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# DYNAMIC EXTRA BUSES SCHEDULING STRATEGY IN PUBLIC TRANSPORT 


#### Abstract

This paper presents a dynamic extra buses scheduling strategy to improve the transit service of transit routes. In this strategy, in order to decide when to dispatch an extra bus, the service reliability of transit route is assessed firstly. A model aimed at maximizing the benefit of the extra buses scheduling strategy is constructed to determine how many stops extra buses need to skip from the terminal to accommodate passengers at the following stops. A heuristic algorithm is defined and implemented to estimate the service reliability of transit route and to optimize the initial stop of extra buses scheduling strategy. Finally, the strategy is tested on two examples: a simple and a real-life transit route in the Dalian city in China. The results show that the extra buses scheduling strategy based on terminal stops with a reasonable threshold can save $8.01 \%$ waiting time of passengers.


## KEY WORDS

transit; service reliability assessment; extra buses scheduling strategy; heuristic algorithm;

## 1. INTRODUCTION

### 1.1 Background

Along with the urbanization and motorization, traffic congestion in urban areas in most countries has become a serious problem for residents living and travelling. In most cities in China, a huge number of passengers could travel by riding every day, especially at peak hour. As a consequence, it has been very often witnessed that the passengers have to experience large intervals and a crowded travelling environment both at stops and in vehicles [1].

As the service of public traffic does not hold much appeal to passengers in most cities, the public transportation does not account for a large market in passenger transport; only about $20 \%$. To improve the transit service, one of the most valuable methods is to improve the reliability of transit route according to the questionnaire survey in our work. The reliability in-
cludes on-time performance, short passengers waiting times and so on. The reliability of transit operation is always disturbed by some stochastic factors, such as weather and incidents. To eliminate the influence of the disruptions in transit operation, the implementation of various control strategies is the main task for the transit operators.

Types of control strategy studies include expressing, and holding strategies, both single and in combination [2]. The real-time dispatching strategies commonly contain holding strategy, stop-skipping strategy, deadheading and short-run strategy [3]. These strategies are available methods to improve transit service when the operation of the transit route is unreliable. There has been some research on real-time dispatching strategy in the recent two decades [4-8].

A lot of researchers did some researching about bus service reliability [9-14]. They tried to assess the reliability of transit service with different methods. The literature on dispatching strategy mainly concentrated on the implementation of the strategy and the improvement of the effectiveness of dispatching [2, 15]. There is a lot of literature about the holding strategy [14, 16-21]. For the improvement of the effectiveness of dispatching, many researchers forecasted bus arriving time and built models [2, 22-25]. There are some studies that considered the improvement of dispatching effectiveness [23, 26-27]. In these studies, some decision-support models were built to improve the applicability of the real-time dynamic dispatching control.

### 1.2 Contributions

The public transportation operation environment is very complicated, interferred by various factors. When some accidents happen, some small-scale scheduling strategy cannot return the operations to normal. Thus, some extra buses have to be dispatched. That scheduling strategy belongs to large-scaled ones due to increasing the number of operating buses. To the best of our knowledge, there is not much in literature discussing when an extra vehicle needs to be dispatched. To decide if a dispatching strategy is implemented or not, the service reliability of a route needs to be estimated first, and it is defined by Eq. (1). Yu et al. [1] assessed the service reliability of bus stops in their partway deadheading dispatching strategy.

The locations of extra buses are essential to the final scheduling strategy. This paper assumes that extra buses are at initial stops or terminal stops to simplify the problem. This paper makes two contributions to the literature. The extra buses scheduling strategy can be implemented in each direction when the transit service is not reliable; not only in the peak direction. This paper uses two kinds of rolling horizons: the temporal
rolling horizon (the rolling horizon of a certain amout of considered vehicles) and the spatial rolling horizon (considered rolling horizon of stops), to evaluate the reliability of the stops and the route. The improvement can greatly increase the accuracy of reliability assessment as well as convenience.

The remainder of this paper is structured as follows. The assessment of service reliability of the route and extra buses scheduling strategy are given in Section 2. Section 3 presents a heuristic algorithm. Section 4 reports on computational results, and the conclusions are discussed in Section 5.

## 2. EXTRA BUSES SCHEDULING STRATEGY DEVELOPMENT

When the service reliability of the route is poor, extra buses are needed to provide the service. Before extra buses are dispatched, the route operation condition should be assessed first. Then the stops of the route on which extra buses provide their service need to be determined.

### 2.1 Service reliability assessment of transit route

Transit operation is always disrupted by some stochastic factors leading to unreliable service to passengers. If some extra buses need to be dispatched to enhance the service reliability, then the service reliability needs to be assessed to determine whether the route needs to be scheduled. As waiting time can be acted as feasible and convenient index to describe the transit reliability, this paper takes the waiting time of passengers at a stop to measure the transit reliability. The reliability of a route is decided by the performance of every stop. And in this paper, we assume that no passenger should miss two buses consecutively. In other words, if a passenger could not get on the first bus, they must get on the second one.

## The temporal-spatial rolling horizon of the stop

A temporal-spatial rolling horizon of consecutive buses on following stops is considered as the effective criterion to assess the service irregularity of the stops. This paper takes rolling horizon of vehicles as the temporal rolling horizon, and takes rolling horizon of stops as the spatial rolling horizon. Therefore, the several latest vehicles and the several following stops are chosen to compute the service irregularity of the stop. That means that only the information of the vehicles in the temporal rolling horizon (vehicles $v, \ldots, v+n-1$ ) is used to assess the service irregularity of the stop ( $n$ is the length of temporal rolling horizon), while the information beyond the rolling horizon is skipped. Moreover, the spatial horizon is rolled forward with ve-
hicles running (stops $j, \ldots, j+m-1$ ) ( $m$ is the length of spatial rolling horizon). For the spatial horizon, only the information of the stops in the spatial rolling horizon is used to assess the service irregularity of the stop, while the information beyond the rolling horizon is also skipped.

