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PROBABILISTIC MODELS OF REDUCTION IN ULTIMATE STRENGTH OF A DAMAGED SHIP

Summary

The aim of this paper is to develop a probabilistic model of the reduction in the bending moment capacity of an oil tanker following grounding and collision accidents. The approach is based on the Monte Carlo (MC) simulation using the probability distributions of damage parameters proposed by the International Maritime Organization (IMO). The reduction in ultimate strength in the case of grounding is calculated by applying existing design equations using the concept of the grounding damage index (*GDI*) and assuming grounding on a conically shaped rock. Design equations for collision damage are originally developed in the present paper by assuming rectangular box damage. The modified Paik-Mansour method is employed for residual strength assessment when developing the design equations. A case study is presented for an Aframax tanker resulting in the Weibull probability distributions fitted to the histograms of residual strength obtained with MC simulations. The obtained probability distributions are intended for structural reliability assessment of damaged ships.

Key words: *damaged tanker, ultimate strength, regression equation*

1. Introduction

The structural failure of an oil tanker may occur due to unfavourable environmental conditions or due to human errors during the design or operation of the ship. The most frequent forms of ship accidents are collision with another ship or grounding. In the case of such accidents, the ship strength could be significantly reduced while still water loads may increase and could become a considerable cause of structural overloading. A damaged ship may collapse after a collision or grounding if she does not have adequate longitudinal strength. Such a collapse can occur when the hull's maximum residual load-carrying capacity (the ultimate hull girder strength, the bending moment capacity) is insufficient to sustain the corresponding hull girder loads applied [1-3].

Ship structural designers are unavoidably faced with the question as to how the ship structure would behave in the case of an accident. The aim is to avoid breaking of the ship in two parts and sinking of the ship even if the ultimate bending moment capacity is reduced because of the damage. However, from the *a priori* perspective of the ship designer, ship damage may occur in a number of ways, while the parameters used to describe damage, the so-called damage parameters, are random quantities. Consequently, the ultimate longitudinal strength of the damaged vessel is also a random value depending on the probability distributions of the damage parameters (damage size and damage location). The International Maritime Organisation (IMO) [4] proposed such probability distributions for the cases of collision and grounding of oil tankers, based on available tanker casualty statistics. Strictly speaking, the probability distributions proposed by the IMO are intended to assess the acceptability of alternative oil tanker designs with regard to the prevention of oil outflow. It is reasonable to assume, however, that damage causing oil outflow and residual strength have the same physical and statistical origin. Therefore, in the present study, the IMO probabilistic models of damage parameters are employed for the residual strength assessment, although these models are not explicitly intended for that purpose. The same assumption is adopted in [3].

In the present study, a probabilistic description of the ultimate longitudinal strength of a double-hull oil tanker damaged by grounding and collision is investigated. In the case of grounding, the methodology proposed by Paik et al. [5] is adopted, assuming that grounding is caused by a conically shaped rock. The reduction in ultimate strength is then calculated by applying the design equations developed by Kim et al. [6] and using the concept of the grounding damage index (*GDI*). For the case of collision, damage is assumed to have a shape of a rectangular box. Design equations are developed in this paper, based on the calculation of the residual ultimate hull girder strength for different sizes of damage boxes. The modified Paik-Mansour method is employed for the residual strength assessment of the damaged ship [7].

In a recent study [8], several design formulas are proposed to predict residual strength of a ship in a damaged condition. Pure incremental algorithm was employed considering the rotation of the neutral axis in damaged conditions. With respect to this, the present study is similar to the studies [5], [6] and [8]. The main difference between these studies and the present one is that design equations are used herein to develop probabilistic models of ultimate strength reduction following collision and grounding accidents.

The procedure of probabilistic models development consists of sampling damage parameters by means of the MC simulation, considered as independent random variables, complying with the probability distributions proposed by the IMO. Once the damage parameters have been defined for each random outcome of the MC simulation, residual ultimate longitudinal strength is determined by using corresponding design equations. The appropriate probability distribution functions are then fitted to the histograms representing random loss of the intact ultimate bending capacity. Finally, accuracy of the simulation and confidence in the results are discussed using appropriate statistical methods.

The contribution of the present paper is the definition of the probabilistic models for residual strength of a damaged double hull oil tanker which are then intended for structural reliability assessment. Such reliability assessments have mostly been performed considering the assumed damage [1],[9],[10]. In [3] probabilistic descriptions of vertical still water and wave bending moments and hull girder residual strength following grounding damage has been used within a Bayesian network framework. That study has contributed to the development of a more rational treatment of accidental conditions in the structural design of the ship. Structural reliability assessment (SRA) of damaged ships has also been performed in [9],[10],[11],[12] while an extensive review is given in [13]. The final goal of the SRA is the application of risk-based methods in different areas: a) design of marine structures [14], b) emergency response actions after accidents [15] and c) in safety assessment of maritime transportation within sensitive, enclosed and coastal waters [16].

2. Calculation methodology

2.1 Grounding

Residual strength assessment of a ship damaged in a grounding accident is performed in [1], where damage to the double bottom structure around the centre line is assumed. Damage is considered to be a rectangular box, where normal and major damage consider the inner bottom plate intact and damaged, respectively. The damage configuration used in the most of the studies is defined by the ABS Guidance notes and it includes damage to the bottom shell plating and bottom longitudinal girders but not to the inner bottom plating [17]. The Harmonized Common Structural Rules propose grounding damage much larger than the ABS [17], but still with no damage to the inner bottom plating [18]. The IMO probabilistic model, however, allows for some small probability that the inner bottom plating will be damaged as well [4]. Such a possibility is taken into account in [3] and [8] by assuming that damage is a rectangular box, i.e. that the breadth of the damage is equal in outer and inner bottom shells. In [5], however, it is assumed that grounding damage is caused by a conically shaped rock. In such a way, the extent of the damage to the inner bottom plate is less compared to the outer bottom, but both types of damage are still correlated by the shape of the rock. This approach, which is fairly realistic from an engineering point of view, is adopted also in the present study.

The procedure proposed in [5] and [6] is based on the damage description using the grounding damage index (*GDI*) and the calculation of the ultimate strength reduction by the nonlinear design equations depending only on the *GDI*. The grounding damage parameters are defined as non-dimensional transverse damage location (x_1), non-dimensional damage height (x_2), non-dimensional damage breadth (x_3) and angle of the rock ($x_4 = \varphi$), as shown in Figure 1. Damage breadth in the outer bottom is calculated from the variable x_3 ($r_1 = x_3 \cdot B$), while the damage to the inner bottom (r_2) is calculated for the assumed conical shape of the rock (Figure 2), applying the following expression:

$$r_2 = r_1 - 2h_{DB} \tan\frac{\phi}{2} \tag{1}$$

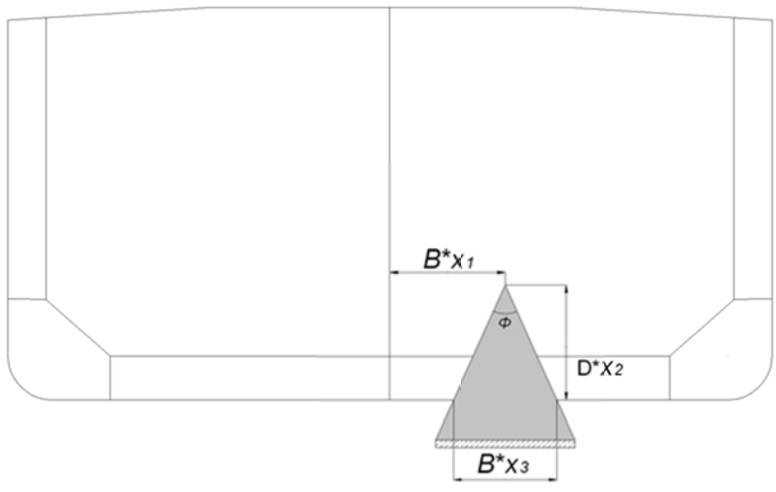

Fig. 1 Location and extent of the grounding damage [5]

Generally, the variable Φ representing the angle of the rock is between 15° and 150° [6]. However, in order to get credible results, it is also necessary to limit the maximum value of Φ for each damage scenario given by a combination of the damage breadth $x_3 \cdot B$ at the outer bottom and the damage height $x_2 \cdot D$:

$$\phi_{max} = 2\tan^{-1}\frac{x_3 \cdot B}{2x_2 \cdot D} \quad (2)$$

In such a way, some kind of correlation is established between the damage parameters and the rock that caused the damage. In case that Φ is independent of the damage parameters, unrealistic results are produced, e.g. very high and narrow damage is caused by a wide rock, which is obviously unrealistic.

