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ANALYSIS OF THE VIBRATION SIGNAL USING TIME-FREQUENCY METHODS

Summary

Condition monitoring (CM) is an essential technique used for monitoring various systems and system components and is supplemented with other techniques, such as video monitoring. Due to the fact that the acceleration signal is suitable for acquisition and is of non-stationary nature, CM is used to monitor various parameters of a monitored system. In order to process the acquired vibration signal and draw valid conclusions a time-frequency analysis has to be performed. In this paper, the advantage of the wavelet application is shown with an example and manifolds of wavelets reported in this application are reviewed. The case study implies that the optimal wavelets are db8 and db7 and the optimal level of decomposition is the fifth level.

Key words: *condition monitoring, wavelet transform, vibration signal, time-frequency domain, diagnostic tool*

1. Introduction

Condition monitoring is an invaluable modern diagnostic technique for monitoring ship engine. Vibration signal is a basic signal source, which is analysed by signal analysis techniques. The signal acquisition, i.e. the sensors that are used in the acquisition, is non-destructive and robust to harsh environmental applications. In shipbuilding and maritime industry, it is used at the test bed in the shipyard, during the maiden voyage, and continuously during the ship's lifetime. In industry, i.e. in turning machines, it can provide information about knife wear and tear, as well as about the time needed to replace the tool [1].

This paper is organized as follows. The second section provides a review of related literature. The third section presents concepts of the vibration signal analysis. In the fourth section, an example illustrates practical applications of wavelets for the acceleration signal analysis and what this means for velocity and position signal analysis. A comparative study is performed for the case study. The last section provides conclusions.

2. Literature review

Time-frequency (TF) methods include [2 – 4] the short-term Fourier transform (STFT), the wavelet transform (WT) and its variants (complex, continuous, discrete, wavelet tree), the Wigner-Ville distribution (WVD), the Hilbert-Huang transform (HHT), the Empirical Mode

Decomposition (EMD), the Ensemble Empirical Mode Decomposition (EEMD), the Wigner-Ville Distribution with the Choi-Williams kernel (WV-CW), etc.

Furthermore, many transforms have been invented in the last two decades, which are in essence integral transforms [5]. Some of them are built upon the above mentioned transforms. However, different authors report on different success rates of these methods. Therefore, it is necessary to systematize some of the methods. Furthermore, it is not certain that the frequency domain methods do not produce similar results in every case. So, the advantages of the TF methods should be carefully examined in order to reach appropriate conclusions for a specific application.

A fault diagnosis of fixed or steady state mechanical failures is performed by the FT. However, wind turbine transmissions or the mechanical arms used in 3C assemblies generate non-stationary signals, which is useful to be analysed by the STFT [6]. Back propagation neural networks (BPNNs) and the TF order spectrum methods have been used to verify the fault diagnosis in non-stationary signals of gear-rotor systems [6]. An intelligent diagnostic technique based on the integration of the EMD, the kernel independent component analysis (KICA), the Wigner bi-spectrum and the support vector machine (SVM) has been used in [7]. The success rate of the proposed algorithm was above 94% [7]. The concept of the Independent Component Analysis (ICA), and, in this case, the KICA, can be defined as in Definition 2.1.

Definition 2.1. Assume the measured variable data matrix $x = [x_1\ x_2 \cdots x_n]^T \in \Re^n$ is measured by different sensors. The matrix x is a linear combination of m (m ≤ n) unknown ICs in the matrix $s = [s_1\ s_2 \cdots s_m]^T \in \Re^m$. Then, and only then, the relationship between $[x]$ and $[s]$ can be expressed by:

$$[x] = [A][s] \tag{1}$$

where $[A] \in \Re^{n \times m}$ is the mixing matrix. The ICA must find a transformation matrix $W \in \Re^{m \times n}$ so that $\hat{s} = Wx \approx s$. This must make the projection $\hat{s}$ statistically independent.

The application of some processing techniques for the vibration analysis in working conditions of two different types of marine flexible couplings for boat propulsion has been presented in [8].

Limiting factor in applications is the fact that TF methods can be implemented only in the Matlab environment [9]. A system, which can be executed separately from Matlab, has been developed [9] for three analytic methods using a programming combination of Delphi and Matlab.

