amount of loans. Besides, the reaction of the optimal rate on time deposits is always negative if the Lagrange multiplier is always negative.

In addition, the binding of capital rule[8]always decreases the amount of risky loans and securities to be invested. Thus, the binding of capital rule decreases the profit of banks due to the limited amount of assets that can be invested. Therefore, banks are less exposed to risky and default assets.

THE VALUE FUNCTION

The method of dynamic programming, as suggested by Bellman (1957), can be used to solve the value function. In order to solve the first period problem, value function for period 2 must be written, which is the maximized profit as a function of all stated variables of the problem. Note that at the optimum level, the regulatory constraint binds and is not explicitly visible in the value function.

$$V_{j2}(K_{j2}, Z_{j2}) = r_{Lj2}qL_{j2} + r^e_{S2}S_{j2} - r_{D2}D_{j2} - r_{Tj2}T_{j2} - i_2B_{j2} \tag{20}$$

where $K_{j2} = \{\omega_{j2}, v_{j2}, i_2, \varepsilon_{j2}\}$ contains the realizations of shocks to time deposits, loan demand, interbank rate and demand deposits. Securities must be replaced with the balance sheet identity and obtains

$$V_{j2}(K_{j2}, Z_{j2}) = (qr_{Lj2} - r^e_{S2})L_{j2} + r^e_{S2}K_{j2} + (r^e_{S2} - i_2)B_{j2} + r^e_{S2}[(1-\alpha)D_{j2} + T_{j2}] - r_{D2}D_{j2} - r_{Tj2}T_{j2} - (1-q)L_{j2} \tag{21}$$

where B_{j2} is a linear function of K_{j2}.

More importantly, the value function of period 2 can be decomposed into two components, which separates the components that depend on K_{j2} from those that do not. Since only B_{j2} depends on K_{j2}, the following function is written:

$$V_{j2}(K_{j2}, Z_{j2}) = K_{j2}\left[r^e_{S2} + (r^e_{S2} - i_2)\left(\frac{1}{\mu} - \delta_S\right)\frac{1}{\delta_S}\right](r^e_{S2} - i_2) \left[(\alpha + (1-\alpha)\delta_S)\frac{D_{j2}}{\delta_S} + (\delta_L - \delta_S)\frac{L_{j2}}{\delta_S} + \delta_S\frac{T_{j2}}{\delta_S}\right] + (qr_{Lj2} - r^e_{S2})L_{j2} + r^e_{S2}[(1-\alpha)D_{j2} + T_{j2}] - r_{D2}D_{j2} - r_{Tj2}T_{j2} - (1-q)L_{j2} \tag{22}$$

$$V_{j2}(K_{j2}, Z_{j2}) = \Gamma_1 K_{j2} + \Gamma_2 \tag{23}$$

where,

$$\Gamma_1 = \left[r^e_{S2} + (r^e_{S2} - i_2)\left(\frac{1}{\mu} - \delta_S\right)\frac{1}{\delta_S}\right] \tag{24}$$

The value function is useful for the next section.

PROFIT MAXIMIZATION IN PERIOD 1

The profit function of period one is given by:

$$\pi_{j1} = (qr_{Lj1} - r_{S1} - (1-q))L_{j1} + r_{S1}K_{j1} + (r_{S1} - i_1)B_{j1} + r_{S1}[(1-\alpha)D_{j1} + T_{j1}] - r_{D1}D_{j1} - r_{Tj1}T_{j1} \tag{25}$$

and subject to constraint:

$$K_{j1} \geq \mu(R_{j1} + \delta_S S_{j1} + \delta_L L_{j1}) \tag{26}$$

$V_{j1}(K_{j1}, Z_{j1})$ is defined as the maximized value of the objective function at time 1 given an initial capital stock of assets K_{j1}. $V_{j2}(K_{j2}, Z_{j2})$ is the maximized value of the objective function at time 2 given an initial capital stock of assets K_{j2}. In other words, the objective function for the two-period problem is defined at the start of period one and greatly simplified in a recursive form using the Bellman equation as follows:

$$V_{j1}(K_{j1}, Z_{j1}) = \max_{r_{Lj1}, r_{Tj1}, B_{j1}} \pi_{j1} + \beta V_{j2}(K_{j2}, Z_{j2}) \tag{27}$$

where the realized π_{j1} is equal to K_{j2}.