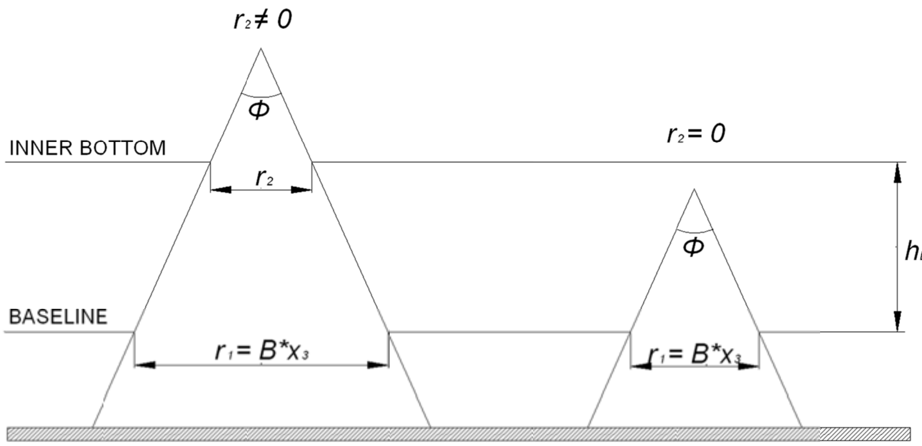

Fig. 2 Conical shape of the rock for cases when double bottom is penetrated (left) and without penetration (right)

Generally, grounding in double-bottomed structures occurs in both the outer-bottom and the inner-bottom structures. Therefore, the *GDI* should identify the extent and the location of grounding damage to both the inner and the outer bottom structures, as it is expressed by equation (3). It includes a correction factor (∝), see equation (4), to reflect the contribution of the inner bottom structure to the ultimate longitudinal strength of the ship [5].

$$GDI = \frac{A_{ro}}{A_{oo}} + \propto \frac{A_{ri}}{A_{oi}} \quad (3)$$

where A_{oi}, A_{oo} are original (intact) areas of the inner and the outer bottom, respectively; A_{ri}, A_{ro} are reduced (damaged) areas of the inner and the outer bottom, respectively; $A_{ro}/A_{oo} = r_1/B = x_3$; $A_{ri}/A_{oi} = r_2/B$.

The correction factor (∝) is determined by the ratio of the slopes (i.e. direction coefficients) of the curves (approximately straight lines) which are representing the influence of the inner and the outer bottom structures (A_{ri}/A_{oi} and A_{ro}/A_{oo}) for the various damage cases on the ultimate longitudinal strength of the ship (M_u/M_{u0}), as shown in equation (4):

$$\propto = {\theta_{IB}}/{\theta_{OB}} \quad (4)$$

where θ_{IB} is the slope of the curve of the ultimate longitudinal strength ratio (M_u/M_{u0}) versus the amount of grounding damage for the inner bottom (A_{ri}/A_{oi}), θ_{OB} is the slope of the curve of the ultimate longitudinal strength ratio (M_u/M_{u0}) versus the amount of grounding damage for the outer bottom (A_{ro}/A_{oo}), M_u and M_{u0} are ultimate strengths of the damaged and the intact ship, respectively. The definition of the above mentioned slopes is shown in [5].

In this paper, the values for the correction factor α are taken from [6], calculated by applying the ALPS/HULL Intelligent Supersize Finite Element Method (ISFEM) [19] for an Aframax tanker. For a hogging condition, the slopes of the curves for the inner and the outer bottom are θ_{IB} = -0.189 and θ_{OB} = -0.253, respectively. Therefore, the correction factor reads ∝=0.747.

For a sagging condition, the slopes of the curves for the inner and the outer bottom are θ_{IB} = -0.057 and θ_{OB} = -0.174, respectively. Therefore, the correction factor reads $\propto$= 0.3276.

The result of the above described calculation is one curve of common influence of the damaged inner and the outer bottom structures on the ultimate longitudinal strength (M_u/M_{u0} - *GDI)* instead of separate curves of dependency between the ultimate longitudinal strength (M_u/M_{u0}) and the double-bottom structures (A_{ri}/A_{oi} and A_{ro}/A_{oo}) for the various damage cases. Regression equations using the *GDI* as the main parameter (equations 5 and 6) are derived from the M_u/M_{u0}-*GDI* dependency for hogging and sagging, respectively, as [5]:

$$M_\mathrm{u}/M_\mathrm{u0} = -0.0036GDI^2 - 0.3072GDI + 1.0 \quad (5)$$

$$M_\mathrm{u}/M_\mathrm{u0} = -0.1941GDI^2 - 0.1476GDI + 1.0 \quad (6)$$

2.2 Collision

The residual ultimate strength assessment of double hull oil tankers damaged in a collision accident is performed in [20], where design equations are proposed as 3rd order polynomials. The assumption adopted in [20] is that the collision damage starts from the deck at the side, which includes the side shell and the side longitudinals and not the deck and the longitudinal bulkhead attached to the damaged zone. Similar assumptions are adopted in [2], where the residual strength index (RIF) is defined as a linear function of the ratio of damage size to ship depth. In [8], damage is divided into damage with minimum and damage with maximum penetration. The former is when only the outer shell is damaged, while the latter represents situations when both outer and inner shells are damaged. For the collision, damage may occur anywhere between main deck and ship bottom. Regarding the correlation between the types of damage to outer and inner shells, it is assumed that the damage extent is the same in both shells. Design equations are provided for the case of an Aframax tanker, where residual strength is given as a function of reduced areas and reduced moments of inertia of the damaged section [8].

Considering that there are only few design equations available in the literature for residual strength following a collision accident, a new set of such equations is developed in the present study. Design equations for ultimate strength are developed for damage that starts from the main deck, considering different damage sizes (from 0% to 100% of the ship height measured from the main deck). Two sets of equations are derived – those for damage to the outer shell only and those for damage to both the outer and the inner shell. Since it is difficult to establish a rational correlation of damage sizes of outer and inner shells, as it was done for the outer and the inner bottom using a conically shaped rock, the same damage size is assumed for outer and for inner shells. Such an assumption is conservative, as it is very likely that damage to the inner shell will be lower than damage to the outer shell. However, it is not possible to justify any other less conservative assumption.

Ultimate strength calculations are performed by using the modified Paik-Mansour method [7]. The method is an extension of the original Paik-Mansour method, which is based on the presumed stress distribution over the hull cross section at the ultimate limit state [21], i.e. yield stress σ_x^Y is assumed for the outer bottom panel or the deck and ultimate stress σ_x^U for the deck or the outer bottom panel together with vertical structural elements, depending on the ship condition (sagging or hogging). The modified Paik-Mansour method assumes different bending stress distributions at the ultimate limit state for the yielded area, i.e. the vertical structure elements close to the tension flange may also have yielded before the hull girder reaches the ultimate limit state. The modified method involves two unknowns, i.e. the height of the buckled element region (h_C) and the height of the yielded element region (h_Y). The condition that the summation of axial forces over the entire cross-section of the hull under a vertical bending moment becomes zero is insufficient to determine two unknowns, and thus an iteration process is required to determine the heights h_C and h_Y. The method is considered as very practical for conceptual studies like the present one.

The collision damage parameters are defined by the parameter non-dimensional transverse damage extent (x_1), non-dimensional vertical damage extent (x_2) and non-dimensional vertical damage location (x_3) as shown in Figure 3.

