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Robust Adaptive Control for a DC Servomotor with wide Backlash Nonlinearity

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Original scientific paper

In this paper, the problem of driving angular position of a direct current servomotor system with unmodeled wide backlash nonlinearity is addressed. In order to tackle this problem, a control scheme based on an adaptive super twisting algorithm is proposed. In order to implement the proposed controller, information about angular velocity is estimated by means of a robust differentiator. Based on a simplified model of the system, the proposed scheme increases robustness against unmodeled dynamics as backlash, as not all the parameters of the system nor the bounds of the perturbations are required to be known. Experimental results considering a wide backlash angle near to 2π , illustrate the feasibility and performance of the proposed control methodology.

Key words: Robust control, Adaptive control, Servomotor, Backlash nonlinearity

Robusno adaptivno upravljanje istosmjernim servomotorom s nelinearnom širokom zračnosti. U ovom radu bavi se problemom kutnog pozicioniranja istosmjernog sevomotora s nemodeliranom nelinearnošću široke zračnosti. Za rješenje tog problema predlaže se korištenje upravljačke sheme bazirane na algoritmu adaptivnog uvijanja. Kako bi se implementiralo predloženo upravljanje, kutna brzina estimira se korištenjem robusnog diferencijatora. Bazirana na pojednostavljenom modelu sustava, predložena shema povećava robustnost u odnosu na nemodeliranu dinamiku kao što je zračnost. Pritom nije potrebno poznavanje svih parametara sustava niti očekivane granice smetnji. Eksperimetalni rezultati, koji uzimaju u obzir široki kut zračnosti od skoro 2π , ilustriraju izvodljivost i učinkovitost predloženog algoritma upravljanja.

Ključne riječi: robusno upravljanje, adaptivno upravljanje, servomotor, nelinearnost zračnosti

1 INTRODUCTION

Backlash can be defined as the play between adjacent movable parts, which is present in many mechanical systems, typically those with gears, e.g. the drive train in cars, rolling mills, printing presses and industrial robots. Backlash or backlash-like characteristic, is also common in control systems such as servomechanisms, electronic relay circuits and electromagnetic devices with hysteresis.

Control of systems with backlash nonlinearity is an important and challenging area of control system research. It imposes serious limits to performance, rendering inaccuracies in the position and velocity of a machine and undesirable delays and oscillations, which can even lead to instability.

With respect to backlash in servomechanisms, the motor losses contact with the load during a time instant. This may happen when a disturbance acts on the load, or when the motor applies a corrective action in the opposite sense regarding the load position. Furthermore, when backlash gaps open, the movement of the load is free. Therefore, the torque generated by the motor does not acts over the load.

In order to tackle backlash control problem, several approaches have been designed. For instance, in [1], a survey including classical PI, linear controllers, non-linear and adaptive control approaches is presented. In the aforementioned work, the backlash nonlinearity is not completely compensated, despite basic backlash models were incorporated in order to improve the performance. Moreover, the control design requires an important analytical effort.

In [2, 3], a feedback control is designed and compensated by means of backlash approximations given by neural networks and fuzzy logic systems. However, this approach demands intensive calculations.

A smooth inverse of backlash was developed in [4], to compensate the nonlinearity through a backstepping approach, where the derivation of the control input was used to get the controller. Nonetheless, this is not always possible.

In [5], an adaptive robust control for nonlinear system with unknown input backlash is presented. Nevertheless,

this methodology is based on a linear model of the backslash. Moreover, in [6], an adaptive PID controller is designed for a motor system with backlash, which eliminates the vibration caused by the backslash. However, this work is based on a backlash model and it depends on the system properties according to engineering experience.

Some works [7, 8] deal with nonlinear control under unknown dead zones and backlash problems. However, the effectiveness of the approach is demonstrated only by numerical simulation.

On the other hand, sliding mode control is used in many applications; in nonlinear plants, it enables high gain accuracy tracking and insensitivity to disturbances and plant parameter variations. The main drawback of sliding mode techniques is the chattering effect, which can damage the actuator due to the high frequency commutation. A sliding mode controller is the super-twisting control algorithm, this controller is designed to converge in a finite-time and ensures the robustness of the system under uncertainties. However, this controller needs to know the bounds of uncertainties and perturbations present on the system. Adaptive super twisting approach (see [9]) represents an alternative to cope with uncertainties as it is not necessary to know their bounds.