This allows the Value Function in period 1 to be rewritten as

$$V_{j1}(K_{j1}, Z_{j1}) = \max_{r_{Lj1}, r_{Tj1}, B_{j1}} \pi_{j1} + \beta(\Gamma_1\pi_{j2}, \Gamma_2) \tag{28}$$

The next step is to maximize this objective function with respect to r_{Tj1}, r_{Lj1}, B_{j1}. Then, take a derivative of the entire right hand side (RHS) of the Bellman operator, where the solution maximization of the value function of period 1 is given as follows:

$$\frac{\partial\pi_{j1}}{\partial\pi_{Lj1}} = \begin{bmatrix} b_0q + (b_1 - b_2)\bar{r}_{L1}q - 2b_1r_{Lj1}q \\ + v_{j1}q + b_1r_{S1} + (1-q)b_1 \\ -\left(\mu - \frac{1}{\delta_S}\right)b_1r_{S1}(\delta_L - \delta_S) \end{bmatrix}[1 + \beta\Gamma_1] = 0 \tag{29}$$

$$\frac{\partial\pi_{j1}}{\partial\pi_{Tj1}} = \begin{bmatrix} d_1r_{S1} + (d_1 - d_2)\bar{r}_{T1} - d_0 \\ -2d_1r_{Tj1} - \omega_{j1} + r_{S1}\left(\mu - \frac{1}{\delta_S}\right)d_1\delta_S \end{bmatrix}[1 + \beta\Gamma_1] = 0 \tag{30}$$

$$\frac{\partial\pi_{j1}}{\partial\beta_{Lj1}} = \left[(r_{S1} - i_1) + r_{S1}\left(\mu - \frac{1}{\delta_S}\right)\delta_S\right][1 + \beta\Gamma_1] = 0 \tag{31}$$

The derivative of r_{Tj1}, r_{Lj1}, B_{j1} in the first period is realized with the addition of the expected value of the second period, which depends on the first period. In this case, β is a discount factor that can be formulated as $\frac{1}{i+\rho}$, in which ρ is a premium rate and i is an interbank rate.

Optimal interest rates in period 1

$$r^*_{Lj1} = \frac{\begin{bmatrix} b_0q + (b_1 - b_2)\bar{r}_{L1}q + v_{j1}q \\ + b_1r_{S1} + (1-q)b_1 - \left(\mu - \frac{1}{\delta_S}\right)b_1r_{S1}(\delta_L - \delta_S) \end{bmatrix}}{2b_1q} \tag{34}$$

$$r^*_{Tj1} = \frac{\left[(d_1 - d_2)\bar{r}_{T1} - d_0 - \omega_{j1} + d_1r_{S1} + r_{S1}\left(\mu - \frac{1}{\delta_S}\right)d_L\delta_S\right]}{2d_1} \tag{35}$$

and note that

$$\frac{\partial r^*_{Lj1}}{\partial i_1} = \frac{e_1}{2q} - e_1\left(\mu - \frac{1}{\delta_S}\right)\left(\frac{(\delta_L - \delta_S)}{2q}\right) = 0 \tag{36}$$

$$\frac{\partial r^*_{Tj1}}{\partial i_1} = \frac{e_1}{2} + e_1\left(\mu - \frac{1}{\delta_S}\right)\left(\frac{\delta_S}{2}\right) = 0 \tag{37}$$

The effect of regulatory constraint in period one is shown by $e_1\left(\mu - \frac{1}{\delta_S}\right)$. Since $\mu = 0.08$ is the capital adequacy ratio and a small percentage if compared with $\frac{1}{\delta_S}$, the effect of regulatory constraint is always negative. During this first period, the constraint of capital plays an important role in influencing the response of the interest rates to loans and time deposits. If regulatory constraint is assumed to always be negative, the response of interest rate on loans during period 1 to a policy rate has a positive effect if $\delta_L > \delta_S$, otherwise if $\delta_L < \delta_S$. The first period predictions are similar to the second period predictions without taking into account the constraint. However, only the constraint makes the predictions difference. In addition, if the effect of regulatory constraint is assumed to be negative, the response of interest rates on time deposits during period 1 to policy rate is negatively predicted. Only the risk factor of securities influences the optimal rate of time deposits.

