EXPERIMENTALİ VIBRATION TESTING AND NONDESTRUCTIVE TESTING

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ABSTRACT *- An overview of the Vibration and Experimental Modal Analysis (EMA) and system identification method and possible applications in non-destructive testing (NDT) is provided. The system identification in time, frequency and scale-time domain methods are presented. The illustrative examples of stiffness, mass and damping identification are provided. The theoretical and experimental basis of for mechanical system condition assessment is given. Displayed NDT method is based on the residual force vector and dynamic properties of the system. Theoretically, the method is applicable to any structure that can be accurately modeled using the finite element method and whose frequencies and mode shapes can be reliably obtained. To confirm the method numerically, it is applied to the numerical model of story frame, where the damage was simulated for three different cases: change in mass, change in stiffness and both change in mass and change in stiffness simultaneously. The practical, experimental, validity of the method is demonstrated by applying it to the free-free beam. It is shown that the practical application of the method requires additional actions, without which the method can be difficult to implement. This is a model update with which it is possible to achieve reliable modal parameters and the model reduction is used with which enables the equality of degrees of freedom between numerical and experimental model to be obtained.*

1. INTRODUCTION

The non-destructive testing correlates with Vibration Testing in a number of aspects. As far as the vibration is influenced by the system properties (stiffness, mass and damping) the structural changes that involve these properties can be observed throw dynamic-vibration system behaviour.

Vibration testing comprises a very wide area that includes Experimental modal analysis (EMA), operational mode analysis (OMA), System identification, Dynamic testing with deferent focuses including fatigue tests, functionality tests, Force detection,... The vibration testing area can additionally divided by the tested object to structure and machine dynamics. The machine includes moving parts and consequently intrinsic excitation sources what requires specific tests correlated to the machine speed... etc.

In what follows an overview of the EMA with NDT application will be given.

2. EMA and NDT

EMA basically detects system modal parameters i.e. natural frequencies, natural modes and modal damping. The modal properties are functions of the stiffness, mass and energy dissipation densities. The changes in these densities will affect modal parameters and structural change will be noted. The next steps are detection of location of structural change and further step is detection of amount of this structural change.

How do we perform EMA? Modal parameters we can detect in specially designed experiments when we excite the structure at some points with so called vibration exciters and measure structure response at some points with accelerometers (or velocimeters). By the ways we excite the structure (type of excitation forces) we can divide the EMA tests to impact test (impact excitation), random excitation, modal excitation,... However, it is also possible to detect modal parameters during regular operation (when the structure is excited by operational forces). The EMA test can be presented by input-output scheme. Depending on the number of inputs and outputs we divide tests to: SISO (single input single output), SIMO (single input multiple output), SIMO, MISO, MIMO,... SISO and SIMO tests with random excitation prevail.

In the EMA we can distinguish segments: a) experiment and data acquisition and b) data processing and system identification. The first segment deals with the experiment i.e. excitation and measurement technique and battle with noise. The second segment involves advanced techniques of identification of the system parameters.
Although there are considerable amount of software that are specialized in EMA, there is always need to use specially designed procedures to handle specific tasks.

3. MODEL VALIDATION AND MODEL UPDATE

Before using FE models for simulation process, those models should be correlated with experimental data to ensure that dynamic properties of models are related with the real structures. If dynamic properties of the model are not related to the real structure, then model must be updated so that its dynamic response more closely matches the dynamic of the real structure. Hence, mass and stiffness of FE model are modified.

While FE model has numerous degrees of freedoms (DOFs), including rotational DOFs, experimental analysis for practical reasons is limited to just on subset of the translational DOFs. To overcome this difficulty, model reduction process is needed.

For example let FE model of free-free beam be updated. Model reduction should be used to eliminate at least all rotational DOFs. Genetic algorithm can be used to minimize object/cost functions.

4. FE MODEL AND SEREP MODEL REDUCTION

In this example, free-free beam is used to update mass of beam elements. The length of the beam is 1.196 m, density 7800 kg/m3, modulus of elasticity 2.1e11 N/m2, rectangular cross-section with dimensions 60.7 by 12.2 mm. The beam is divided into 10 beam elements, with 4 DOFs per element, two translations and two rotations. Simulated structural dynamic modification is mass added at fourth element, 20% of element mass.