This paper addresses the angular position control of a DC servomotor system with wide backlash nonlinearity. With the aim of solving the control problem, an adaptive super twisting control approach is applied. Furthermore, in order to implement the proposed controller, necessary information about angular velocity is estimated through a sliding mode differentiator. Due to its robustness properties, the proposed control scheme is able enough to compensate parametric uncertainties and unmodeled dynamics as backlash. Furthermore, while in the literature, to the best of the authors knowledge, it has been considered a narrow backlash angle [1, 10]; in this work, a wide backlash angle near to 2π has been considered, i.e. a one tooth gear mechanism. Experimental results illustrate the performance of the proposed control scheme.

The layout of this paper is as follows: Section 2 deals with a system description. In section 3, an Adaptive Super Twisting Controller is derived with the aim of providing robustness under parametric uncertainties and unmodeled dynamics. Furthermore, in order to implement the proposed controller, angular speed is estimated by a Robust Differentiator presented in section 4. Experimental results given in section 5 illustrate the effectiveness of the proposed scheme. Finally, conclusions are drawn.

2 SYSTEM DESCRIPTION

In this section, a DC servo motor system moving an inertia load with a couple of gears with backlash is consid-

ered. A schematic view of this system can be seen in the Figure 1.

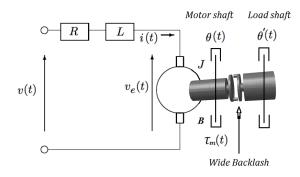


Fig. 1. Electromechanical system diagram.

Electrical and mechanical dynamics of a DC motor can be described by following equations

$$\frac{di}{dt} = \frac{1}{L} \left(-Ri + V - K_e \frac{d\theta}{dt} \right), \tag{1}$$

$$\frac{d\theta}{dt} = \frac{1}{J} \left(K_m i - B \frac{d\theta}{dt} \right), \qquad (2)$$

where *i* represent motor current and *V* the input voltage. θ denotes the angular position and $\dot{\theta}$ the speed of the rotor. K_e denotes the back electromechanical torque. *L* corresponds to motor armature inductance, *B* stands for viscous friction and *R* to the resistance of armature winding, while *J* represent the inertia moment of the moving parts. K_m describes the coefficient of electromechanical torque.

Based on physical considerations, we introduce the following assumption:

Assumption A1. Taking into account small size DC motors, a standard consideration is given by $L \ll R$. Then, the motor inductance can be neglected.

Thus, equations (1) and (2) can be written in a fin control form as follows

$$\dot{x}_1 = x_2
\dot{x}_2 = f(x)x_2 + g(x)u + \Delta(x, u, t),
f(x) = -\frac{BR + K_e K_m}{RJ} < 0 , \quad g(x) = \frac{K_m}{RJ} > 0$$
(3)

where $x = (x_1, x_2)^T$ correspond to $(\theta, \dot{\theta})^T$, respectively. *u* denotes the motor input voltage. Since backlash is considerated as an unmodeled dynamic, a backlash model is not addressed. Therefore, $\Delta(x, u, t)$ represent the unmodeled dynamics including external disturbance lumped together.

3 ADAPTIVE SUPER-TWISTING CONTROLLER

In this section, the synthesis of a control law based on Adaptive Super Twisting Approach (ASTA) to track a desired angular reference (θ_d) , is addressed. The main advantage of the proposed algorithm is that, it combines chattering reduction and the robustness of high order sliding mode approach. Moreover, the bounds of the disturbances are not required to be known.

Consider the following uncertain nonlinear system

$$\dot{x} = f(x,t) + g(x,t)u, \tag{4}$$

where $x \in \Re^n$ is the state, $u \in \Re$ the control input, $f(x,t) \in \Re^n$ is a continuous function.

Then, consider the following adaptive super twisting controller

$$u = -K_1 |s|^{1/2} \operatorname{sign}(s) + v,$$

$$\dot{v} = -K_2 \operatorname{sign}(s), \qquad (5)$$

where u represents the control signal, K_1, K_2 are the adaptive control gains and s is a sliding variable.

Adaptive super twisting algorithm increases the control gains K_1 and K_2 until the 2-sliding mode establishes. Then, the gains decreases.

