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AN ANALYSIS OF VEHICULAR TRAFFIC FLOW USING LANGEVIN EQUATION

ABSTRACT

Traffic flow data are stochastic in nature, and an abundance of literature exists thereof. One way to express stochastic data is the Langevin equation. Langevin equation consists of two parts. The first part is known as the deterministic drift term, the other as the stochastic diffusion term. Langevin equation does not only help derive the deterministic and random terms of the selected portion of the city of Istanbul traffic empirically, but also sheds light on the underlying dynamics of the flow. Drift diagrams have shown that slow lane tends to get congested faster when vehicle speeds attain a value of 25 km/h, and it is 20 km/h for the fast lane. Three or four distinct regimes may be discriminated again from the drift diagrams; congested, intermediate, and free-flow regimes. At places, even the intermediate regime may be divided in two, often with readiness to congestion. This has revealed the fact that for the selected portion of the highway, there are two main states of flow, namely, congestion and free-flow, with an intermediate state where the noise-driven traffic flow forces the flow into either of the distinct regimes.

KEY WORDS

Langevin equation; traffic dynamics; Brownian motion; traffic regimes; traffic flow; stochastic forces; drift; diffusion;

1. INTRODUCTION

In this study, stochastic traffic data are modelled using a stochastic differential equation, known as Langevin equation. By Langevin equation, deterministic and random terms are estimated non-parametrically. The authors assume a similarity between vehicular traffic flow and random movement of interacting pollen grains suspended in a fluid (Brownian motion). Even though vehicles do not correspond to pollen grains per se, vehicular speed differences in one dimension represent the Brownian motion. Random interactions of agents/particles are seen explicitly in underlying dynamics of a stochastic system like vehicular traffic flow. Similar to the Brownian motion, vehicular traffic flow phenomenon generally shows complex multi-particle interactions and non-linearity. Considering dynamic systems, an attempt is made to obtain the time evolution and the dynamics of traffic flow by including delta-correlated Gaussian distributed white noise. Thus the information about the stochastic process of traffic flow by means of experimental data is extracted. The desired formulation of the process in this study would be compatible with this stochastic differential equation.

Phase transitions between different traffic states would also be considered. Langevin equation is composed of deterministic and stochastic forces which include the Wiener process. The work deals with the drift and the diffusion coefficients, which are the Kramers-Moyal coefficients [1]. It is too difficult to build exact (deterministic) relations due to the nature of vehicular traffic flow. Deterministic models in traffic field (e.g. single regime and multi-regime parametric models) are also well-studied and their analytical tractability is strong [2]. However, the measured traffic data generally involve noise and randomness. It is intended to focus on obtaining a stochastic model to capture the traffic flow dynamics. It is assumed that the deterministic and stochastic forces are not explicitly time-dependent, and the traffic flow remains stationary within the observation period. It is believed that the Langevin equation is quite a versatile tool in the analysis of stochastic time series data such as the traffic flow.

There is a large variety of microscopic and macroscopic approaches to model traffic flow in physics and transportation literature. Some of the techniques used in traffic flow modelling such as hydrodynamics models, Lighthill–Whitham model, gas-kinetic models, car-following models, optimal velocity models and cellular automata models can be found in [3]. There are numerous examples of traffic modelling studies including stochastic elements. For example, Boel and Mihaylova [4] consider consecutively connected road sections (cells) and their interactions and present a stochastic compositional model for traffic flow dynamics on freeways. The authors point out that the model could be applied to on-line estimation, routing and ramp metering control at large freeway networks. The studies by Kim and Zhang [5] and Li et al. [6] are also representative of stochastic analysis.

Alperovich and Sopasakis [7] categorize the models describing traffic behaviour. According to the categories, deterministic type models (without random effects) have no descriptive capabilities especially to capture transient behaviour. To do this, stochastic models would be more feasible. Likewise, Liebe et al. [8] emphasize that the description of flow dynamics would be incomplete without stochastic forces.

Three recent and relevant articles are worth mentioning. In the study of Laval and Leclercq [9], Hamilton-Jacobi partial differential equations were used to form a flow surface. For the triangular flow density diagram, solution methods were shown. It is claimed that the study generates new models and acts as a unifying model. But it lacks an additive noise term to approximate the traffic flow. A stochastic traffic flow model is offered in Jabari and Liu [10]. This renders accounts stochastic for the uncertainty in driver gap choices. Random headway distributions were generated in order to ensure non-negative flow densities. Instead of adding a random noise to the traffic flow, a variety of distributions could be incorporated as a novelty, and as an example, the distribution is a Gaussian in Jabari and Liu [11].

Research using Langevin equation is far and wide, but there are only a few studies in transportation literature. Some of the related Langevin equation studies will now be discussed. Friedrich et al. [12] perform observed data and numerically formulate dynamic model equation. The study analyzes tremor data of the three different tremor groups and compares them. They utilize two-dimensional Langevin equation which includes measured time series and created time series by a time-delay method. In the study the numerical algorithm is first applied to data from a chaotic electric circuit. Afterwards, the study uses the dataset collected at a sampling rate of 800 Hz and applies the method. According to their analysis, deterministic parts of the dynamics of physiological and essential tremor are similar in terms of having a fixed point, but different in terms of damping and rotation. However, Parkinson's disease dynamics is formed with a limit cycle. The study points out that there might be other possible scientific applications especially in the time series of non-linear, complex systems.

