

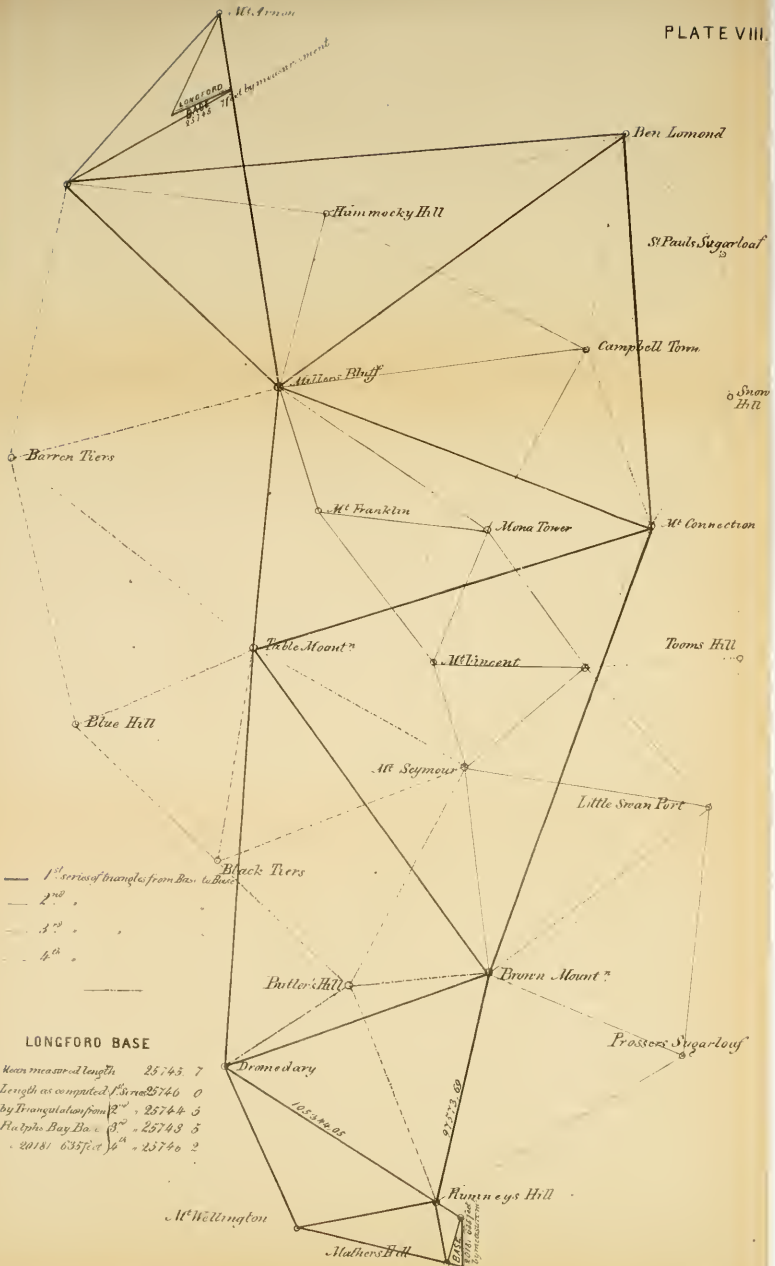
XI.—*On the Trigonometrical Survey of Van Diemen's Land.* By MAJOR COTTON, H. E. I. C. S.; Deputy Surveyor-General. [*Read 10th May, 1854.*]

ON the 21st April, 1852, I made a report on the measurements of the two Base Lines, and the result of the connecting Triangulation, of which the following is a copy:—

“ Measurement of the two Base Lines, and result of the connecting Triangulation.

21st April, 1852.

The Rods used for the measurement of the two Base lines were of old Baltic fir, about fifteen feet in length and two inches square: they were saturated with boiling oil, and varnished, rolled in flannel and packed in sawdust, in coffers six inches square, closed at the ends, but leaving the rods free to contract and expand. The rods were supported centrally in the coffers by means of blocks of wood; the coffers aided by these blocks serving to truss the rods. To the ends of the rods were attached Brass Caps, rising to the level of the surface of the coffer, and bearing on their upper surface the scales, by means of which their lengths were determined. One cap bore a zero mark only, and the other a vernier scale 19-20ths of an inch divided into twenty parts. The only standard then in the colony was a four-foot steel standard, divided into inches and fortieths, and the vernier scales were made to accord with these divisions, determining the measurements to 1-400th of an inch. Besides the vernier scale attached to the cap of the rod, three similar scales were laid on the surface of the coffer at intervals, and their several distances



and the total length of the rod measured by the four-foot standard. Each rod when in use was supported on two tressels fitted with screw lifts, affording the means of raising and depressing them into their position.

The rods were made in damp weather, and were used during damp weather in the first measurements of the Base at Ralph's Bay in 1849. They were measured as soon as completed, and from time to time during the operation, and the variation in length was scarcely appreciable. Since this measurement was made, a ten-foot steel bar has been received from England, by which the four-foot standard has been tested and found to be very true, so that the measurement requires no reduction on that account, and any difference found to exist between it and subsequent re-measurements must be imputed to the apparatus used and to carefulness of operation. The Base was marked out with pickets, placed in line by means of a transit instrument, and divided into convenient gradients or hypotenuses, permanent marks, over which the altitude and azimuth instruments could be placed, being established at each extremity.