## Waiting time of passengers at a stop

Irregularity of transit operation can be estimated by using the variation of the intervals between a vehicle and its following vehicle. According to the study by van Oort and van Nes [21], the service reliability of a stop in this study is also described by using the percentage of irregularity.

The definitions of variables used throughout the model formulation are the following:
$P_{j}^{\text {stop }}$ - Service reliability at stop $j$.
$\rho_{j}^{\text {stop }}$ - Service irregularity at stop $j$.
$\bar{H}$ - Scheduled headway of the transit route, constant for every pair of stops (min)
$H_{i, j}$ - Actual headway between vehicle $i$ and vehicle $i-1$ at stop $j$. (min)
$t_{i, j}$ - Arrival time of vehicle $i$ at stop $j$.
$\bar{t}_{j}^{\text {waiting }}$ - Average waiting time of passengers at stop j. (min)
$\Gamma^{\text {route }}$ - Service reliability of the route.
$q_{j}$ - Total number of passengers at stop $j$.
Q - Total passengers at all stops of the route.
$\Omega_{j}$ - Vehicle set that is used to compute the service irregularity after vehicle $v$ on stop $j$. $\Omega=\left\{\Omega_{j} \mid j=v, v+1, \ldots, v+n-1\right\}$, where $n$ is the length of the temporal rolling horizon (the number of vehicles) at stop $j$.
$\pi_{j}$ - Stop set that is used to compute the service reliability after stop $j$. $\pi=\left\{\pi_{j} \mid j=j+1, \ldots, J+m-1\right\}$, where $m$ is the length of the spatial rolling horizon (the number of stops) after stop $j$.
$\lambda_{i j}$ - Weighting parameter which denotes the weight of vehicle $i$ in the vehicle set $\Omega_{j}$ for the service irregularity.
$w_{j}$ - Weighting parameter which denotes the weight of stop $j$ in the stop set $\pi_{j}$ for the service irregularity at stop $j, w_{j} \in[0,1]$.
Taking stop $k$ as the research object, the service reliability function of stop $k$ can be computed as in Eq. (1).
$P_{k}^{\text {stop }}=\sum_{j \in \pi_{k}} w_{j} \times \rho_{j}^{\text {stop }}$
$\rho_{j}^{\text {stop }}=\sum_{i \in \Omega_{j}} \lambda_{i j} \times\left|\frac{\bar{H}-H_{i, j}}{\bar{H}}\right|$
s.t. $H_{i, j}=t_{i, j}-t_{i-1, j}$

The more recent data will be settled in a larger weight in Eq. (2). $\lambda_{i j}$ is defined as Eq. (4):
$\lambda_{i j}=\left\{\begin{array}{cc}2^{-i} & 1 \leq i \leq n-1 \\ 2^{-i+1} & i=n\end{array}\right.$
Also, the following stops by the objective stop will be given a larger weight in the formula. $w_{j}$ denotes the weight of stop $j$ in the stop set for the service irregularity at stop j . It is computed as in Eq. (5):
$w_{j}=\left\{\begin{array}{cc}2^{-j} & 1 \leq j \leq m-1 \\ 2^{-j+1} & j=m\end{array}\right.$
If we assume that the scheduled headway of the transit route is constant, the waiting time of passengers at stop j can be computed in Eq. (6) [21]:
$\bar{t}_{j}^{\text {waiting }}=\frac{1}{2} H_{i j} \times\left[1+\left(P_{j}^{\text {stop }}\right)^{2}\right]$

## Service reliability assessment of a route

The expected waiting time of passengers at a stop can be used to estimate the service reliability of the stop, but it does not reflect the service reliability of the entire route. Since the number of passengers at different stops has great variability, the expected waiting time of passengers at large stops is more important in the service reliability of an entire route. Therefore, when assessing the service reliability at a route, the number of passengers at each stop is considered as the relative weight to the service reliability of the entire route.

$$
\begin{equation*}
\Gamma^{\text {route }}=\frac{1}{\sum_{j=1}^{N} \frac{q_{j}}{Q} \times \bar{t}_{j}^{\text {waiting }}} \tag{7}
\end{equation*}
$$

The higher the $\Gamma^{\text {route }}$, the better the service reliability of the route. If $\Gamma^{r o u t e} \geq \theta$ ( $\theta$ is a threshold value), it indicates that the service is reliable. Otherwise, the service is unreliable and some real-time control strategies should be used to eliminate the disruptions.

### 2.2 Extra buses scheduling strategy formulation

If the service of the route is unreliable and some extra buses need to be dispatched to provide the services, then the initial stop of the new service for the dispatching service should be determined. If the initial stop for the new service is determined inappropriately, extra buses scheduling strategy can also affect the service quality negatively. In this study, the determination of the initial stop of the new service for the dispatching service aims to maximize the efficiency of the extra buses scheduling strategy.

The objective of the optimization model for extra buses scheduling strategy is to achieve the trade-off between the benefits of the saving passenger waiting time and extra running time of the dispatched bus.