An EMD method has been proposed in [10] with improved multiscale median filtering for the extraction of the TF feature of telemetry vibration signals under interference of impulse noise. The result is better TF feature extraction performance under the impulse noise interference condition. Infinite impulse response (IIR) digital filters have been used in [11] to identify nonlinear vibrations. The advantage of the TF analysis procedure over the FT analysis has been shown.

A multicriteria method for the TF representation selection has been proposed in [12], which was tested in synthetic and real environments. This method can also be applied in the analysis of the vibration signal. If we want to find an optimal and adaptive TF representation, it is proposed that it is necessary to maximize the measure:

$$C_x(t,p) = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left| D_p(\tau,\Omega) w(\tau,\Omega) \right|^4 d\tau d\Omega}{\left(\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left| D_p(\tau,\Omega) w(\tau,\Omega) \right|^2 d\tau d\Omega \right)^2} \tag{2}$$

where D_p is the TF representation with a single parameter p, w is the 1D window function, C_x is a measure of the TF representation, t and τ times, and Ω is the frequency. The authors proposed a weighting multicriteria performance measure:

$$SQM = \sum_{i=1}^{n} W_i EF_i \tag{3}$$

with: $0 < SQM \le 1$, $\sum_{i=1}^{n} W_i = 1$, $W_i \ge 0$, and $EF_i \le 1$, where SQM is the structural multicriteria quality factor, W_i is the eighting factor, and EF_i is the evaluation factor.

A procedure for the parameterized TF method for the analysis of the nonstationary vibration signal of varying speed rotary machinery has been proposed in [13]. An isotropic and anisotropic diffusion method has been proposed in [14] for the vibration signal analysis.

2.1 Wavelets in the vibration signal analysis

A very popular method for the signal analysis is the WT. The WT has been successfully used in many applications. Hence, it is not unexpected that there are many studies on the WT analysis of the vibration signal.

The study in [15] used experimental data obtained in a turning machine and the WT analysis of the vibration signal.

A fault diagnosis of a water hydraulic motor has been analysed in [16]. The diagnosis was obtained from the vibration signal by using an adaptive wavelet analysis (AWA). The AWA used a linear combination of wavelets adapted for the particular vibration signal, and incorporated the parametric optimization by a generic algorithm.

The wavelet analysis of modal shapes in the vibration signal as a non-destructive method to assess damage has been presented in [17]. It was found that the effectiveness of an algorithm dependeds greatly on the type of the applied wavelet. A combination of a multiobjective meta-optimization was proposed to select wavelet parameters for an optimal solution in order to be able to have real time condition monitoring. It is well known that some advanced techniques, such as the complex WT, that maybe could produce better results, are not applicable in some applications since they are time consuming, i.e. they are not useful in real time monitoring.

The use of wavelets for the control of the multi-frequency rotor vibration in real time has been investigated in [18]. An adaptive wavelet was used for time-varying signals. Experiments were used to deduce the best wavelet basis for the evaluation of the operating conditions. An interesting result was that a rotor impact test at 20 Hz resulted in the detection of detail's coefficients of WT at the second level of decomposition of the measured vibration signal in the x-axis direction. The intention was to design a control algorithm that would measure the vibration response of the rotor during a cycle and then produce the required control force to minimize the response during the following cycle. Several wavelets were used in the analysis of the vibration signal. The authors concluded that the rate of convergence that can be achieved decreases with a decrease in the wavelet order and the length of the wavelet support. The lower order wavelets are not accurate enough to approximate the measured vibration signals using fewer scale levels.

A selection of wavelets has been presented in [19] for the vibration signal analysis of a rotational mechanical system. The proposed selection method was successful in a way that the selected wavelets were better in the sense of structural defect detection.

The complex WT has been used for the vibration signal analysis in [20]. The proposed algorithm included a combination of the complex WT and the wavelet packet (WP) approach. The authors concluded that wavelets are highly suitable for finding misalignment, unbalance, mechanical looseness, shaft crack, rubs, etc. This is possible due to the non-stationary nature of the vibration signal.

The WT value and effectiveness in the detection of the health status of a bridge structure has been shown in [21].

The WP energy flow of the vibration signal has been explored in [22]. It used the manifold learning technique. The proposed method was confirmed in case studies of the fault classification of machines.

The study presented in [23] used the WT and showed that the WT is especially suitable for non-stationary vibration measurements obtained from accelerometer sensors. The WT detects abnormal change in the measured data. It was found that in this investigation the best wavelet was the Daubechies wavelet of 4th order (db4). Approximation coefficients at 4th level of decomposition were used on the vibration signal of the mass unbalance at the speed of 1200 rpm.

