

IMPROVED SUPPORT VECTOR CLUSTERING ALGORITHM FOR COLOR IMAGE SEGMENTATION

Y. Q.Wang^{1,2*} – X. Liu³

¹ Department of Computer Science and Applications, Zhengzhou Institute of Aeronautical Industry Management, Zhengzhou 450015, China

² Henan aviation economics research center, collaborative innovation centers of aviation economic development & aeronautic materials technology, Zhengzhou 450015, China

³ College of Information & Business, Zhongyuan University of Technology, Zhengzhou 450007, China

ARTICLE INFO

Article history:

Received: 05.04.2014.

Received in revised form: 14.07.2014.

Accepted: 23.07.2014.

Keywords:

Image processing

Color image segmentation

Support vector clustering

MEB algorithm

Massive data

Abstract:

Color image segmentation has attracted more and more attention in various application fields during the past few years. Essentially speaking, color image segmentation is a process of clustering according to the color of pixels. But, traditional clustering methods do not scale well with the number of training samples, which limits the ability of handling massive data effectively. With the utilization of an improved approximate Minimum Enclosing Ball algorithm, this article develops a fast support vector clustering algorithm for computing the different clusters of given color images in kernel-introduced space to segment the color images. We prove theoretically that the proposed algorithm converges to the optimum within any given precision quickly. Compared to other popular algorithms, it has the competitive performances both on training time and accuracy. Color image segmentation experiments performed on both synthetic and real-world data sets demonstrate the validity of the proposed algorithm.

1 Introduction

With the rapid popularization and development of computer vision, image processing has achieved lots of successful applications in more and more fields, such as biological feature recognition, information retrieval, and so on. Being one of the primal problems in image processing, image segmentation can be divided into two main types according to the different strategies, gray image segmentation and color image segmentation. Due to the fact that color images contain more information to describe the

real-world more vividly than gray images, along with the rapid increasing of computer processing capacity, color image segmentation has attracted more attention than gray image segmentation during the past few years.

Traditional color image segmentation methods can be implemented with different ideas, such as edge segmentation, region segmentation, segmentation based on threshold, segmentation based on artificial neural network, segmentation based on wavelet, segmentation based on active contour model, segmentation based on genetic algorithm, clustering

* Corresponding author. Tel.: +86 158 383 23560

E-mail address: wyq-yongqing@163.com

segmentation, and so on. As the essence of color image segmentation is a kind of clustering according to the color of pixels, large number of clustering segmentation methods appeared in recent years.

But, traditional clustering methods do not scale well with the training sample size, which limits the ability of color image segmentation to handle massive data effectively. With the utilization of an improved approximate Minimum Enclosing Ball (MEB) algorithm, we develop a fast support vector clustering algorithm for computing the different clusters of given color images in kernel-introduced space to segment the color images. We prove theoretically that the proposed algorithm converges to the optimum within any given precision quickly. Compared to other popular algorithms, it has the competitive performances both on training time and accuracy. Color image segmentation experiments performed on both synthetic and real-world data sets demonstrate the validity of the proposed algorithm.

The rest of this paper is organized as follows. Section 2 provides a review on Color Image Segmentation. Support Vector Clustering problem is presented in Section 3. Section 4 proposes the improved Support Vector Clustering algorithm in detail. Section 5 conducts the experimental results on both synthetic and real-world data set, and Section 6 ends this paper with a conclusion.

2 Color Image Segmentation

Comparing by the principle of segmentation, color image segmentation and gray image segmentation are the same techniques since they are both based on pixel numerical similarity and proximity of input space, except for the transition of the investigation on the pixel attribute and feature extraction technology from one dimension space to high dimension space. With the rapid increasing of computer processing capacity in recent years, color image segmentation has attracted increasingly more attention. According to different strategies adopted in handling methods, color image segmentation can be divided into categories as follows.