The definitions of variables used throughout the model formulation are as follows:
$r_{j}$ - Constant average passenger arrival rate at stop j per time unit period.
$q_{j}$ - Proportion of passengers exiting the bus at stop $j$, related to all passengers riding the bus. It is a fixed constant computed by history data for the stop.
$B_{i, j}$ - Number of passengers boarding bus $i$ at stop $j$.
$\hat{A}_{i, j}$ - Number of passengers exiting from bus $i$ at stop $j$.
$\hat{L}_{i, j}$ - Departure load of bus $i$ from stop $j$.
$\hat{R}_{i, j}$ - Number of passengers who are left by bus $i$ at stop $j$ and have to wait for bus $i+1$.
$\hat{t}_{i, j}^{a}$ - Arrival time of bus $i$ at stop $j$.
$\hat{t}_{i, j}^{s}$ - Dwell time of bus $i$ at stop $j$.
$\hat{t}_{i, j}^{d}$ - Departure time of bus i from stop $j$.
VC - Rated capacity of a standard bus.
M - Total number of buses.
$N$ - Total number of stops.
Textra - Total waiting time of passengers with extra buses schedule operation.
Tnormal - Total waiting time of passengers without extra buses schedule operation.
$C^{w}$ - Unit cost associated with waiting time of passenger.
$C^{r}$ - Unit cost associated with running time of bus.

## Numbers of passengers

When computing the departure time of bus $i$ from stop $j$, the dwelling time of the bus at the stop should be estimated first. The dwelling time of a bus is determined by the number of boarding or exiting passengers at the stop. The number of passengers exiting bus $i$ at stop $j$ can be computed as in Eq. (8):

$$
\begin{equation*}
\hat{A}_{i, j}=q_{j} \times \hat{L}_{i, j-1} \tag{8}
\end{equation*}
$$

The passengers who expect to board a bus can be divided into three groups: passengers $\hat{R}_{i-1, j}$ left by bus $i-1$, passengers $\hat{g}_{i, j}^{-}$arriving during the time of bus $i-1$ departing to bus $i$ arriving and passengers $\hat{g}_{i, j}^{+}$arriving during the dwelling time of the bus at the stop. Thus, as bus $i$ is arriving, the number of passengers who expect to board the bus is equal to $\left(\hat{R}_{i-1, j}+\hat{g}_{i, j}^{-}\right)$at stop $j$.
$\hat{\mathrm{g}}_{i, j}^{-}=r_{j} \times\left(\hat{t}_{i, j}^{a}-\hat{t}_{i-1, j}^{d}\right)$
Let us assume that all boarding takes place at the front door and exiting takes place at the rear door. The estimated dwelling time $\hat{t}_{i, j}^{u d}$ for passenger boarding and exiting time is equal to the longer one between the total boarding time and the total exiting time. The estimated dwelling time could be computed as in Eq. (10)
$\hat{t}_{i, j}^{u d}=\max \left[\bar{u} \times\left(\hat{g}_{i, j}^{-}+\hat{R}_{i-1, j}\right), \bar{d} \times \hat{A}_{m, j}\right]$
where $\bar{u}$ and $\bar{d}$ denote average times of the boarding and alighting per passenger, respectively.

Then, the number of passengers $\hat{g}_{i, j}^{+}$arriving while bus $i$ is dwelling at stop $j$ can be computed as in Eq. (11).
$\hat{g}_{i, j}^{+}=r_{j} \times \hat{t}_{i, j}^{u d}$
Thus, according to the bus capacity, the number of boarding passengers $B_{i, j}$ can be computed as follow.
$B_{i, j}=\left\{\begin{array}{l}\hat{g}_{-, j}^{-}+\hat{R}_{i-1, j}+\hat{g}_{i, j}^{+} \text {if } \hat{g}_{-, j}^{-}+\hat{R}_{i-1, j}+\hat{g}_{i, j}^{+} \leq V C-\left(\hat{L}_{i, j-1}-\hat{A}_{i, j}\right) \\ V C-\left(\hat{L}_{i, j-1}-\hat{A}_{i, j}\right) \\ \text { otherwise }\end{array}\right.$

Now the load of bus $i$ departing from stop $j$ can be updated and the number of the exiting passengers by the bus can also be obtained as follows:
$\hat{L}_{i, j}=\hat{L}_{i, j-1}+B_{i, j}-\hat{A}_{i, j}$
$\hat{R}_{i, j}=\max \left[\hat{g}_{i, j}^{-}+\hat{R}_{i-1, j}+\hat{g}_{i, j}^{+}-B_{i, j, 0}\right]$
After obtaining the number of the boarding passengers, the dwelling time of bus $i$ at stop $j$ can also be determined.
$\hat{t}_{i, j}^{s}=\max \left[\bar{u} \times B_{i, j}, \bar{d} \times \hat{A}_{i, j}\right]$
Substituting the arrival time and the dwelling time of bus $i$ at stop $j$ in the following equation, the departure time of the bus from the stop can be obtained.
$\hat{t}_{i, j}^{d}=\hat{t}_{i, j}^{a}+\hat{t}_{i, j}^{s}$
In addition, when a bus arrives at a stop and the preceding bus does not depart from it, we assume that the preceding bus departs from the stop immediately as the current one arrives.

## Benefit of passengers of the following stops

The benefit of the dispatching operation is the reduction of passenger waiting times from the initial stop of the new service to all the following stops on the route. Furthermore, since the dispatched bus will be inserted behind bus $v-1$, waiting times of the passengers, who will wait for the buses before bus $v-1$ will not vary either. The total waiting times of the passengers at stops $k$ to $N$ with and without the termi-nal-based extra buses scheduling operation should be computed. The saving waiting time would be the benefit of the terminal-based extra buses scheduling operation.