Gradisek et al. [13] extend the method for analysis of stochastic processes (i.e. Langevin equation) to the class of periodically forced stochastic processes. The study shows how to estimate deterministic term, and the method is illustrated performing both synthetic and experimental data. The authors emphasize the necessity of forcing period T and stroboscopic data since the deterministic term has periodic dependency.

In the most relevant study, Kriso et al. [1] investigated the iterative dynamics of traffic flow with respect to deterministic and stochastic forces by considering Langevin equation. The authors point out that their method provides more insight in traffic dynamics rather than presentation of fundamental diagrams. That study also presents the fundamental diagrams in which density and flux states of the cars are included. The authors deal with one- (velocity) and two-dimensional (flux-velocity) dynamics of the traffic. The diffusion part has provided a thorough understanding of the traffic dynamics as it is emphasized in the study. The stable and unstable fixed points are obtained for traffic phase transitions. The sampling rate is rather precise, but it would be problematic in processing the traffic flow data when no traffic is present for a given lane.

In this research, the sampling rate is kept deliberately a little longer for the concerns mentioned above. Also, no manuscript thus far has provided an in-depth interpretation of drift and diffusion diagrams. Here, all the positive and negative values of drift data have been expounded and qualified. Additionally, the meaning of the slopes of the aforementioned diagrams is elaborated.

2. METHOD

Since Langevin equation has a stochastic term, it would be proper to talk first of the white noise and its characteristics. White noise is an idealization in that it spans the complete frequency range. It has a uniform power spectrum. The white noise is usually considered stationary and normally distributed. It is delta-correlated in time [14].

$$R_{\eta\eta}(t_2 - t_1) = (N_0 / 2)\delta(t_2 - t_1) \qquad (1)$$

where $R_{\eta\eta}$ is the autocorrelation function, N_0 is a real positive constant and δ is the well-known Dirac delta function.

Power spectral density of white noise η is uniform and defined by $S_{\eta\eta}(f)=N_0/2$ for $f\in R$.

The Wiener Process is a continuous time stochastic process with $w(t)$ as the integral of white noise:

$$w(t)=\int_0^t \eta(\lambda)d\lambda \tag{2}$$

Every independent increment in the Wiener process is normally distributed with zero mean and variance $(N_0/2)t$. Since the Wiener process involves independent increments, it satisfies Markovian property presented here by mathematical expectation:

$$E\{w(t)\}=0 \tag{3}$$

$$E\{w^2(t)\}=(N_0/2)t \tag{4}$$

In Langevin equation (Eqn. 6), $X(t)$ is the vehicle speed and has Markovian property since $X(t)$ at time t depends on the preceding time $t-\tau$. By Markov description, the process has no memory prior to $t-\tau$. Thus, one-step conditional Markov density functions are expressed as follows

$$P\left(X_{k+1}(t)\,|\,X_k(t-\tau)\right) \tag{5}$$

$$\frac{d}{dt}X(t)=g(X(t))+h(X(t))\eta(t) \tag{6}$$

In Langevin equation, the time derivative of $X(t)$ equals the sum of the deterministic and stochastic components where $g(X(t))$ is the deterministic component, $h(X(t))$ is the amplitude matrix and $\eta(t)$ is the Gaussian distributed white noise. This work could also have been based on a band-limited white noise assumption, without loss of generality.

Adapting Ito's definition [1] by taking forward difference, the following is obtained:

$$g(X(t))=\left\langle\left(X(t+\tau)-x\right)\right\rangle\Big|_{X(t)=x} \tag{7}$$

$$h((X))h^T((X))=\left\langle\left(X(t+\tau)-x\right)\left(X(t+\tau)-x\right)^T\right\rangle\Big|_{X(t)=x} \tag{8}$$

Here T represents the matrix transpose, the angle brackets represent the conditional moments. In our case, $X(t)$ variable is one-dimensional and the transpose is not implemented.

The deterministic component of the Langevin equation $g(X(t))$ is known as drift and it is basically a speed difference term. The stochastic component of the Langevin equation $h(X(t))$ is the diffusion term, which could also be considered as standard deviation observed at a given speed. Please note that the diffusion term is not an exact square of the drift term.

The angle brackets in Eqn. 7 and Eqn. 8 define the averaging. This averaging is performed as follows. For a given speed value, the successive speed differences, which followed at exactly tau seconds later, interspersed in fifteen-day period have been checked. All these differences for that given speed value are computed, which happens to be the reference value. Then all those instances are counted, and the summation of differences is divided by this count.