SEREP method [4] is used to reduce mass and stiffness matrices to translational DOFs

$$\tilde{\mathbf{M}} = \mathbf{T}^T\mathbf{M}\mathbf{T}, \tilde{\mathbf{K}} = \mathbf{T}^T\mathbf{K}\mathbf{T} \qquad \mathbf{T} = \begin{bmatrix}\mathbf{\Phi}_m \\ \mathbf{\Phi}_s\end{bmatrix}\mathbf{\Phi}^{\dagger}_m \qquad \mathbf{\Phi}^{\dagger}_m = \left(\mathbf{\Phi}^T_m\mathbf{\Phi}_m\right)^{-1}\mathbf{\Phi}^T_m \qquad (1)$$

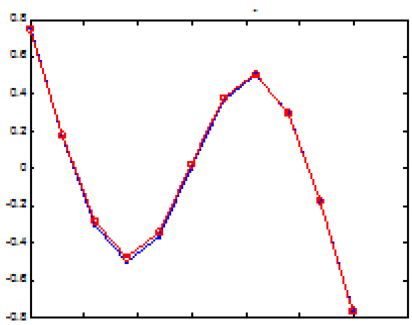
where Φ_m and Φ_s together make mode shape matrix in which subscript m and s relate to master and slave coordinates. $\Phi^{\dagger}_m$ is the Moore-Penrose pseudo inverse of the matrix. ***T*** *is transformation matrix.* ***M*** *and* ***K*** *are full mass and stiffness matrices.*Φ

5. FE MODEL UPDATE METHOD

It is too complex to make update directly to components of the mass and stiffness matrices. A more practical approach is to change physical properties of the finite elements and translate those changes into mass and stiffness changes. Some kind of measure between FE model and real structure has to be defined. The following two functions connect properties of FE model (a) and real structure (e) and can be minimized to update model

$$f_1(x) = \sum_{k=1}^{n} \frac{\lambda_a(k) - \lambda_e(k)}{\lambda_e(k) MAC(k)} \qquad (2)$$
$$\mathbf{f}_2(x) = \left(\tilde{\mathbf{K}}_a + \lambda_e(k)\tilde{\mathbf{M}}_a\right)\varphi_e(k)$$

where $\lambda(k)$ and $\varphi(k)$ are kth eigenvalue and eigenvector. MAC(k) is Modal Assurance Criterion between mode shapes for mode k.
Second mode shape for FE model and target FE model is shown in figure 1.

1 

	1th mode	2th mode	3th mode	4th mode	5th mode
Model	45.49	125.43	246.05	407.3	610.1
Target	45.18	123.67	245.59	402.2	603.2
Upd.f1	45.18	123.66	245.59	402.3	603.2
Upd.f2	45.11	123.47	245.17	401.6	602.1

Figure 1. Second mode shape, model (x) – target (o) *Table 1. Natural frequencies*

Table 1 shows first five natural frequencies for FE model, target FE model, update FE model using f1 function and update FE model using f2 function.

It follows an application of NDT based on EMA in structure health monitoring.

6. NDT – Delta pseudo residual force method

The serious approach to the structural health monitoring based on vibration techniques requires accurate numerical model of the structure (typically FEM model). As usually, many data for the model are not available (or are from not reliable sources) the very important step in the building of the model is model validation. The model is typically validated using MAC (modal assurance criteria) and other criteria. The next step is model update. The model update is procedure in which we match our numerical model to the tested structure.

7. THEORETICAL BACKGROUND

Modal properties of structure can be measured experimentally or simulated using finite element method. When modal properties for structures are known in their original state and changes are detected, changed regions can be located. Motion equation of undamped dynamic problem is:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(\mathbf{t}) \qquad (3)$$

*where **M** is the mass matrix, **K** is the stiffness matrix, and x are vectors representing accelerations and displacements, **f**(t) is excitation force vector. Free vibration are characterized with force **f**(t) equals zero. To compute the natural frequencies and mode shapes eigenvalue problem have to be solved*

$$(\mathbf{K} - \lambda_i \mathbf{M})\boldsymbol{\varphi}_i = 0 \qquad (4)$$

where λ_i is the ith mode eigenvalue, and φ_i is ith mode shape. The λ_i and φ_i can be considered ith modal property. To locate changed regions, difference in modal properties between two states has to be determined. Assuming eigenvalue problem for changed state can be written as

$$(\mathbf{K}_c - \lambda_{ci} \mathbf{M}_c)\boldsymbol{\varphi}_{ci} = 0 \qquad (5)$$

where subscript c indicates changed state. Stiffness and mass matrices in changed state $\mathbf{K}_c$ and $\mathbf{M}_c$ are defined as

$$\mathbf{K}_c = \mathbf{K} + \Delta\mathbf{K}$$
$$\mathbf{M}_c = \mathbf{M} + \Delta\mathbf{M} \qquad (4)$$