With this aim, let us introduce a domain $|\sigma| \leq \iota$. Thus, as soon as this domain is reached, the gains K_1 and K_2 start dynamically reducing until the system trajectories leave the domain. Then, the gains start dynamically increasing in order to force the trajectories return to the domain in finite time. The adaptive gains are formulated into the following theorem [9].

Theorem 1 Consider the system (4) in closed-loop with the control (5), expressed in terms of the sliding variable dynamics (7). Then, for given initial conditions x(0) and s(0), there exists a finite time $t_F > 0$ and a parameter ι , as soon as the condition

$$K_1 > \frac{(\lambda + 4\epsilon_*)^2 + 4\delta_4^2 + 4\delta_4(\lambda - 4\epsilon_*^2)}{16\epsilon_*\lambda}$$

holds, if $|s(0)| > \iota$, so that a real 2-sliding mode, i.e. $|s| \le \eta_1$ and $|\dot{s}| \le \eta_2$, is established $\forall t \ge t_F$, under the action of Adaptive Super-Twisting Control Algorithm (5) with the adaptive gains

$$\dot{K}_1 = \begin{cases} C_{\alpha} \operatorname{sign}(|s| - \iota), & \text{if } K_1 > K_*, \\ K_*, & \text{if } K_1 \le K_*, \\ K_2 = 2\epsilon_* K_1, \end{cases}$$
(6)

where ϵ_*, λ are arbitrary positive constants, C_{α} is the adaptation velocity, K_* is a minimal value for adaptive gain (small) and $\eta_1 \geq \iota, \eta_2 > 0.$

In order to apply the aforementioned controller the following assumptions must be satisfied.

B1. The sliding variable $s = s(x,t) \in \Re$ is designed so that the desired compensated dynamics of the system (4) are achieved in the sliding mode s = s(x,t) = 0.

B2. The relative degree of the system (4) is equal to 1 with respect to the sliding variable s, and the internal dynamics of s must be stable.

Therefore, the dynamics of the sliding variable s is given by

$$\dot{s} = a(x,t) + b(x,t)u.$$
 (7)

where $a(x,t) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x}f(x,t)$ and $b(x,t) = \frac{\partial s}{\partial x}g(x)$.

B3. The function $b(x,t) \in \Re$ is unknown and different from zero $\forall x$ and $t \in [0,\infty)$. Furthermore, $b(x,t) = b_0(x,t) + \Delta b(x,t)$, where $b_0(x,t)$ is the nominal part of b(x,t) which is known, and there exists γ_1 an unknown positive constant such that $\Delta b(x,t)$ satisfies

$$\left|\frac{\Delta b(x,t)}{b_0(x,t)}\right| \le \gamma_1.$$

B4. There exist δ_1, δ_2 unknown positive constants such that the function a(x, t) and its derivative are bounded

$$|a(x,t)| \le \delta_1 |s|^{1/2}, \quad |\dot{a}(x,t)| \le \delta_2.$$
 (8)

The objective of ASTA approach is to design a continuous control without overestimating the gain, to drive the sliding variable s and its derivative \dot{s} to zero in finite time, under bounded additive and multiplicative disturbances with unknown bounds γ_1 , δ_1 and δ_2 .

Now, in order to implement the proposed controller, it is necessary to know the values of (x_1, x_2) . Then, to overcome this difficulty, and due to only angular position is available, the estimation of unmeasurable terms is addressed in next section.

4 SLIDING MODE ROBUST DIFFERENTIATOR

Since only load shaft position is available and in order to implement the proposed controller, angular velocity is estimated by means of a Robust Differentiator (see Fig. 2). With this aim, some basics are introduced in this section in order to build a differentiator for computing the real-time derivative of an output function with finite-time convergence.