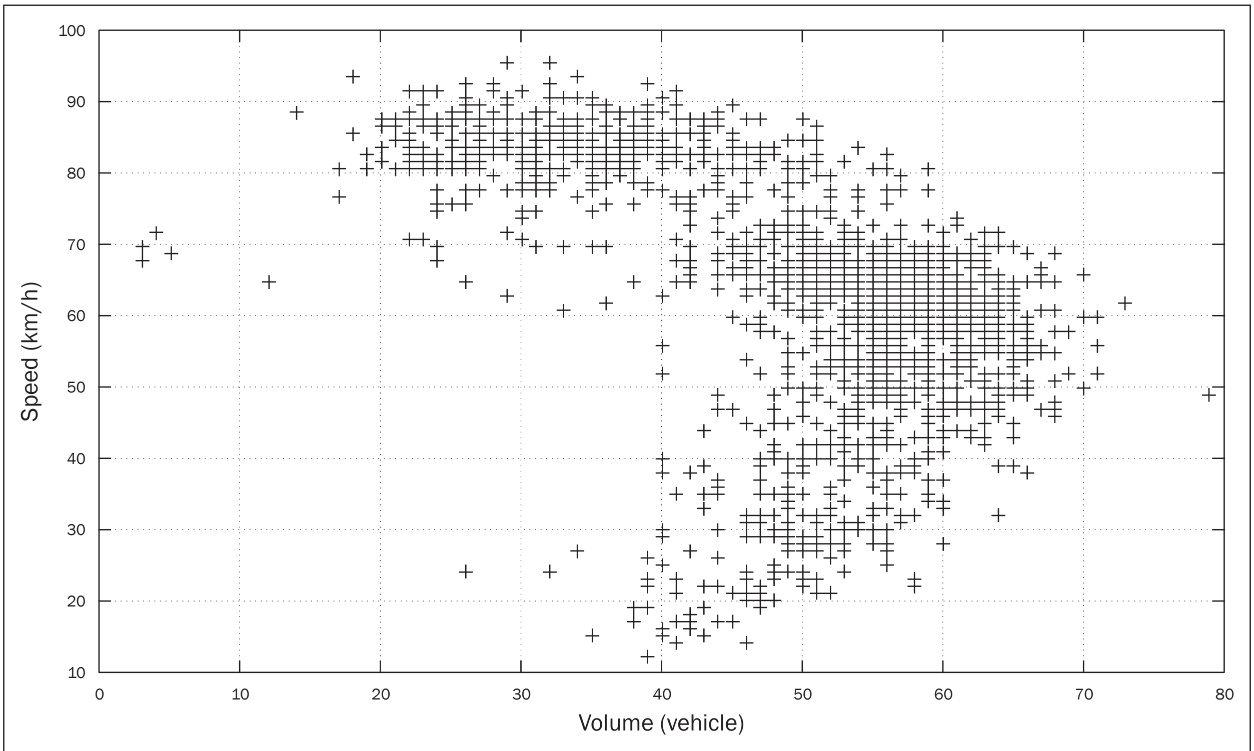

Figure 1 – Speed-volume diagram of the slow lane

3. DATA ANALYSIS RESULTS AND DISCUSSION

The authors have obtained traffic volume and speed data for one traffic observation point at the Istanbul highway. The data are from only one direction at three lanes for 15 consecutive days in 2012. The data at 2-minute intervals from 5:00 p.m. to 9:00 p.m. are processed. The highway is a link to Istanbul TEM (Transit European Motorway) and the traffic observation point at the selected portion of the highway is near a large number of residential and commercial areas.

This study first obtains the fundamental diagram (*Figure 1*) of traffic flow (speed-volume) at the slow lane. The spread of the speed-volume quantities would provide a general idea about traffic characteristic of the uninterrupted flow on the selected highway. Afterwards, the study evaluates the dynamics of the traffic flow on the highway. To this end, a sampling interval, total measurement time and step size are specified. After obtaining the deterministic part and noise amplitude, the data are simulated implementing Langevin equation in M-file developed by the authors in MATLAB environment.

To further grasp the traffic flow dynamics of the selected highway, the drift and diffusion diagrams of all the three lanes in the light of Langevin equation are constructed. Here, $X(t)$ is the speed value. Hence, drift $g(X(t))$ is the speed difference $X(t+\tau)-X(t)$, Eqn. 7, and diffusion $h(X(t))$ is the absolute value of the speed difference $(X(t+\tau)-X(t))(X(t+\tau)-X(t))^T$, Eqn. 8.

After obtaining the drift and diffusion diagrams, the drift diagram is evaluated in terms of the slope of the drift curve. For example, when the drift curve decreases (negative slope), the traffic congestion rate diminishes. In contrast, the positive slope in drift curve indicates an upward trend in traffic congestion.

In other words, for the negative slope in the drift diagram the vehicles in the previous interval have higher average speed than the vehicles in the current interval; thus the traffic congestion rate decreases and the spacing values between successive vehicles are greater. Conversely, in the positive slope, the vehicles in the previous interval have lower speed and the spacing between successive vehicles is shorter. Thus, the vehicles would be slower and the traffic congestion occurs. In drift and diffusion diagrams the data points are not necessarily successive counts. Instead, drift and diffusion diagrams are representative of traffic regimes composed of data points scattered in time.

Kriso et al. [1] examine three different flow regimes in drift and diffusion diagrams for the highway close to Köln-Nord in Germany. For example, for lane C (fast lane) it is seen that there are regular drift values, i.e. there are no large deviations in the drift diagram. Thus, it can be interpreted that the highway has generally heavy traffic. However, in our study, there are mainly irregular successive drift values as shown in *Figure 2a* since a series of zigzags can be seen and the slopes fluctuate in the drift diagram. Exceptionally, drift values remain horizontally more regular between the speed values of 45 km/h and 85 km/h in the diagram. In this speed range, the fact that the slope is stable and almost zero indicates that the traffic flow is regular. Notably, in the speed range of 12 km/h to 45 km/h, the slope changes from positive to negative or vice versa. Hence, the traffic flow occasionally oscillates between congested and uncongested in that speed range.