The rods were then placed on their stands, and arranged into the vertical plane of the Base line, and at the inclination of the first hypotenuse, the first rod within a few inches of the terminus, and two others in succession at similar intervals, the zero mark on one rod being antagonist to the vernier scale of the next. The intervals between the rods were measured by means of a small scale engraved for the purpose, which with the vernier indicated inches, twentieths, and four-hundredths of inches. The inclination of each hypotenuse was ascertained by levelling, the rise and fall being entered in the field book in feet and decimals. The horizontal value of each hypotenuse was then obtained, and the requisite reductions to the level of the sea computed.

In the summing up of the measurements much labour was occasioned by the divisions of the scale being in inches and four-hundredths, instead of decimals; but at the time I had no good dividing apparatus, and considered it necessary rather to depend on the divisions of the four-foot standard, (which I had the means of copying on to the scale used for measuring the intervals), the vernier scale alone being divided by such imperfect means as I could obtain. I pursued a better system in the measurement of this Base and the measurement of the base of verification on Norfolk Plains.

In 1851 the Base at Ralph's Bay was measured. The same rods were used and the same means of determining their lengths, but I had obtained a good dividing apparatus, and I engraved scales, the divisions of which were decimal parts of a foot. These with the verniers read to four places of decimals, the fourth decimal ambiguous, so that the reading was true to the 5000th part of a foot.

The steel bar, which with other instruments for the trigonometrical operations had been obtained from England by the Lieutenant-Governor, through the Astronomer-Royal, is one of those which were employed as Standards in the measurement of the Base line in Ireland, and the Base line here will therefore be referred to the same standard of measurement.

The only other improvement introduced was that of attaching a telescope and sights to the rods for keeping them more accurately to the line, and more attention to the piling under the feet of the tressels, where the ground was inclined to shake from the tread of the men in laying the rods.

The Field Book of the second measurement was much more simple, and the labour of computation much reduced,

from the adoption of the decimal scale. The reductions to the horizontal and to the level of the sea were made as before. A third measurement was undertaken immediately on the completion of this with the same means and the same careful attention.

The three several measurements gave the following results :—

Measurement of 1849.....20182·484496 feet.

First measurement of 1851.....20181·692922 ,,

Second measurement of 1851....20181·577215 ,,

The difference between the measurement in 1849 and the mean of those in 1851 is ·85 foot, or 10½ inches; that between the two in 1851 only ·115 feet, or 1⅓ inch. I consider this latter accordance to prove that the slight variations in the length of the rods were truly valued, and the apparatus in all respects sufficiently true, and that Mr. Sprent, who conducted the whole operation, has obtained the nearest results obtainable by its means.

It also seems satisfactorily to show that the first measurement may be rejected, though (considering the comparative inferiority of means) not very greatly differing from what will be adopted as the actual length of the Base, viz., the mean of the two last measurements.

The Base of verification on Norfolk Plains has since been measured twice by the same apparatus, and to the same standard, by Mr. Sprent; the results of which are as follows :—

First measurement, reduced to the

level of the lowest point.....25746·019443

Second ditto ditto25746·304833

Difference..... 285,390 = 3½ in.

The difference of 3½ inches in nearly five miles is almost

as good an accordance as resulted from the two last measurements of the Base at Ralph's Bay.

I must mention one other circumstance connected with these operations, and which at first led me to doubt the safety of dependence upon the deal rods.

The measurement of their length by the four-foot standard during the operations, especially at Norfolk Plains, indicated a small amount of contraction and expansion not to be expected, and in no way to be accounted for; but it appeared that these measurements were made in the extreme heat of mid-day, and arose from the steel standard being slow in following the changes of temperature indicated by a detached thermometer. The temperature of the metal, in fact, was not ascertained at a high temperature of the atmosphere; and as the rods embedded in their coffers, and screened from the sun, could undergo no such sudden changes, I rejected the mid-day measurement of their length, and adopted those taken early, when no great allowance was required to be made for the effect of temperature on the standard. The Base at Ralph's Bay being measured in cooler weather, this difficulty did not arise.

For the angular observations from the main triangles, an Altitude and Azimuth instrument had been obtained from England, with a repeating table of excellent finish, both very portable, and at the same time efficient.

The horizontal arc of the instrument is twelve inches in diameter, graduated to 10."

Many repetitions were made of every angle with reverse observations, and every possible attention to ensure an accurate mean; and the result has been most satisfactory.

The greatest error in the sum of the angles of one triangle was 3.3 seconds, and this was in a triangle of nearly forty miles sides.

The Bases are situated nearly 100 miles asunder. The computed length of the Longford Base, taken from the measured length of that at Ralph's Bay, and carried up through thirteen triangles, compared with its measured length, gives the following results, viz.:—

Mean of two measurements.....25,746·2 feet

Reduction to level of sea5

Reduced measured length25,745·7

Computed length.....25,746·0

Differencefeet .3 or about $3\frac{1}{2}$

inches. The instruments are decidedly good of their kind, but not of course possessing the perfection of construction or minuteness of division of those used in the great surveys of Europe, India, and America. The observations have been entirely in the hands of one individual, Mr. J. Sprent, whose scientific knowledge, together with untiring perseverance and patient endurance, has enabled him, single-handed, to effect what would in other countries have been shared by many equally qualified for the work. But the result is such as he will, I am sure, from the interest he takes in this work of science, feel no small recompense for his efforts.

The actual distance from hill to hill, extending for nearly fifty miles in some of the triangles, and the total distance from Base to Base being determined to a very minute degree of accuracy, a foundation of the framework of the whole map of the island is established, and the future operations will be based upon it with security and confidence.

“ (Signed) “ H. C. COTTON.”