An efficient technique for the wavelet spectrum analysis of truncated vibration signals has been proposed in [24]. It was found that the proposed technique is efficient for the incipient machine fault detection.

The wavelet method for a fault diagnosis of the wind turbine planetary gearbox components has been proposed in [25].The emphasis was on the signal processing of the acquired vibration signal to extract features of value to the diagnostics. It was concluded that the best wavelet for the fault detection was the so called Morlet wavelet. The Morlet wavelet was the best wavelet for matching behaviours of hidden impulses. Kurtosis was used for the detection of fault symptoms, because it was sensitive to sharp variant structures. This measure was defined as:

$$Kurt\,(x) = E\left(x^4\right) - 3\left[E\,(x^2)\right]^2 \tag{4}$$

where x is the sampled time series, E the mathematical expectation of the series and $Kurt\ (x)$ is the kurtosis of the signal. The procedure to find the adaptive wavelet filter was to vary the parameters of the Morlet wavelet to produce different daughter wavelets. The value of the kurtosis for each daughter wavelet was compared to find the best kurtosis. Kurtosis was a parameter that was sensitive to the shape of the signal. To determine where the defect was, time-domain features were used, such as the crest factor.

3. Vibration signal analysis

Vibration signal can be obtained by three basic measurements: position, velocity and acceleration. Position sensors provide an indication of the stress. In long lifetime systems, such as ships, the vibration signal analysis can be replaced by the infrared periodic examination. Velocity sensors provide the status of the monitored system in the sense of wear and tear. This indicator is also not practical for on-line diagnostics of an operating system. In simple cases, accelerometers are force indicators due to proportionality relation. However, multibody systems could be discussed, which are more complex. Force provides an insight into the system's dynamics. As a consequence, accelerometers provide time-varying information about the system's status. This makes it convenient for the on-line diagnostics of the operating system, i.e. the ship main engine. Fig.1 shows an illustration of the vibration signal application.

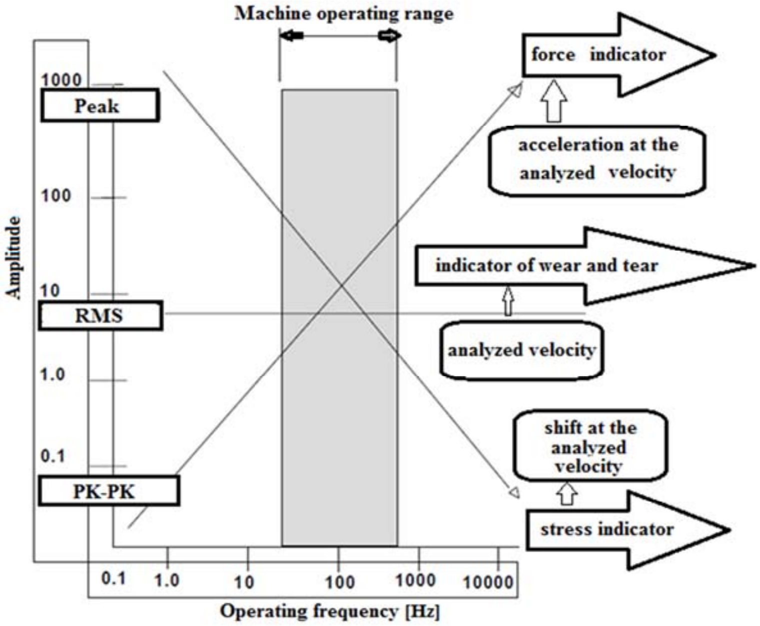


Fig. 1 Position, velocity and acceleration as indicators of force, wear and tear and stress