2.1 The method based on neighborhood

According to the same or similar characters of pixels, the method based on neighborhood can

connect the neighboring pixels to achieve image segmentation. It includes all sorts of region growing methods, watershed segmentation method, and the Markov Random Field (MRF) method. This method can make a full use of space information and the correlation between pixels. However, some priori information is needed at the same time, such as the seed pixels and a variety of criteria to define the color target boundaries, which are difficult to get in some cases.

1) Region growing method

Region growing method (including region splitting-and-merging technology), initializes from several seed points or seed regions, according to certain growth standards, discriminates and connects the points of neighborhood pixels, until all pixel points have been connected [1-3]. This method defines three principles in Euclid distance of RGB color space, color identity principle, principle based on the color similarity and space neighborhood, global identity principle based on the color similarity, which leads to the subjectivity in the selection of the corresponding thresholds, and not suitable for image segmentation with shadow area.

2) Watershed segmentation method

Watershed segmentation method [4-7] is based on gradient, whose advantage is that it can get one pixel wide, which can result in a closed connected precise outline. However, it is difficult to select a proper label in watershed segmentation method, and improper labeling always leads to an over-segmentation result.

3) Markov Random Field method

Markov random field method [8-10] is one of the most commonly used statistical methods in image segmentation, especially in the widespread application of the texture image. Its essence is regarding the color value of each point in image as a random variable with certain probability distribution, where the probability of a pixel point taking some color is determined by its neighborhood other than the global information of the image. Technologies based on random field model can provide more accurate image area domain feature. When confronted with complex image area, or difficulty to divide the image through simple technology, the methods based on random field model will always achieve very good segmentation results. However, it needs a lot of calculation, and the related algorithms are very complicated, so the balance between computing

complexity and making good segmentation results is a challenging topic.

2.2 The method based on histogram threshold

The method based on histogram threshold [11-13] uses the valley value between the two adjacent peaks in color histogram as a threshold to segment images. Unlike gray images, color images have three color components, and the histogram is a 3-d array, by which it is more difficult to determine the thresholds. The most used solving method includes adopting three two-dimensional spectrum subsets for RGB space, and choosing three main spectrum subsets for images with more spectrum subsets by the use of principal component transform. Histogram threshold-based method does not need any priori information, and the amount of calculation is small. However, the segmented areas based on color segmentation may be incomplete, and the results for images without obvious peaks are poor.

2.3 The method based on clustering

The method of color image segmentation based on clustering [14-18], represents the pixels in the image space with the corresponding feature space points, segments the feature space points according to their clustering in feature space, and then maps them back to the original image space to get the segmentation result.

The clustering-based color image segmentation method can be classified into two categories, multi-dimensional color clustering and multi-dimensional extension of histogram threshold, both of which have its own merits and shortcomings. There exists some dependent losses and correlation among multi-dimensional data in multi-dimensional color clustering, but it cannot correctly represent data clustering. However, the multi-dimensional extension of histogram threshold method has high efficiency in calculation; but on the other hand, it cannot represent the color feature space very well. The former has higher computation costs than the latter, but it can represent the color space better.

At present, there are many strategies combining multidimensional threshold segmentation with other methods, such as 3-d histogram growth method, scale spatial clustering method, the dynamic clustering algorithm, etc., which can overcome the

phenomenon of excessive segmentation with simple process of classification, and it is easy to implement. However, it has also some disadvantages. Firstly, there is the problem of how to determine the number of colors in color image segmentation. Secondly, the characteristics are usually independent of the images, and there is the question of how to select features in order to get satisfactory separation results. Thirdly, there is no good use of spatial information, which is very useful for image segmentation.

2.4 The method Combined with other theories

Except for the methods mentioned above, there are some methods combined with other theories, which can be listed as follows [19-22]. The color image segmentation technology based on fuzzy set theory, the color image segmentation technology based on wavelet analysis, the color image segmentation technology based on neural network, and the method based on physical model, all of which are suitable for some certain kind of application, and a unified framework which is to be further studied.