Since the date of the above Report I have made further computations for the verification of the work, comparing

the length of the Longford Base as computed by other triangles with its measured length, (the measured length of the four mile base at Ralph's Bay being the given side of the first triangle in each case), and the results are as follows :—

Measured Length of Longford Base reduced to the Level of the Sea.	Feet. 25,745·7	Difference in Feet.
Length as computed by 1st series of triangles	25,746·0	+ ·3
Do. do. 4th series	25,746·2	+ ·5
Do. do. 2nd series	25,744·5	— 1·2
Do. do. 3rd series	25,743·5	— 2·2
1st Series computed by Mr. Sprent	25,746·01	+ ·29
1st Series varied in one triangle computed by Mr. Sprent.....	25,745·35	— ·35

The accompanying diagram exhibits the character of the triangles comprising each series; and it will be observed that the series No. 1, composed of the largest and best triangles, give a result the most nearly in accordance with the measurement. The series No. 4 is that from which the next best result was to be expected from the character of the triangles, and it accords most nearly with the first.

The results of the other two series are very satisfactory, and give abundant proof that both the measurements and the observations are good. With the exception of those at Mona Tower, all the angles used in these computations are taken from the centre of the stations. The angles there were observed out of the centre and reduced.

In the triangulation to which I have adverted, and in about three hundred other calculated triangles, the nearness with which the sum of the triangles corresponds with the sum of 180°, and the spherical excess, gives the greatest proof of

the accuracy of the angular observations, the error rarely amounting to more than four or five seconds, and generally not exceeding two seconds. The observations have been all made by Mr. Sprent, and with the same twelve-inch instrument.

By means of the repeating table each angle is observed under a series of repetitions, and the mean obtained is exceedingly near the truth.

At every principal station Angles of Elevation and Depression for determining relative altitudes have been observed, and at several stations astronomical observations made for the determination of the true meridian.

The observations taken for this purpose are extreme Elongations of circumpolar stars;—both the east and west elongation of one or more stars, with single elongations east or west of others;—observing their azimuthal angle from another station, or from a fixed lamp whose position with reference to some other station is known.

The observations have been made with the principal instrument at sixty-eight main stations, and with an eight-inch theodolite at sixty-five secondary stations, those at the latter being confined to the horizontal angles. Mr. Sprent is continuing the observations for the main triangulation, during the summer, in the north-west portion of the island, and preparations are made for those in the south-west next summer. In the meantime, I am in the expectation of being able shortly to appoint two other parties for carrying on the secondary and minor triangulation; but the extreme and urgent demand at present for surveys of small blocks of land, and the want of strength in the establishment, delays this work.

Besides the calculations connected with the two Base lines, and the four series of triangles between them, to which I have already adverted, about three hundred triangles have

been calculated, and the latitudes at several main stations, both from the local observations and by the triangulation commencing at Hobart Town, have been computed with the true bearings of lines meeting at the same stations.

With a view to show more particularly the nature of the field operations, and of the computations and the results, I append to this Report extracts from the Field Books and examples of the method of calculation adopted. I have given also a comparison of the results of the calculations for latitude and true bearings, as derived from the local observations at each station, and as derived from the triangulation, commencing with the latitude and meridian of Church Street, Hobart Town, where Mr. Sprent took a series of observations, and calculated his latitude with great care, as will be seen by his Report attached. For the purpose of forming an accurate map of the island, and fixing its geographical position, the operations are proved to be exceedingly true, and I believe the results equal any operation of the kind in any part of the world; it being remembered that they are not intended for the measurement of an element in the dimensions or figure of the earth.

The Bases, measured nearly one hundred miles asunder, verify the operations with exceeding perfectness; and though it would be satisfactory to measure two other base lines on the east and west coast, it cannot be considered necessary, as by multiplying the calculations of the length of every line by various chains of triangles branching from those directly verified, any accidental error must be discovered, and a very accurate mean taken, so that when the main triangulation is completed, the main stations will be most accurately fixed in position, both relatively and geographically, and the inferior triangulation and filling in will be true in detail under its control. It will be perceived that the latitudes of

several of the main stations, calculated from entirely independent observations at those stations, differ in some measure from those derived by means of the triangulation from the observations at Hobart Town. Respecting this difference I must observe,—

1st. That the observations on the mountain cannot be considered so trustworthy as those taken in a secure observatory in town.

2nd. The probable deviation of gravitation from the true vertical direction may slightly throw out the observations of Elongations taken on the mountain.

3rd. The number of observations at each station is not sufficient to give so sure a mean as those taken for the same purpose at Hobart Town.

Under these circumstances, and considering the satisfactory proofs of accuracy of the triangulation, the latitudes, so far as they have been hitherto calculated by its means, from the initial latitude at Hobart Town, may be depended on in preference to those derived from the local observations. I must mention further, that while all the triangles have been calculated by two or more different computers, those for the latitudes and bearings have not yet gone through this proof, having been only done by myself, and by formulæ not giving the most minute results, but such merely as I consider commensurate with the degree of accuracy to be expected from the character of the observations.

H. C. COTTON.

1st January, 1854.

SECOND MEASUREMENT OF LONGFORD BASE.