*where **ΔK** and **ΔM** is change of original stiffness and mass matrices **K** and **M**. Substituting the last equation into eigenvalue problem and rearranging residual force vector is*

$$\mathbf{R}_i = \Delta\mathbf{K}\boldsymbol{\varphi}_{ci} - \lambda_{ci}\Delta\mathbf{M}\boldsymbol{\varphi}_{ci} = (-\mathbf{K} + \lambda_{ci}\mathbf{M})\boldsymbol{\varphi}_{ci} \qquad (6)$$

Residual force vector $\mathbf{R}_i$ can be simple determined if is known natural frequencies and mode shapes. Subscript i denote ith degree of freedom. From the last equation it is observed that residual force for ith degree of freedom is zero if changed in stiffness and mass matrices are zero. Furthermore, only members of residual force which belong changed elements are different from zero.

Theoretically, only one natural frequency and mode shape is needed to locate changed element. In real world, measured data usually include certain amount of errors which may lead to inaccuracies in residual force vector. Therefore, it is needed several natural frequencies and mode shapes. If it is obtained p measured frequencies and mode shapes, pseudo residual force vector may be expressed as

$$\mathbf{R} = \{r_1, r_2, \ldots, r_n\}^T \ (7) \qquad \text{where } r_j \text{ is} \qquad r_j = \left[\prod_{i=1}^{p} \left(|r_j|_i\right)\right]^{\frac{1}{p}} \qquad (8)$$

in which $|r_j|_i$ is the absolute value of the jth entry of $\mathbf{R}_i$. In obtained pseudo residual force vector the real signals are amplified, and pseudo residual force vector is more accurate.

To determine the amount of changes, natural frequencies and mode shapes in changed state are defined as

$$\lambda_{ci} = \lambda_i + \Delta\lambda$$
$$\boldsymbol{\varphi}_{ci} = \boldsymbol{\varphi}_i + \Delta\boldsymbol{\varphi} \qquad (9)$$

Now, the eigenvalue equation can be updated as

$$(\mathbf{K}-\lambda_i\mathbf{M})\boldsymbol{\varphi}_i+(\mathbf{K}-\lambda_i\mathbf{M})\Delta\boldsymbol{\varphi}_i+(\Delta\mathbf{K}-\lambda_i\Delta\mathbf{M})\boldsymbol{\varphi}_i-\Delta\lambda_i\mathbf{M}\boldsymbol{\varphi}_i+ \\ +(\Delta\mathbf{K}-\lambda_i\Delta\mathbf{M})\Delta\boldsymbol{\varphi}_i-\Delta\lambda_i(\mathbf{M}\Delta\boldsymbol{\varphi}_i+\Delta\mathbf{M}\boldsymbol{\varphi}_i+\Delta\mathbf{M}\Delta\boldsymbol{\varphi}_i)=0 \tag{10}$$

The first term is equal to zero, moreover, neglecting higher order terms last two terms are equal to zero. Pre-multiplying all remaining terms by φ_I^T, second term becomes equal to zero. Furthermore, if φ_I is normalized so that $\varphi_I^T M \varphi_I=1$, last equation can be reduced to

$$\boldsymbol{\varphi}_i^{\mathrm{T}}\Delta\mathbf{K}\boldsymbol{\varphi}_i-\lambda_i\boldsymbol{\varphi}_i^{\mathrm{T}}\Delta\mathbf{M}\boldsymbol{\varphi}_i=\Delta\lambda_i \tag{11}$$

Values of ΔK and ΔM matrices are nonzero only at location where degrees of freedom are associated with changed elements. Reducing ΔK and ΔM to $\Delta K'$ and $\Delta M'$ matrices that contains only nonzero rows and columns associated with changed elements, and reducing φ_I to $\varphi_I^{\,I}$ that contains values only at corresponding changed elements, the last equation can be expressed as

$$\boldsymbol{\varphi}_i'^{\mathrm{T}}\Delta\mathbf{K}'\boldsymbol{\varphi}_i'-\lambda_i\boldsymbol{\varphi}_i'^{\mathrm{T}}\Delta\mathbf{M}'\boldsymbol{\varphi}_i'=\Delta\lambda_i \tag{12}$$