As can be seen in the drift diagrams, the following

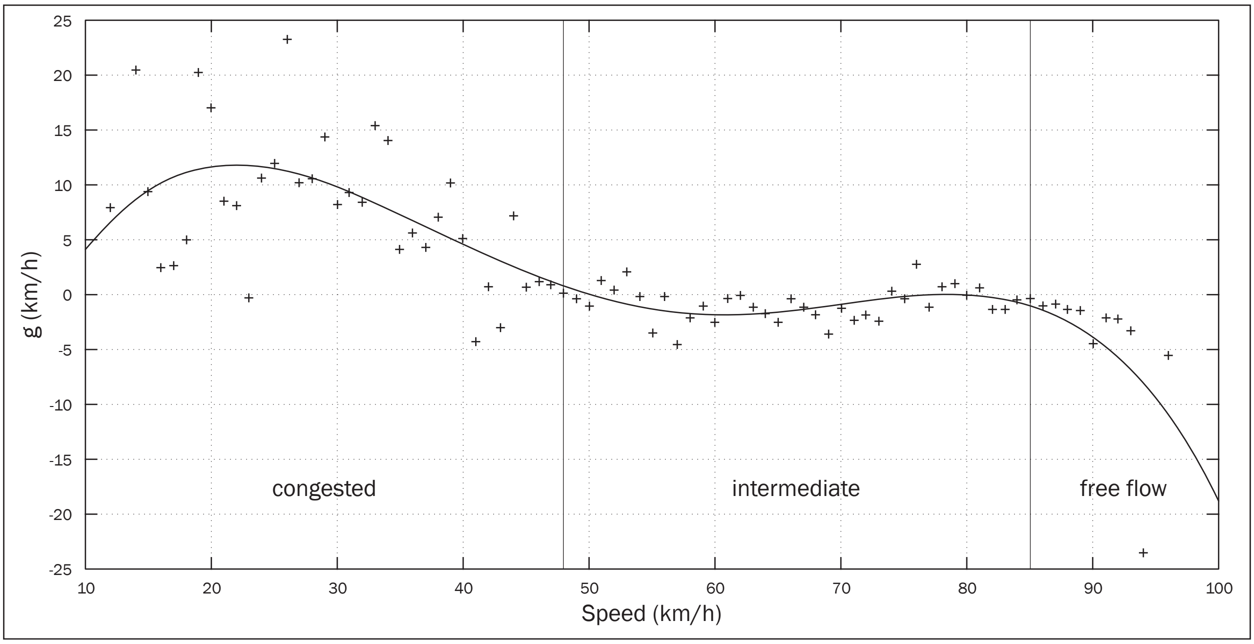

Figure 2a – Drift diagram of the slow lane per two-minute interval conclusions can be deduced:

- There are congested traffic flow regimes up to 45 km/h both for the slow lane and fast lane, but up to 50 km/h for the middle lane (*Figures 2a*, *2b*, *2c*).
- The traffic speed is almost stable between 45 km/h and 85 km/h (*Figure 2a*).
- The traffic flow characteristic changes after 85 km/h and results in a free-flow regime.
- The drift values enter into the negative zone and the congestion dissolves (*Figure 2a*).

Incidentally, even though the terms such as congested, intermediate, etc. are obvious, it would be timely to explain these terms in connection with drift diagrams. Since the drift term is actually a speed difference term, a large positive value would hint a faster following vehicle approaching a jam. When the drift is roughly zero, this would indicate the same speed traffic flow. From the drift diagrams it is clear that there is a transition from a certain regime to another when the drift assumes zero values. This transition occurs after 45 km/h and another one could be observed at about 85 km/h. These three distinctive regimes are called congested, intermediate and free-flow by the authors based only on the drift characteristics.