Measurement of the Rods by the Four-foot Steel Standard.		Date.	No of Hypothe-nuse.	Difference of Level.		Rod.	Measurement of Interval. Feet.	Total of Hypothenuse. Feet.	Reduced Hori-zontal Measure-ment — Feet.
Thermo-meter.	Rods.			Measurement in Fortieths of Inches.	Measurement reduced to Feet.				
60	A	7255.2	15.1150			A ¹	.1588	175.3137 } Deduction .013642 } 175.300058	
60	-	7255.2	15.1150			B ¹	.3328		
61	B	7263.2	15.1316			C ¹	.3456		
61	-	7263.2	15.1316			A ²	.3790		
61	C	7250.3	15.1048			B ₂	.3390		
61	-	7250.3	15.1048			C	.3400		
						A ₃	.3808		
						B ₃	.3370		
						C ₃	.3540		
						A ⁴	.3752		
						B ⁴	.3238		
						C	.3442		
						Part of } C	5.0042		
						4 A	60.4592		
						4 B	60.5260		
						3 C	45.3141		

$$\frac{2.187^2}{175.3137 \times 2} = .0136 = \text{approximate deduction.}$$

$$\frac{2.187^2}{157.3137 \times 2} = .013642 = \text{correct deduction.}$$

When the difference of level is not greater than $\frac{1}{30}$ of the hypothenuse the approximate deduction is true to the 7th decimal.

The reduction made to correct the length given by the Steel Standard is .000007 feet per foot of length for each degree of temperature above or below the mean temperature of 62°.

$$4 A = 15.1148 \times 4 = 60.4592$$

$$4 B = 15.1315 \times 4 = 60.5260$$

$$3 C = 15.1047 \times 3 = 45.3141$$

FIRST MEASUREMENT OF LONGFORD BASE REDUCED TO THE LEVEL OF THE LOWEST POINT.

Numbers of Hypothenuses.	Nos. of Classes.	Sum of Hypothenuses of each Class reduced to Horizontal Feet.	Mean Altitude above Lowest Point. Feet.	Calculated Multiplier for Reduction, $\frac{r}{r+a}$	Each Class Reduced, Feet.	Total Length of Base reduced to Level of the Lowest Point. Feet.
From No. 12 to No. 20.....	...	5399·917769	Under 25 feet	No reduction required	5399·91776900	
From No. 1 to No. 11 and From No. 21 to No. 31.....	1	12501·603354	... 25 feet	99999880325	12501·58889270	
From No. 31 to No. 50.....	2	5414·243631	... 50 feet	99999760650	5414·24235510	
From No. 51 to No. 61.....	3	2430·283744	... 110 feet	99999473430	2430·27094685	25746·019443

Note.—If r = Earth's radius + elevation of lowest point above the sea.

a = elevation of each class above the lowest point.

b = sum of each class reduced to horizontal.

c = each class reduced to its value at the lowest point.

$e = \frac{r}{r+a}b$ and $\frac{r}{r+a}$ is the multiplier calculated for each class.

EXTRACT from Field Book of 8-inch Theodolite, as adopted
by Mr. Sprent at the Secondary Stations.

GORDON'S SUGAR LOAF.

Name of Station observed to.	Zero Point.	Reading of Arc, Three Verniers.	Mean of Reading.	Mean Angle.
	° ' "	° ' "	° ' "	° ' "
Rumney's Hill ...	359 58 50			
Brown Mountain...		67 59 0 58 30 60 0	67 59 10	68 0 20
		6 Repe- titions. 48 0 30 47 59 0 48 0 30	48 0 0	68 0 11
Rumney's Hill.....	359 58 50			
Grass Tree Hill ...		21 5 30 3 0 4 0	21 4 10	21 5 20
		9 Repe- titions. 189 46 30 49 0 46 30	189 47 20	21 5 23

Note.—At many of the Secondary Stations a connection by traverse survey with some natural feature or other permanent object in the neighbourhood is also recorded.

EXTRACT from Field Book of 12-inch Altitude and Azimuth Instrument, as adopted by Mr. Sprent at the Main Stations.

DROMEDARY STATION.

Names of Stations and Stars observed to.	No of Repetitions	OBSERVATIONS.						
		Horizontal Angles.						
		Reading, Degrees & Minutes.	Reading of Three Verniers Seconds		Mean Reading.		Angle.	
		° "	"		° ' "		° ' "	
Butler's Hill. Platform Peak.	...	47 57	75 65 25	47 57 55.0				
	1	290 46	10 30 25	290 46 21.6		117 11 33.4		
	7	307 36	40 40 45	307 36 41.6		117 11 34.2		
		Elongations.						
		Reading, Degrees & Minutes.	Reading of Three Verniers Seconds.		Mean Reading.	Reading of Levels each 2 Seconds.	Date.	
		° ' "	"		° ' "			
Brown Mt. η Argus E.		89 59	60 60 40	89 59 53.3		39 29	1851 June 5	
		155 56	90 50 70	155 57 10.0		35 32		
Brown Mt. α Crucis E. Brown Mt.		89 59	60 55 35	89 59 50				
		161 19	55 10 30	161 19 31.3		43 26		
		89 59	70 65 45	90 0 0.0		43 26		
		Elevations and Depressions.						
		Barometer.	Thermometer.		Reading Degrees and Minutes	Reading of Vernier Seconds	Mean of Verniers	Angle.
			Attd.	Detd.				
Rumney's Hill, Top of Stones.	27.016	46	46	88 46 74	40	57	1 13 3	
				1 12 36	24		30	1 12 30
				88 46 74	42		58	1 13 2
				0 12 38	24		31	1 12 31
				88 46 74	44		59	1 13 1
				1 12 38	24		31	1 12 31
						1 12 45.6		

EXTRACT FROM BOOK OF TRIANGLES.