3 Support Vector Clustering

As a kind of common tool of data analysis and unsupervised machine learning methods, clustering aims at dividing the data set into several classes (or clusters), keeping the maximum similarity between the data of the same class, and the maximum difference between the data of each pair of different class [23]. According to the basic ideas, the clustering algorithms can be roughly divided into five types [24], the partition clustering, the hierarchical clustering, the density-based clustering, the grid-based clustering, and the model-based clustering. At present, the research into the clustering algorithm is deepening, and the kernel clustering and spectral clustering are two methods that have attracted much attention recently [25].

The main idea of kernel clustering method is adopting a nonlinear mapping φ , such that the data points in input space can be mapped into a high-dimensional feature space, selecting appropriate Mercer kernel function instead of nonlinear mapping of the inner product to cluster in the feature space. The kernel clustering method is universal, and has great improvement over the classical clustering methods. The adopted nonlinear

mapping can increase the linear separable probability on input data points, which can achieve more accurate clustering, and faster convergence speed, as well. On condition that the classical clustering algorithms fail, the kernel clustering algorithms can always work. The kernel trick idea in kernel clustering method can be illustrated in Figure 1 below, where the left is the original input space, and the right is the kernel-induced space.

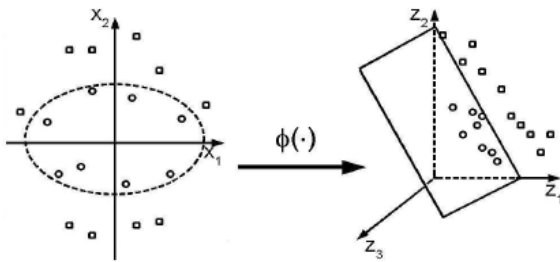


Figure 1. The kernel trick: Nonlinear problems are linear in kernel-induced space

Support Vector Clustering (SVC) belongs to the method of kernel clustering, whose foundational tool for clustering is Support Vector Machine (SVM) [26]. Based on the Support Vector Domain Description (SVDD) algorithm [27], Ben-Hur (2001) proposed an unsupervised nonparametric clustering algorithm, called SVC [28], whose basic idea is formulated as follows.

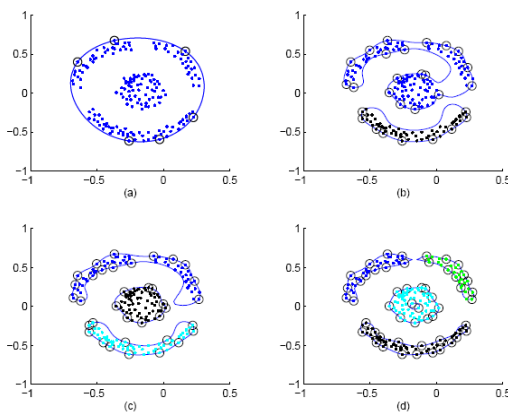


Figure 2. The clustering procedure of SVC on 183 points with $C=1$

The data points are mapped from input space to a high dimensional feature space using a Gaussian kernel, and seeking for the smallest sphere that encloses all the image of the data in feature space. This sphere is mapped back to data space, where it

forms a set of contours which enclose the data points. These contours are interpreted as cluster boundaries. Points enclosed by each separate contour are associated with the same cluster. As the width parameter of the Gaussian kernel is decreased, the number of disconnected contours in data space increases, leading to an increasing number of clusters. Since the contours can be interpreted as delineating the support of the underlying probability distribution, SVC algorithm can be viewed as one identifying valleys in this probability distribution [28, 29]. The shape of the enclosing contours in input space is governed by two parameters: q , the scale parameter of the Gaussian kernel, and C , the soft margin constant. Figure 2 above demonstrates the effects of these two parameters [28], where the support vectors are designated by small circles, and cluster assignments are represented by different grey scales of the data points. (a): $q = 1$ (b): $q = 20$ (c): $q = 24$ (d): $q = 48$. SVC algorithm has two significant advantages. The first is that SVC can handle clusters with the boundary in arbitrary shape, and the second one can analyze the noise data points and separate the overlapping clusters, in which circumstances many other clustering algorithms are hard to tackle. But there is still a bottleneck of weak scalability with the number of training sample data size in SVC algorithm, so many new SVC algorithms are designed to improve the computing efficiency [30, 31].