The regulatory constraint in periods 1 and 2 are predicted depending upon the regulatory parameters and constraints for both periods. The impact of interest rates on loans and time deposits to a policy rate more or less occur during the first period when banks face a shock of capital rule from the start of period 1.

CONCLUSIONS

The overall result of the predictions demonstrates how essential it is to analyse, in detail, a dynamic model of a bank lending channel in a monopolistic competition market. During period one, the regulatory constraint always demonstrates a negative value. This implication is true for both periods of time predicted. In other words, the prediction of period one and two are shown to have the same effect and the only difference is the constraint. The impact of interest rates on loans and time deposits on a policy rate more or less occur during the first period when banks face a shock of capital rule from the start of period one. Thus, the effect of optimal rates on the policy rate is felt greater or less during the first period than the second period, which means tightening capital requirements increases or decreases the risks of assets and banks taking a higher or lower risk during the first period than during the second period. This is consistent with the analysis performed by Ahmad (2006).

Two principal limitations exist for the present study. First, data are not calibrated or estimated in accordance with the predictions of the present study. However, the research will be expanded by calibrating and estimating a banks data from the first period of implementation of the Basel II regulatory constraints until more recent years in order to determine the effect of new regulatory constraints on the optimal decisions predicted in the present study. The effect of new regulatory constraints will depend upon the regulatory parameters predicted in the first and second periods. Second, the model presented in the present study is only a partial equilibrium model. Therefore, the results can be of greater interest if the role of other agents is included, such as government, households and firms,in order to obtain a full general equilibrium.

NOTES

1 Exposure to sovereign, and so on as stated in the Basel II regulatory constraint.

2 Sources: Prudential Financial Policy Department, Central Bank of Malaysia.

3 If no money is left over, the bank is extinguished and the problem ends. Limited liability exists, so bank owners are not forced to cover unfulfilled claims. However, this problem is side stepped by assuming that the distributions of the relevant random variables are such that some positive profit will always occur in period 1. This issue is discussed further in the author's PhD thesis (on file with the author), which demonstrates the difference of capital ratios and capital adequacy ratios (known as excess of capital) for all banks (to show whether banks are well-capitalized or less-capitalized); and also in the appendix of descriptive analysis that demonstrates that all banks have a positive equity. If the excess of capital is positive (negative), banks are classified as well capitalized (less capitalized). While this does not indicate that those banks with a negative excess of capital have negative equity, their capital ratio is less than their capital adequacy ratio (8%).

4 The present study assumes that $v_t \sim N(0,\Omega)$; where Ω is a scalar. The interest rate on private securities reflects the production possibilities frontier and is also assumed to respond to monetary policy via the interbank rate.

5 The shock does not include bank run, in which case the error will only give a positive value. So the distribution is only positive and $\varepsilon_t \sim N(0,\Omega)$ where Ω is a scalar.

6 In Basel II, the delta can be varied because assets depend on credit risk. Therefore δ_L, δ_S>1, which can be divided into high and low credit risks concerning loans and securities.

7 Toxic asset is a popular term for certain financial assets whose value has fallen significantly and for which a functioning market no longer exists. Such assets cannot be sold at a price satisfactory to the holder.

8 The binding capital rule refers to the capital regulatory constraint regulated by the Basel Committee on Banking Supervision (BCBS) in Basel II.

9 This configuration is close to the average configuration of the balance sheet in the data of the present study. It is not identical because the balance sheet in the model is a simplification of the actual data.

REFERENCES

Ahmad, R. 2006. Capital requirement and financial regulations in banking: Are they effective? *Banker's Journal Malaysia* 130: 13-21.

Baglioni, A. 2007. Monetary Policy Transmission under different banking structures: The role of capital and heterogeneity. *International Review of Economics and Finance* 16: 78-100.

Bellman, R. 1957. *Dynamic Programming.* New Jersey: Princeton University Press, Princeton, New Jersey.

Blum, J. 1999. Do capital adequacy requirements reduce risks in banking? *Journal of Banking and Finance* 23: 755-771.

Boivin, J., Kiley, M. and Mishkin, F. 2010. How has the monetary transmission mechanism evolved over time? NBER Working paper Series 15879.

Honda, Y. 2004. Bank capital regulations and the transmission mechanism. *Journal of Policy Modelling* 26: 675-688.

Jacques, K. T. 2008. Capital shocks, bank asset allocation, and the revised Basel Accord. *Review of Financial Economics* 17: 79-91.