No. of Triangle.	Angular Points.	No. of Repetitions.	Spherical Excess.	Observed Angle.	Corrected Angle.	Reduced from Spherical Excess.	Sine of Angle	Log of Sides.	Sides in Feet.
No. 11.	Mount Arnon. } Miller's Bluff. } Dry's Bluff. }	28	3.1	55 37 41.2	41.5	55 37 40.5	9.9166585	5.1042901	127142.31
		10		42 5 38.8	39.5	42 5 38.5	9.8263011	5.0139327	103260.14
		11		82 16 41.6	42.1	82 16 41.0	9.9960438	5.1836754	152642.46
				$\begin{array}{r} 1.6 \\ 3.1 \\ \hline \text{Error... } 1.5 \end{array}$	$\begin{array}{r} 3.1 \\ \hline 180 \quad 0 \quad 0.0 \end{array}$				

THE spherical excess has been calculated by multiplying the area in square miles by .0132, and the area is derived from a plotting of the triangles. One-third of the spherical excess is the correction applied to each angle after the error of observation is removed, in proportion to the weight of each angle.

EXTRACT from Book of Calculations of Latitudes and Bearings.

- E Observed greatest Elongation of a Star.
 E E Angle subtended by point of reference and greatest elongation east.
 W E Ditto ditto ditto west.
 A Elevation of a Star at its greatest Elongation.
 P D South Polar distance of star.
 L Latitude of place of observation.

DROMEDARY STATION.

<p>Observation. — a Argus = 179° 29' 16" 6 Brown Mountain = 121 35 53.3</p> <hr style="width: 50%; margin-left: 0;"/> <p>E E = 54 53 23.3</p> <p>Mean of W E = 166 19 47.9</p> <hr style="width: 50%; margin-left: 0;"/> <p>2) 111 26 24.6</p> <hr style="width: 50%; margin-left: 0;"/> <p>E = 55 43 12.3</p>	<p>a Argus west.... = 247° 10' 3" 3 Brown Mountain = 80 15 16.6</p> <hr style="width: 50%; margin-left: 0;"/> <p>W E... = 166 19 46.7</p> <hr style="width: 50%; margin-left: 0;"/> <p>a Argus W..... = 181 24 25.0 Brown Mountain = 15 4 35.8</p> <hr style="width: 50%; margin-left: 0;"/> <p>W E = 166 19 49.2</p>
<p>Cos. L. = $\frac{\text{Sin. P D}}{\text{Sin. E}}$</p>	<p>a Argus P D = 37 23 1.8 E = 55 43 12.3 L = 42 42 38.8</p>

Assuming this to be the Latitude of the Dromedary Station and Sin. A = $\frac{\text{Sin. L}}{\text{Cos. P D}}$ and Sin. E = $\frac{\text{Sin. P D}}{\text{Cos. L}}$, E is calculated from each observed elongation.

a Crucis P D = 27 43 32.2		
L = 42 42 38.8	Sin. = 9.8314204	Cos. = 9.8661616
PD = 27 43 32.2	Cos. = 9.9470343	Sin. = 9.6676750
Nat. Sin. A = .77	= 9.8843861	E = 39 17 1.1
		Sin. = 9.8015134

<p>Observation. — Brown Mountain..... 89 59 55.0 a Crucis..... 161 19 31.3</p> <hr style="width: 50%; margin-left: 0;"/> <p>71 19 26.3</p> <p>Correction for level..... + 13.1</p> <hr style="width: 50%; margin-left: 0;"/> <p>71 19 49.3</p> <p>E 39 17 1.1</p> <hr style="width: 50%; margin-left: 0;"/> <p>Bearing of Brown Mountain S 110 36 50.4 E</p>	<p>Levels. 43 26 43 26</p> <hr style="width: 50%; margin-left: 0;"/> <p>43 — 26 = 17 + .77</p> <hr style="width: 50%; margin-left: 0;"/> <p>119 119</p> <hr style="width: 50%; margin-left: 0;"/> <p>13.09</p>
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In like manner the bearing of Brown Mountain is calculated from each of the other observations at the Dromedary with the following results:—

Results of Eastern Elongations. Results of Western Elongations.

<i>a</i> Crucis ...	110° 36' 50.4"	
Ditto ...	110 36 46.7	
<i>η</i> Argus ...	110 36 54.0	
Ditto ...	110 36 50.8	
Ditto ...	110 36 54.0	
<i>α</i> Argus ...	110 36 36.4	110° 36' 33.1"
Ditto		110 36 32.2
<i>β</i> Centauri	110 36 11.9	
<i>α</i> Eridani	110 36 43.6	110 36 58.6
Means...	110 36 43.6	110 36 41.3
Mean of all.....	S. 110° 36' 42.4 E.	

At Brown Mountain the bearing of the Dromedary from three Eastern and four Western elongations is—

	S. 69° 6' 25.2" W.	
Convergence of Meridians	0 16 42.4	
	69 23 7.6	
	180 0 00	
	S. 110 36 52.4 E.	{ Bearing referred to the meridian of Dromedary. { Do. do. from observations at Dromedary.
	S. 110 36 42.4	
	10.0	Difference.

The convergence of the meridians of the different stations has been computed by the formula $36549.2 \cos. \text{Lat.} - 305.8 \cos. 3 \text{ Lat.} + 4 \cos. 5 \text{ Lat.} = \text{length of 1 degree of parallel at the latitude of the Station, (their difference giving the convergence in feet due to the difference of latitude), and}$

$$\frac{\text{Convergence in feet}}{\text{Difference of latitude in feet}} = \text{Tan. of angle of convergence}$$

or at once by the formula.

$$Z - z = 180 - \frac{w}{a} \times d. \sin. z \sec. l \sin. \frac{1}{2}(L + l) \sec. \frac{1}{2}(L - l)$$

where *Z* and *z* are the azimuths at the two extremities of the line from Station to Station.