4 Improved MEB Algorithm for Clustering

Utilizing the concept of Core Set (seen in Figure 3), the MEB algorithm adopted in references [28, 29, 32, 33, 34] has the time and space complexities of $O(\frac{m}{\epsilon^2} + \frac{1}{\epsilon^4})$ and $O(\frac{1}{\epsilon^2})$, respectively.

However, we found that the final core vectors obtained for formulating the final decision function is always more than necessary in implementations, which results in some redundancies in the process of storing and training. This claim can be demonstrated clearly in Fig. 4 above, where the number of square points is larger than 3, the necessary number of points in 2-D space to determine the most appropriate circle.

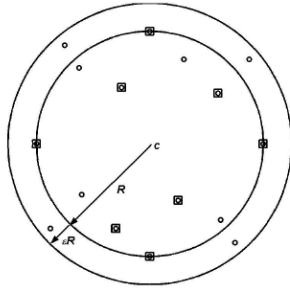


Figure 3. Inner circle is the exact MEB contains all the square points, whose $(1 + \varepsilon)$ expansion contains all the points, so the square points is the core set of all points

4.1 The SMEB algorithm

We formulate the SMEB algorithm in Table 1 below.

Table 1. The improved MEB algorithm in detail

SMEB	Outputs a $(1 + \varepsilon)$ -approximation of MEB(S)
0	Given an $\varepsilon > 0$, pick any $p \in S$, $S_0 \leftarrow \{p\}$;
1	Outputs S_0, c_0, R_0 ;
2	Terminate if there is no training point z such that $\varphi(z)$ falls outside the $(1 + \varepsilon)$ -ball $B(c_t, (1 + \varepsilon)R_t)$;
3	Find $z \in S \setminus S_t$, such that $\varphi(z)$ is furthest away from c_t , set $S_t = S_t \cup \{z\}$;
4	Find new MEB(S_t), set $c_t = c_{MEB(S_t)}, R_t = r_{MEB(S_t)}$, and increment t by 1, if $t < 48/\varepsilon^2 - 2$, go back to Step 2; otherwise go to Step 5;
5	Find $y \in S_t$ such that $\ \varphi(y) - c_t\ < R_t$, set $S_t = S_t \setminus \{y\}$;
6	Increment t by 1 and go back to Step 2.

4.2 The analysis on time and space complexities

We conclude the analysis on time and space complexities in Theorem 1 to 4 below.

Theorem 1. In the process of SMEB Algorithm, when the iteration satisfies $i \geq \frac{48}{\varepsilon^2} - 2$, if one point q falls into the interior of current MEB, i.e., $\|q - c_i\| < r_i$, it will fall into the interior of subsequence MEBs, i.e., $\|q - c_{i+j}\| < r_{i+j}$, $j \in \mathbb{Z}^+$.

Theorem 2. SMEB Algorithm can achieve a $(1 + \varepsilon)$ -approximate MEB for training data set S within $O(\frac{1}{\varepsilon^2})$ iterations.

Theorem 3. In the iterations of SMEB Algorithm, there exists a subset $P \subset S$, whose points are at distance at most $(1 + \varepsilon)r_{B(S)}$ from center $c_{B(S)}$, and the size of P is $O(\min\{\frac{1}{\varepsilon^2}, d\})$.

Theorem 4. The time and space complexities of SMEB Algorithm are $O(d^4 + d^2m + \frac{d^3}{\varepsilon^2} + \frac{dm}{\varepsilon^2})$ and $O(\min\{\frac{1}{\varepsilon^4}, d^2\})$.

The detailed proofs of these theorems are omitted here for conciseness, interested readers can refer to Wang [35, 36].

5 Experimental Results

Experiments are performed on five synthetic data sets, which follow the uniform distribution on the interval $(0, 10)$ (seen in Table 2).