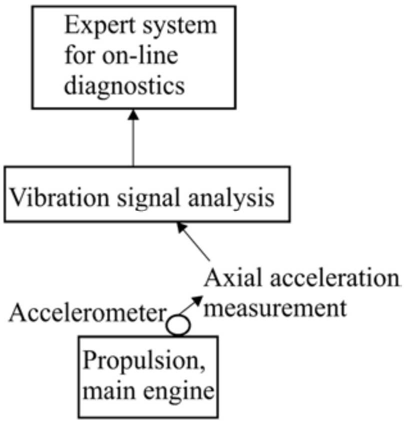
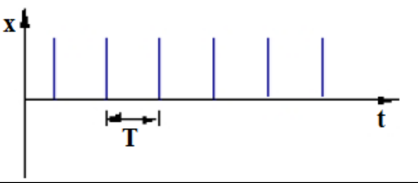
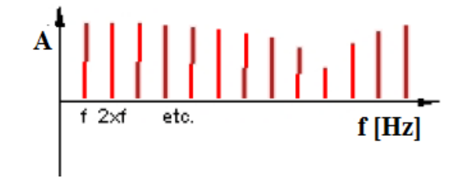
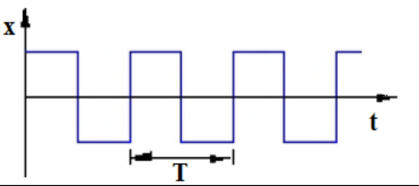
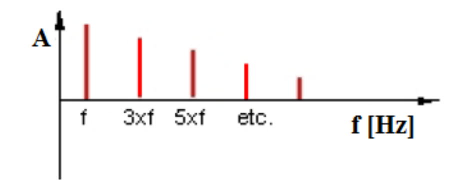
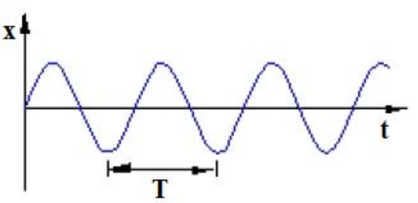
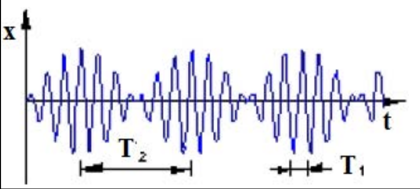
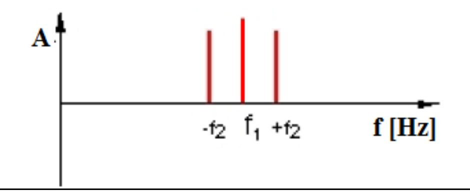
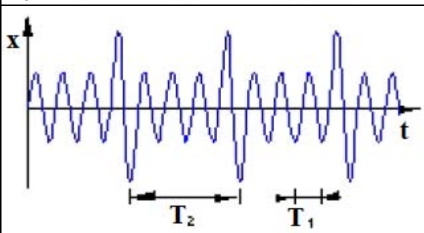
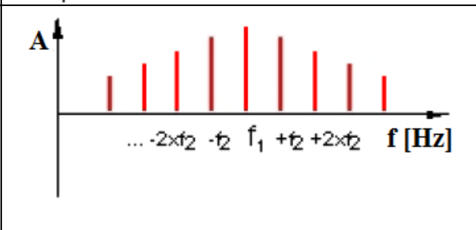


Fig. 2 Role of the vibration signal analysis in fault diagnostics

However, the analysis of the vibration signal has to be seen in a wider perspective to understand the purpose of this research area. Fig. 2 shows the role of the vibration signal analysis in the case of a ship main propulsion engine. Sensors are placed on the engine to monitor axial vibrations under acceleration. The number and type of the sensors depend on the application and desired measurements (Fig. 2, accelerometer or accelerometers). The signal analysis is performed with the obtained sensor data. It can be, for example, the FT analysis. In our case, the WT analysis is used. The analysis gives an insight into different domain representations of the original signal. In some instances, it is only necessary to transform the original time-domain signal into the frequency-domain signal. The question whether it is necessary to perform the frequency or the time-frequency analysis can be answered at a higher level of expert knowledge. This knowledge is used to design an on-line diagnostic expert system. Hence, it can be seen that the role of the vibration signal analysis is in obtaining an input to the expert system, which completes the diagnosis of the fault.

Table 1 shows how the frequency-domain signal can be used to diagnose some fault states of the engine. It is only a simple guideline. There are many details that could be discussed. For example, the row “sin wave“ is used to detect unbalance. This is simplification, because there are dynamical unbalances, where wing frequencies appear, and frequency ratios (f_1/f_2 and f_1/f_3, where f_1 is the central and f_2 and f_3 are the wing frequencies) are used.