L. 1. Their latitudes.

a Earth's radius in feet.

d Distance or length of line in feet.

w 206264.8 seconds.

The convergence of the meridians of Brown Mountain and Dromedary is computed as follows:—

Latitude of Dromedary ...	42° 42' 38.8"	from local observation.	
Lat. of Brown Mountain ...	42° 36' 7.7"	ditto	ditto
Length of line.....	117825 feet.		
Bearing	69° 6' 25".2		
Log. w =	206264.8 =	5.3144251	
Log. $\frac{1}{a}$		2.6801158	
Log. d =	117825	5.0712374	
Sin. z =	69 6 25.2	9.9704622	
Sec. l =	42 42 38 8	0.1338385	
Sin. $\frac{1}{2}(L+l)$	42 39 20.5	9.8309677	
Sec $\frac{1}{2}(L-l)$	0 3 16.7	0.0000002	

Log.	1002.4	= 3.0010469	
	= 0° 16' 42.4"	Angle of convergence.	

The above calculations of Azimuths and Latitudes wholly depending on the local observations, I proceed to give the calculations for the same, derived from the latitude of Hobart Town, through the triangulation. The following is a copy of Mr. Sprent's Report on the observations made by him at his Observatory in Church-street.

MR. SPRENT'S REPORT.

“ The following results are derived from observations upon σ Octantis, for ascertaining the latitude of my observatory. They are obtained from 108 elevations of that star taken with the Altitude and Azimuth circle. The error of the chronometer was ascertained by transits of the sun or stars on my meridian, taken as near the times of observation as convenient.

The observations were taken in groups of four, viz.: four with instrument direct or reading Altitudes, and four with the instrument inverted or reading Zenith distances. They have been calculated in sets of two direct and two

inverted ; the mean of these giving one of the results stated below. The whole form 27 sets of four Altitudes each.

o	i	"
42	52	26
,,	,,	28.4
,,	,,	24.9
,,	,,	26.8
,,	,,	27.9
,,	,,	26.2
,,	,,	26.5
,,	,,	27.1
,,	,,	25.3
,,	,,	27.2
,,	,,	23.8
,,	,,	25.6
,,	,,	26.3
,,	,,	25.6
,,	,,	24.7
,,	,,	24.3
,,	,,	24.7
,,	,,	26.5
,,	,,	26.1
,,	,,	24.1
,,	,,	23.6
,,	,,	23.1
,,	,,	24.5
,,	,,	27.9
,,	,,	25.3
,,	,,	26.1
42	52	25.66

In August 1851, I took several elongations of α Centauri, α Crucis, η Argus, for fixing my Meridian mark, and from these I find the latitude. They give results as follows,—

α Centauri	Lat	$\hat{4}2$	$\hat{5}2$	$\hat{2}8.1$	S
α Crucis.....		,,	,,	28.5	
η Argus		,,	,,	29.5	
Mean		42	52	28.7	S

It is to be observed that in the observations from which the above results were obtained, refraction forms no function in the elements for the calculation.

The elongations gave for my Meridian mark the following results,—

Instrument at zero on the mark, or	0 0 0
Reading of the instrument South	180 0 44.9
That is the mark is to the Eastward of } North.....	4."97 of arc

I took this month, (August 1853), 36 angles of η Argus with the Meridian mark, using the chronometer. They gave the following results,—

Instrument direct.	Instrument inverted.
12 Sets 180 0 8.3	180 0 4.2
12 „ 3.4	4.0
12 „ 4.98	5.9
36 Sets 180 0 5.56	180 0 4.7
„ „ 4.7	
Mean 180 0 5.13	

Agreeing very nearly with those derived from the greatest elongations.

By a series of small triangles connected with the main triangulation, I find the difference of latitude between my Observatory and the Magnetic Observatory — $30''.41$, hence $42^\circ 52' 25''.66 - 30''.41 = 42^\circ 51' 55''.25 =$ latitude of Magnetic Observatory, and the difference of latitude between my Observatory and the Hobart Town Semaphore is $+ 43.48$, hence $42^\circ 52' 25''.66 + 43.48 = 42^\circ 53' 9''.14$ latitude of Hobart Town Semaphore.

Bearing of Meridian mark from my Observatory...	N. $\overset{\circ}{0} \overset{'}{0} \overset{''}{5}$ E.
Angle of mark with Rumney's Hill.....	81 3 31.2
Bearing of Rumney's Hill from my Observatory...	N. 81 3 36.2 E.

Bearing of Meridian mark from my Observatory ...	N.	0	0	5	E.
Angle of Meridian mark with Wellington.....		108	5	55.6	
		N. 108 5 50.6			W.
Bearing of Wellington from my Observatory.....	S.	71	54	9.4	W.
		(Signed)			
30th August, 1853.		JAMES SPRENT.			

Taking then the latitude of Hobart Town to be $42^{\circ} 52' 25''.66$, and the bearing of Mount Wellington from the Observatory in Church-street to be $S. 71^{\circ} 54' 9''.4 W.$

The diff. of latitude $L-l = \left(\frac{w}{a} d \cos. z - \frac{w d^2}{3 a} \text{Sin.}^2 Z \text{Tan. } l. \right)$ Seconds

Here $d = 23486.7$ = distance by the triangulation.
 $z = 71 54 9.4$ = Azimuth.
 $l = 42 52 25.66$ = Latitude.
 $w = 206264.8$
 24856×5280
 $a = \frac{\quad}{6.28318}$ = Earth's radius in feet.