We use Matlab 7.0 on a PC with Pentium-4 3.20 GHz CPU, 1GB of RAM running Windows XP to implement our experiments.

Table 2. Data sets used in the experiments

Data	data	data 2	data	data 4	data 5
Dim.	2	2	2	2	2
Num.	10	100	1000	10000	100000

We compare the algorithms of CVM, SCVM and SMEB on Optimum Bias Ratio and Training Time at different values of ε on the five data sets, where the Optimum Bias Ratio (OBR) is defined as $OBR = \frac{\|\text{experimental value} - \text{optimum}\|}{\|\text{optimum}\|}$.

Fig. 4 – Fig. 8 demonstrate the different performances with different values of ε for different data listed in Table 2, from which we can see that SMEB algorithm is usually faster than CVM and SCVM on the same ε with comparable accuracies, which implies that SMEB is more suitable for solving larger data problems (seen in Figure 8). By decreasing ε , the SMEB tends to be closer to the exact optimal solution, but at the expense of higher time and space complexities. Such a tradeoff between efficiency and

approximation quality is typical of all approximation schemes.

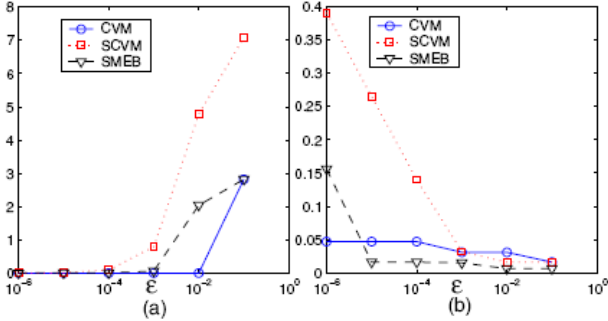


Figure 4. Performance with different value of ε for data 1, (a) for Optimum Bias Ratio (%), (b) for training time (s)

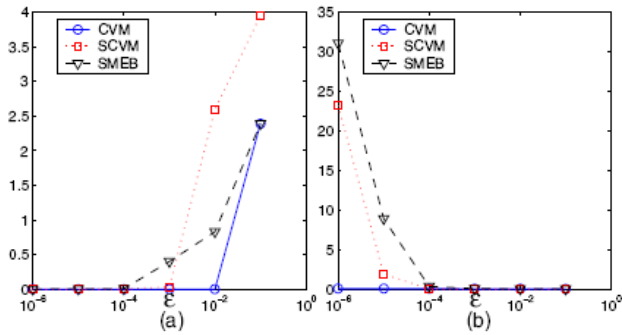


Figure 5. Performance with different value of ε for data 2, (a) for Optimum Bias Ratio (%), (b) for training time (s)

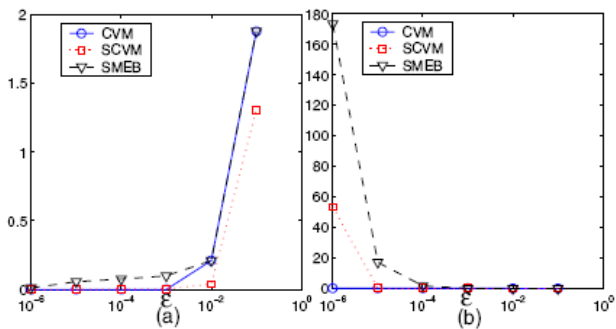


Figure 6. Performance with different value of ε for data 3, (a) for Optimum Bias Ratio (%), (b) for training time (s)

Utilizing the SMEB algorithm, we can tackle the image segmentation problem based on color. All of the original images to be segmented are chosen from internet randomly, including the flower image, the car image, the sunset image, and the plane image. The segmentation results are demonstrated in Fig. 9 to Fig.12, where the left images stand for the original color images, the middle ones stand for

the segmented color images under the algorithm of mean-shift, and the right images stand for the segmented color images under the SMEB algorithm we proposed. From the comparison results we can see that the SMEB-based Clustering algorithm can achieve good performances in color image segmentation.