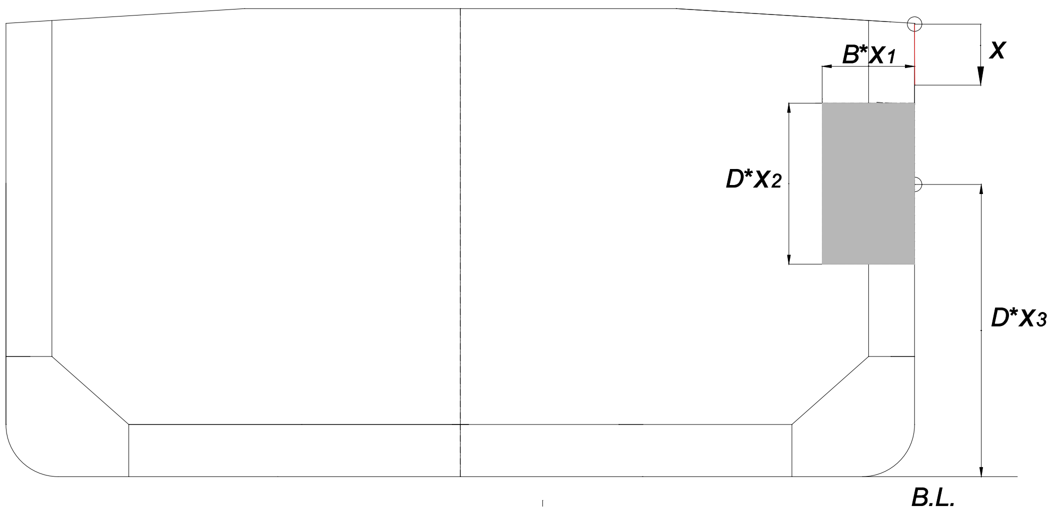

Fig. 3 Location and extent of the collision damage

The ultimate strength calculations were performed by modifying only one parameter – the length of the damaged area (x) presented as a percentage of the ship depth (D), i.e. $x=(l_v/D)\cdot 100$ (%), where l_v is the vertical extent of the damage. Furthermore, it is assumed that the damage starts at the main deck, representing the upper limit of the damaged area. The analysis is performed for two cases: 1) damage to the outer shell and 2) damage to the inner and the outer shell. In the former case, all longitudinal elements between the outer and the inner shell are considered as being damaged.

As a result of the described procedure, a diagram (Figure 4) and regression equations (8-11) are obtained. The results are presented as a percentage of the loss of the ultimate longitudinal strength of the intact ship, i.e.:

$$M_{\mathrm{uloss\%}}(x) = \left(1 - \frac{M_{\mathrm{u}}(x)}{M_{\mathrm{u0}}}\right) \cdot 100 \tag{7}$$

where $M_{\mathrm{u}}(x)$ and M_{u0} are the ultimate strengths of the damaged and the intact ship, respectively.

For the outer shell damage for hogging and sagging, respectively:

$$M_{\mathrm{uloss\%}}(x) = 0.0000298x^3 - 0.0045967x^2 + 0.2233738x \tag{8}$$

$$M_{\mathrm{uloss\%}}(x) = 0.0000107x^3 - 0.0030543x^2 + 0.2912896x \tag{9}$$

For the outer and the inner shell damage for hogging and sagging, respectively:

$$M_{\mathrm{uloss\%}}(x) = 0.0000417x^3 - 0.0068157x^2 + 0.3760094x \tag{10}$$

$$M_{\mathrm{uloss\%}}(x) = 0.0000118x^3 - 0.0042945x^2 + 0.4809863x \tag{11}$$

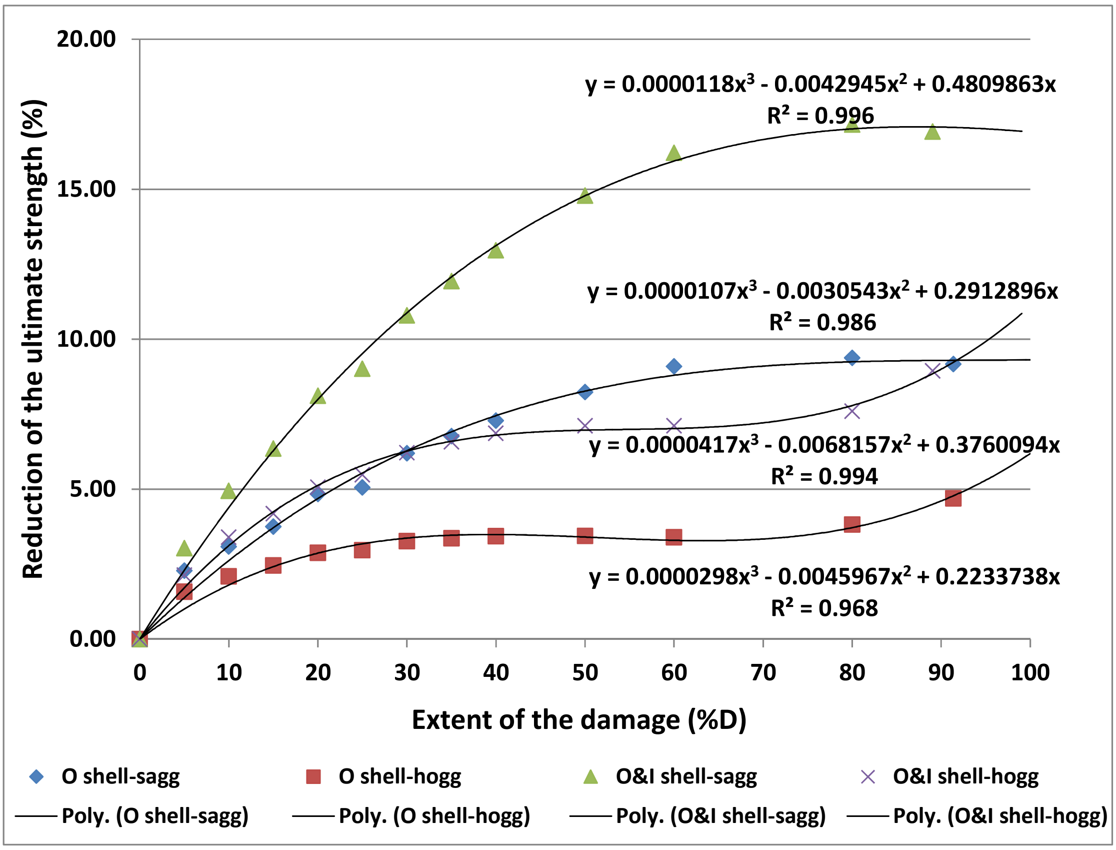

Fig. 4 Curves of reduction in ultimate strength for hogging and sagging

The obtained regression equations are valid for the damage to the outer shell and both the outer and the inner shell starting from the deck at the side. However, damage may occur anywhere between the baseline and the main deck. In such a case, design equations may be approximately employed using the following reasoning. If we assume the function $M_{uloss\%}(x)$ representing ultimate strength loss for damage up to the level x starting from the main deck, then first-order Taylor expansion of the function around this value enables to approximate $M_{uloss\%}$ $(x+\Delta x)$ as

$$M_{\mathrm{uloss\%}}(x+\Delta x) = M_{\mathrm{uloss\%}}(x) + \frac{\partial M_{\mathrm{uloss\%}}(x)}{\partial x}\Delta x \tag{12}$$

where $M_{\mathrm{uloss\%}}$ $(x+\Delta x)$ is the ultimate strength of the damage starting from the main deck, where damage of the size x is increased by Δx. The second term of the right hand side represents the contribution of the damage with size Δx starting at a distance x from the main deck - $M_{uloss\%}(\Delta x)$. Therefore, that value may be considered as approximation of the damage that does not start from the main deck. It may be calculated from design equations (8) to (11) as:

$$M_{\mathrm{uloss\%}}(\Delta x) = M_{\mathrm{uloss\%}}(x+\Delta x) - M_{\mathrm{uloss\%}}(x) \tag{13}$$

To illustrate the applicability of the proposed procedure, damage of between 25% and 75% starting from the main deck may be considered. The ultimate strength loss may then be calculated as:

$$M_{\mathrm{uloss\%}}(25\% - 75\%) = M_{\mathrm{uloss\%}}(75\%) - M_{\mathrm{uloss\%}}(25\%)$$

where $M_{\mathrm{uloss\%}}(75\%)$ and $M_{\mathrm{uloss\%}}(25\%)$ are determined from design equations (8-11).