This can be used to solve for changes in original stiffness and mass matrices. To reduce the number of unknowns that need to be solved from the last equation, matrices and are expressed as function of original element stiffness and mass matrices by following equations

$$\Delta\mathbf{K}'=\sum_{j=1}^{n_c}\mathbf{K}_j^e\delta k_j \\ \Delta\mathbf{M}'=\sum_{j=1}^{n_c}\mathbf{M}_j^e\delta m_j \tag{13}$$

where $\boldsymbol{K}^e_j$ and $\boldsymbol{M}^e_j$ are stiffness and mass matrices of the jth changed element, n_c is the number of changed element and δk_j and δm_j are stiffness and mass proportional modification factors for changed element j, on this one way number of unknowns is reduced to two per changed element. Now we have:

$$\mathbf{A}\delta\mathbf{k}+\mathbf{B}\,\delta\mathbf{m}=\Delta\boldsymbol{\lambda} \tag{14}$$

*where **A** and **B** are p x n_c matrices, δk_j and δm_j column vector of size n_c , and $\Delta\lambda$ is column vector of size p, given by*

$$\mathbf{A}_{ij}=\boldsymbol{\varphi}_i'^{\mathrm{T}}\mathbf{K}_j^e\boldsymbol{\varphi}_i' \\ \mathbf{B}_{ij}=-\lambda_i\boldsymbol{\varphi}_i'^{\mathrm{T}}\mathbf{M}_j^e\boldsymbol{\varphi}_i' \tag{15}$$

$$\delta\mathbf{k}=\left[\delta k_1,\delta k_2,\dots,\delta k_{n_c}\right]^{\mathrm{T}} \\ \delta\mathbf{m}=\left[\delta m_1,\delta m_2,\dots,\delta m_{n_c}\right]^{\mathrm{T}} \tag{16}$$

$$\Delta\boldsymbol{\lambda}=\left[\Delta\lambda_1,\Delta\lambda_2,\dots,\Delta\lambda_p\right]^{\mathrm{T}} \tag{17}$$

Values $\varphi_I^{\,I}$, $\boldsymbol{K}^e_j$, $\boldsymbol{M}^e_j$ and $\Delta\lambda$ can be obtained by measuring or modelling using finite element method structures in their original state. For unique solution only $p=2n_c$ eigenvalues needed to be determined. As far as measured data usually include certain errors it is necessary to have more natural frequencies than $2n_c$.

Further simplification of the equation can be:

$$\mathbf{L}\cdot\mathbf{D}=\Delta\boldsymbol{\lambda} \quad \text{where} \quad \mathbf{L}=[\mathbf{A},\mathbf{B}] \text{ and } \mathbf{L}=[\delta\mathbf{k},\delta\mathbf{m}] \tag{18}$$

To solve equation this equation an optimization procedure is needed. The least square method is used to obtain acceptable solution. If the error is denoted as

$$\mathbf{E}=\mathbf{L}\cdot\mathbf{D}-\Delta\boldsymbol{\lambda} \tag{19}$$

the condition from least square method that is needed to be satisfied is

$$\frac{\partial\mathbf{E}^T\mathbf{E}}{\partial d_k}=0 \tag{20}$$

d_k is the kth entry of vector D. Combining the last two equations one can obtain

$$\mathbf{D}=\left(\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{L}^T\Delta\boldsymbol{\lambda} \tag{21}$$

The last equation can be used to solve for all the unknown stiffness and mass proportional modification factors which indicate amount of changes.

8. NUMERICAL EXAMPLE

The numerical example is performed on the story steel frame.
The results are obtained using finite element method, each node has three degrees of freedom – two translational and one rotational, stiffness and mass element matrices are given by Chandrupatla and Belegundu. The algorithm is developed and coded in order to perform computations and present results.
*A two connected five-story steel frame shown in **figure 2** is analyzed. It has 31 elements and 24 nodes. For the first node degrees of freedom are labeled as 1, 2, 3, for second 4, 5, 6 and etc. All 31 elements have area moment of inertia 2e-10 m4 and cross section 9e-5 m2, elastic modulus used for steel was 210 GPa and density 7800 kg/m3.*
*The first four natural frequencies are listed in **table 2** and corresponding eigen modes are shown on **figure 3**.*