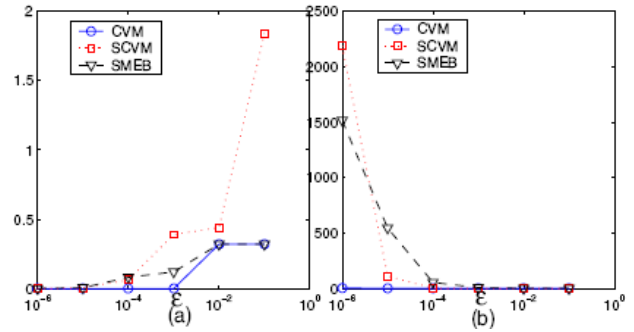


Figure 7. Performance with different value of ε for data 4, (a) for Optimum Bias Ratio (%), (b) for training time (s)

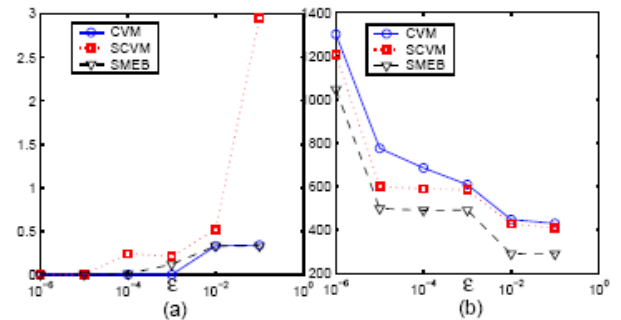


Figure 8. Performance with different value of ε for data 5, (a) for Optimum Bias Ratio (%), (b) for training time (s)

6 Conclusion

We have developed a $(1 + \varepsilon)$ -approximate support vector clustering algorithm in this article for computing the clusters of a given points set quickly. We have proved theoretically that the proposed SMEB algorithm converges to the optimum within any precision in $O(1/\varepsilon)$ iterations. The SMEB has time complexity of $O(m/\varepsilon^2 + 1/\varepsilon^3)$, which is linear in the number of training samples m for a fixed ε , and space complexity of $O(1/\varepsilon^2)$, which is independent of m for a fixed ε . Compared to related algorithms, it has the competitive performances both on training time and accuracy.

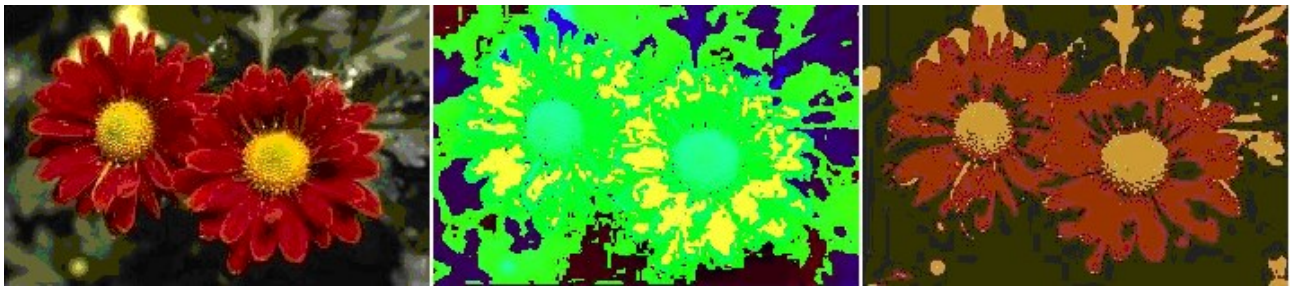


Figure 9. Original color image (left) vs. segmented image under mean-shift (middle) vs. segmented image under SMEB (right) for flower image



Figure 10. Original color image (left) vs. segmented image under mean-shift (middle) vs. segmented image under SMEB (right) for car image



Figure 11. Original color image (left) vs. segmented image under mean-shift (middle) vs. segmented image under SMEB (right) for sunset image