From equation (12), one may conclude that expression (13) will be more accurate for minor damage, while the accuracy of the procedure will decrease by increasing the extent of the damage.

The verification of the above given formula is performed for three damage cases between the baseline (BL) and the deck. The specification of the damage and the values of the reduction in ultimate strength calculated by equation (12) for both types of damage (only the outer shell or both the outer and the inner shell) for hogging and sagging are given in Table 1.

Table 1 Reduction in ultimate strength for damage between BL and deck (in %)

Damage limit from deck (%D)		Length of damage	Outer shell damage				Outer+inner shell damage			
			$M_{uloss\%}$ hogging		$M_{uloss\%}$ sagging		$M_{uloss\%}$ hogging		$M_{uloss\%}$ sagging	
x (%)	$x+\Delta x$	L (%)	Eq. (8&13)	Direct anal.*	Eq. (9&13)	Direct anal.*	Eq. (10&13)	Direct anal.*	Eq. (11&13)	Direct anal.*
25	75	50	0.29	0.83	3.64	4.33	1.66	1.36	7.37	7.87
40	60	20	0.00	0.00	1.34	1.57	0.23	0.00	2.82	2.92
10	30	20	1.56	0.92	3.66	3.00	3.15	2.02	6.49	5.71
70	90	20	1.26	0.26	0.18	0.27	1.81	0.58	0.43	0.60

* Direct analysis means direct application of the modified P-M method with damage extent as specified in the table

Some differences between the described approximate procedure using equations (8-13) and the direct analysis may be noticed in Table 1. However, taking into account the simplicity of the approach and the overall level of uncertainties, differences seem to be reasonable.

In the new International Association of Classification Societies (IACS) harmonized common structural rules (H-CSR) two methods for assessment of ultimate longitudinal capacity of an intact structure for a double hull oil tanker are used: the one-step method, where non-linear finite element method can be used, and the incremental-iterative method based on the progressive collapse analysis (PCA) [18] . The ultimate strength of the same Aframax tanker is calculated in [22] by the computer program MARS using the implemented PCA. A comparison of the PCA and the modified Paik-Mansour (P-M) method results is given in the Table 2. A comparison of two methods recommended by the IACS is performed in [23], indicating that the one step method gives 2% lower ultimate strength compared to the PCA.

Table 2 Comparison of ultimate strength calculated by PCA and P-M method

Ship condition	Ultimate strength (MNm)	
	MARS	P-M
hogging	10893	9094
sagging	-8470	-8744

One additional remark regarding the accuracy of the proposed approach is that the rotation of the neutral axis due to the side damage is neglected in the present study. The background for this potentially un-conservative assumption is the conclusion from the PhD thesis [23], stating that the reduction ratio of the residual hull girder strength due to the rotation of the neutral axis is almost negligible for the case of oil tankers having suffered outer shell damage. The effect of rotation of the neutral axis has been studied also in [8].

Therefore, in most of the cases, the effect of the rotation of the neutral axis will not have significant influence. In rare cases, when the inner hull is breached and with a large vertical extent of the damage, the described approach may be un-conservative, which should be taken into account in the safety assessment.

2.3 Main particulars of the studied ship

The main particulars of the double-hull oil tanker analysed in the present study are presented in Table 3.

Table 3 Main particulars of the oil tanker

Dimension	Unit (m, dwt)
Length between perp., L_{pp}	234
Breadth, B	42
Depth, D	20
Draught, T	14
Deadweight, DWT	105000
Double bottom height, h_{DB}	2.3
Breadth of inner bottom, b_{DB}	16.4
Position of inner shell from CL	18.95

Definition of the actual double-bottom height (h_{DB}) at the transverse damage position is given by the following expressions:

h_{DB} = 2.3 m (from CL to 16.4 m)

h_{DB} = 5.3 m (from 18.95 m to 21 m i.e. from the inner to the outer shell) (14)

$h_{DB} = 2.3 + \frac{2.3-5.3}{16.4-18.95}(y-16.4)$, from the side girder at 16.4 m to the inner hull,

where y is a variable of the hopper geometry definition i.e. horizontal coordinate which at the same time defines the distance of the damage from CL in the case of grounding.

Definition of the actual double-hull breadth (b_{DH}) at the vertical damage position is given by the following expressions:

b_{DH} = 2.05 m (from 5.3 m to the deck)

b_{DH} = 4.6 m (from the bottom to the double bottom height 2.3 m) (15)

$b_{DH} = \frac{B}{2} - \left(16.4 + \frac{16.4-18.95}{2.3-5.3}(z-2.3)\right)$, from the double bottom to the double-hull girder at 5.3 m,

where z is a variable of the hopper geometry definition i.e. vertical coordinate which at the same time defines the vertical distance of the damage from BL in the case of a collision.

3. Probabilistic ultimate strength reduction for grounding damage

3.1 Probabilities of grounding damage parameters according to IMO

The probability density functions provided by the IMO [4] for the damage parameters are shown in Figures 5 to 7. Cumulative distribution functions (CDF), calculated by integrating the PDFs, are shown in the same figures. They are adopted as reasonable damage scenarios in terms of non-dimensional transverse damage location (x_1), non-dimensional damage height (x_2) and non-dimensional damage breadth (x_3). The actual damage location, height and breadth are obtained by multiplying x_1 and x_3 by the ship breadth B, while x_2 is to be multiplied by the ship depth D.

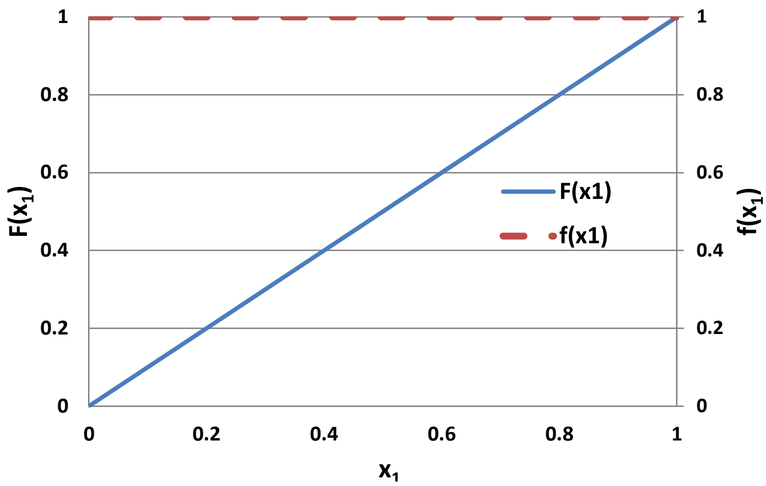

Fig. 5 Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional transverse location of grounding damage (x_1) [4]

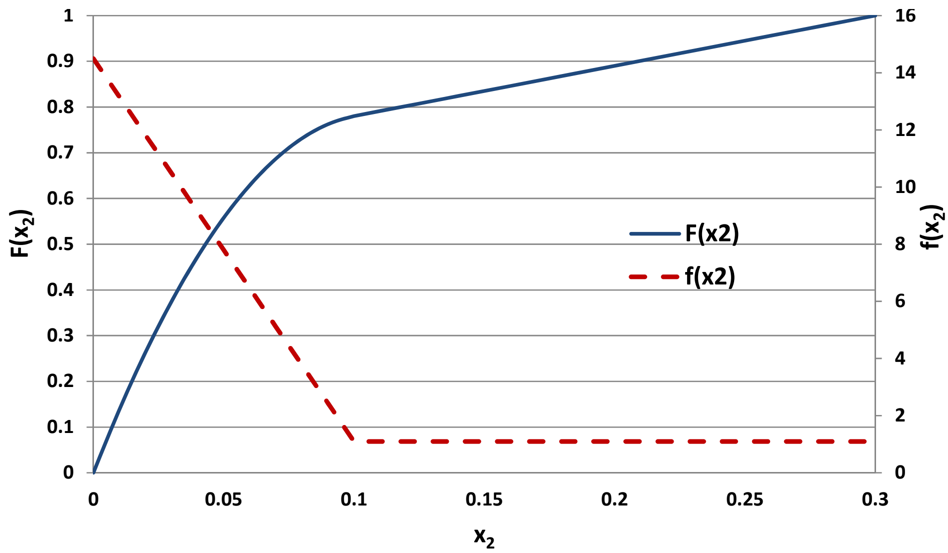

Fig. 6 Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional height of grounding damage (x_2) [4]

