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SIMPLE NUMERICAL METHOD FOR KINETICAL INVESTIGATION OF PLANAR MECHANICAL SYSTEMS WITH TWO DEGREES OF FREEDOM

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ABSTRACT

The aim of this article is to demonstrate the application of a simple numerical method which is suitable for motion analysis of different mechanical systems. For mechanical engineer students it is important task.

Mechanical systems consisting of rigid bodies are linked to each other by different constraints. Kinematical and kinetical analysis of them leads to integration of second order differential equations. In this way the kinematical functions of parts of mechanical systems can be determined. Degrees of freedom of the mechanical system increase as a result of built-in elastic parts. Numerical methods can be applied to solve such problems.

The simple numerical method will be demonstrated in MS Excel by author by the aid of two examples. MS Excel is a quite useful tool for mechanical engineers because easy to use it and details can be seen moreover failures can be noticed. Some parts of results obtained by using the numerical method were checked by analytical way.

The published method can be used in higher education for mechanical engineer students.

KEY WORDS

motion analysis, mechanical systems, numerical method, initial-value problems, nonlinear vibration

CLASSIFICATION

ACM: D 4. 8.

JEL: O35

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INTRODUCTION

The studying of motion analysis is important part in the education of mechanical engineers. Kinematic and dynamic analysis of mechanical systems is a fundamental chapter in motion analysis [1-4]. Multi DOF mechanical systems can be described by second order differential equations. Analytical solution of them in most cases is quite difficult or impossible.

In such cases the application of numerical methods can be advantageous. There are different numerical algorithms which are suitable for solving differential equations. The results obtained in this way can be plotted in different kinematical diagrams. This method can help engineer students better learning of school-work and connections among different physical quantities. In this article the results of kinetic analysis of two mechanical systems will be demonstrated.

PROPOSED SIMPLE NUMERICAL METHOD

In general cases motion equations of two degree-of-freedom mechanical systems as homogenous differential equation-system are

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{S}\mathbf{q} = 0 \tag{1}$$

where **M** is mass matrix, **S** spring stiffness matrix and **q** vector of generalized coordinates. Let us suppose that the mechanical system can be described by $\mathbf{q} = q(x, \varphi)$ generalized coordinates. Physical quantities $\dot{x}_o, x_o, \dot{\varphi}_o, \varphi_o$ describe the initial state of the system. Time step is $t_{i+1} - t_i$. Applied algorithms in MS Excel can be seen in Table 1.

Table 1. Applied algorithms for solving differential-equation system (Example 1).

t	$\ddot{x}(\dot{x}, x, \dot{\varphi}, \varphi)$	$\dot{x}$	x	$\ddot{\varphi}(\dot{x}, x, \dot{\varphi}, \varphi)$	$\dot{\varphi}$	φ
t_o	$\ddot{x}_o(\dot{x}_o, x_o, \dot{\varphi}_o, \varphi_o)$	$\dot{x}_o$	x_o	$\ddot{\varphi}_o(\dot{x}_o, x_o, \dot{\varphi}_o, \varphi_o)$	$\dot{\varphi}_o$	φ_o
t_1	$\ddot{x}_1(\dot{x}_1, x_1, \dot{\varphi}_1, \varphi_1)$	$\dot{x}_1 = \dot{x}_o + \ddot{x}_o(t_1 - t_o)$	$x_1 = x_o + \dot{x}_1(t_1 - t_o)$	$\ddot{\varphi}_1(\dot{x}_1, x_1, \dot{\varphi}_1, \varphi_1)$	$\dot{\varphi}_1 = \dot{\varphi}_o + \ddot{\varphi}_o(t_1 - t_o)$	$\varphi_1 = \varphi_o + \dot{\varphi}_1(t_1 - t_o)$
t_2	$\ddot{x}_2(\dot{x}_2, x_2, \dot{\varphi}_2, \varphi_2)$	$\dot{x}_2 = \dot{x}_1 + \ddot{x}_1(t_2 - t_1)$	$x_2 = x_1 + \dot{x}_2(t_2 - t_1)$	$\ddot{\varphi}_2(\dot{x}_2, x_2, \dot{\varphi}_2, \varphi_2)$	$\dot{\varphi}_2 = \dot{\varphi}_1 + \ddot{\varphi}_1(t_2 - t_1)$	$\varphi_2 = \varphi_1 + \dot{\varphi}_2(t_2 - t_1)$
t_3	….	….	….	….	….	….
t_4	….	….	….	….	….	….