Let us assume that each bus starts from terminal 1 , goes through the destination terminal $N / 2$ and returns to terminal 1 ( $N$ refers to the total number of bus stations on the bus route). Assuming that when a bus completes a service cycle, its mark number would be increased by one. For example, when No. 1 bus finishes its transit service in the reverse direction and returns to the starting terminal 1, its mark number would be changed into $M+1$ ( $M$ is the total amount of vehicle fleet on one bus route).
(1) Total waiting time of passengers without extra buses schedule operation ( $T_{\text {normal }}$ )

If extra buses schedule operation is not implemented, each bus of the entire fleet provides regular transit service in both directions. Let us assume that the bus queue of the entire fleet is $\{i-1, i+1, \ldots, M+i-2\}$. Then, the total waiting time of the passengers waiting for buses $i-1$ to $M+i-2$ can be defined as in Eq. (17).

$$
\begin{equation*}
T_{\text {normal }}=\sum_{m=i-1}^{M+i-2} \sum_{j=1}^{N}\left(B_{m, j} \times \frac{h_{m, j}}{2}+\hat{R}_{m-1, j} \times h_{m, j}\right) \tag{17}
\end{equation*}
$$

(2) Total waiting time of passengers with extra buses schedule operation ( Textra)
If implementing the extra buses schedule operation, as the deadheaded bus begins the new service, the bus queue will be varied. Assuming that the current vehicle is bus i , since the deadheaded bus i ' is inserted behind bus $v-1$, the bus queue will be $\left\{i-1, i+1, \ldots, v, i^{\prime}, v+1, \ldots, M+i-2\right\}$. Let $i^{\prime}-1$ be the first bus (i.e. the initial bus $i-1$ ), $i^{\prime}$ be bus $i-1$, and so on, the bus queue can be renumbered as $\left\{i^{\prime}-1, i^{\prime}, \ldots, M+i^{\prime}-2\right\}$. Thus, the total waiting time of the passengers can be computed by implementing the extra buses schedule operation.

$$
\begin{equation*}
T_{\text {extra }}=\sum_{m=i^{\prime}-1}^{M+i^{\prime}-2} \sum_{j=1}^{N}\left(B_{m, j} \times \frac{h_{m, j}}{2}+\hat{R}_{m-1, j} \times h_{m, j}\right) \tag{18}
\end{equation*}
$$

## Objective function

In this study, the objective of extra buses scheduling optimization strategy is to maximize the total benefit of the transit system. The objective function integrates the benefit of the passengers at the following stops on the transit route, the additional waiting time of the passengers at the skipping stops and the extra running time of the dispatched bus.
$\max F=C^{w} \times\left(T_{\text {normal }}-T_{\text {extra }}\right)-C^{r} \times T_{k}^{\text {shortest }}$,
where $F$ denotes the total benefit of transit system. $T_{k}^{\text {shortest }}$ is used to denote the shortest running time from terminal 1 to stop $k$ (the shortest running time can be computed with history data, $k=1,2, \ldots, N$ ).

### 2.3 Prediction of the locations of vehicles on the route

Let us assume that the extra bus begins transit service from bus stop $k$. $T_{k}^{\text {shortest }}$ is the shortest running time from the starting station (terminal 1) to stop $k$. Then the arrival time of extra bus $i$ at its initial stop $k$ of the new transit service can be defined as in Eq. (20).

$$
\begin{equation*}
\hat{t}_{i, k}^{\prime a}=\hat{t}_{i, 1}^{d}+T_{k}^{\text {shortest }} \tag{20}
\end{equation*}
$$

where $\hat{t}_{i, k}^{\prime}$ is the arrival time of bus $i$ from stop 1 to $k$.

When an extra bus begins its service, it is inserted between two consecutive buses (e.g., v-1 and v). Bus $v-1$ is the latest bus that has reached or passed through stop $j$, while bus $v$ will be the nearest bus that has not reached the stop. Furthermore, the arrival times of buses $v-1$ and $v$ should satisfy the following constraint (Eq. (21)).
$\hat{t}_{v, k}^{a} \leq \hat{t}_{i, k}^{a}<\hat{t}_{v-1, k}^{a}$

## 3. SOLUTION ALGORITHM

For the implementation of the extra buses scheduling strategy in the previous sections, we proposed a solution algorithm to assess service reliability of the route and to determine the initial stop of the new service for the extra buses scheduling strategy in this section.
Step 1. Initialization
Determine the length of the rolling horizon $(n, m)$, the service reliability assessment threshold value (k) of the route and the time interval (TI) of estimating the route service reliability;

Set Time=0;
Step 2. Loop assessment of service reliability of the route

If (Time $\% T I)=0$, go to step 3, otherwise finish.
Step 3. Compute service reliability of the route as Eq. (7)

Step 3.1 Roll the temporal time horizon at each stop
As vehicle arrives at a stop, the first arrived vehicle in the horizon is taken out of the temporal time horizon and the current vehicle is inserted into the horizon.

Step 3.2 Compute the service irregularity at each stop according to Eq. (2).

Step 3.3 Roll the spatial horizon at each stop
The spatial rolling horizon of the stop has to be considered when computing average waiting time of passengers at a stop. The current stop and its following m -2 stop (except the terminals) are inserted into the horizon.

Step 3.4 Compute the average waiting time of passengers at each stop according to Eq. (6)
Step 3.5 Compute the service reliability of the route according to Eq. (7)
If $\Gamma^{\text {route }} \geq \theta$ go to Step 2 ; otherwise go to Step 4 .
Step 4. Optimize the initial stop for the new service of the extra buses scheduling strategy

Compute the shortest running time from the terminal to each stop $\left(\bar{T}_{k}^{\text {shortest }}(k=1,2, \ldots, N)\right)$.