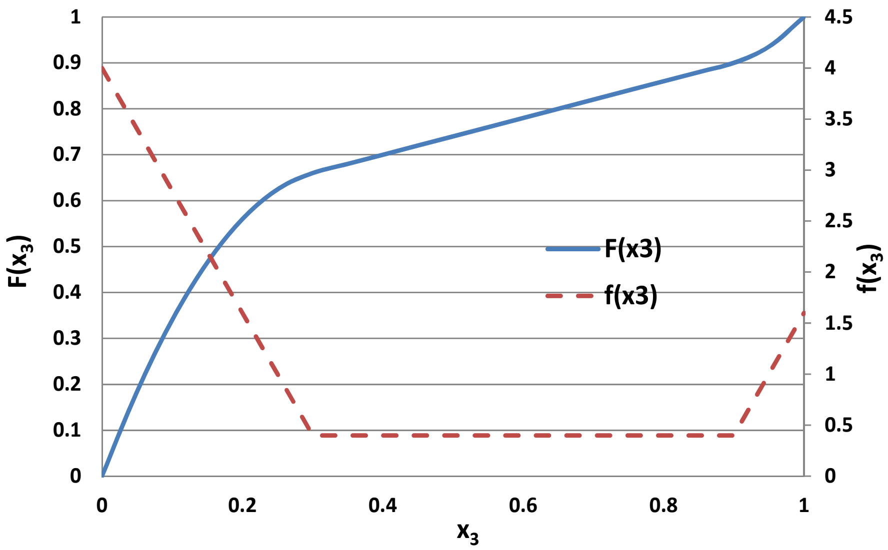

Fig. 7 Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional grounding damage breadth (x_3) [4]

3.2 Monte Carlo (MC) simulation for grounding

Using the principal dimensions of the oil tanker given in Section 2.3 together with the IMO's probability density functions and the assumptions of the rock's shape, grounding damage parameters are simulated by the MC simulation method. Variables x_1, x_2 and x_3 are drawn from their corresponding probability distributions F_1, F_2 and F_3. After that, a random rocking angle is drawn from the normal distribution as described in the previous paragraph. For each outcome of the random damage scenario, the grounding damage index *GDI* is calculated by equation (3) and then the residual ultimate bending moment by equations (5) and (6).

The actual double-bottom height depends on the transverse position of the damage, i.e. it takes the sloped longitudinal bulkhead and the lowest stringer of the double hull into account. Definition of the actual double-bottom height (h_{DB}) at the transverse damage position is given by equations (14).

It is further assumed that the rocking angle is a normally distributed random variable with the mean value $\Phi_{mean}=(15+\Phi_{max})/2$, and the standard deviation $\sigma = (\Phi_{max} - \Phi_{mean})/2$. Thus, the maximum value is 2 standard deviations away from the mean rocking angle used in each simulation.

Another dependency is introduced in the cases when a half of the damage breadth ($x_3 \cdot B/2$) added to the transverse damage location ($x_1 \cdot B$) exceeds a half of the ship breadth. In such cases, the damage breadth is reduced to the maximum permissible value to avoid unrealistic damage outside the ship's breadth.

1000 MC simulated grounding damage scenarios are calculated. For the known CDFs denoted by $F(x)$ appropriate damage parameters values are calculated as the inverse transformation $x=F^{-1}(u)$, where u is a random number from a uniform distribution in the interval $[0-1]$.

The calculation procedure can be summarised as follows:

1. Simulation of the transverse damage location (x_1) from the cumulative distribution functions shown in Figure 5;
2. Simulation of the damage height extent (x_2) from the cumulative distribution functions shown in Figure 6;
3. Simulation of the transverse damage extent (x_3) from the cumulative distribution functions shown in Figure 7;
4. Reduction in the transverse damage extent x_3 is formulated as follows:

   Introduce the condition: $B \cdot x_3/2 > B/2 - B \cdot x_1$

   Damage extent if the condition is fulfilled,
   i.e. damage is outside of the ship: $B/2 - B \cdot x_1$

   Damage extent if the condition is not fulfilled: $B \cdot x_3/2$
5. Simulation of the assumed angle of the rock (x_4) according to the normal distribution with the following parameters:

$$\Phi_{max} = 2\tan^{-1}\frac{Bx_3}{2Dx_2};\ 15 \le \Phi_{max} \le 150$$

$$\Phi_{mean} = \frac{15+\Phi_{max}}{2};\ \sigma = \frac{\Phi_{max} - \Phi_{mean}}{2}$$

6. Check to see if the actual double-bottom height is penetrated. Calculation of r_2 according to equation (1), A_{ro}/A_{oo} and A_{ri}/A_{oi}.
7. Calculation of the grounding damage index (*GDI*) according to equation (3).

8. Calculation of residual strength for the damaged ship in hogging and sagging condition by applying regression equations (5) and (6).
9. Steps 1-8 are repeated N=1000 times.
10. Analysis of the results of the simulation i.e. damage cases using the probability density functions.
11. Fitting of the distribution functions to the MC simulation data.

3.3 Results of the analysis for the ship damaged by grounding

The percentages of loss of ultimate strength $M_{\mathrm{uloss\%}} = \left(1\text{-}\frac{M_{\mathrm{u}}}{M_{\mathrm{u0}}}\right)\cdot 100$ for hogging and sagging condition are presented in form of histograms (Figures 8 and 9). Based on the shape of the histograms, either exponential or 2-parameter Weibull distributions are considered as good candidates to approximate histograms by the theoretical probability function. The method of moments is employed to fit the theoretical distributions. Thus, the Weibull distribution is fitted by matching the distribution moments to the sample average and standard deviation. The Weibull probability density and cumulative distribution function are given as:

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \cdot e^{-\left(\frac{x}{\lambda}\right)^{k}} \qquad\qquad F(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^{k}} \tag{16}$$

respectively, where k, λ are the shape and scale parameters of the Weibull probability distribution function, while x is the random variable representing percentage of the loss of ultimate strength, i.e. $x = (1\text{-}M_{\mathrm{u}}/M_{\mathrm{u0}})\cdot 100$. It should be noted that exponential distribution is a special case of the Weibull distribution, with exponent k=1.

Mean value μ and variance σ^2 of the Weibull distribution are given as [11][9]:

$$\mu = \frac{1}{\lambda}\Gamma\left(1+\frac{1}{k}\right) \qquad\qquad \sigma^2 = \frac{1}{\lambda^2}\Gamma\left(1+\frac{2}{k}\right) - \mu^2 \tag{17}$$

Gamma functions, $\Gamma(2) = 1$ and $\Gamma(3) = 2$, so obvious for the exponential distribution mean value and the standard deviation are the same and read $\mu = \sigma = 1/\lambda$. Thus, the parameter λ of the exponential distribution is determined from the average value $\bar{x}$ of the random variable x calculated from the histogram as $\lambda = 1/\bar{x}$.

Fitting of the Weibull distribution is not so simple, as there are 2 unknown parameters k and λ that cannot be directly determined. The procedure of calculating these two parameters requires their optimization so that the sample mean value $\bar{x}$ and the variance σ_x^2 calculated from the simulation process are equal to the mean value and the variance of the Weibull distribution given by equations (17). The procedure may be efficiently performed by using the Solver option in MS Excell.

The theoretical distributions fitted to the data are presented in Figures 8 and 9 for hogging and sagging, respectively. As it is not obvious from the figures which of the two theoretical distributions represent better approximation, the analysis using χ^2-test is performed. The results of the χ^2-test did not confirm adequacy of the probability distributions used. However, the conclusion is that the Weibull function provides a better fit for hogging, while the exponential function is more suitable for sagging.