EXAMPLE 1: CRANK DRIVE WITH OSCILLATING MASS (2 DEGREES OF FREEDOM)

In Figure 1 a sketch of a simple mechanical system (two degrees of freedom) can be seen. There is a mass *m* and a crank drive in between spring and viscous damping. Disc on the left side driven by moment *M* can rotate around horizontal axis denoted by *A*. Moment of inertia is marked by J_A.

There is a frame between disc and mass is driven by slider secured on the disc can move along horizontal axis *x*. Spring stiffness of linear spring and damping factor of viscous damping are denoted by *s* and *k*. Unloaded length of spring marked by l_o.

The motion of the crank drive mechanism can be described by the following second order differential equation system:

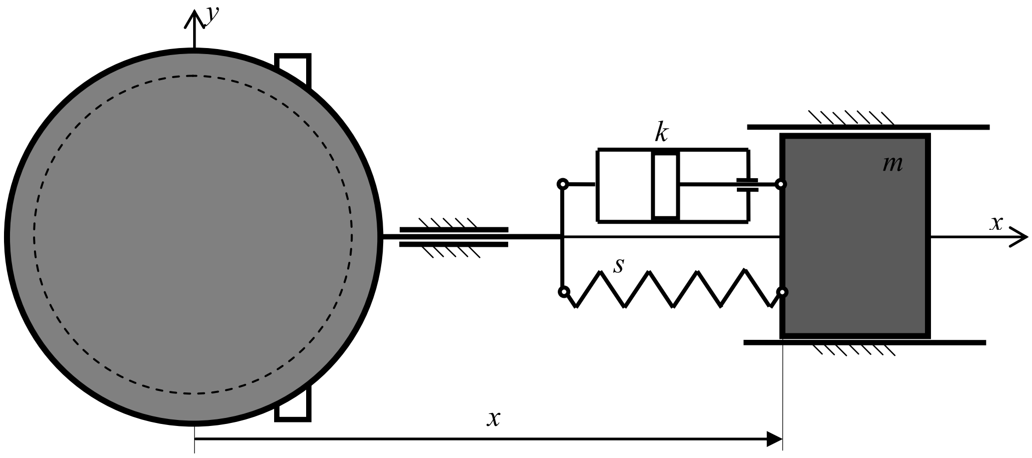


Figure 1. Sketch of crank drive mechanism and oscillating mass.

$$m\ddot{x}+k(\dot{x}+r\dot{\varphi}\sin\varphi)+s(x-l_o-r\cos\varphi)=0\,, \tag{2}$$

$$J\ddot{\varphi}-M+\left[k(\dot{x}+r\dot{\varphi}\sin\varphi)+s(x-l_o-r\cos\varphi)\right]r\sin\varphi=0\,. \tag{3}$$

Applying numerical integration method the kinematical functions of disc and mass can be plotted and studied in case of different physical properties of elements of moving structure.

During application of proposed simple numerical method the initial conditions of rotating disc and translating mass can be varied optionally. Further effects for example friction and external loads can be taken into consideration.

In first case the structure starts from rest position i.e. $\dot{x}_o=0$ m/s, $x_o=$ 0 m, $\dot{\varphi}_o=0$ rad/s , $\varphi_o=0$ rad. Data: M = 20 Nm, J_A = 4 kg m^2, m = 8 kg, r = <u>AB</u> = 0,2 m, l_o = 1,0 m, s = 2000 N/m, k = 200 Ns/m, (time step: 0,001 s, time interval: $0 \le t \le 5$ s).
After two-time numerical integration of motion equations the kinematical functions can be seen in Figure 2.

EXAMPLE 2: TWO MASSES SPRING IN BETWEEN MOVE IN CROSS DIRECTION

In Figure 4 a sketch a special mechanical system can be seen. There are two mass points marked by m_1 and m_2 moreover linear spring in between. Mass points can slide vertically and horizontally.

The mass of spring between mass points is neglected. As it can be seen in Figure 4 positions of mass points are determined by coordinates y_1 and x_2. Friction between mass points and surfaces is neglected.