Set $S^{N o}=1$, where $S^{N o}$ denotes the stop number.

Set $S^{\text {Optimal }}=0$, where $S^{\text {Optimal }}$ denotes the current optimal initial stop for the new service

Set Max $=0$, where $\operatorname{Max}=F\left(S^{\text {No. }}\right)$
Step 4.1 Update the average speed of each road segment between two consecutive stops.

Step 4.2 Terminating optimal initial stop for the new service.
If Set $S^{N o .}>N-1$, go to Step 5; otherwise go to Step 4.3.

Step 4.3 Compute the saving waiting time of passengers by $T_{\text {normal }}-T_{\text {extra }}$, according to Eq. (17) and Eq. (18).
Step 4.4 Compute the efficiency of the extra buses scheduling strategy according to Eq. (19).
Step 4.5 If $F\left(S^{N o .}\right)>$ Max, $\operatorname{Max}=F\left(S^{\text {No. })}\right.$ and $S^{\text {optimal }}=S^{\text {No. }}$.
Step 4.6 $S^{\text {No. }++ \text {, go to Step 4.2. }}$
Step 5. Implement the extra buses scheduling with $S^{\text {optimal }}$.
Step 6. Terminating check of the extra buses scheduling strategy operating;

If exceeding the maximum time of the total running times that have been gained, then stop; otherwise, go to Step 2.

## 4. NUMERICAL TEST

The extra buses scheduling strategy proposed in this study has been tested on two examples. The first
one is designed to illustrate the validity and feasibility of the strategy on a simple transit route with six stops in each direction. The second one aims at testing the performance of the strategy for improving the service of the real transit route in the Dalian city in China.

### 4.1 Test 1

The simple route consists of six stops in each direction where the first (the twelfth) and the sixth (the seventh) stops are the terminals, and the others are the intermediate stops. The location and information of the stops are shown in Figure 1. The passenger volume at every stop is set as $\overline{q_{i}}$. We supposed that the number of extra buses is available.

We randomly create speeds on the segments of 5 vehicles, as shown in Table 1. The length of temporal rolling horizon $(n)$ is set at 2 , the length of spatial rolling horizon $(m)$ is set at 2 , and the threshold value ( $k$ ) of service reliability assessment is set at 0.6.

Each stop has been already reached by at least two buses in the first 12 minutes. The arrival times at the stops are shown in Table 2. Inside the $12^{\text {th }}$ minute, we compute the service irregularity of the stops in every time interval (3 min.). In Table 2, the numbers with shading are the arrival times of the vehicles in the rolling horizon. The service reliability of the route is about 0.62 in the first $12^{\text {th }}$ minute and it is higher than the threshold value. This indicates that the service of the route is still reliable at the time. The system continues operating and the arrival times at the stops before the $15^{\text {th }}$ minute are shown in Table 3. The service reliability of the route is about 0.59 , which is lower than


Figure 1 - The location and information of the stops of a simple route

Table 1 - Vehicle speed on each segment (km/h)

|  |  | Segment $1 \rightarrow 2$ | $\begin{gathered} \text { Segment } \\ 2 \rightarrow 3 \end{gathered}$ | $\begin{gathered} \text { Segment } \\ 3 \rightarrow 4 \end{gathered}$ | Segment $4 \rightarrow 5$ | $\begin{gathered} \text { Segment } \\ 5 \rightarrow 6 \end{gathered}$ | $\underset{7 \rightarrow 8}{\text { Segment }}$ | $\begin{gathered} \text { Segment } \\ 8 \rightarrow 9 \end{gathered}$ | Segment $9 \rightarrow 10$ | $\begin{gathered} \text { Segment } \\ 10 \rightarrow 11 \end{gathered}$ | $\begin{aligned} & \text { Segment } \\ & 11 \rightarrow 12 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\frac{0}{0}}{\frac{0}{2}}$ | 1 | 18 | 11 | 18 | 11 | 18 | 18 | 11 | 18 | 11 | 18 |
|  | 2 | 14 | 12 | 16 | 12 | 14 | 14 | 12 | 16 | 12 | 14 |
|  | 3 | 16 | 18 | 16 | 18 | 16 | 16 | 18 | 16 | 18 | 16 |
|  | 4 | 14 | 13 | 15 | 13 | 14 | 14 | 13 | 15 | 13 | 14 |
|  | 5 | 13 | 15 | 14 | 15 | 13 | 13 | 15 | 14 | 15 | 13 |

Table 2 - Vehicle arrival times at the stops at the $12^{\text {th }}$ minute

|  |  | Stop 1 | Stop 2 | Stop 3 | Stop 4 | Stop 5 | Stop 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \frac{0}{O} \\ & \stackrel{0}{\bar{\omega}} \\ & > \end{aligned}$ | 1 | 0 | 1.17 | 3.35 | 4.35 | 6.53 | 7.70 |
|  | 2 | 3 | 4.50 | 6.50 | 7.63 | 9.63 | 11.13 |
|  | 3 | 6 | 7.31 | 8.65 | 9.77 | 11.10 | 12.42 |
|  | 4 | 9 | 10.50 | 12.35 | 13.55 | 15.39 | 16.89 |
|  | 5 | 12 | 13.62 | 15.22 | 16.50 | 18.10 | 19.72 |

Table 3 - Vehicle arrival times at the stops at the $15^{\text {th }}$ minute

|  |  | Stop 1 | Stop 2 | Stop 3 | Stop 4 | Stop 5 | Stop 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \frac{0}{.0} \\ & \frac{0}{0} \\ & > \end{aligned}$ | 1 | 0 | 1.17 | 3.35 | 4.35 | 6.53 | 7.70 |
|  | 2 | 3 | 4.50 | 6.50 | 7.63 | 9.63 | 11.13 |
|  | 3 | 6 | 7.31 | 8.65 | 9.77 | 11.10 | 12.42 |
|  | 4 | 9 | 10.50 | 12.35 | 13.55 | 15.39 | 16.89 |
|  | 5 | 12 | 13.62 | 15.22 | 16.50 | 18.10 | 19.72 |

the threshold value. This indicates that the service of the route is unreliable at the time and the extra buses scheduling strategy needs to be applied to improve the service.