Log. w	= 206264.8	= 5.3144251	
Log. d	= 23486.7	= 4.3708232	
Log. $\frac{1}{a}$	= $\frac{6.28318}{24856 \times 5280}$	= 2.6801158	
Cos. z	= 71 54 9.4	= 9.4922478	"
		1.8576119	= 72.046 = 1st Term.
Log. w	= 5.3144251	
Log. d^2	= 2 Log. d = 8.7416464	
Log. $\frac{1}{a^2}$	= 2 Log. $\frac{1}{a}$ = 5.3602316	
Log. $\frac{1}{3}$	= 9.5228787	
Sin. $^2 z$	= 2 Log. Sin. z ...	= 9.9559316	
Tan. l	= 42 52 25.66 ..	= 9.9677377	
		8.8628511	= 0.073 = 2nd Term.
Difference of Latitude	= 71.973	= $0^{\circ} 1' 11''.97$

Latitude of Church-street	42	52	25.66	
Difference of Latitude.....		1	11.97	
		Latitude of Wellington		42 53 37.63
$\frac{1}{2}$ Sum of Latitudes.....	42	53	1.64	
$\frac{1}{2}$,, Difference ditto			35.98	

$\frac{w}{a}$ Constant Log ...	7.9945409
$d = 23486.7 \dots = 4.3708232$	
Sin. $z = 71 \ 54 \ 9.4 = 9.9779658$	
Sec. $l = 42 \ 52 \ 25.66 = 0.1349824$	
Sin. $\frac{1}{2}$ Sum $42 \ 53 \ 1.64 = 9.8328368$	
Sec. $\frac{1}{2}$ Difference $35''98 = 00.00000$	
Log.....204''7	<u>$= 2.3111491$</u>

$= 3 \ 24.7 =$ Angle of convergence.

71 54 9.4 Bearing of Wellington from Church-street.

N. 71 57 34.1 E. Bearing of Church-street from Wellington.

97 51 41.6 Angle at Wellington; - Church-street,—Dromedary.

N. 25 54 7.5 W. = Bearing of Dromedary at Mount Wellington.

74244.02 Dist. by triangulation from Mt. Wellington to Dromedary.

$\frac{w}{a}$ Constant Log	$= 7.9945409$
Log. $d=74244.02$	$= 4.8706615$
Cos. $z=25 \ 54 \ 7.5$	$= 9.9540214$
	<u>$2.8192238 = 659.51$</u>

$\frac{w}{3 a^2}$ Constant Log	$= 0.1975354$
Log. $d^2 = 2 \text{ Log. } 74244.02 = 9.7413230$	
Sin. $^2 z = 2 \text{ Sin. } 25 \ 54 \ 7.5 = 9.2806338$	
Tan. $l = 42 \ 53 \ 37.63 = 9.9680415$	
	<u>$9.1875337 = .15$</u>
	<u>$659''36$</u>

Difference of Latitude..... $= 10 \ 59.36$

Latitude of Wellington ... $= 42 \ 53 \ 37.63$

Latitude of Dromedary ... $= 42 \ 42 \ 38.27$

$\frac{1}{2}$ Sum. of Latitudes $= 42 \ 48 \ 7.95$

$\frac{1}{2}$ Difference $= 5 \ 29.68$

$\frac{w}{a}$ Constant Log.	$= 7.9945409$
$d = 74244.02$	$= 4.8706615$
Sin. $z = 25 \ 54 \ 7.5$	$= 9.6403169$
Sec. $l = 42 \ 53 \ 37.63$	$= 0.1351231$
Sin. $\frac{1}{2}$ Sum. $= 42 \ 48 \ 7.95$	$= 9.8321701$
Sec. $\frac{1}{2}$ difference $= 0 \ 5 \ 29.68$	$= 0.0000006$
Log. 297.04	<u>$= 2.4728131$</u>

= Angle of convergence	= 0° 4' 57.04"
Bearing of Dromedary at Mount Wellington	} = 25 54 8.5
Bearing of Wellington at Dromedary = S.	25 49 11.46 E.
Angle;—Table Mt.,—Dromedary =	152 9 31.7
	177 58 43.16
	180

Bearing of Table Mountain at Dromedary N. 2 1 16.84 E.

Distance by triangulation from Dromedary to Table Mountain 173875 feet.

$\frac{w}{a}$ Constant Log	= 7.9945409	
Log. $d = 173875$	= 5.2402390	
Cos. $z = 2 1 16.84$	= 9.9997297	
	17 15.9	
	= 3.2345096	

Note,
2nd term of no value.

= Difference of Lat. =	28 35.9
Lat. of Dromedary =	42 42 38.27
Lat. of Table Mt. ...	42 14 2.37
$\frac{1}{2}$ Sum.	42 28 20.32
$\frac{1}{2}$ Diff.	14 17.95

$\frac{w}{a} d$ Log.	= 3.2347799	
Sin. $z = 2 1 16.84$	= 8.5474275	
Sec. $l = 42 42 38.27$	= 0.1338376	
Sin. $\frac{1}{2}$ Sum. 42 28 20.32	= 9.8294542	
Sec. $\frac{1}{2}$ Diff. 14 17.95	= 0.0000038	
	Log. 55'65	= 1.7455030

= Angle of Convergence	55'65
Bearing of Table Mountain at Dromedary.....	} 2 1 16.84
Bearing of Dromedary at Table Mountain.....	} 2 0 10.2
Angle { Dromedary,— Miller's Bluff }	} 177 6 11.8
Bearing of Miller's Bluff, at Table Mountain ...	} 175 6 0.6
	180 0 0.0