Unloaded length of spring is marked by l_o, its actual length is

$$l=\left(\sqrt{y_1^{\,2}+x_2^{\,2}}\right), \tag{4}$$

the spring force,

$$R=s(l_o-l)=s\left(l_o-\sqrt{y_1^{\,2}+x_2^{\,2}}\right). \tag{5}$$

Algebraic sign of spring force of pressed spring is positive. Motion equations of mass points are:

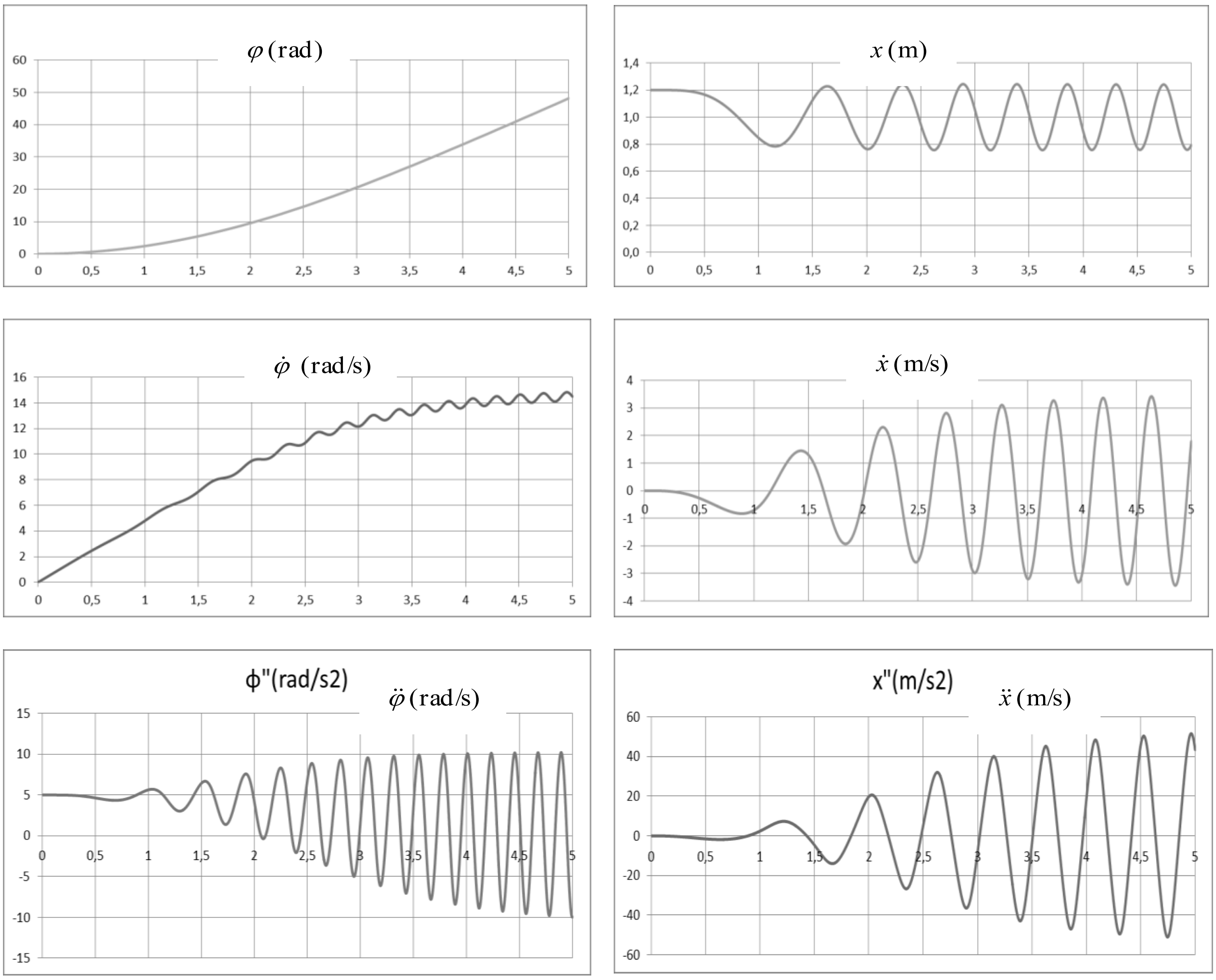


Figure 2. Kinematical diagrams of rotating disc and translating mass (example 1, first case).

$$-m_1 g + R\sin\varphi = m_1\ddot{y}_1\,, \tag{6}$$

$$R\cos\varphi = m_2\ddot{x}_2\,. \tag{7}$$

Taking into consideration that

$$\sin\varphi = \frac{y_1}{\sqrt{{y_1}^2 + {x_2}^2}} \qquad \text{and} \qquad \cos\varphi = \frac{x_2}{\sqrt{{y_1}^2 + {x_2}^2}}\,,$$

the form of motion equations yields the following:

$$-m_1 g + s\left(l_o - \sqrt{{y_1}^2 + {x_2}^2}\right)\frac{y_1}{\sqrt{{y_1}^2 + {x_2}^2}} = m_1\ddot{y}_1\,, \tag{9}$$

$$s\left(l_o - \sqrt{{y_1}^2 + {x_2}^2}\right)\frac{x_2}{\sqrt{{y_1}^2 + {x_2}^2}} = m_2\ddot{x}_2\,, \tag{10}$$

from which after rearrangement

$$\ddot{y}_1 = \frac{s y_1}{m_1}\left(\frac{l_o}{\sqrt{{y_1}^2 + {x_2}^2}} - 1\right) - g\,, \qquad \ddot{x}_2 = \frac{s x_2}{m_2}\left(\frac{l_o}{\sqrt{{y_1}^2 + {x_2}^2}} - 1\right).$$