Based on the prediction of vehicle locations on the route, the terminal for dispatching the bus should be determined. According to our numeral experience, if the distance between the initial stop of the new service and the terminal in the forward direction is less than the one with the terminal in the reverse direction, the terminal in the forward direction should be chosen, and vice-versa.

According to Eq. (19), the efficiencies in two cases are 42.86 and $21.43\left(C_{w}\right.$ is $2.7 \mathrm{RMB} / \mathrm{h}$ and $C_{r}$ is 1.5 $R M B / h$ ), respectively, where stop 3 and stop 4 are the initial stops for the new service. It is obvious that the efficiency is the best at implementing the extra buses scheduling operation at stop 3 . The result of the simple example also shows the advantages of the extra buses scheduling strategy, which can serve more passengers and fast entering transit service.

### 4.2 Test 2

In the second test, the data of route number 19 in the Dalian city in China have also been used to evaluate the extra buses scheduling strategy. The transit route No. 19 goes from 19 stops and 14.5 km per di-
rection. The headway and the average travel speed of the route are 2.5 minutes and about $12-18 \mathrm{~km} / \mathrm{h}$. The number of the running buses on the route is 48 , and the number of all the buses is 55 (there are 7 extra buses). The number of total passengers was 5,810 persons on April $17^{\text {th }}, 2012$. The bus operational data, such as bus cycle times, passenger demands at stops, and traffic conditions on links, have been collected from the analyzed route [1, 28-31].

### 4.2.1 Length of the rolling horizon $m$

The assessment of the length of the rolling horizons ( $m$ and $n$ ) is the basic work for the service reliability of the route computation. The value of $n$ has been assessed as 4 in Yu et al., 2012 [1]. In this part, the length of rolling horizon $m$ is to be computed. And in the tests of this paper, to simplify the computing process the values of the length of the two types of rolling horizon were considered to be the same. When the length of the rolling horizon is 2 to 5 ( $m=2 \sim 5$ ), the influence of different rolling horizons on the service irregularity of the stops is compared under different rolling horizons. From time zero, we compute the service irregularity of the stops every time interval (1 min.) under various rolling horizons. The average service irregularity according to Eq. (2) of each stop is shown in Figure 2.


Figure 2 - The average service irregularity at stops

It can be observed that the average service irregularity of each stop is similar when $m=3 \sim 5$. Moreover, when $m=4$ or 5 , the difference of the average service irregularity of each stop is less than $1 \%$. This indicates that the increase of the length of the rolling horizon has little significance for the computation of service irregularity when $m=4$. Therefore, the value of the considered rolling horizon m is set at 4 in this study.

### 4.2.2 Threshold value of service reliability of the route

After the determination of the considered rolling horizon, the service reliability of the route can be computed. Figure 3 shows the computation results in an hour.

Before the implementation of the extra buses scheduling strategy, the threshold value of the service reliability assessment should also be determined. The large threshold of reliability value means that more extra buses scheduling strategies will be implemented. On the contrary, there are fewer strategies. Generally,
frequent strategies can overlap and not reach the expected effect, while this can also influence the transit service of the reverse direction. Therefore, in this study, before the extra buses scheduling strategy is entirely implemented (the dispatched vehicle enters the transit service), new extra buses scheduling operation is not permitted.

From Figure 3, the threshold value of the service reliability assessment should not be more than 0.16 (as the threshold value is set at 0.16 , the service of only $30 \%$ is probably unreliable) to avoid frequent extra buses scheduling. When the threshold value of service reliability of the route is very small, the extra buses scheduling operation cannot provide its utility. Thus, the range of the threshold value of service reliability of the route $(k)$ is determined from 0.14 to 0.16 .

The service reliabilities of the route under various threshold values are computed. To further compare the threshold values, Figure 4 shows the average service reliability of the route under four degrees in which the strategy with $\theta=0$ means no control.

It can be observed that after comparing the no control strategy with the three extra buses scheduling


Figure 3 - Service reliability of the route in an hour


Figure 4 - The average service reliability of the route under various threshold values
strategy, the extra buses can improve the service reliability of the route, especially for the strategies with $\theta=0.16$ and $\theta=0.15$. The strategy with $\theta=0,16$ improves unreliability slightly better than the one with $\theta=0.15$. Considering the efficiency and the frequencies of the extra buses scheduling strategies, the threshold value of service reliability of the route is set at $\theta=0.15$ in this study.

### 4.2.3 Extra buses scheduling strategy

To validate the performance of the extra buses scheduling strategy, the strategy with the determined parameters continues to be tested 10 times under the same conditions. In each test, the period is set to two hours. Under no control, the total waiting time of passengers and the times implementing extra buses scheduling operations are shown in Figure 5. This paper simplifies the times of forward dispatching (FD) operations, and the times of reverse dispatching (RD) operations.