Let $f(t) \in [0, \infty)$, consisting of a bounded Lebesguemeasurable noise with unknown features and $f_0(t)$ an unknown basic signal, whose k-th derivative has a known Lipschitz constant $\tilde{L} > 0$. Thus, the problem of finding real-time robust estimations of $f_0^{(i)}(t)$, for i = 0, ..., k; being exact in the absence of measurement noises, is known

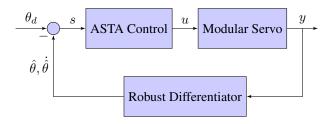


Fig. 2. Proposed control scheme.

to be solved by the robust exact differentiator (see [11] for more details.), which is given by

$$\dot{z}_{0} = -\lambda_{k} \tilde{L}^{\frac{1}{k}} |z_{0} - f(t)|^{\frac{k-1}{k}} sign (z_{0} - f(t)) + z_{1}$$

$$\vdots$$

$$\dot{z}_{j} = -\lambda_{k-j} \tilde{L}^{\frac{1}{k-j}} |z_{j} - z_{j-1}|^{\frac{k-j-1}{k-j}} sign (z_{j} - z_{j-1})$$

$$+ z_{j+1}$$

$$\dot{z}_{k-1} = -\lambda_{1} \tilde{L} sign (z_{k-1} - z_{k-2}), \qquad (9)$$

for j=0,...,k-2; where $z_0, z_1, ..., z_j$ are estimates of the *j*-th derivatives of f(t). In order to assure the initial differentiator convergence, one can take a voluntarily large constant parameter \tilde{L}_0 and switch it to the given variable value \tilde{L} after the convergence [12].

Then, according to (9), the homogeneous differentiator for (3) is given by

$$O: \begin{cases} \dot{z_0} = -\lambda_3 \tilde{L}^{\frac{1}{3}} |z_0 - \sigma|^{\frac{2}{3}} sign(z_0 - \sigma) + z_1 \\ \dot{z_1} = -\lambda_2 \tilde{L}^{\frac{1}{2}} |z_1 - \nu_0|^{\frac{1}{2}} sign(z_1 - \nu_0) + z_2 \\ \dot{z_2} = -\lambda_1 \tilde{L} sign(z_2 - \nu_1), \end{cases}$$
(10)

where σ is the output measurable, $\hat{e}(t) = x - z$ is the estimation error and $Z = (z_0, z_1)^T$, is the estimated state vector.

Consider the system (3) in closed-loop with the adaptive super-twisting controller (5), using the estimates obtained by the differentiator (9). Then, the trajectories of the system (3) converge in finite-time to the reference signal $\theta_d(t)$.

Remark 1 Since the observer converges in finite-time, the control law and the observer can be designed separately, *i.e.*, the separation principle is satisfied. Thus, if the controller is known to stabilize the process then the stabilization of the system in closed-loop is assured whenever the differentiator dynamics are chosen fast enough to provide an exact calculation of the estimations θ and $\dot{\theta}$.

5 EXPERIMENTAL RESULTS

In this section, experimental results carried out on the Modular Servo System (MSS) platform (see Fig. 3) are provided to illustrate the feasibility of the proposed methodology. The MSS experimental platform consist of

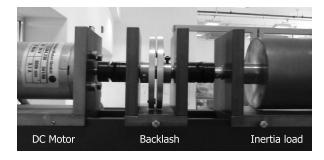


Fig. 3. Experimental setup.

a DC motor with several modules, arranged in a chain, mounted on a metal rail and coupled with small clutches. Modules as backlash and inertia load are attached to the chain (see Fig. 3). The measurement system is based on RTDAC/PCI acquisition board equipped with A/D converters. The angle of the load shaft is measured using an incremental encoder on load side and thus the system has no inner feedback for dead zone compensation. The accuracy of angle measurement is 0.1%. Angular velocity of the DC motor is measured through a tachogenerator. The control signal is normalized to ± 1 , corresponding to $\pm 24V$ (see [13] for further information).

On the other hand, controller and observer algorithms were developed in the MATLAB/Simulink environment, while the associated executable code was automatically generated by the RTW/RTWI rapid prototyping environment, with a sampling time of 0.4kHz using Euler solver. Parameters of the DC motor can be seen in the Table 1.

Parameter	Value	Unit
Rated voltage V	[-24, 24]	Volts
Rated current i	3.1	Ampere
Armature resistance R	2	Ohm
Rotor inertia J	63.41	oz-in ²
Torque back emf K_e	1	ms
Torque constant K_m	13	ms

Table 1. DC Servomotor parameters.

Additionally, controller and observer parameters are displayed in the Table 2. Furthermore, in order to provide a comparative study, a PID controller and the Super Twisting Algorithm (STA, see [11]) were also considered. The experiment, which consist in two cases, will be described in sequel.