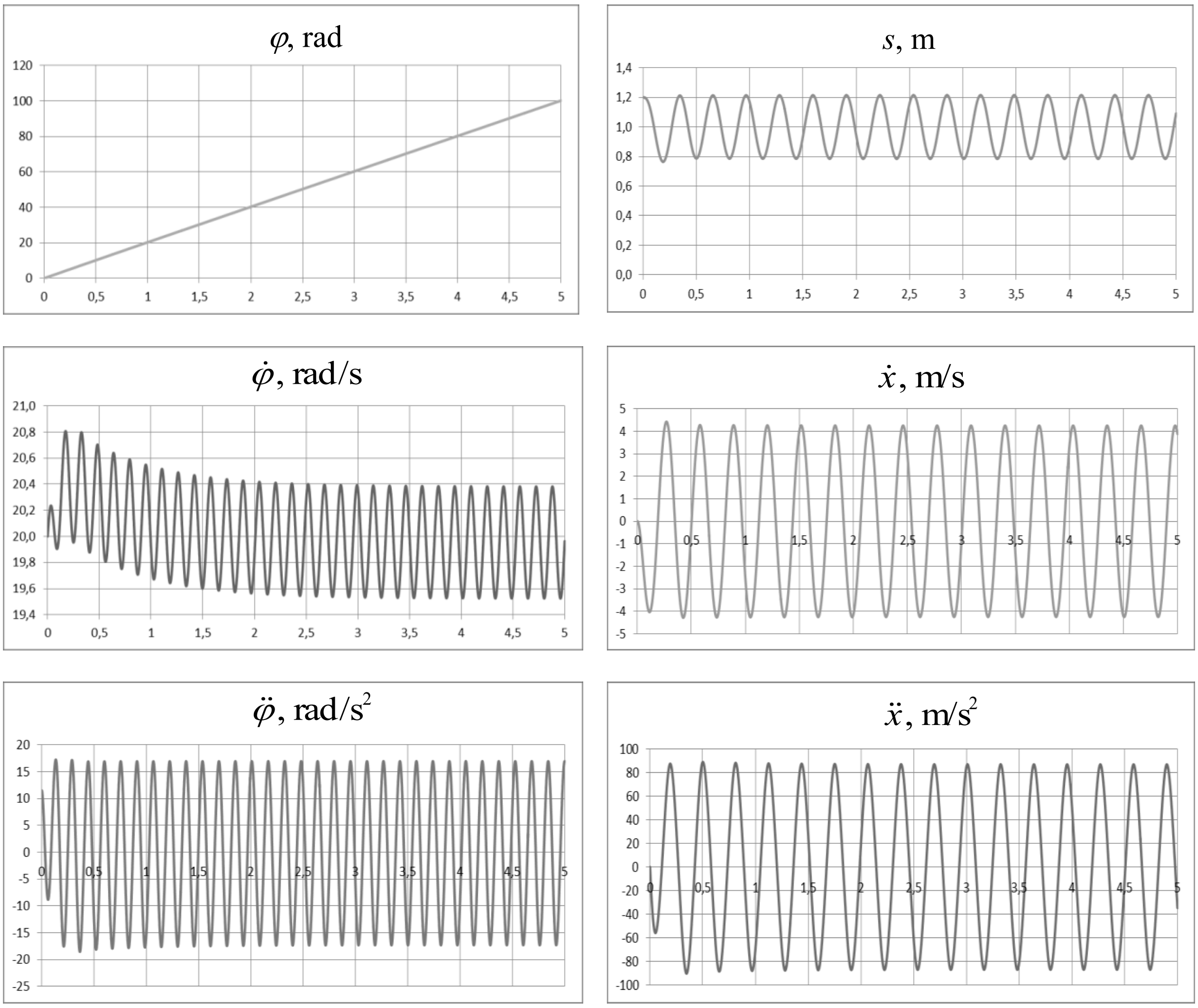


Figure 3. Kinematical diagrams of rotating disc and translating mass (example 1, second case).