From Figure 7, it is obvious that the performance of the extra buses scheduling strategy is greatly better than the one of no control. In the 10 runs, the average times of the extra buses scheduling strategies are 5.3 and the average saving time of passengers can be reduced by $8.01 \%$ (minutes) compared with the one of no control. This indicates that the extra buses scheduling strategy can obviously improve the transit service. Moreover, in Figure 7, the times of forward dispatching operations account for a large proportion in the whole dispatching operations, while the reverse dispatching operations are applied just a few times. It is because of this that the additional waiting time and extra running cost of reverse dispatching operations is larger than the one of forward dispatching operations.

To test the method of reliability assessment proposed in this paper, we compared it with the method based on K-Nearest Neighbor prediction (k-NN prediction) [1]. Both methods are applied in the 10 computation tests to determine the dispatching operation times, as well as the error dispatching times in each test. Figure 8 and Figure 9 show the compared results, respectively. Compared with the operation times when applying $k$-NN in computation shown in Figure 8, the dispatching operation times based on the model proposed in this paper are higher (Figure 9). Besides, it can be found that the rate of error dispatching times applying the $k$-NN prediction-based method is larger than the one of our method (the rates of the two methods are $21.33 \%$ and $8.24 \%$, respectively). This indicates that the model proposed in this paper makes less error dispatching rate than the $k$-NN method. This may be because the k-NN prediction-based method was lacking in accuracy for not considering the influence between the stops nearby.


Figure 5 - Illustration of forward terminal-based stop-skipping dispatching strategy


Figure 6 - Illustration of reverse terminal-based stop-skipping dispatching strategy


Figure 7 - The performance of the extra buses scheduling strategy


Figure 8-Operation times based on k-NN prediction


Figure 9-Operation times based on the approach of this paper

The performance of four types of operation methods: the one without extra buses strategy, the one with the forward extra buses scheduling dispatching strategy, the one with the reverse extra buses scheduling dispatching strategy, and the one with the terminalbased dispatching strategy, are evaluated respectively. In general, the additional waiting time of passengers for the reverse dispatching is larger than for the forward dispatching. The results are evaluated in terms of the average total waiting time of passengers during the test periods, which is shown in Figure 10.


Figure 10 - The average waiting time of passengers with four different dispatching strategies

The performance of the proposed dispatching strategy in this paper is the best among the four strategies. The total average waiting time using our strategy is less than the other strategies by $8 \%, 3 \%$ and $5 \%$, respectively. The performance of the no control strategy is the worst among the four strategies due to no operation to remedy the unreliable service. As to the total waiting time, the performance of the forward dispatching strategy is better than the one of the reverse dispatching strategy.

The performance of the three extra buses scheduling strategies is compared. The forward extra buses scheduling strategy and the extra buses scheduling dispatching strategy based on terminal stops can bring less passengers waiting time than the reverse extra buses scheduling strategy. The extra waiting time of passengers of the extra buses scheduling dispatching strategy is the least among the four strategies. Considering the total cost in three dispatching strategies, it is obvious that there are some invalid dispatching operations in the forward dispatching strategy and the reverse dispatching strategy, especially in the reverse dispatching strategy. This indicates that there is high risk of error dispatching if not justifying transit service statement during real-time control.

## 5．CONCLUSION

In this paper，an extra bus scheduling strategy is proposed to determine when and how to dispatch the buses of an unreliable service route．To determine whether to implement extra buses scheduling opera－ tion or not，service reliability of the route is first as－ sessed based on the average waiting time of passen－ gers．In service reliability，the temporal and spatial rolling horizon is introduced to improve the accuracy of the assessment．As the transit service is unreliable， a bus could be dispatched to implement extra buses scheduling operation from a terminal．Also，an optimi－ zation model aiming to maximize the efficiency of the strategy is designed to determine the initial stop of the new service for the dispatched vehicle．Then，a heu－ ristic algorithm has been defined and implemented to address the operation．

Finally，the extra buses scheduling strategy pro－ posed in this study is tested with two examples：a simple route and a real－life route in the Dalian city in China．The simple example is used to describe the pro－ cess of service reliability assessment and extra buses scheduling strategy．From the results of the real－life route case，the performance of the extra buses sched－ uling strategy is validated and the strategy can reduce the waiting time of passengers by $8.01 \%$ ．

This paper presents a useful tool for estimating the service reliability of the route and an effective strategy for improving the transit service of the route．The com－ putational results are satisfactory and also show the effectiveness of the proposed strategies．

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## 摘要

## 动态加车调度策略在公共交通中的应用

本文提出一个用于提高运输服务水平的动态公交加车调度策略。该策略首先对公交线路服务可靠性进行评价，从而决定何时派遣一辆加车；然后，建立了一个以线路总效益最大为目标的模型来确定加车开始服务的站点。为了对该问题进行求解，本文设计一个启发式算法来评估线路公交服务的可靠性以及优化公交加车调度策略的起始站。

最后，通过两个算例：一个简单算例和大连市的一个实际公交线路数据，对该方法进行测试。结果表明，本文提出的公交加车调度策略在合理阈值约束下可以节省 $8.01 \%$ 的乘客等待时间。