C_{α}	λ	ι	ϵ_*	λ_1	λ_2	λ_3	Ĩ
0.022	0.3	0.1	0.01	10.5	10	0.02	100

Table 2. Gains of ASTA controller and differentiator.

5.1 Nominal case.

The control task consists in tracking a square signal of amplitude 40 rad and frequency of 0.1Hz without disturbances, *i.e.*, there is not backlash and thus the motor shaft and the load shaft have the same angular position.

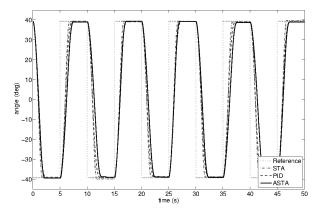


Fig. 4. Angular response (nominal case).

The angular response can be seen in the Figure 4. In this case, all the tested controllers have similar tracking performance.

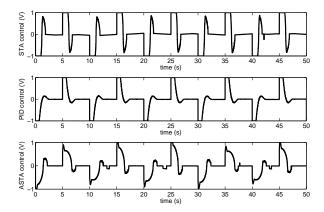


Fig. 5. Control signals (nominal case): (top) STA, (center) PID, (bottom) ASTA.

On the other hand, in the Figure 5 the corresponding control signals are shown, where it is possible to see no saturation in ASTA controller due to adaptation of gains. Moreover, in the Table 3, Mean Square Error (MSE), Integral Time Absolute Error (ITAE), Norm of the Error (||e||)

and Norm of the Control Signal (||u||) illustrate the performance of the controllers. As can be seen, ASTA controller requires less control effort.

Control	MSE	ITAE	$\ e\ _2$	$ u _{2}$
STA	1528.94	3.52×10^{6}	3029.05	40.96
PID	1526.60	3.51×10^{6}	3026.74	39.23
ASTA	1527.52	3.51×10^{6}	3027.65	30.31

Table 3. Controllers performance for nominal case.

Additionally, in the Figure 9-(top) adaptation of ASTA gain is presented, as can be seen gain values fluctuate due to time varying reference.

5.2 Wide backslash case

With the aim of testing the robustness of the proposed controller, a backlash module has been attached between the servo and the inertia load in the experimental platform; *i.e.*, the position of motor shaft is different to the load shaft. In this case, the backlash is near to 2π .

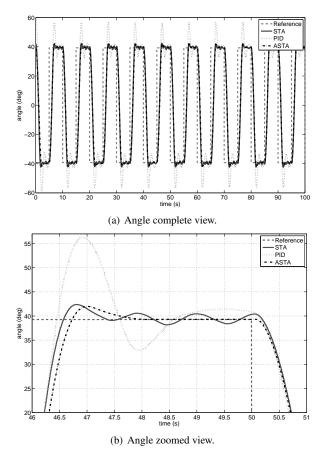


Fig. 6. Angular response (backlash case).

Angular profiles for the second test are shown in the Figure 6.a. Furthermore, Fig. 6.b shows details of the be-

havior, where it can be observed that the proposed controller shows a better response, while for PID and STA angular oscillations are present. Control signals are shown

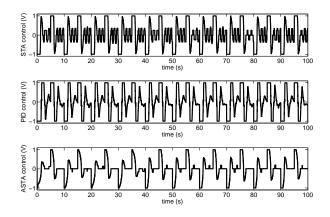


Fig. 7. Control signal (backlash case): (top) STA, (center) PID, (bottom) ASTA.

in the Figure 7, where ASTA presents a smoother behavior while satisfying the tracking of the desired angle. Furthermore, in the Table 4 the performance of the controllers according to several indexes for backlash case is illustrated. From the indexes it can be observed, the proposed controller held the best tracking performance and required less control effort among the tested controls.

Control	MSE	ITAE	$\ e\ _2$	$ u _{2}$
STA	991.54	1.58×10^{7}	4453.29	85.12
PID	985.90	1.78×10^{7}	4440.62	97.06
ASTA	958.94	1.54×10^{7}	4379.48	66.91

Table 4. Controllers performance for backlash case.