Table 1 Representation in time and frequency domain of usual failures

Signal type	Time-domain signal	Frequency-domain signal	Fault
Impact impulses of the same frequency			Rubbing of rotor
Rectangle vibration			Mechanically labile
Sin wave			Unbalance
AM sin wave			Eccentricity of gear (general problem)
AM sin wave			Broken gear (local problem)

4. WT analysis of the vibration signal – an example of comparison analysis for Daubechies wavelets

In this section, we will illustrate why the WT is well suited for the acceleration signal and not so well suited for the velocity or position analysis. Figure 3 shows an example of the vibration signal (μm). It can be seen that the position changes are the slowest in the time domain. The velocity signal is more sensitive to small changes due to the properties of the differentiation. Finally, the acceleration signal is the most non-stationary. Its shape can exhibit behaviour of the signal with several harmonics.

Figure 4 shows the FT analysis of the vibration signal. It can be seen that the FT is well suited for the postion analysis. However, the most important signal, i.e. acceleration, exhibits almost a continious spectrum, which makes it not well suited for feature extraction.

The most important part in the wavelet analysis is to determine weather it is useful to use details, approximations or wavelet tree. Figure 5 illustrates that it is not possible to combine details and approximation, because they are not of the same order of magnitude. This is a correct conclusion for all three types of the vibration signal.

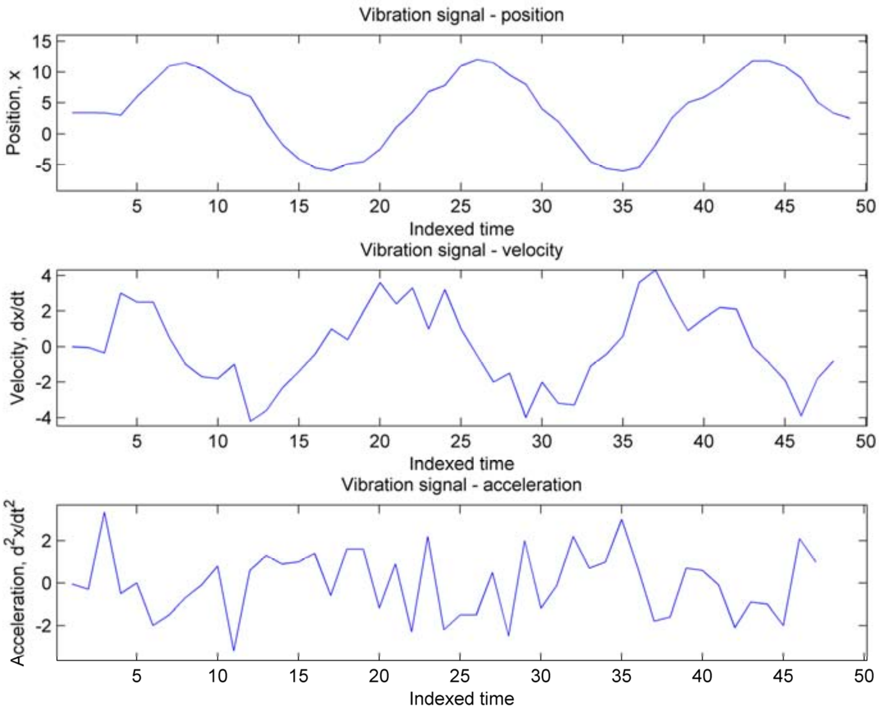


Fig. 3 An example of vibration signal for position, velocity and acceleration (not in physical units due to division by indexed time and not physical time)