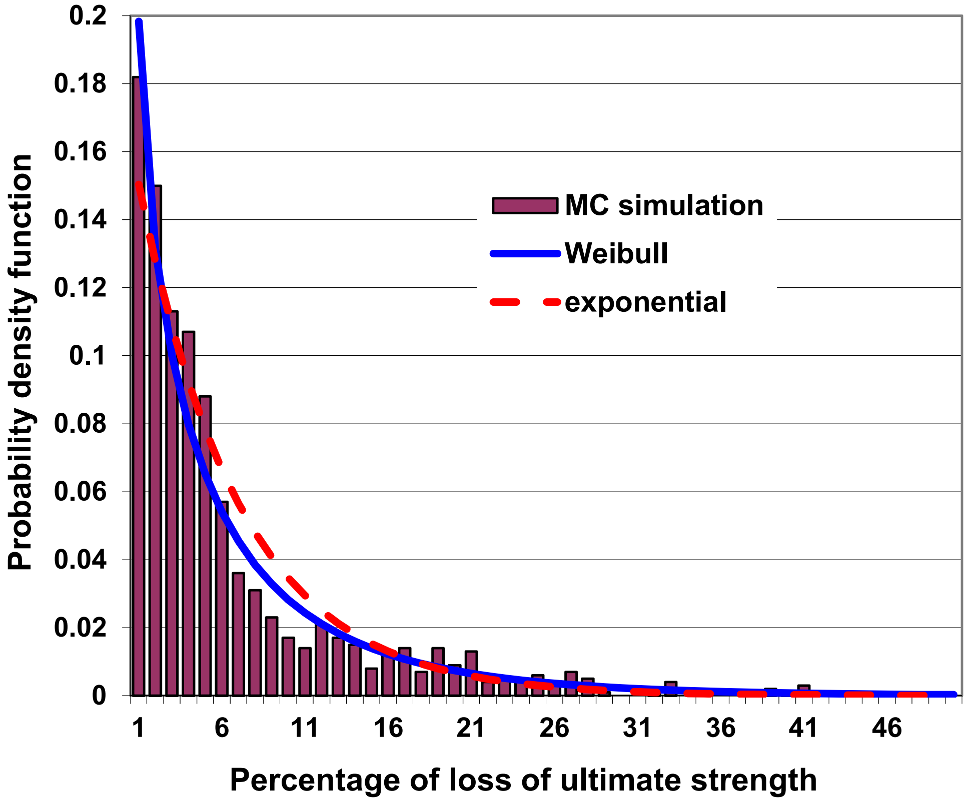

Fig. 8 Histogram of loss of ultimate strength for hogging condition with fitted functions (Histogram-mean value: $\bar{x} = 6.157$, Weibull-shape and scale parameter: $k = 0.809$, $\lambda = 5.624$)

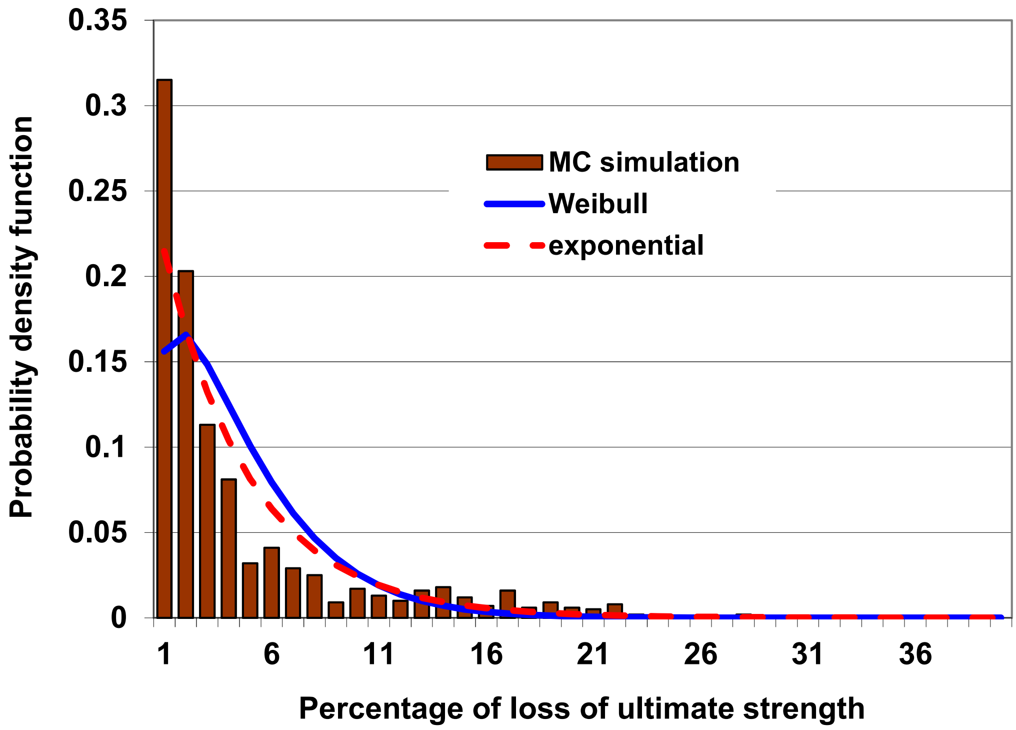

Fig. 9 Histogram of loss of ultimate strength for sagging condition with fitted functions (Histogram-mean value: $\bar{x} = 4.160$, Weibull-shape and scale parameter: $k = 1.232$, $\lambda = 4.445$)

4. Probabilistic ultimate strength reduction for collision damage

4.1 Probabilities of collision damage parameters according to IMO

The probability density functions provided by the IMO [4] are shown in Figures 10 to 12. Cumulative distribution functions (CDF), calculated by integrating PDFs, are shown in the same figures. They are adopted as reasonable damage scenarios in terms of non-dimensional transverse damage penetration relative to the ship's breadth (x_1), non-dimensional vertical damage extent relative to the ship's depth (x_2) and non-dimensional vertical distance between the baseline and the centre of the vertical extent x_2 relative to the distance between the baseline and the deck level (normally the ship's depth) (x_3). The actual damage location, height and breadth are obtained by multiplying x_1 by the ship's breadth B, while x_2 and x_3 are to be multiplied by the ship's depth D.

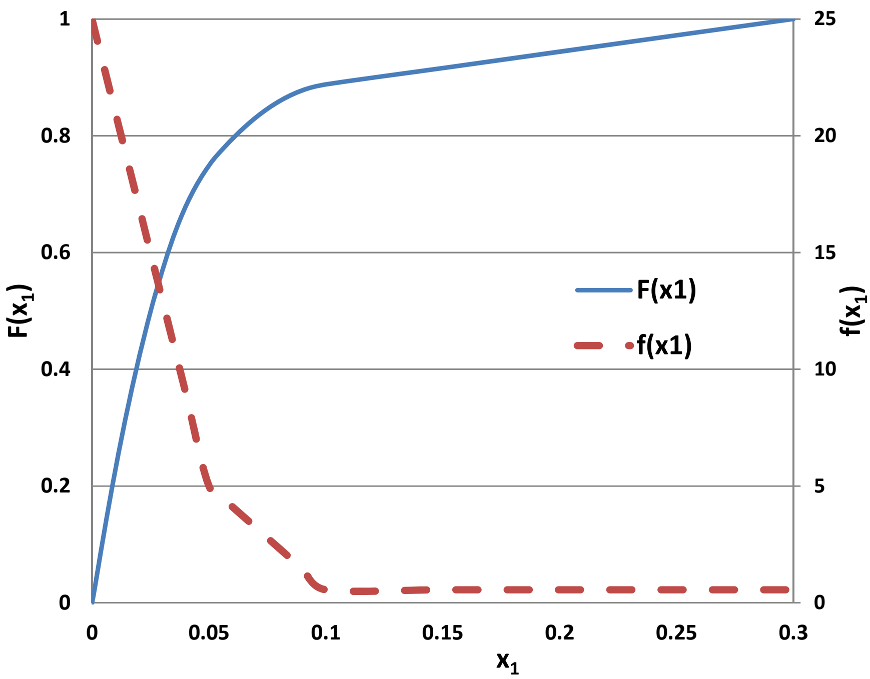

Fig. 10 Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional transverse extent of collision damage (x_1) [4]