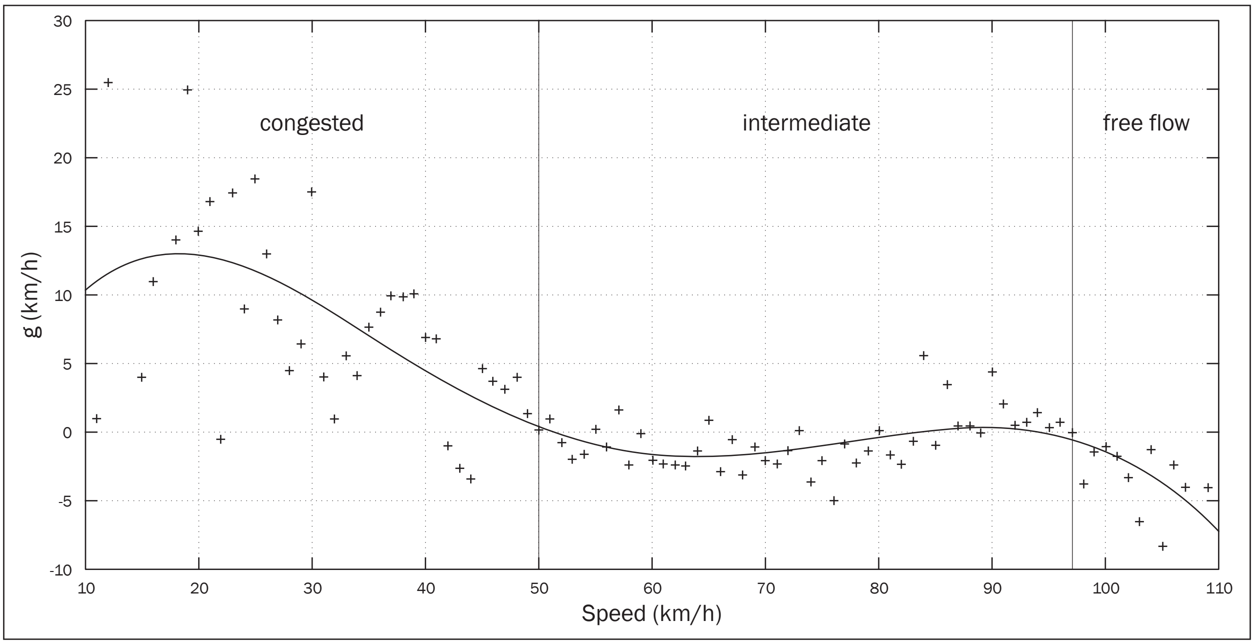

Figure 2b – Drift diagram of the middle lane per two-minute interval

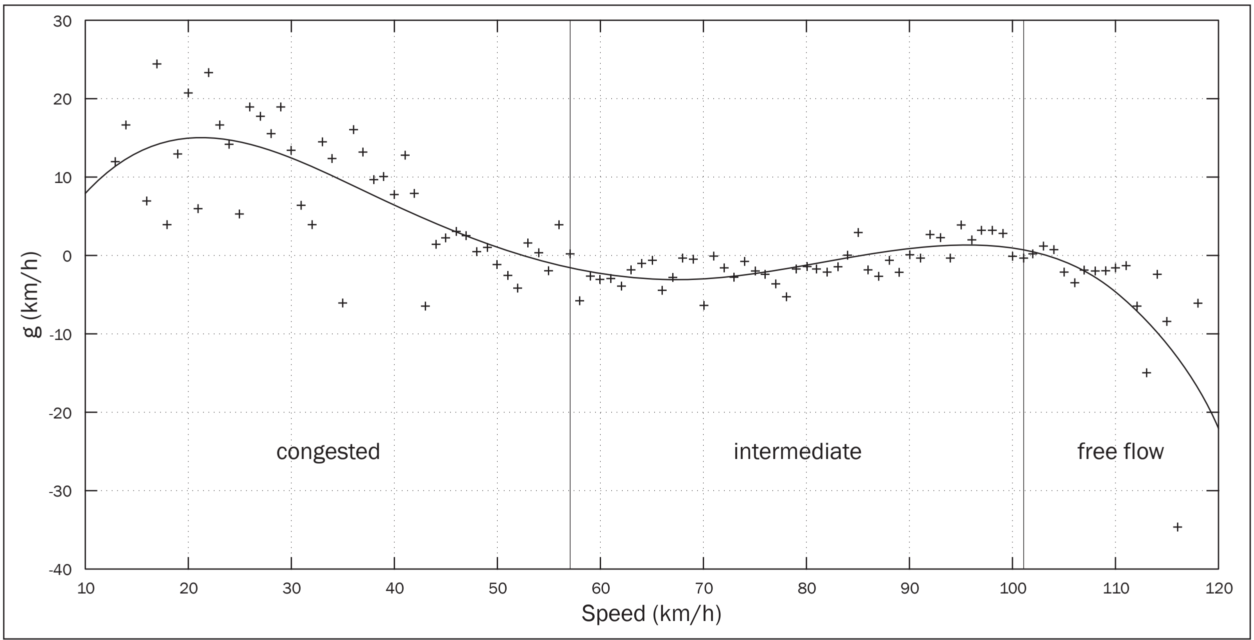

Figure 2c – Drift diagram of the fast lane per two-minute interval

When the diffusion equation (Eqn. 8) is considered in our study, the sign of the difference between successive speed values disappears and only the absolute values are meaningful since diffusion values could also be considered as a standard deviation for the designated drift value in the traffic flow. The outcomes obtained from diffusion diagrams (*Figures 3a*, *3b*, *3c*) corroborate the outcomes of the drift diagrams. For example, between 45 km/h and 85 km/h in *Figure 2a*, the speed values of the vehicles are very close in the traffic flow. After 85 km/h in the diagram, the difference between the speed-values increases and free-flow regime occurs. However, before almost 45 km/h, the congested flow regime occurs. The speed-volume curve (*Figure 1*) also upholds the congested regime at before 45 km/h since the forced-flow conditions have already started.

As shown in *Figure 2a*, the traffic flow has a tendency to concentrate in the middle zone (approximately between 45 km/h and 85 km/h). Furthermore, a stable flow can be observed in the range of 60-65 km/h on the highway. According to the observations, 82% of the total vehicles traversing at the surveillance point on the highway have average speed values within 45 km/h – 85 km/h limits. This supports the idea of the concentration of the traffic flow in the middle zone. In the drift diagram (*Figure 2a*), the speed values 45 km/h and 85 km/h can denote the boundaries to the neighbour states.