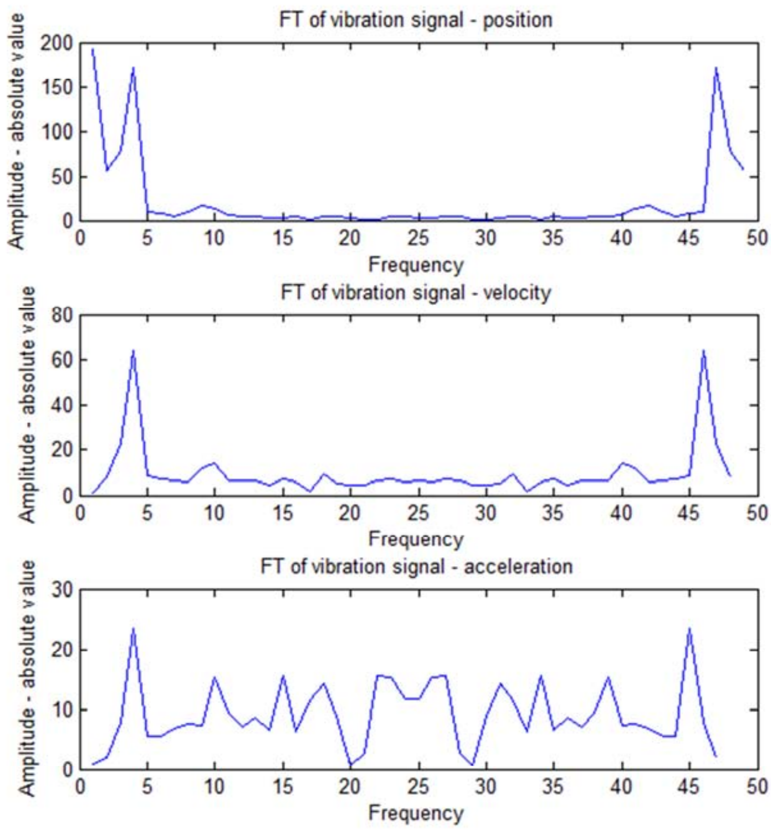
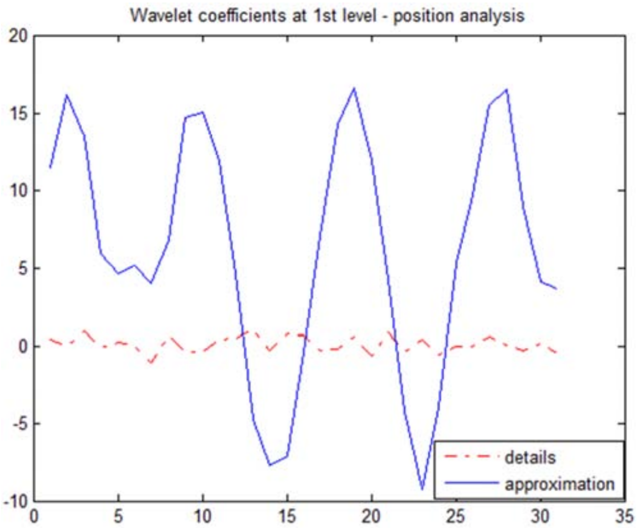
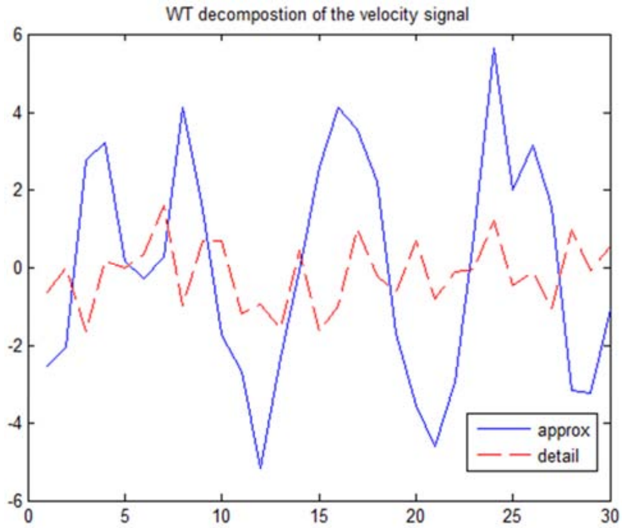


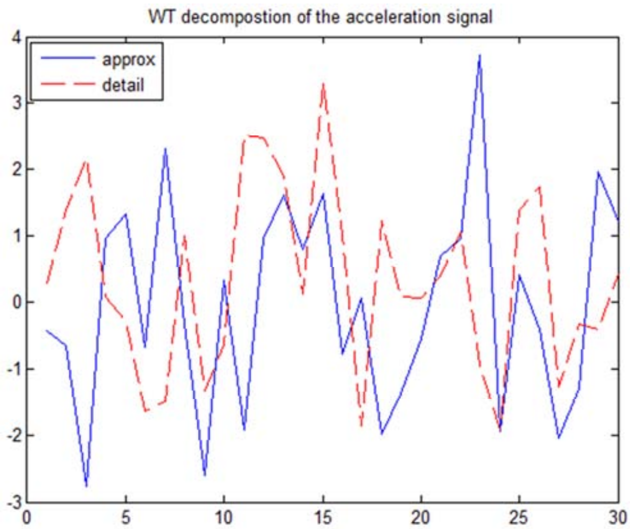
Fig. 4 FT analysis of position, velocity and acceleration



a)



b)



c)