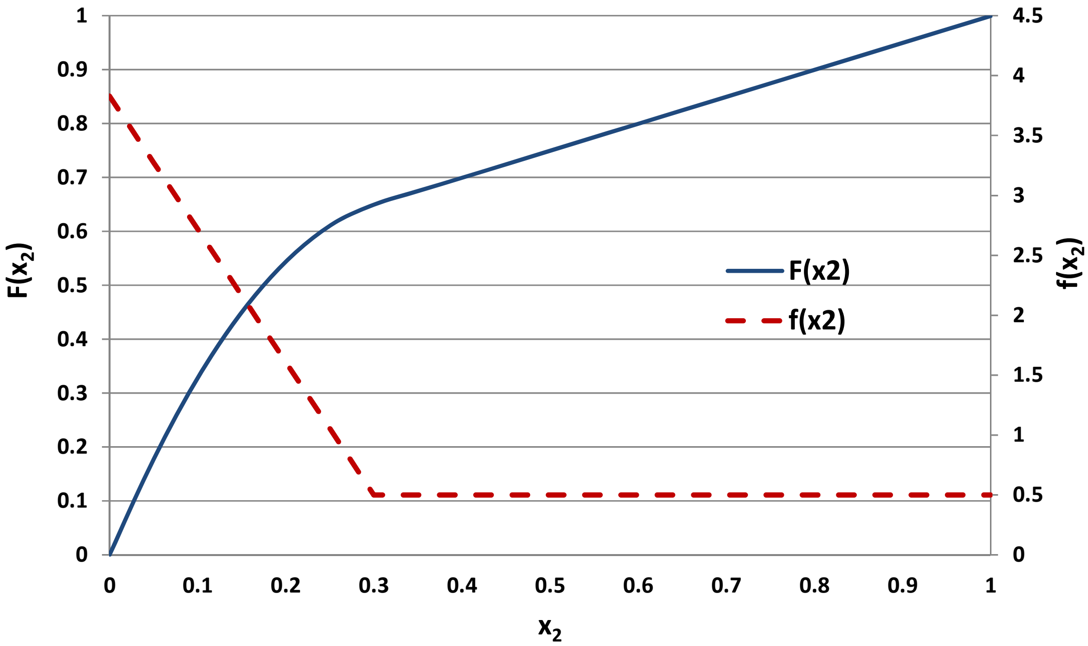

Fig. 11 Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional height of collision damage (x_2) [4]

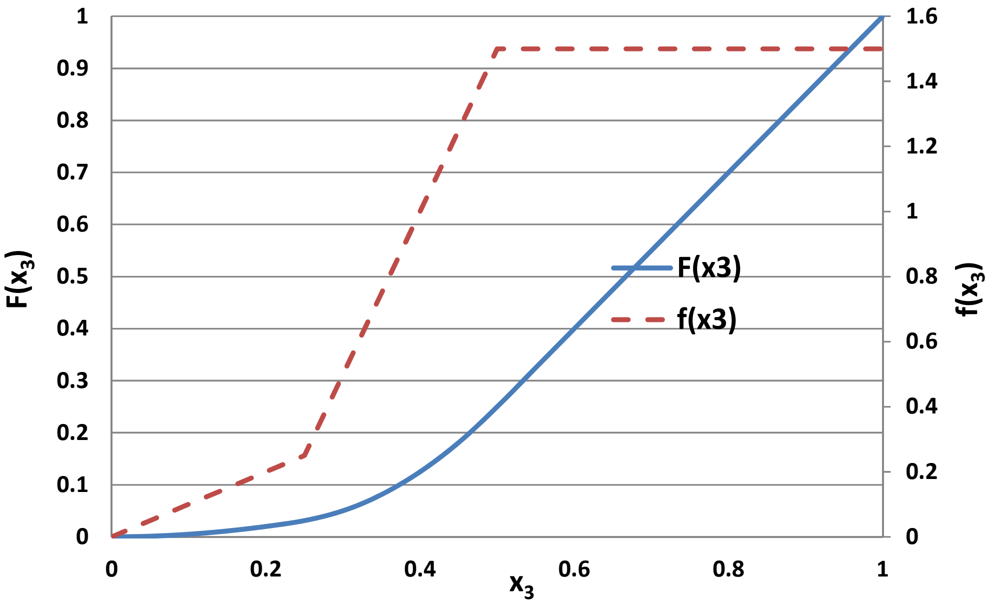

Fig. 12 Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional vertical location of collision damage (x_3) [4]

4.2 Monte Carlo (MC) simulation for collision

Using the principal dimensions together with the IMO's probability density distributions collision damage parameters may be simulated by applying the MC simulation method. Variables x_1, x_2 and x_3 are drawn from their corresponding cumulative probability distributions F_1, F_2 and F_3. For each outcome of the random damage scenario the loss of the ultimate bending moment is calculated by equations (8-11).

The actual double-hull breadth depends on the vertical position of the damage, i.e. it takes the slope of the hopper into account. Definition of the actual double-hull breadth (b_{DH}) at the vertical damage position is given by equations (15).

1000 MC-simulated collision damage scenarios are calculated. For the known CDFs denoted by $F(x)$ from the interval 0-1 appropriate damage parameters values are calculated as the inverse transformation $x=F^{-1}$.

The calculation procedure is similar to the procedure for grounding and can be summarised as follows:

1. Simulation of the transverse damage extent (x_1) from expressions relating to curves in Figure 10
2. Simulation of the vertical damage extent (x_2) from expressions relating to curves in Figure 11
3. Simulation of the vertical damage location (x_3) from expressions relating to curves in Figure 12
4. Definition of damage types relating to the location of boundary edges of the damage:
   - If the upper edge is above the deck, i.e. $x_2/2 + x_3>1$
     Damage extent if the condition is fulfilled: $(1- x_{3+}\, x_2/2)\cdot D$
     Damage extent if the condition is not fulfilled: $x_2 \cdot D$
   - If the lower edge is below BL, i.e. $x_3 - x_2/2<0$
     Damage extent if the condition is fulfilled: $(x_3 + x_2/2)\cdot D$
     Damage extent if the condition is not fulfilled: $x_2 \cdot D$
   - If both edges are between BL and deck or only the lower edge is below BL, we introduce d_1 and d_2 as a distance of the lower and the upper edge of the damage from BL, respectively
     $d_1 = (x_3 - x_2/2)\cdot D$
     $d_2 = (x_3 + x_2/2)\cdot D$
5. Calculation of loss of ultimate strength for the damaged ship in hogging and sagging condition by applying regression equations (8-11) and equation (13).
6. Steps 1-8 are repeated N=1000 times.
7. Interpretation of the results for 1000 random generated variables i.e. damage cases, using probability density functions.
8. Fitting of the exponential and 2-parameter Weibull distribution functions to the MC probability density function.

4.3 Results of the analysis for the ship damaged by collision

The percentages of loss of ultimate strength $M_{\mathrm{uloss\%}} = (1\text{-}\, M_{\mathrm{u}}/M_{\mathrm{u0}})\cdot 100$ for hogging and sagging conditions are presented in form of histograms. The same procedure of fitting the exponential and the Weibull distribution functions to the data as in the case of grounding damage is applied for the collision damage (Figures 13 and 14), too. It is obvious from the figures that there is no difference between the two theoretical distributions for both conditions of the ship. The analysis using $\chi 2$-test was performed showing that both the exponential and the Weibull functions represent a better estimate for sagging compared to hogging.