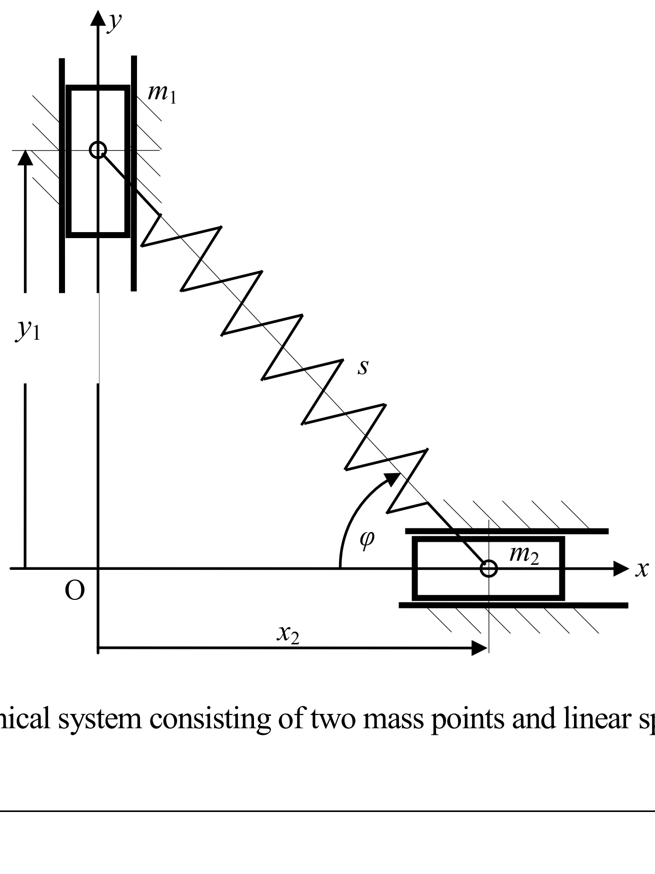


Figure 4. Sketch of mechanical system consisting of two mass points and linear spring in between.

Data and initial conditions for the numerical solution: unloaded length of spring, l_o=0,5 m, spring stiffness, s = 2000 N/m, gravital acceleration g = 9,81 m/s^2, m_1 = 10 kg, m_2 = 6 kg, $y_{1o} = 0{,}4$ m, $\dot{y}_{1o} = 0$ m/s, $x_{2o} = 0{,}3$ m, $\dot{x}_{2o} = 0$ m/s. Timeinterval: 0 s $\le t \le$ 2 s, time step: 0,0001 s.

Kinematic functions of mass points and the spring force in function of time can be seen in Figure 5-8.

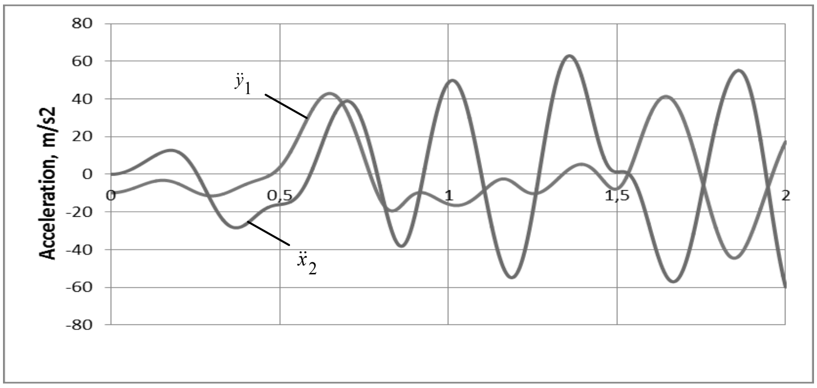


Figure 5. Acceleration functions of mass points.

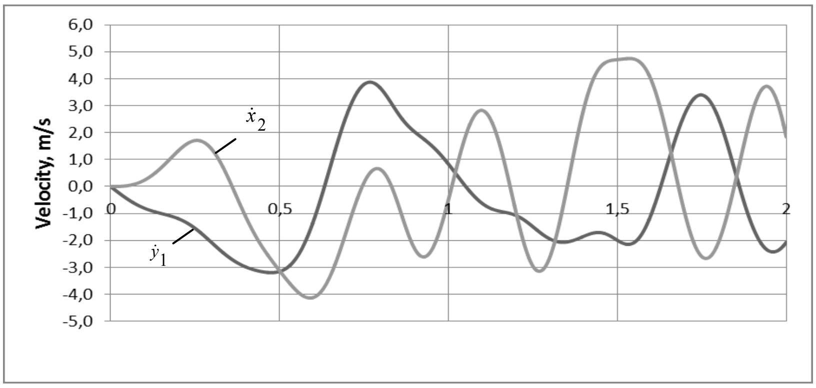


Figure 6. Velocity functions of mass points.