*Three different types of change are investigated on element 9 and the same changes on element 28: case 1: 15% stiffness reduction; case 2: 15% mass reduction; and case 3: 15% stiffness and 15% mass reduction. Obtained pseudo force vector for degrees of freedom which belong to the changed element are in range 10, for other degrees of freedom pseudo force vector is less than 10-8. Example of the pseudo residual force vector for the case of change on element 9 is shown on **figure 4**. The **table 3** presents the results of detected amount of change for three above cases.*

Mode	Frequencies (Hz)
1	0.6496
2	1.5019
3	2.0748
4	3.2737

***Table 2** Natural frequencies of 5-story frame*

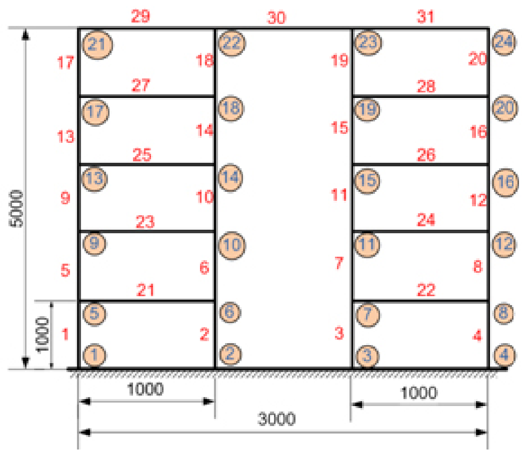


***Figure 2** Model of the structure*

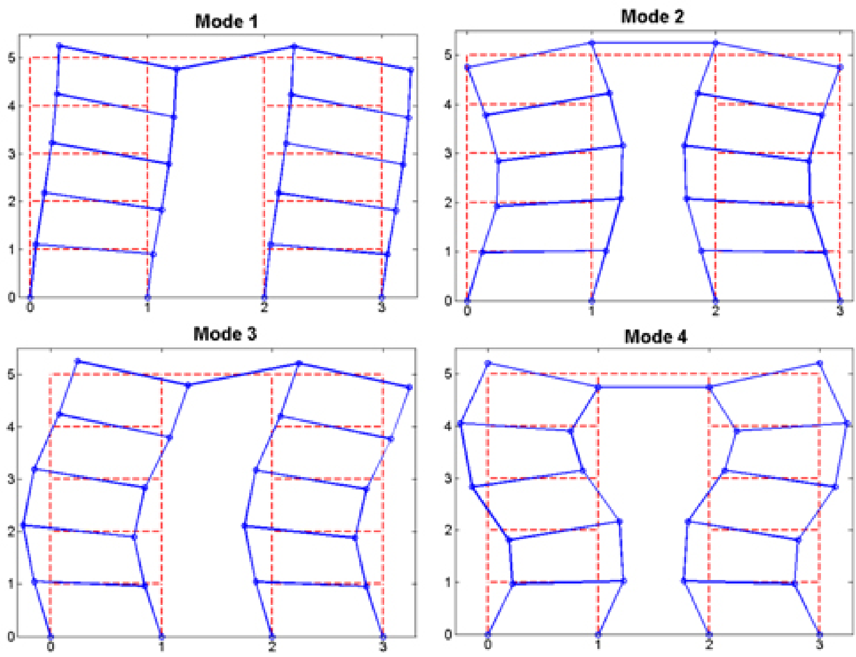


***Figure 3** The first four eigen mode of 5-story frame*

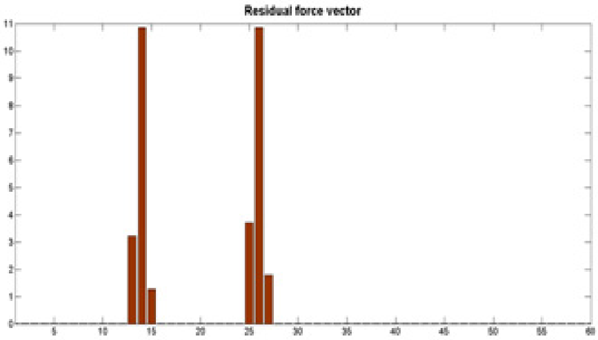


Figure 4 *Residual force vector - element 9 is change*

	Change element 9	Change element 28
Case 1	δm=0.18 δk=-17.1	δm=0.04 δk=-16.2
Case 2	δm=-15.3 δk=0.09	δm=-15.04 δk=0.01
Case 3	δm=-15.5 δk=-16.9	δm=-15.1 δk=-16.2

Table 3 *Detected amount of change (%) of 5-story frame*

So, we find the method presented very reliable and efficient. The calculated values are very close to actual changes in all three cases. Minimum deviations are the result of numerical errors and errors due to neglecting higher order terms.