Fig. 5 Details and approximation of WT – comparison of amplitudes:
a) position signal, b) velocity signal, c) acceleration signal

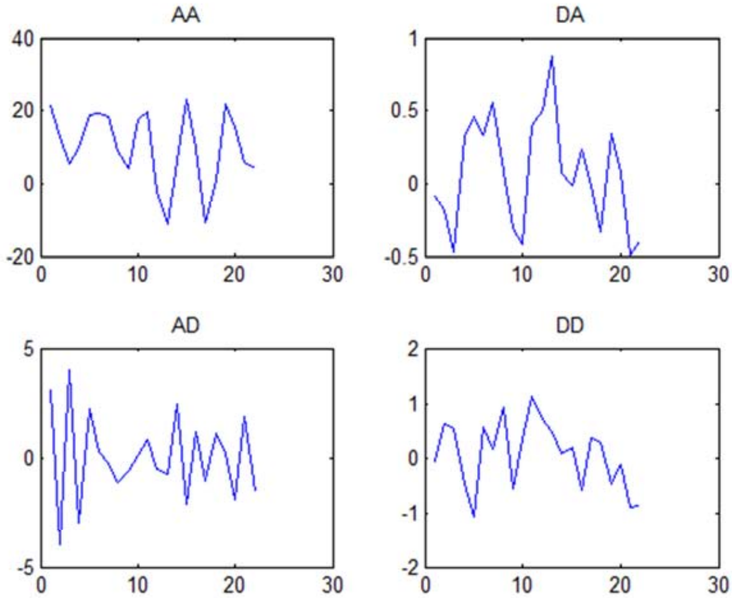


Fig. 6 Second level wavelet analysis of position vibration signal: approximation of the approximation of first level (left upper corner, AA), AD – details of the approximation of first level, DA – approximation of first level details, DD – details of the first level details

Figure 6 shows the second level decomposition by the WT. In the wavelet tree analysis, expert knowledge is used to determine which branch of the WT will be used for specific application.

Figure 7 shows the obtained components of the vibration signal by the WT at 4th level of decomposition. The original signal is decompossed to basic signal components.

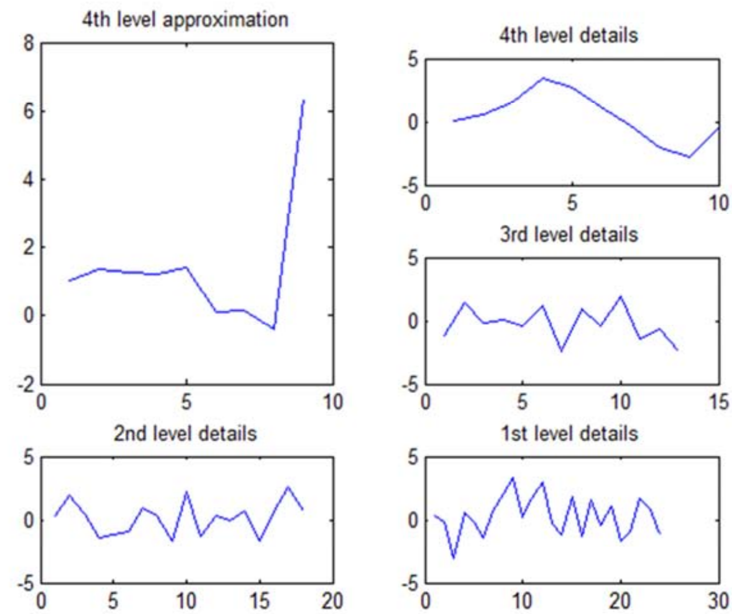


Fig. 7 Acceleration signal analysis by WT at 4th level of decomposition with Daubechies wavelet of 4th order (Matlab notation: db4)