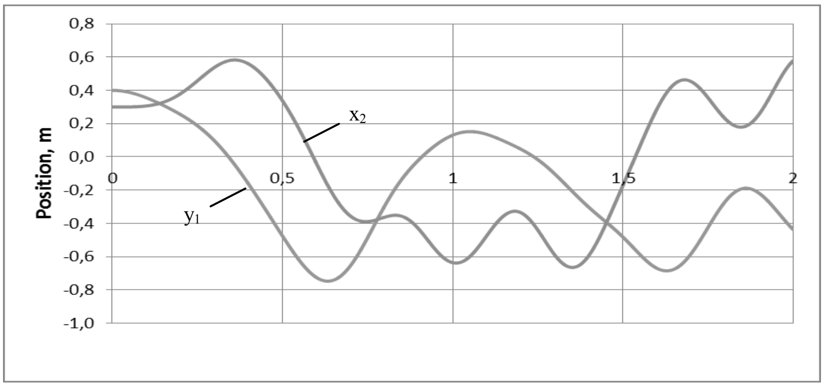


Figure 7. Position functions of mass points.

The level of mechanical power of the moving system in function of time is suitable to check the reliability and accuracy of applied numerical method. Because of the fact that the investigated mechanical system is conservative the level of mechanical power has to be constant. The difference of the level of mechanical power from its initial value is not significant as it can be seen in Figure 9.

In nonlinear case the characteristic of the spring can be described in function of deformation according to next equation, i. e.:

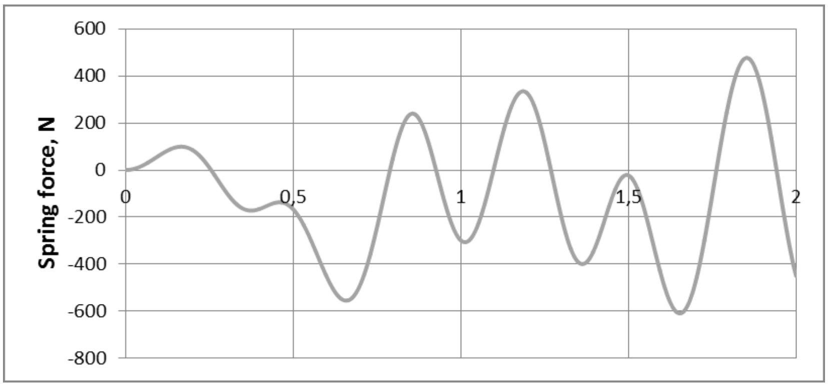


Figure 8. Spring force in function of time.

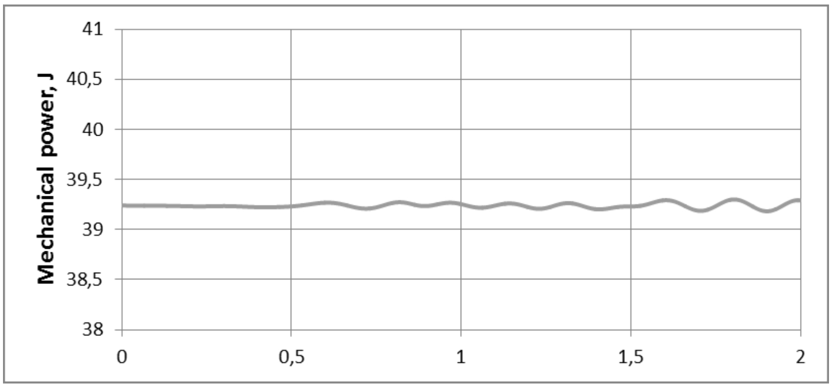


Figure 9. Mechanical power of the mechanical system in function of time.

$$R = \left(l_o - \sqrt{y_1^2 + x_2^2}\right) + b\left(l_o - \sqrt{y_1^2 + x_2^2}\right)\left|l_o - \sqrt{y_1^2 + x_2^2}\right| + c\left(l_o - \sqrt{y_1^2 + x_2^2}\right)^3, \quad (12)$$

where *a, b* and *c* are parameters. By modification of these parameters the real nonlinear characteristic of the spring can be approximated with the demanded accuracy.