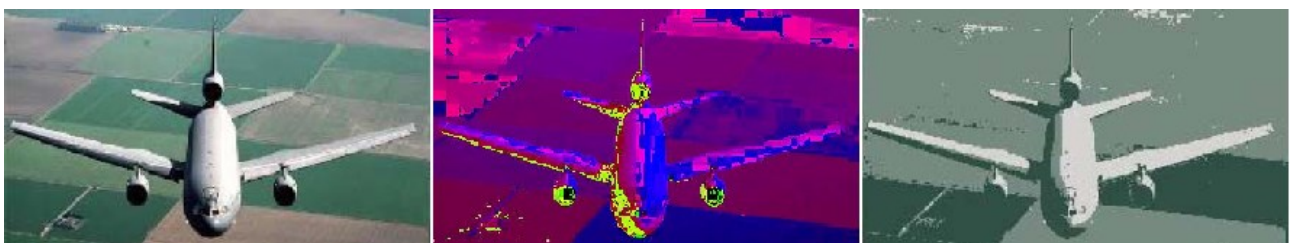


Figure 12. Original color image (left) vs. segmented image under mean-shift (middle) vs. segmented image under SMEB (right) for plane image

Besides, using the proposed SMEB algorithm, we can handle Color Image Segmentation problems effectively. Experiments performed on both

synthetic and real-world data sets demonstrate the validity of the algorithm we proposed.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (Grant No. 41001235), the Aeronautical Science Foundation of China (Grant No. 2011ZC55005), the Project of Henan Provincial Audit Office (Grant No. 20120922), the Scientific & Technological Research Key Project of Henan Provincial Education Office (Grant No. 13A520404), and the Project of Science and Technology Bureau of Zhengzhou (Grant No. 20120435 and 20130713). The authors are also grateful to the anonymous referees for their valuable comments and suggestions to improve the presentation of this paper.

References

- [1] Celenk, M.: *A color clustering technique for image segmentation*, Computer Vision, Graphics, and Image Processing, 52 (1990), 2, 145-170.
- [2] Tremeau, A., Borel, N.: *A region growing and merging algorithm to color segmentation*, Pattern Recognition, 30 (1997), 7, 1191-1203.
- [3] Cheng, H. D., Sun, Y.: *A hierarchical approach to color image segmentation using homogeneity*, IEEE Trans. on Image Processing, 9 (2000), 12, 2071-2082.
- [4] Lezoray, O., Cardot, H.: *Cooperation of color pixel classification schemes and color watershed: a study for microscopic images*, IEEE Transactions on Image Processing, 11 (2002), 7, 783-789.
- [5] Shafarenko, L., Petrou, M., Kittler, J.: *Automatic watershed segmentation of randomly textured color images*, IEEE Trans. on Image Processing, 6 (1997), 11, 1530-1544.
- [6] Shiji, A., Hamada, N.: *Color image segmentation method using watershed algorithm and contour information*, International Conference on Image Processing, 4 (1999), 305-309.
- [7] Ma, L. H., Zhang, Y., Deng, J. P.: *A target segmentation algorithm based on opening-closing binary marker on watersheds and texture merging*, Journal of Image and Graphics, 8 (2003), 1.
- [8] Celeux, G., Forbes, F., Peyrard, N.: *EM procedures using mean field-like approximations for Markov model-based image segmentation*, Pattern Recognition, 36 (2003), 1, 131-144.
- [9] Zhang, Y., Brady, M., Smith, S.: *Segmentation of brain MR images through a hidden Markov random field model and the expectation-maximization algorithm*, IEEE Transactions on Medical Imaging, 20 (2001), 1, 45-57.
- [10] Huang, C. L., Cheng, T. Y., Chen, C. C.: *Color images segmentation using scale space filter and Markov random field*, Pattern Recognition, 25 (1992), 10, 1217-1229.
- [11] Guo, G. D., Yu, S., Ma, S. D.: *Unsupervised segmentation of color images*, Proc. ICIP'98, 3 (1998).
- [12] Kurugollu, F., Sankur, B., Harmanci, A. E.: *Color image segmentation using histogram multi-thresholding and fusion*, Image and Vision Computing, 19 (2001), 13, 915-928.
- [13] Liu, C. Q., Cheng H.: *An efficient clustering method for color image segmentation*, Pattern Recognition and Artificial Intelligence, 8 (1995), A01, 133-138.
- [14] Carevic, D., Caelli, T.: *Region-Based Coding of Color Images Using Karhunen-Loeve Transform*, Computer Vision, Graphics, and Image Processing, 59 (1997), 1, 27-38.
- [15] Wang, X. W., Shen, L. S., Wei, B. G.: *The focus segmentation of color ophthalmologic image based on modified K-means clustering and mathematical morphology*, Chinese Journal of Biomedical Engineering, 5 (2002).
- [16] Zahid, N., Limouri, M., Esseaid, A.: *A new cluster-validity for fuzzy clustering*, Pattern Recognition, 32 (1999), 7, 1089-1097.
- [17] Lin, K. Y., Xu, L. H., Wu, J. H.: *A fast fuzzy C-means clustering for color image segmentation*, Journal of Image and Graphics, 9 (2004), 2.
- [18] Chen, T. Q., Lu, Y.: *Color image segmentation-An innovative approach*, Pattern Recognition, 35 (2002), 2, 395-405.
- [19] Sammouda, M., Sammouda, R., Niki, N.: *Segmentation and analysis of liver cancer pathological color image based on artificial neural networks*, ICIP'99, 3 (1999), 392-396.
- [20] Sammouda, M., Sammouda, R.: *Improving the performance of Hopfield neural network to segment pathological liver color images*, Proceedings of the 17th International Congress and Exhibition, London, June 25-28, 2003.