Kashyap, A. K. & Stein, J. C. 1995. Impact of monetary policy on bank balance sheets. *Cornegie-Rochester Series on Public Policy* 42: 151-195.

Kashyap, A. K., & Stein, J. C. 2004. Cyclical implications of the Basel II capital standards. *Economic Perspectives, Federal Reserve Bank of Chicago* Q I: 18-31.

Kishan, R. P. & Opiela, T. P. 2000. Bank size, bank capital and the bank lending channel. *Journal of Money, Credit and Banking* 32: 121-141.

Miyake, A. & Nakamura, T. 2007. A dynamic analysis of an economy with banking optimization and capital adequacy regulations. *Journal of Economics and Business* 59: 14-27.

Naceur, S. B. & Kandil, M. 2009. The impact of capital requirements on banks' cost of intermediation and performance: The case of Egypt. *Journal of Economics and Business* 61: 70-89.

Repullo, R. & Suarez J. 2004. Loan pricing under Basel capital requirements. *Journal of Financial Intermediation* 13: 496-521.

Stein, J. C. 1998. An adverse selection model of bank asset and liability management with implications for the transmission of monetary policy. *RAND Journal of Economics* 29(3): 466-86.

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APPENDIX A

PROFIT MAXIMIZATION

Profit Maximization of Second Period, Given $K_{j2} > 0$

The objective function is rewritten as

$$\mathcal{L}_{j2} = \begin{bmatrix} E_q r_{Lj2}\, qL_{j2} + E_{rS} r_{S2} S_{j2} - r_{D2} D_{j2} - r_{Tj2} T_{j2} - \\ i_2 B_{j2} - E_{1-q}(1-q)L_{j2} + \lambda_{j2}\{\mu_{j2}(R_{j2} + \delta_S S_{j2} \\ + \delta_L L_{j2}) - K_{j2}\} \end{bmatrix} \quad (39)$$

where $\lambda_{j2} \leq 0$. Now the balance sheet equality is used to eliminate securities:

$$S_{j2} = (1-\alpha)D_{j2} + T_{j2} + K_{j2} + B_{j2} - L_{j2} \quad (40)$$

Capital constraint becomes:

$$\frac{K_{j2}}{\mu} \geq \alpha D_{j2} + (1-\alpha)\delta_S D_{j2} + \delta_S T_{j2} + \delta_S K_{j2} + \delta_S B_{j2} - \delta_S L_{j2} + \delta_L L_{j2} \quad (41)$$

$$(\alpha + (1-\alpha)\delta_S)D_{j2} + (\delta_L - \delta_S)L_{j2} + \delta_S T_{j2} + \delta_S B_{j2} \leq \left(\frac{1}{\mu} - \delta_S\right)K_{j2} \quad (42)$$

Therefore:

$$\mathcal{L}_{j2} = \begin{bmatrix} E_q r_{Lj2}\, qL_{j2} + E_{rS} r_{S2} S_{j2} - r_{D2} D_{j2} \\ - r_{Tj2} T_{j2} - i_2 B_{j2} - E_{1-q}(1-q)L_{j2} \\ + \lambda_{j2}\left\{ \begin{matrix} (\alpha + (1-\alpha)\delta_S)D_{j2} + (\delta_L - \delta_S)L_{j2} \\ + \delta_S T_{j2} + \delta_S B_{j2} - \left(\frac{1}{\mu} - \delta_S\right)K_{j2} \end{matrix} \right\} \end{bmatrix} \quad (43)$$

Now, the following first order conditions are obtained:

$$\frac{\partial \mathcal{L}_{j2}}{\partial r_{Lj2}} = qL_{j2} - qb_1 r_{Lj2} + r^e_{S2} b_1 + (1-q)b_1 - \lambda_{j2} b_1 (\delta_L - \delta_S) = 0 \quad (44)$$

$$\frac{\partial \mathcal{L}_{j2}}{\partial r_{Tj2}} = d_1 r^e_{S2} - T_{j2} - d_1 r_{Tj2} + \lambda_{j2} d_1 \delta_S = 0 \quad (45)$$

$$\frac{\partial \mathcal{L}_{j2}}{\partial r_{Bj2}} = r^e_{S2} - i_2 + \lambda_{j2}\delta_S = 0 \quad (46)$$

and

$$\frac{\partial \mathcal{L}_{j2}}{\partial r_{Bj2}} = (\alpha + (1-\alpha)\delta_S)D_{j2} + (\delta_L - \delta_S)L_{j2} + \delta_S T_{j2} + \delta_S B_{j2} - \left(\frac{1}{\mu} - \delta_S\right)K_{j2} = 0 \quad (47)$$