In this case the motion equations in final form are

$$\ddot{y}_1 = \frac{y_1\left\{a\left(l_o - \sqrt{y_1^2 + x_2^2}\right) + b\left(l_o - \sqrt{y_1^2 + x_2^2}\right)\left|l_o - \sqrt{y_1^2 + x_2^2}\right| + c\left(l_o - \sqrt{y_1^2 + x_2^2}\right)^3\right\}}{m_1\sqrt{y_1^2 + x_2^2}} - g, \quad (13)$$

$$\ddot{x}_2 = \frac{x_2\left\{a\left(l_o - \sqrt{y_1^2 + x_2^2}\right) + b\left(l_o - \sqrt{y_1^2 + x_2^2}\right)\left|l_o - \sqrt{y_1^2 + x_2^2}\right| + c\left(l_o - \sqrt{y_1^2 + x_2^2}\right)^3\right\}}{m_2\sqrt{y_1^2 + x_2^2}}. \quad (14)$$

Data and initial conditions are quite the same like in linear case: unloaded length of spring, l_o = 0,5 m, characteristical parameters a = 1000 N/m, b = 30 000 N/m^2, c = 400 000 N/m^3, gravital acceleration g = 9,81 m/s^2, m_1 = 10 kg, m_2 = 6 kg, $y_{1o} = 0{,}4$ m, $\dot{y}_{1o} = 0$ m/s ; $x_{2o} = 0{,}3$ m, $\dot{x}_{2o} = 0$ m/s . Time interval: 0 s $\le t \le$ 2 s, time step: 0,0001 s. Kinematic functions of mass points and the mechanical power in function of time can be seen in Figures 10-13.

The spring force in function of time and deformation (unit: meter) are demonstrated in Figure 14. As it can be noticed the applied spring is strongly nonlinear.

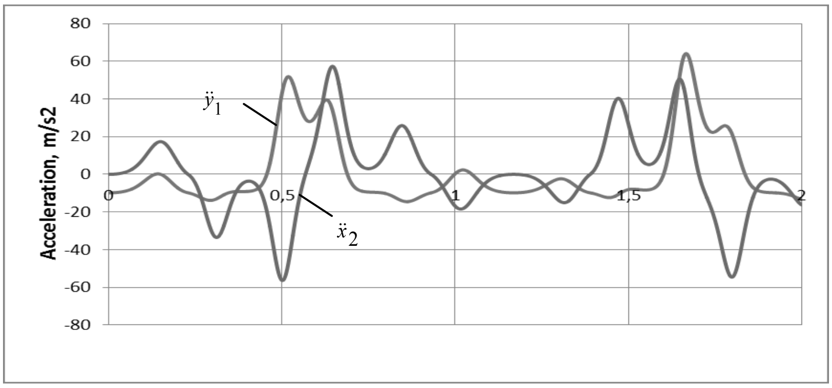


Figure 10. Acceleration functions of mass points.

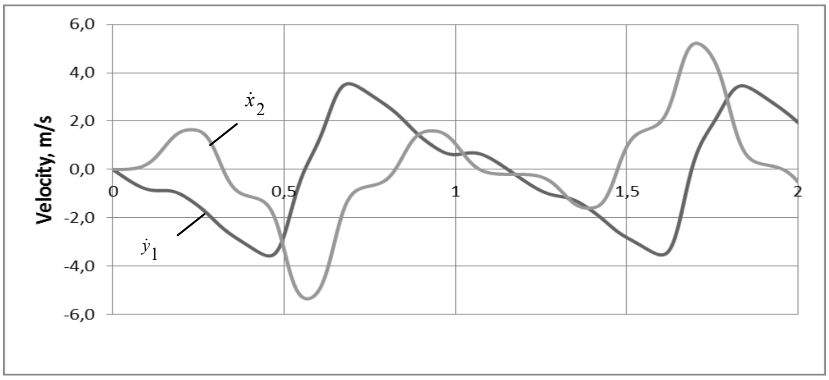


Figure 11. Velocity functions of mass points.

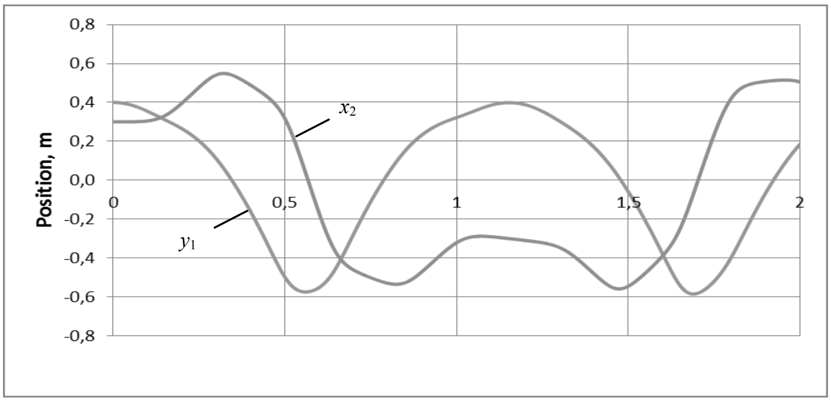


Figure 12. Position functions of mass points.