Both positive and negative drift values are interpreted in the drift diagrams. For example, in *Figure 2a*, the drift values of 20 km/h and 5 km/h are different in terms of congestion rates. Thus, when the drift value is 20 km/h, the traffic tends to get congested more rapidly than the drift value of 5 km/h. When the drift value range is in the neighbourhood of zero, as represented by the intermediate regime, not much could be said of the vehicle speeds. The only explanation is that the distance between the vehicles in traffic flow is fixed. Hence, the experimental data must be inspected to ascertain the traffic flow in zero (or almost zero) drift values. On the other hand when the drift moves towards negative values, the vehicles start breaking away, clearing any possible congestion. As seen in *Figure 2a*, at a drift of -5 km/h, the congestion dissolves more rapidly than it does at -1 km/h. Since the departure from zero drift into a negative one is the indication of a transition from intermediate to free-flow regime, zero drift may not be considered a part of free flow. The free-flow zone also indicates a rapid detachment regime of the traffic flow. The drift diagrams for each three lanes show that both on the fast lane and slow lane, there is a clearer dissolution of the traffic where the drift values are negative in the free-flow regime. As it is also expected, the dissolution in the middle lane is less pronounced.

Since what is said in accordance with drift diagrams holds true for diffusion diagrams as well, we refrain from restating what has already been stated. However, diffusion graphs reveal one interesting fact regarding all the lanes (*Figures 3a*, *3b*, *3c*). If congested regimes are inspected, dispersion among the speeds of the vehicles is greater than the free-flow regime, for example. This could be explained by the fact that in congested regime vehicle speeds range in a wider interval as opposed to the speeds in the free-flow zone. Free-flow zone denotes a traffic flow with less dispersion in speeds resulting in a more consistent traffic.

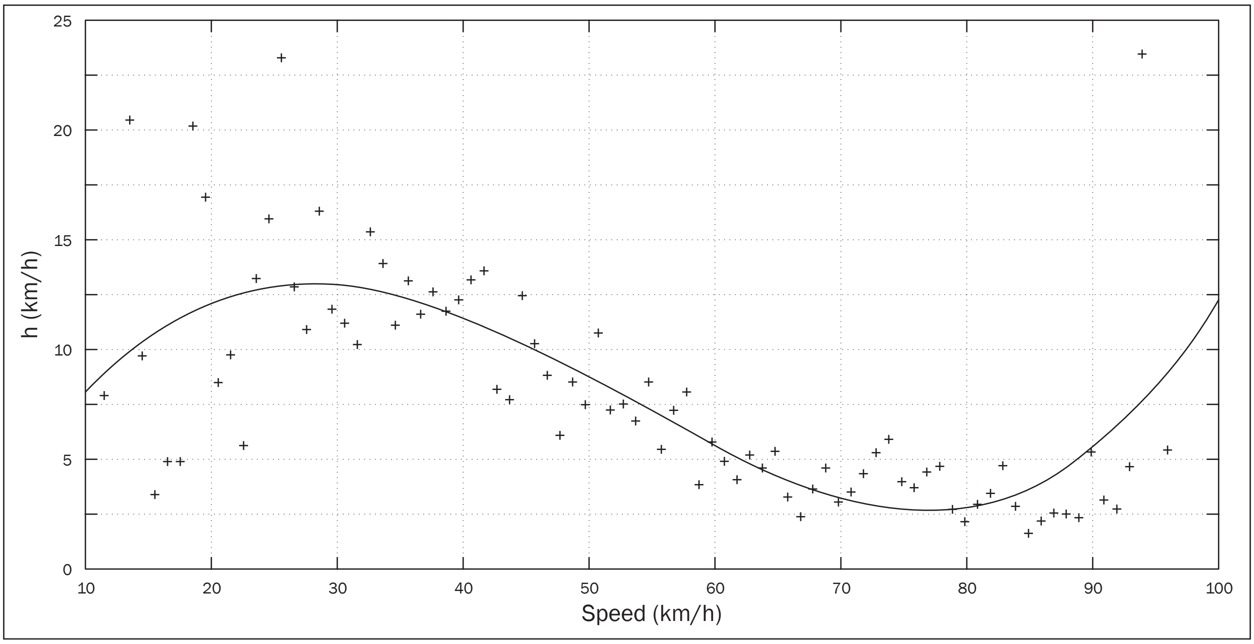

Figure 3a – Diffusion diagram of the slow lane per square root of two-minute interval

The analytical approach to the determination of the regimes is through zero drift transitions. A computer code may automatically cluster the data points with respect to zero drift axis. However, the use of absolute value in the diffusion term renders any regime distinction impossible.

4. CONCLUSION

This manuscript has tried to reveal the underlying dynamics of the selected portion of a highway in the city of Istanbul. Langevin equation is used to account for the stochastic nature of the traffic flow. Langevin equation consists of two parts, the deterministic and the noise-induced terms. Hence, the authors were able to separate the deterministic and random terms of the traffic flow for the given segment. Langevin equation provides an instantaneous view of the traffic flow characteristics. It is seen that the traffic flow tends to get congested fast until about 25 km/h speed for the slow lane, and eases afterwards. This maximum is roughly 20 km/h for the fast lane. Congested regime lasts until about 45 km/h speeds for slow and fast lanes, and 50 km/h for the middle lane. An intermediate regime