Capital constraint is always binding, $\lambda_{j2} \neq 0$:

$$r^*_{Lj2} = \frac{[b_0 + (b_1 - b_2)\bar{r}_{L2} + v_{j2}]q - \lambda_{j2} b_1 (\delta_L - \delta_S) + b_1 r^e_{S2} + (1-q)b_1}{2qb_1} \quad (48)$$

$$r^*_{Tj2} = \frac{[r^e_{S2} + \lambda_{j2}\delta_S]d_1 - d_0 + (d_1 - d_2)\bar{r}_{T2} - \omega_{j2}}{2d_1} \quad (49)$$

$$\lambda_{j2} = \frac{i_2 - r^e_{S2}}{\delta_S} < 0 \quad (50)$$

And note that

$$\frac{\partial r^*_{Lj2}}{\partial i_2} = \frac{e_1}{2q} - \frac{(\delta_L - \delta_S)}{2q}\frac{\partial \lambda_{j2}}{\partial i_2} \quad (51)$$

$$\frac{\partial r^*_{Tj2}}{\partial i_2} = \frac{e_1}{2} + \frac{\delta_S}{2}\frac{\partial \lambda_{j2}}{\partial i_2} \quad (52)$$

CONSISTENCY

It is important to check for consistency. Are these rates positive and is $r_{Lj2} > r_{Tj2}$? The assumption is reasonable that banks always offer the rate of loans higher than the rate of time deposits to ensure that banks have sufficient income or returns to continue operating.

$$r_{Lj2} = \frac{[b_0 + (b_1 - b_2)\bar{r}_{L2} + v_{j2}]q - \lambda_{j2} b_1 (\delta_L - \delta_S) + b_1 r^e_{S2} + (1-q)b_1}{2qb_1}$$

$$r_{Tj2} = \frac{[r^e_{S2} + \lambda_{j2}\delta_S]d_1 - d_0 + (d_1 - d_2)\bar{r}_{T2} - \omega_{j2}}{2d_1}$$

First, the market rates are required.

If all banks are assumed to be equal except for their draws of the shock ω_{j2}, and $E(\omega_{j2}) = \bar{\omega}_{j2}$, the following occurs:

$$\bar{r}_{T2} = \frac{[r^e_{S2} + \lambda_{j2}\delta_S]d_1 - d_0 + (d_1 - d_2)\bar{r}_{T2} - \omega_{j2}}{2d_1}$$

$$\bar{r}_{Tj2} = \frac{[r^e_{S2} + \lambda_{j2}\delta_S]d_1 - d_0 - \bar{\omega}_{j2}}{d_1 + d_2}$$

$$\bar{r}_{T2} = \frac{[r^e_{S2} + \lambda_{j2}\delta_S]d_1}{d_1 + d_2} - \frac{d_0 + \bar{\omega}_{j2}}{d_1 + d_2}$$

$$\bar{r}_{T2} = \frac{d_1}{d_1 + d_2}[i_2] - \frac{d_0 + \bar{\omega}_{j2}}{d_1 + d_2}$$

where r^e_{S2} is the expected rate on securities.

The following realized rate is obtained:

$$r_{Tj2} = \frac{[r^e_{S2} + \lambda_{j2}\delta_S]d_1 - d_0 - \omega_{j2}}{2d_2} + \frac{1}{2}\frac{(d_1 - d_2)}{d_1 + d_2}\left[r^e_{S2} + \lambda_{j2}\delta_S - \frac{d_0 + \bar{\omega}_{j2}}{d_1 + d_2}\right]$$

$$r_{Tj2} = \left[\frac{d_1}{(d_1 + d_2)}\left[r^e_{S2} + \lambda_{j2}\delta_S - \frac{d_0}{d_1}\right] - \frac{1}{2d_1}\left[\omega_{j2} + \frac{d_0 + d_2}{d_1}\bar{\omega}_{j2}\right]\right]$$