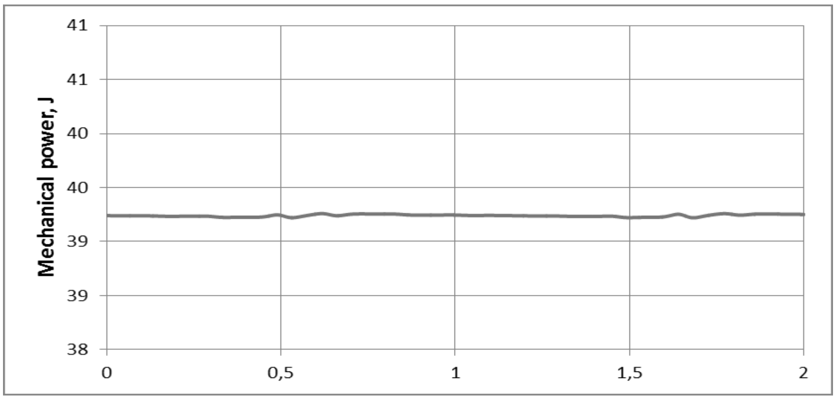
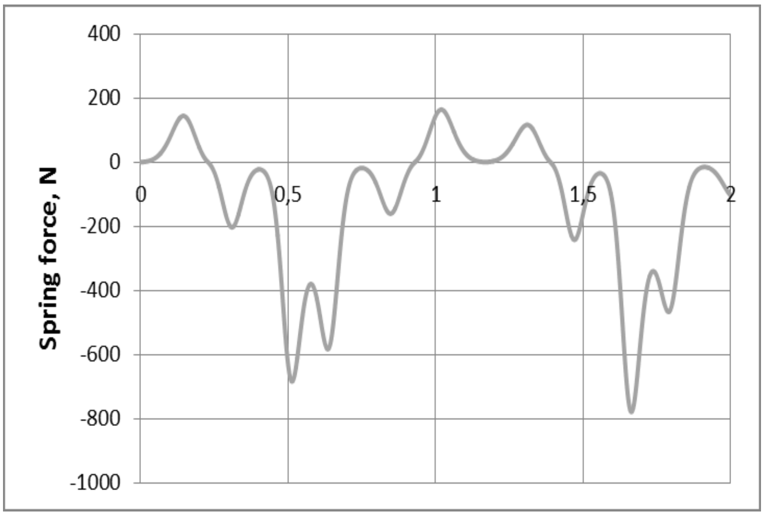


Figure 13. Mechanical power of the mechanical system in function of time.

a)



b)

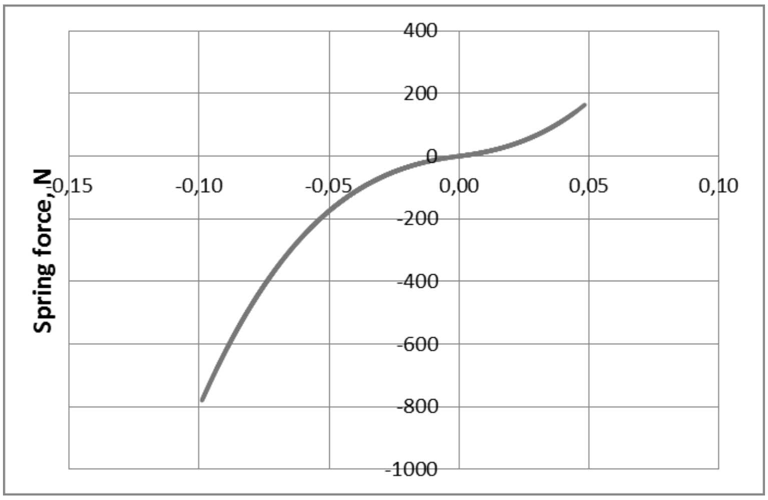


Figure 14. a) Spring force in function of time and deformation and b) algebraic sign of spring force of pressed spring is positive.

CONCLUSIONS

The demonstrated method can be applied easily for engineer students in the higher education. The method is suitable for investigation of similar mechanical systems having one or more degrees of freedom. By consequent modification of data (physical quantities) of systems a wide range of possible structures and their kinematical behavior can be analyzed. For this reason the application of this method can be advantageous for engineer students.

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