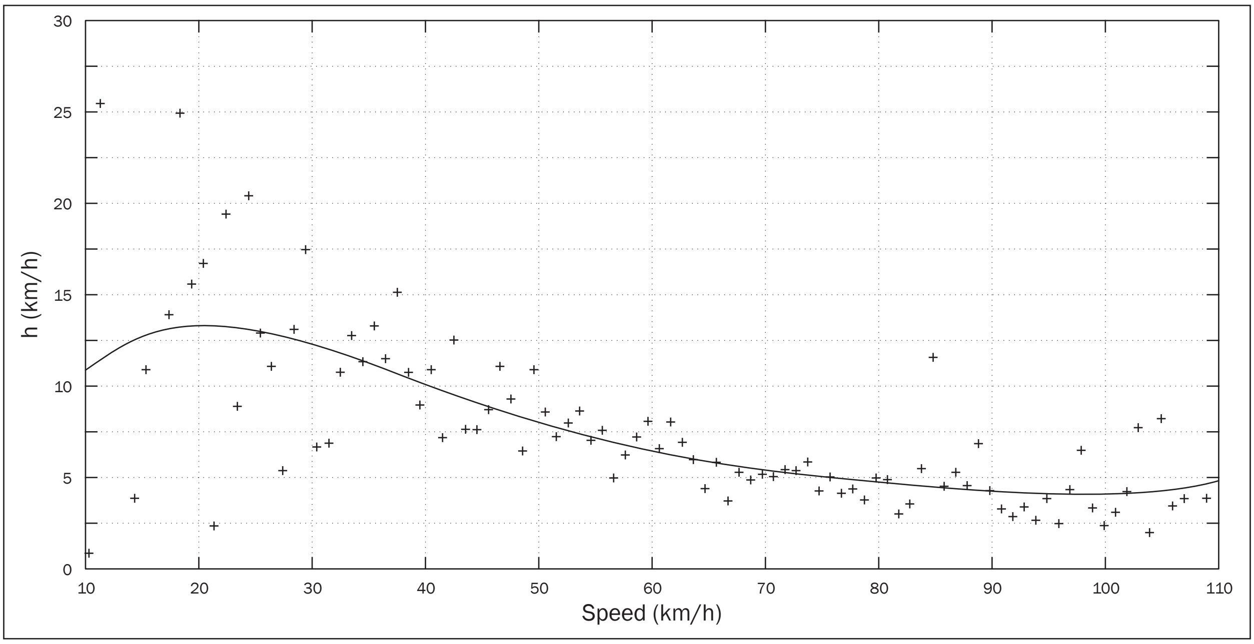

Figure 3b – Diffusion diagram of the middle lane per square root of two-minute interval

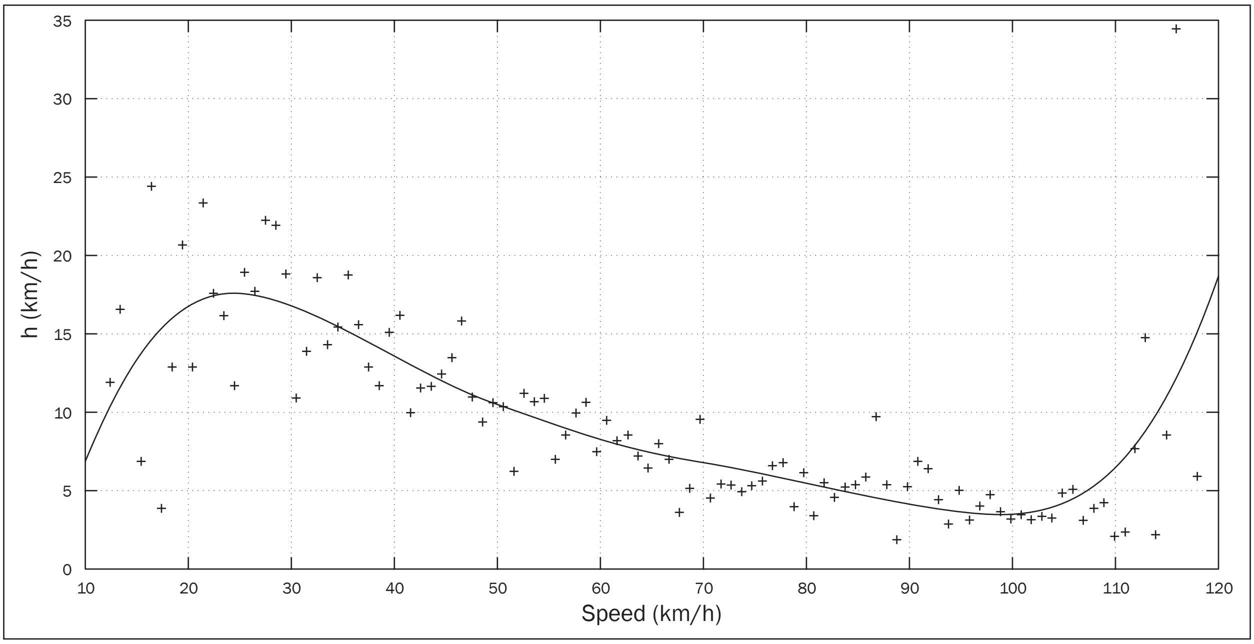

Figure 3c – Diffusion diagram of the fast lane per square root of two-minute interval follows until speeds of 85 km/h, often with a propensity to flow back into the congested state for the slow lane. Vehicles move in synchrony in this intermediate state, as their drift values dwell mostly around zero. As apparent from the diffusion graphs, free-flow regimes start at about 85 km/h, 100 km/h, and 100 km/h, for slow, middle and fast lanes, respectively. We did not mention the compatibility of the white noise assumption of the noise in the traffic flow, even though it seems to be working quite well. An upcoming work may focus on the true nature of the noise embedded deep in the data. A curious behaviour of the traffic data is that occasional breakaways may be seen both in free-flow and congested regimes, even if temporarily. We wish to analyze this behaviour, next, in a broader stochastic perspective.