The same procedure is performed for the loan rate:

$$\bar{r}_{L2} = \frac{b_0 + (b_1 - b_2)\bar{r}_{L2} + \bar{v}_{j2}}{2b_1} + \frac{r^e_{S2}}{2q} - \frac{\lambda_{j2} b_1 (\delta_L - \delta_S)}{2q} + \frac{(1-q)}{2q}$$

$$\bar{r}_{L2} = \frac{[b_0 + \bar{v}_{j2}]q - \lambda_{j2} b_1 (\delta_L - \delta_S) + b_1 r^e_{S2} + (1-q)b_1}{qb_1 + qb_2}$$

$$\bar{r}_{L2} = \frac{[b_0 + \bar{v}_{j2}]}{(b_1 + b_2)} - \frac{\lambda_{j2}}{q}\frac{b_1(\delta_L - \delta_S)}{(b_1 + b_2)} + \frac{b_1 r^e_{S1}}{(b_1 + b_2)} + \frac{(1-q)b_1}{(b_1 + b_2)}$$

Finally, we determine whether the realized interest rates chosen by the bank are both positive and whether (or under what conditions) the loan rate is bigger than the time deposit rate.

The following realized rate is obtained:

$$r_{Lj2} = \frac{b_1}{(b_1 + b_2)}\left[\frac{b_0}{b_1} - \frac{\lambda_{j2}}{q}(\delta_L - \delta_S) + r^e_{S2} + (1-q)\right] + \left[\frac{1}{2b_1}\left(v_{j2} + \frac{(b_1 - b_2)}{(b_1 + b_2)}\bar{v}_2\right)\right]$$

First note that if $(\delta_L > \delta_S)$ and $i_2 < r^e_{S2}$, then $\bar{r}_{L2} > 0$. But, on the other hand, $\bar{r}_{T2}$ will be positive if d_0 is small enough. Both rates appear to be positive without much problem. Now, is $r_{L2} > r_{T2}$?

$$\bar{r}_{L2} - \bar{r}_{T2} = \left[\frac{(b_1 + \bar{v}_2)}{(b_1 + b_2)} + \frac{d_0 + \bar{\omega}}{d_1 + d_2}\right] - \lambda_{j2}\left[\frac{(\delta_L - \delta_S)}{q}\frac{b_1}{(b_1 + b_2)} + \delta_S\frac{d_1}{d_1 + d_2}\right] + \left[\frac{b_1}{(b_1 + b_2)} - \frac{d_1}{d_1 + d_2}\right] r^e_{S2} + \left[\frac{b_1}{(b_1 + b_2)}(1-q)\right]$$

and this is the case.As long as $\left[\frac{b_1}{(b_1 + b_2)} - \frac{d_1}{(d_1 + d_2)}\right]$ is not too negative, the desired result, which is , should be obtained.

OPTIMAL BORROWING

The binding constraint and the balance sheet can be used to find the expression for optimal borrowing:

$$\frac{K_{j2}}{\mu} = R_{j2} + \delta_S S_{j2} + \delta_L L_{j2}$$

$$S_{j2} = (1-\alpha)D_{j2} + T_{j2} + K_{j2} + B_{j2} - L_{j2}$$

and the following equation is obtained

$$B_{j2} = \left(\frac{1}{\mu} - \delta_S\right)\frac{K_{j2}}{\delta_S} - (\alpha + (1-\alpha)\delta_S)\frac{D_{j2}}{\delta_S} - (\delta_L - \delta_S)\frac{L_{j2}}{\delta_S} - \delta_S\frac{T_{j2}}{\delta_S}$$

where T and L are functions of the optimal rates, which, in turn, are functions of the Lagrange Multiplier.It is useful to determine whether suitable values for the different variables (R,S,L,D,T,K,B) yield acceptable values for the missing parameters (δ_S, δ_L). In fact, some freedom exists since only the Basel constraint must be satisfied. The following table illustrates two combinations of deltas that are consistent with the Basel constraint for a given configuration of the balance sheet:[9]

IMPLIED SECURITIES

Given B, S can be constructed as:

$$S_{j2} = (1-\alpha)D_{j2} + T_{j2} + K_{j2} - L_{j2} + \left(\frac{1}{\mu} - \delta_S\right)\frac{K_{j2}}{\delta_S} - (\alpha + (1-\alpha)\delta_S)\frac{D_{j2}}{\delta_S} - (\delta_L - \delta_S)\frac{L_{j2}}{\delta_S} - \delta_S\frac{T_{j2}}{\delta_S}$$

$$S_{j2} = \left[\frac{1}{\mu}\frac{1}{\delta_S}\right]K_{j2} - \left[\frac{\alpha}{\delta_S}\right]D_{j2} - \left[\frac{\delta_L}{\delta_S}\right]L_{j2}$$

and the size of S is determined by the size of loans since the first 3 terms are positive. This quantity will be positive because the combinations of the (b,c,d) parameters are such that the balance sheet configurations can be generated similar to those in Table 1. In such a case, S will take that value by consistency.