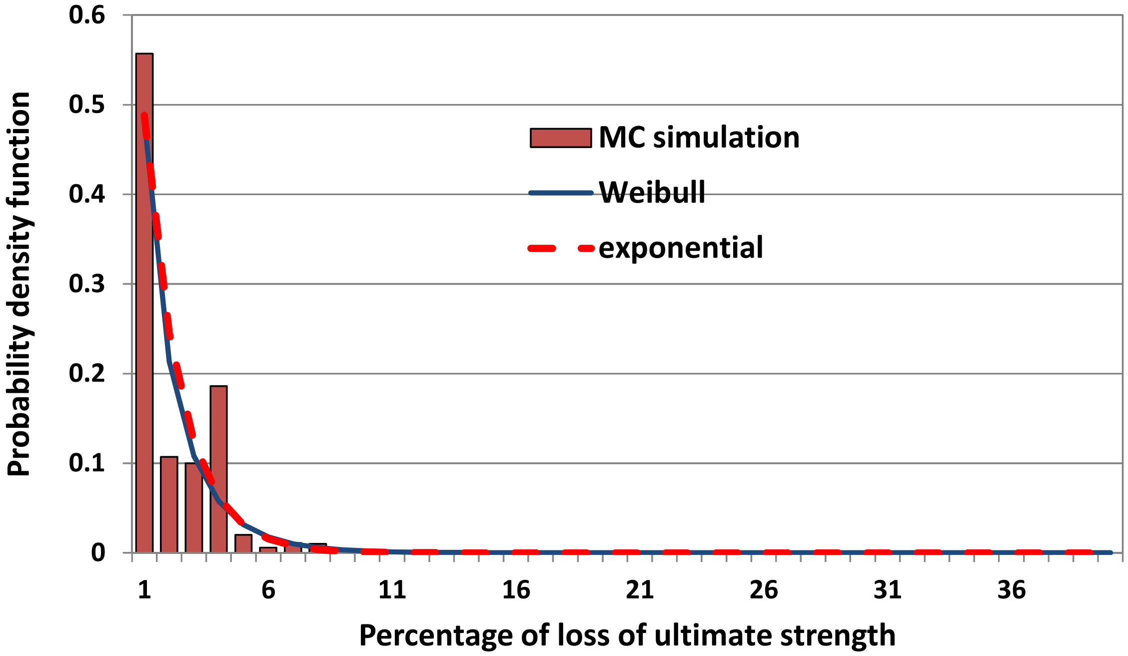

Fig. 13 Histogram of loss of ultimate strength for hogging condition with fitted functions (Histogram-mean value: $\bar{x} = 1.469$, Weibull-shape and scale parameter: $k = 0.876$, $\lambda = 1.349$)

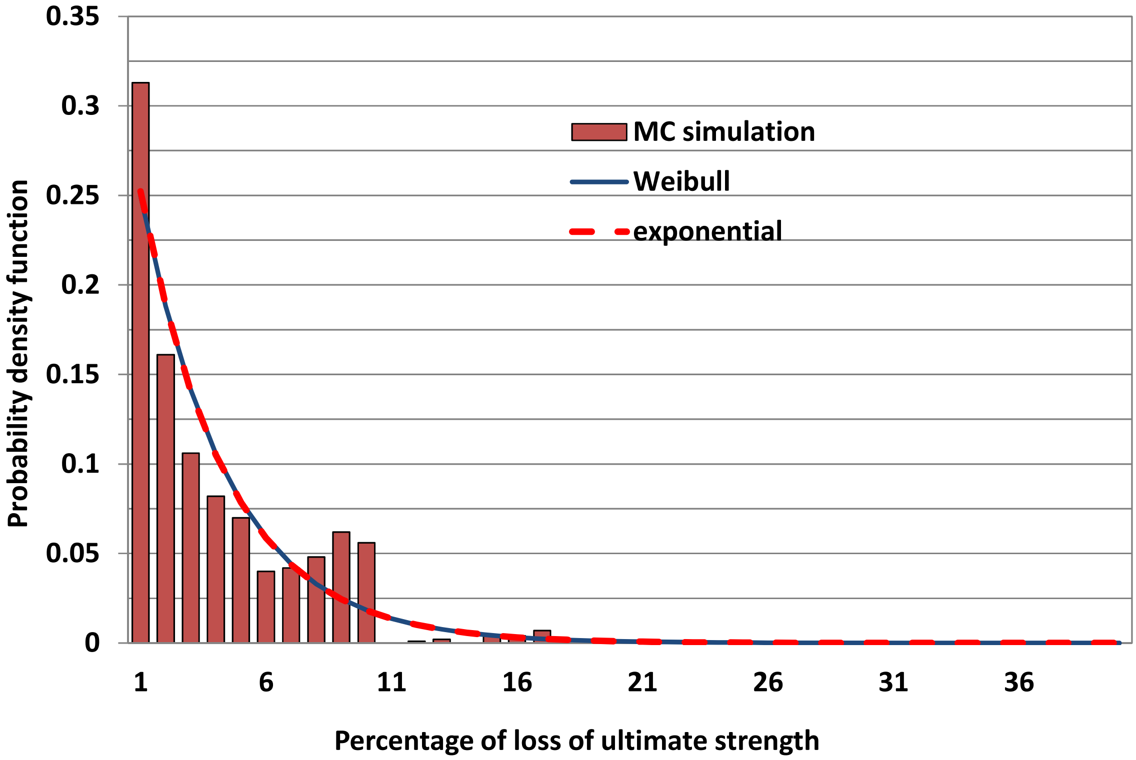

Fig. 14 Histogram of loss of ultimate strength for sagging condition with fitted functions (Histogram-mean value: $\bar{x} = 3.453$, Weibull-shape and scale parameter: $k = 1.007$, $\lambda = 3.451$)

5. Statistical uncertainty and simulation accuracy

Confidence intervals and associated errors of the mean values $\bar{x}$ of the percentage of the loss of the intact bending moment capacity for grounding and collision are performed by using the conventional approach. The 95% confidence interval and the error are given by the following expressions (central limit theorem):

$$95\%\ confidence\ interval = \left(\bar{x} - 1.96\frac{S}{\sqrt{n}}, \bar{x} + 1.96\frac{S}{\sqrt{n}}\right) \tag{18}$$

$$error = \left(\frac{1.96\frac{S}{\sqrt{n}}}{\bar{x}}\right) \cdot 100 \tag{19}$$

where n is the number of simulations (n = 1000), S is the standard deviation of the loss of M_u while 1.96 is a value of a standard normal variate with cumulative probability level related to the 95% confidence level.

Assessment of the accuracy of the simulation process both for hogging and sagging and for grounding and collision damage is given in Tables 4 and 5, by calculating the following parameters: the sample mean of loss of M_u, the standard deviation, the 95% confidence interval and the error on the mean of loss of M_u.

Table 4 Accuracy of the simulation process for grounding

	μ	S	95% confidence interval	error (%)
sagging	4.161	5.62	(3.81, 4.51)	8.37
hogging	6.157	8.05	(5.66, 6.66)	8.10

Table 5 Accuracy of the simulation process for collision

	μ	S	95% confidence interval	error (%)
sagging	3.453	3.63	(3.23, 3.68)	6.52
hogging	1.469	1.69	(1.36, 1.57)	7.13

The results of this statistical analysis indicate that the simulation procedure performed in the present study provides an acceptably accurate estimate of the mean value.

6. Conclusion

Probabilistic models of the loss of ultimate strength of an Aframax oil tanker damaged in collision and grounding accidents are developed based on the MC simulation. The probabilistic models for the transverse location and size of the damage are prescribed by the IMO. It is assumed that grounding is caused by a rock of a conical shape. This assumption is used to correlate damage to the outer and the inner bottom. The angle of the rock is considered as a random variable limited by the height and breadth of the grounding damage. Based on the grounding damage index (*GDI*), ultimate strength of the damaged ship is calculated by the regression equation developed in [6].

The outcome of the MC simulations is the histogram of the loss of ultimate strength showing that most of the damage causes a fairly low reduction in ultimate strength of the ship. The frequency of large losses of ultimate strength is reduced relatively fast. Based on these characteristics, it seems that the uncertainty regarding the loss of ultimate strength may be reasonably described by either the exponential or the 2-parameter Weibull probability distributions. The average loss (in percentage) for the Aframax tanker reads about 6.1% and 4.1% for grounding damage and 1.5% and 3.4% for collision damage for hogging and sagging, respectively.

The main purpose of this study is the development of probabilistic models for the bending capacity of a damaged ship to be used in structural reliability studies within the scope of the safety of the maritime transportation.

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