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ÖZET

LANGEVIN DENKLEMİ KULLANILARAK ARAÇ TRAFİK AKIŞININ ANALİZİ

Trafik akış verisi stokastik yapıya sahiptir ve buna ait oldukça zengin bir literatür vardır. Stokastik veriyi ifade etmenin bir yolu da Langevin denklemiyle olmaktadır. Langevin denklemi iki kısımdan oluşmaktadır. Birinci kısım deterministik sürüklenme terimi, ikinci kısım ise stokastik difüzyon terimi ile bilinmektedir. Langevin denklemi İstanbul trafiğinden seçilen yerin deterministik ve rastgele terimlerini çıkartmakla birlikte trafik akışının dinamiğine de ışık tutmaktadır. Çevreyolunun seçilen yerinde sağ şeritte hızın 25 km/sa, sol şeritte ise hızın 20 km/sa olduğu anlarda trafiğin daha hızlı sıkışma eğiliminde olduğu sürüklenme grafiklerinde görülmektedir. Sürüklenme grafiklerinden sıkışık, ara ve serbest akış rejimleri gibi üç veya dört ayrı rejim elde edilebilmektedir. Hatta ara rejim de kendi içinde ikiye ayrılabilir. Trafik akışı, sıkışık ve serbest akış olmak üzere iki ana rejime ayrılmakta ve trafiği bu rejimlere zorlayan gürültüye dayalı trafik akışının bulunduğu ara rejim verilmektedir.

ANAHTAR SÖZCÜKLER

Langevin denklemi; trafik dinamiği; Brown hareketi; trafik rejimleri; trafik akışı; stokastik güçler; sürüklenme; difüzyon;

REFERENCES

[1] Kriso S, Peinke J, Friedrich R, Wagner P. Reconstruction of dynamical equations for traffic flow. Phys Lett A. 2002;299(2-3):287-291. doi:10.1016/S0375-9601(02)00288-8
[2] Wang H, Li J, Chen QY, Ni D. Speed–density relationship: From deterministic to stochastic. The 88th Transportation Research Board (TRB) Annual Meeting. Washington, DC; 2009. Available from: http://people.umass.edu/ndh/Publications/C21.pdf
[3] Schadschneider A. Traffic flow: A statistical physics point of view. Physica A. 2002;313(1-2):153-187. doi:10.1016/S0378-4371(02)01036-1
[4] Boel R, Mihaylova L. A compositional stochastic model for real time freeway traffic simulation. Transportation Research Part B. 2006;40(4):319-334. doi: 10.1016/j.trb.2005.05.001
[5] Kim T, Zhang HM. A stochastic wave propagation model. Transportation Research Part B. 2008;42(7-8):619-634. doi:10.1016/j.trb.2007.12.002
[6] Li J, Chen QY, Wang H, Ni D. Analysis of LWR model with fundamental diagram subject to uncertainties. The 88th Transportation Research Board (TRB) Annual Meeting. Washington DC; 2009. Available from: http://people.umass.edu/ndh/Publications/C22.pdf
[7] Alperovich T, Sopasakis A. Stochastic description of traffic flow. J Stat Phys. 2008;133(6):1083-1105. doi: 10.1007/s10955-008-9652-6
[8] Liebe C, Mahnke R, Kühne R. From traffic breakdown to energy flow analysis. Transp Res Part C Emerg Technol. 2011; 19(2):172-181. doi:10.1016/j.trc.2010.05.005
[9] Laval JA, Leclercq L. The Hamilton–Jacobi partial differential equation and the three representations of traffic flow. Transportation Research Part B. 2013;52:17-30. doi:10.1016/j.trb.2013.02.008
[10] Jabari SE, Liu HX. A stochastic model of traffic flow: Theoretical foundations. Transportation Research Part B. 2012;46(1):156-174. doi:10.1016/j.trb.2011.09.006
[11] Jabari SE, Liu HX. A stochastic model of traffic flow: Gaussian approximation and estimation. Transportation Research Part B. 2013;47:15-41. doi:10.1016/j.trb.2012.09.004
[12] Friedrich R, Siegert S, Peinke J, Lück St, Siefert M, Lindemann M, Raethjen J, Deuschl G, Pfister G. Extracting model equations from experimental data. Phys Lett A. 2000;271(3):217-222. doi: 10.1016/S0375-9601(00)00334-0
[13] Gradisek J, Friedrich R, Govekar E, Grabec I. Analysis of data from periodically forced stochastic processes. Phys Lett A. 2002;294(3-4): 234-238. doi:10.1016/S0375-9601(02)00060-9
[14] Peebles PZ, Jr. Probability, random variables, and random signal principles. Singapore: McGraw-Hill; 1993.