THE VALUE FUNCTION OF SECOND PERIOD

The value function for period 2, in which B and the optimal rates are linear functions of the policy rate, is written as follows:

$$V_{j2}(K_{j2}, Z_{j2}) = r_{Lj2}qL_{j2} + r^e_{S2}S_{j2} - r_{Dj2}D_{j2} - r_{Tj2}T_{j2} - i_2B_{j2} - (1-q)L_{j2} \tag{53}$$

where $Z_{j2} = \{\omega_{j2}, v_{j2}, i_2, \varepsilon_{j2}\}$ contains the realizations of shocks to time deposits, loan demand, interbank rate and demand deposits. Securities must be replaced with the balance sheet identity, which obtains:

$$V_{j2}(K_{j2}, Z_{j2}) = (qr_{Lj2} - r^e_{S2})L_{j2} + r^e_{S2}K_{j2} + (r^e_{S2} - i_2)B_{j2} + r^e_{S2}[(1-\alpha)D_{j2} + T_{j2}] - r_{D2}D_{j2} - r_{Tj2}T_{j2} - (1-q)L_{j2} \tag{54}$$

where B_{j2} is a linear function of K_{j2}.

More important, the value function of period 2 can be decomposed into two components by separating the components that depend on K_{j2} from those that do not. The following equation can be written since only B_{j2} depends on K_{j2}:

$$V_{j2}(K_{j2}, Z_{j2}) = K_{j2}\left[r^e_{S2} + (r^e_{S2} - i_2)\left(\frac{1}{\mu} - \delta_S\right)\frac{1}{\delta_S}\right](r^e_{S2} - i_2)\left[(\alpha + (1-\alpha)\delta_S)\frac{D_{j2}}{\delta_S} + (\delta_L - \delta_S)\frac{L_{j2}}{\delta_S} + \delta_S\frac{T_{j2}}{\delta_S}\right] + (qr_{Lj2} - r^e_{S2})L_{j2} + r^e_{S2}[(1-\alpha)D_{j2} + T_{j2}] - r_{D2}D_{j2} - r_{Tj2}T_{j2} - (1-q)L_{j2} \tag{55}$$

$$V_{j2}(K_{j2}, Z_{j2}) = \Gamma_1 K_{j2} + \Gamma_2 \tag{56}$$

where,

$$\Gamma_1 = \left[r^e_{S2} + (r^e_{S2} - i_2)\left(\frac{1}{\mu} - \delta_S\right)\frac{1}{\delta_S}\right]$$

The value function is useful for the next section.
Profit Maximization in Period 1, Given $K_{j1} > 0$
The profit function of period one is given by:

$$\pi_{j1} = (qr_{Lj1} - r_{S1} - (1-q))L_{j1} + r_{S1}K_{j1} + (r_{S1} - i_1)B_{j1} + r_{S1}[(1-\alpha)D_{j1} + T_{j1}] - rD_1D_{j1} - r_{Tj1}T_{j1} \quad (57)$$

and subject to constraint:

$$K_{j1} \geq (R_{j1} + \delta_S S_{j1} + \delta_L L_{j1}) \quad (58)$$

Now the balance sheet equality is used to eliminate securities:

$$S_{j1} = (1-\alpha)D_{j1} + T_{j1} + K_{j1} + B_{j1} - L_{j1}$$

The constraint in (3.58) becomes:

$$K_{j1} = \left(\mu - \frac{1}{\delta_S}\right)[(\alpha + (1-\alpha)\delta_S)\, D_{j1} + (\delta_L - \delta_S)L_{j1} + \delta_S T_{j1} + \delta_S B_{j1}] \quad (59)$$

The objective function for the 2-period problem defined at the start of period one is then given by the Bellman equation:

$$V_{j1}(K_{j1}, Z_{j1}) = \max_{r_{Lj1},\, r_{Tj1},\, B_{j1}} \pi_{j1} + \beta V_{j2}(K_{j2}, Z_{j2}) \quad (60)$$

where the realized π_{j2} is equal to K_{j2}.