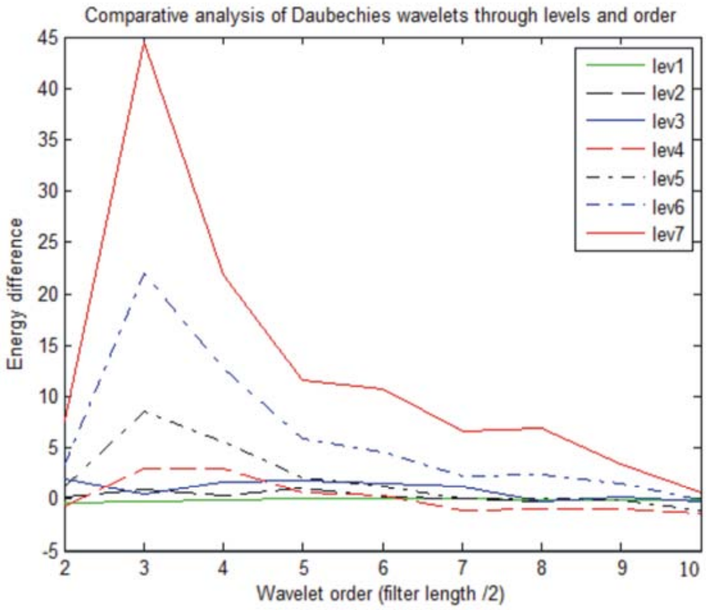


Fig. 8 Results of numerical experiments for Daubechies wavelets: energy difference for levels of decomposition (1-7) and different wavelet orders (Matlab notation db2 – db10)

Figure 8 shows the results of the comparison analysis for 7 levels of decomposition. The comparison measure is the energy difference between the decomposed and the sourced signal. It can be seen that the worst case is around 3rd wavelet order. Furthermore, the increase in the filter length can be traded off with the energy difference.

Table 2 Comparison of the best experimental results by the level of decomposition, length of the approximation coefficients vector and absolute energy difference

Wavelet	Decomposition level	Length of approximation	Absolute energy difference	Note
~~db7~~	~~1~~	~~30~~	~~0.0013~~	~~Best for the level~~
~~db7~~	~~2~~	~~21~~	~~0.0182~~	~~Best for the level~~
db8	5	16	0.0281	Best for the level
db7	5	14	0.0455	
~~db9~~	~~1~~	~~32~~	~~0.0497~~	
~~db6~~	~~1~~	~~29~~	~~0.1013~~	
~~db5~~	~~1~~	~~28~~	~~0.1092~~	
~~db8~~	~~1~~	~~31~~	~~0.1329~~	
~~db4~~	~~1~~	~~27~~	~~0.1331~~	
~~db6~~	~~2~~	~~20~~	~~0.1412~~	
db9	5	17	0.1439	
~~db2~~	~~2~~	~~14~~	~~0.1472~~	
db10	6	19	0.1562	Best for the level
~~db10~~	~~1~~	~~33~~	~~0.1597~~	
~~db9~~	~~2~~	~~24~~	~~0.1787~~	
~~db10~~	~~2~~	~~26~~	~~0.1821~~	

As can be seen from Table 1, the results for levels 1 and 2 are naturally the best, because there is a small difference between the original and the processed approximation. However, heuristically speaking, these levels are not sufficient to process the data satisfactorily. Therefore, these results appear as strikethrough entries in the Table. Since heuristics is not a sufficient criterion of science, it is necessary to find another criterion to discard as many results as possible in order to choose the optimal wavelet combination. This criterion is the length of the coefficient vector, which should be minimal. In this case, the approximation coefficients are used. From the remaining results, two best results are obtained for db8 and db7, both at 5^{th} level of decomposition. Wavelet db8 has a better value of the energy criterion, and db7 has a better value of the length criterion.

5. Conclusions

Signal analysis techniques include signal transformation from the original to the transformed domain. The usual original domain is time domain. The time domain is not suited for a diagnostic tool for any engine, e.g. turning machine or ship propulsion engine. Therefore, scientists developed many methods to transform the time domain signal in a better suited domain. There are different methods to design an engine diagnostic tool. An effective method is also the order tracking method, which is used for variable-speed machines. A special case is the slice analysis. However, our primary concerns were WT methods.

This paper presents advantages of the WT for the acceleration vibration signal analysis due to its non-stationary nature. The advantages are shown on the case presented in Section 4. The results also show that the WT should be carefully used due to the fact that every signal is not best suited for the application of the WT. In some cases, there are better time-frequency transforms, as mentioned in Section 2.

The role of the time-frequency methods is in the preparation of the input to the expert system for on-line fault diagnostics.

The results of the numerical experiments presented in Section 4 show that the wavelets db8 and db7 at 5^{th} level of decomposition are best suited for the analysis of the case signal. Although only the Daubechies wavelets were in the scope of the comparison analysis, the paper demonstrates the suitability of the WT application in the acceleration vibration signal analysis.

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