## 关键词

公交；服务可靠性评价；加车调度策略；启发式算法

## REFERENCES

［1］Yu B，Yang ZZ，Yao BZ．Real－Time Partway Dead－ heading Strategy Based on Transit Service Reliabil－ ity Assessment．Transportation Research Part A． 2012；46（8）：1265－1279．
［2］Eberlein XJ，Wilson NHM，Bernstein B．The Holding Problem with Real－Time Information Available．Trans－ portation Science．2001；35（1）：1－18．
［3］Ceder A．Public Transit Planning and Operation Theory． Modeling and Practice．Oxford，UK：Elsevier； 2007.
［4］Miller EJ，Bunt PD．Simulation model of shared right－ of－way streetcar operations．Transportation Research Record．1987；1152：31－41．
［5］van Oort N，Wilson NHM，van Nes R．Reliability Im－ provement in Short Headway Transit Services：Sched－ ule－Based and Headway－Based Holding Strategies．The 89th Annual Meeting of the Transportation Research Board．Hilton，Cabinet； 2010.
［6］Fu L，Liu Q，Calamai P．Real－Time Optimization Model for Dynamic Scheduling of Transit Operations．Trans－ portation Research Record．2003；1897：48－55．
［7］Strathman JG，Kimpel TJ，Ducker KJ，Gerhart RL，Turner K，Griffin D，Callas S．Bus transit operations control：Re－ view and an experiment involving TRIMET＇s automated bus dispatching system．Journal of Public Transporta－ tion．2001；4：1－26．
［8］Shen S，Wilson NHM．An optimal integrated real－time disruption control model for rail transit systems．In： Voss S，Daduna JR，editors．Computer－aided Schedul－ ing of Public Transport．Lecture Notes in Economics and Mathematical Systems．Springer－Verlag，2001；p． 335－364．
［9］Guo G，Luo H，Lin X，Feng Ch．Headway－based Evalua－ tion of Bus Service Reliability．Proceedings of the 14th International IEEE Conference on Intelligent Transpor－ tation Systems； 2011 Oct 5－7；Washington，DC，USA．p． 1864－1868．
［10］Chen X，Yu L，Zhang Y，Guo J．Analyzing Urban Bus Ser－ vice Reliability at the Stop，Route，and Network Levels． Transportation Research Part A．2009；43（8）：722－734．
［11］Carrasco N．Quantifying Reliability of Transit Service in Zurich，Switzerland．Transportation Research Re－ cord：Journal of the Transportation Research Board． 2012；2274（1）：114－125．
［12］Lin J，Wang P，Barnum DT．A Quality Control Framework for Bus Schedule Reliability．Transportation Research Part E．2008；44（6）：1086－1098．
［13］Casello JM，Nour A，Hellinga B．Quantifying Impacts of Transit Reliability on User Costs．Transportation Re－ search Record．2009；2112（145）：136－141．
［14］Zhao J，Dessouky M，Bukkapatnam S．Optimal Slack Time for Schedule－Based Transit Operations．Transpor－ tation Science．2006；40（4）：529－539．
[15] Ceder A, Stern HI. Deficit Function Bus Scheduling with Deadheading Trip Insertions for Fleet Size Reduction. Transportation Science. 1981;15(4):338-363.
[16] Xuan YG, Argote J, Daganzo CF. Dynamic bus holding strategies for schedule reliability: Optimal linear control and performance analysis. Transportation Research Part B. 2011;45(10):1831-1845.
[17] Sorratini JAP, Liu R, Sinha S. Assessing Bus Transport Reliability Using Micro-Simulation. Transportation Planning and Technology. 2008;31(3):303-324.
[18] Ding Y, Chien SI. Improving transit service quality and headway regularity with real-time control. Transit. 2001;1760: 161-170.
[19] Milla F, Saez D, Cortes CE, Cipriano A. Bus-Stop Control Strategies Based on Fuzzy Rules for the Operation of a Public Transport System. IEEE Transactions on Intelligent Transportation Systems. 2012;13(3):1394-1403.
[20] Teng J, Xu R. Bus Dispatching Strategies in Urban Rail Emergent Events. Journal of the China Railway Society. 2010;32(5):13-17.
[21] Van Oort N, van Nes R. Regularity Analysis for Optimizing Urban Transit Network Design. Public Transp. 2009;1(2):155-168.
[22] Fu LP, Yang XH. Design and implementation of busholding control strategies with real-time information. Transit. 2002;1791:6-12.
[23] Dessouky M, Hall R, Zhang L, Singh A. Real-time Control of Buses for Schedule Coordination at A Terminal. Transportation Research Part A. 2003;37(2):145-164.
[24] Eberlein XJ, Wilson NHM, Barnhart C, Bernstein D. The Real-Time Deadheading Problem in Transit Operations Control. Transportation Research Part B. 1998;32(2):77-100.
[25] Wei M, Jin WZ, Sun B. Ant Colony Algorithm for Regional Bus Scheduling Problem. Journal of Highway and Transportation Research and Development. 2011;28(6):141-145,152.
[26] Rossetti MD, Turitto T. Comparing Static and Dynamic Threshold Based Control Strategies. Transportation Research Part A. 1998;32(8):607-620.
[27] Adamski A, Turnau A. Simulation Support Tool for RealTime Dispatching Control in Public Transport. Transportation Research Part A. 1998;32(2):73-87.
[28] Yu B, Yang ZZ, Yao BZ. Genetic Algorithm for Bus Frequency Optimization. Journal of Transportation Engineering. 2009;136(6):576-583.
[29] Yao BZ, Hu P, Lu XH, Gao JJ, Zhang MH. Transit network design based on travel time reliability. Transportation Research Part C. 2014;43:233-248.
[30] Yao BZ, Hu P, Zhang MH, Jin MQ. A Support Vector Machine with the Tabu Search Algorithm For Freeway Incident Detection. International Journal of Applied Mathematics and Computer Science. 2014;24(2):397-404.
[31] Yao BZ, Hu P, Zhang MH, Tian XM. Improved Ant Colony Optimization for Seafood Product Delivery Routing Problem. Promet - Traffic \& Transportation. 2014;26(1):1-10.