This allows the Value Function in period oneto be rewritten as

$$V_{j1}(K_{j1}, Z_{j1}) = \max_{r_{Lj1},\, r_{Tj1},\, B_{j1}} \pi_{j1} + \beta(\Gamma_1 \pi_{j2} + \Gamma_2) \quad (61)$$

The next step is to maximize this objective function with respect to r_{Tj1}, r_{Lj1}, B_{j1}. Then, take a derivative of the entire right hand side (RHS) of Bellman operator, where the solution maximization of the value function of period 1 is given as below:

$$\frac{\partial RHS}{\partial r_{Lj1}}, \frac{\partial RHS}{\partial r_{Tj1}}, \frac{\partial RHS}{\partial B_{j1}}$$

The differentiation reduces to

$$\frac{\partial \pi_{j1}}{\partial r_{Lj1}}(1 + \beta\Gamma_1) = 0$$
$$\frac{\partial \pi_{j1}}{\partial r_{Tj1}}(1 + \beta\Gamma_1) = 0$$
$$\frac{\partial \pi_{j1}}{\partial B_{j1}}(1 + \beta\Gamma_1) = 0$$

So, the following derivatives are shown:

$$\frac{\partial \pi_{j1}}{\partial r_{Lj1}}\left[\begin{array}{c} b_0 q + (b_1 - b_2)\bar{r}_{L1} q - 2b_1 r_{Lj1} q \\ + v_{j1} q + b_1 r_{S1} + (1-q)b_1 \\ -\left(\mu - \frac{1}{\delta_S}\right) b_1 r_{S1}(\delta_L - \delta_S) \end{array}\right][1 + \beta\Gamma_1] = 0 \quad (62)$$

$$\frac{\partial \pi_{j1}}{\partial r_{Tj1}}\left[\begin{array}{c} d_1 r_{S1} + (d_1 - d_2)\bar{r}_{T1} - d_0 \\ - 2d_1 r_{Tj1} - \omega_{j1} + r_{S1}\left(\mu - \frac{1}{\delta_S}\right) d_1 \delta_S \end{array}\right][1 + \beta\Gamma_1] = 0 \quad (63)$$

$$\frac{\partial \pi_{j1}}{\partial B_{j1}}\left[(r_{S1} - i_1) + r_{S1}\left(\mu - \frac{1}{\delta_S}\right)\right][1 + \beta\Gamma_1] = 0 \quad (64)$$

The derivative of r_{Tj1}, r_{Lj1}, B_{j1} during the first period is realized with the addition of the expected value of the second period, which depends upon the first period. In this case, β is a discount factor that can be formulated as $\frac{1}{i+\rho}$, in which ρ is a premium rate and i is an interbank rate.

OPTIMAL INTEREST RATES IN PERIOD 1

$$r^*_{Lj1} = \frac{\left[\begin{array}{c} b_0 q + (b_1 - b_2)\bar{r}_{L1} q + v_{j1} q + b_1 r_{S1} + (1-q)b_1 \\ -\left(\mu - \frac{1}{\delta_S}\right) b_1 r_{S1}(\delta_L - \delta_S) \end{array}\right]}{2b_1 q} \quad (65)$$

$$r^*_{Tj1} = \frac{\left[(d_1 - d_2)\bar{r}_{T1} - d_0 - \omega_{j1} + d_1 r_{S1} + r_{S1}\left(\mu - \frac{1}{\delta_S}\right) d_1 \delta_S\right]}{2d_1} \quad (66)$$

and note that

$$\frac{\partial r^*_{Lj1}}{\partial i_1} = \frac{e_1}{2q} - e_1\left(\mu - \frac{1}{\delta_S}\right)\left(\frac{(\delta_L - \delta_S)}{2q}\right) = 0 \quad (67)$$

$$\frac{\partial r^*_{Tj1}}{\partial i_1} = \frac{e_1}{2} + e_1\left(\mu - \frac{1}{\delta_S}\right)\left(\frac{\delta_S}{2}\right) = 0 \quad (68)$$