4 53 59.4

Distance by triangulation from Table Mountain to Miller's Bluff }
109893.82 feet. }

$\frac{w}{a}$ Constant Log.	= 7.9945409	
Log. $d = 109893.82$	= 5.0409733	
Cos. $z = 4 53 59.4$	= 9.998400	
Log. 1081.25	= 3.0339242	

$\frac{w}{3 a^2}$ Constant Log.	=	0.1975354
Log. $d^2 = 2$ Log. d	=	0.0819466
Sin. ² $z = 2$ Sin. 4 53 59.4	=	7.8630584
Tan. $l = 42$ 14 2.37	=	9.9580041
Log. .013	=	8.1005445
1081.25		

Difference Latitude ... 1081.24	=	0° 18' 1.24"
Latitude of Table Mountain ...	=	42 14 2.37
Latitude of Miller's Bluff	=	41 56 1.13
$\frac{1}{2}$ Sum	=	42 5 1.75
$\frac{1}{2}$ Diff	=	9 0.62
$\frac{w}{a} d$ Log.	=	3.0355142
Sin. $z = 4$ 53 59.4	=	8.9315292
Sec. $l = 42$ 14 2.37	=	0.1306452
Sin. $\frac{1}{2}$ Sum. 42 5 1.75	=	9.8262156
Sec. $\frac{1}{2}$ Diff. 0 9 0.62	=	0.0000015
Log. 83.9	=	1.9239057

z Angle of convergence	0° 1' 23.9"
Bearing of Miller's Bluff at Table Mountain	} N. 4 53 59.4 E.
Bearing of Table Mountain at Miller's Bluff.....	} S. 4 52 35.5 W.

Distance by Triangulation from Dromedary to Brown Mountain	
117825.21 feet.	
Bearing of Mount Wellington at Dromedary	S. 25 49 11.46 E.
Angle;—Wellington,—Brown Mountain	84 47 17.70
Bearing of Brown Mountain at Dromedary	110 36 29.16 = Z.

$\frac{w}{a}$ Constant Log.	=	7.9945409
Log. d	=	117825.21 = 5.0712383
Cos. z	=	110 36 29.16 = 9.5465107
Difference of Latitude	=	2.6122898 = 409.6 = 0° 6' 49.6"

Note.—2nd term of no value.

Difference of Latitude	0° 6' 49.6"
Latitude of Dromedary	42 42 38.27
Latitude of Brown Mountain ...	42 35 48.67
$\frac{1}{2}$ Sum	= 42 39 13.47
$\frac{1}{2}$ Differences	= 0 3 24.80

Log. $\frac{w}{a} d$	° ' "	= 3·0657791	
Sin. z	= 110 36 29·16	= 9·9712805	
Sec. l	= 42 42 38·27	= 0·1338375	
Sin. $\frac{1}{2}$ Sum.	= 42 39 13·47	= 9·8309517	
Sec. $\frac{1}{2}$ Diff.	= 0 3 24·8	= 0·0000002	
		= 3·0018490	= 1004·24 = 16 44·2 Angle of convergence.

Angle of convergence	=	0 16 44·2	
Bearing of Brown Mountain } at Dromedary.....	=	110 36 29·2	
		110 53 13·4	
		180 0 0·0	

Bearing of Dromedary at Brown } Mountain.....	S.	69 6 46·6	W.
Angle; — Dromedary,— } Table Mountain.....		72 23 46·6	
Bearing of Table Mountain } at Brown Mountain S.	141 30 33·2	W.
		180 0 0·0	
	Z. = N.	38 29 26·8	W.

Distance by Triangulation from Brown Mountain to Table Mountain }
d = 168374·26 }

Log. $\frac{w}{a}$ Constant	= 7·9945409	
Log. d	= 5·2262757	
Cos. z = 38 29 26·8	= 9·8936008	
	3·1144174	= 1301·4

Log. $\frac{w}{3 a^2}$ Constant	= 0·1975354	
Log. d^2 = 2 Log. d	= 0·4525514	
Sin. ² z = 2 Sin. 38 29 26·8	= 9·5881234	
Tan. l = 42 35 48·67	= 9·9635262	
	0·2017364	= 0·16

Difference of Latitude = 1301" 24	=	0° 21' 41"·24
Latitude of Brown Mountain		42 35 48·67
Latitude of Table Mountain		42 14 7·43

In like manner of computation the Brown Mountain at Table Mountain is S. 38° 13' 38"·6 E.

COMPARISON of Latitudes and Bearings obtained from Local Observations with those obtained by the Triangulation commencing at Hobart Town.

Stations.	Latitudes from Local Observations.			Latitudes from the Triangulation from Hobart Town.		
	°	'	"	°	'	"
Dromedary Station....	42	42	38·8	42	42	38·27
Brown Mountain.....	42	36	7·7	42	35	48·67
Table Mountain.	42	14	28·9	42	14	7·43
Miller's Bluff.	41	56	18·2	41	56	1·13

Lines.	Bearings from Local Observations.			Bearings by Triangulation from Hobart Town.		
	°	'	"	°	'	"
Dromedary,—Brown Mt.	110	36	40·6	110	36	29·2
Brown Mt.,—Table Mt.	38	29	48·2	38	29	26·8
Table Mt.,—Brown Mt.	38	13	46·8	38	13	38·6
Miller's Bluff,—Table Mt.	4	52	21·0	4	52	35·5

(Signed)

H. C. COTTON.