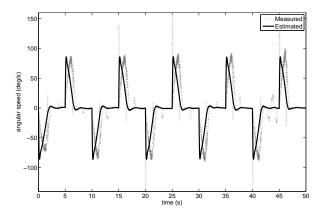


Fig. 8. Angular speed (backlash case).

A comparison between estimated angular speed on the load shaft and the rotor speed measured by the tachogenerator is illustrated in the Figure 8. The use of the robust differentiator is justified as it is not possible to get information about angular velocity on the load shaft. Due to

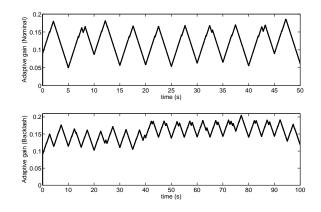


Fig. 9. Adaptive gains: (top) Nominal case, (bottom) backlash case.

the backlash presence, adaptive gain increase their magnitude with respect to the nominal case, as can be seen in the Figure 9.

6 CONCLUSIONS

In this paper, an adaptive super-twisting control for driving the angular position of a direct current servomotor system with backslash nonlinearity has been designed. With the aim of implementing the proposed controller, a robust differentiator was designed for estimating angular velocity. The proposed control scheme has been compared against a PID and the super twisting algorithm, demonstrating its advantages for dealing with hard nonlinear unmodeled dynamics as backlash. Furthermore, among the tested controllers, the proposed scheme required less control effort and held the best tracking performance. Experimental results demonstrated the robustness and efficiency of the proposed control methodology.

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REFERENCES

- M. Nordin, P. Gutman, "Controlling mechanical systems with backlash a survey," *Automatica*, vol. 38, no. 10, pp. 1633–1649, 2002.
- [2] J.O. Jang, M.K. Son, H.T. Chung, "Friction and output backlash compensation of systems using neural network and fuzzy logic," in *Proceedings of American Control Conference*, (Boston, USA), pp. 1758–1763, 2004.

- [3] C.Y. Su, M. Oya, H. Hong, "Stable adaptive fuzzy control of nonlinear systems preceded by unknown backlash-like hysteresis," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 1, pp. 1–8, 2003.
- [4] J. Zhou, C. Zhang, C. Wen, "Robust adaptive output control of uncertain nonlinear plants with unknown backlash nonlinearity," *IEEE Transactions on Automatic Control*, vol. 53, no. 3, pp. 503–509, 2007.
- [5] J. Guo, B. Yao, C. Qingwei, X. Wu, "High performance adaptive robust control for nonlinear system with unknown input backlash," in *Joint 48th IEEE Conference on Decision* and Control and 28th Chinese Control Conference, (Shanghai, P.R. China), pp. 16–18, December 2000.
- [6] J. Dong, B. Mo, "The adaptive pid controller design for motor control system with backlash," in 2013 Fourth International Conference on Intelligent Control and Information Processing (ICICIP), (Beijing, China), pp. 9–11, June 2013.
- [7] X. Zhang, Y. Lin, J. Mao, "A robust adaptive dynamic surface control for a class of nonlinear systems with unknown Prandtl–Ishilinskii hysteresis," *International Journal of Robust and Nonlinear Control*, vol. 21, pp. 1541–1561, 2011.
- [8] Y. Li, S. Tong, Y. Liu, T. Li, "Adaptive fuzzy robust output feedback control of nonlinear systems with unknown dead zones based on a small-gain approach," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 1, pp. 164–176, 2014.
- [9] y. Shtessel, M. Taleb, F. Plestan, "A novel adaptive-gain supertwisting sliding mode controller: methodology and application," *Automatica*, vol. 48, no. 5, pp. 759–769, 2012.
- [10] R. Merzouki, J.A. Davila, L. Fridman, J.C. Cadiou, "Backlash phenomenon observation and identification in electromechamical system," *Control Engineering Practice*, vol. 15, no. 4, pp. 447–457, 2007.
- [11] A. Levant, "High order sliding modes, differentiation and output feedback control," *International Journal of Control*, vol. 76, no. 9, pp. 924–941, 2003.
- [12] A. Levant, M. Livne, "Exact differentiation of signals with unbounded higher derivatives," *IEEE Transactions on Automatic Control*, vol. 57, no. 4, pp. 1076–1080, 2012.
- [13] Inteco, "Modular servo system user manual," tech. rep., 1st ed. Inteco Ltd, 2006.



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