9. EXPERIMENTAL IMPLEMENTATION

Practical implementation was realized on a free-free beam, ***Figure 5****. The beam is mounted with two cords and measurement was carried out in a direction perpendicular to the cords in order to avoid its impact. The beam is divided into ten elements, or 11 nodes. In order to implement the presented algorithm, the numerical model of beam is reduced (SEREP) to match the measured model. In the numerical model only those DOFs are left which are being measured. The measurement was performed in two steps: the original (unchanged) beam and beam with added mass at 6th element. The amount of added mass is 55.1% of the mass of the element itself.*

Before practical applications, the effect of reduction on the numerical model of beam is tested. The numerical model of beams which is used is equal to the experimental model. The figure 6 shows the residual vector before any reduction. As in the case of frame a method determines the location of the damage without any problem. Four values jumped out, those that belong to the nodes which surround the damaged element, and each node has two degrees of freedom.

Figure 7 *shows the effect of reduction. The numerical model has been reduced in a way that only the translations DOFs in every tenth node are left. The remaining translation DOFs and all rotational DOFs were removed. The blurring of the residual vector is due to the reduction, but the area where the damage occurs is still clearly visible.*

Figure 5 *Practical implementation*

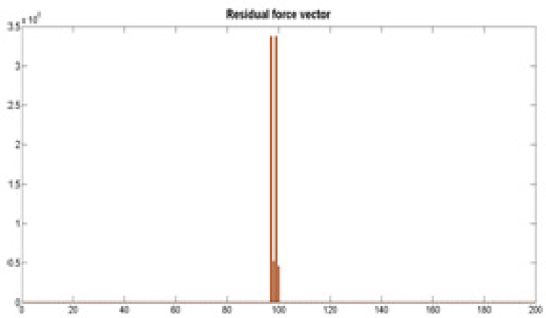


Figure 6 *Residual force vector, numerical example of beam, reduction*

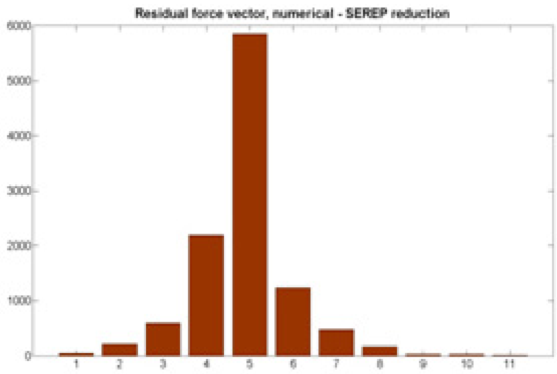


Figure 7 *Residual force vector, numerical example of beam, without reduction*

Observing the pseudo-residual force vector calculated without model update we find considerable amount of noise which hinders the detection of damaged elements. Despite all, the vector has the highest value at members 6 and 7, which belong to the changed element (element 6). Applying the model update and calculating the delta pseudo-residual vector situation has significantly improved. Although not eliminated, the noise is significantly reduced with modal update and it is quite clear which element of the structure is damaged.
*Also, the detection of added mass is made according to the above mentioned algorithm and the results are shown in **Table 4**. **Table 4** shows excellent detection of added mass, while the stiffness which should be equal to zero is slightly increased.*

Actual change	Detected change
$\delta m=55.1$ $\delta k=0$	$\delta m=56.2$ $\delta k=6.9$

Table 4 *Detected amount of change (%) free-free beam*

On this practical example the method proved to be very effective with using model updates and model reduction.

10. CONCLUSION

The possibilities of the application vibration testing to NDT and structural health monitoring are discussed. In this paper NDT and health monitoring methods that involve structural changes in stiffness and mass are presented. This approach typically requires knowledge of the observed system/structure. The structure is modelled numerically (typically FEM model) and model is validated and updated according experiments to prove that model is reliable.

Here, we presented and compared FE model update for two different object functions. Object functions are tested on free-free beam model. Genetic algorithm is used to minimize the object functions.

The pseudo-residual force and delta pseudo-residual force approach is presented as NDT in structural health monitoring. The numerical simulation of the whole damage detection procedure is used to illustrate practical application of the method. Additionally, the accuracy and reliability of the method in practical applications are discussed.

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