- [21] Ong, S. H., Yeo, N. C., Lee, K. H.: *Segmentation of color images using a two-stage self-organizing network*, Image and Vision Computing, 4 (2002).
- [22] Papamarkos, N., Strouthopoulos, C., Andreadis, L.: *Multithresholding of color and gray-level images through a neural network technique*, Image and Vision Computing, 18 (2000), 3, 213-222.
- [23] Xu, R., Wunsch, D.: *Survey of Clustering Algorithms*, IEEE Transaction on Neural Networks, 16 (2005), 3, 645-678.
- [24] Han J., Kamber M.: *Data Mining: Concepts and Techniques*, Second Edition, Morgan Kaufmann, San Francisco, 2006.
- [25] Filippone M., Camastra F., Masulli F.: *A Survey of Kernel and Spectral Methods for Clustering*, Pattern Recognition, 41 (2008), 1, 176-190.
- [26] Burges C. J. C.: *A Tutorial on Support Vector Machines for Pattern Recognition*, Data Mining and Knowledge Discovery, 2 (1998), 2, 121-167.
- [27] Tax D. M. J., Duin R. P. W.: *Support Vector Domain Description*, Pattern Recognition Letters, 20 (1999), 11-13, 1191-1199.
- [28] Ben-Hur A., Horn D., Siegelmann H. T., Vapnik V.: *Support Vector Clustering*, Journal of Machine Learning Research, 2 (2001), 12, 125-137.
- [29] Scholkopf B., Williamson R., Smola A., Shawe-Taylor J., Platt J.: *Support Vector Method for Novelty Detection*, Advances in Neural Information Processing System, 12 (2000), 582-588.
- [30] Lee J., Lee D.: *An Improved Cluster Labeling Method for Support Vector Clustering*, IEEE Transactions on Pattern Analysis and Machine Intelligence, 27 (2005), 3, 461-464.
- [31] Camastra F., Verri A.: *A Novel Kernel Method for Clustering*, IEEE Transactions on Pattern Analysis and Machine Intelligence, 27 (2005), 5, 801-805.
- [32] Tsang I. W., Kwok J. T., and Cheung P. M.: *Core vector machines: Fast SVM training on very large data sets*, Journal of Machine Learning Research, 6 (2005), 363-392.
- [33] Asharaf, S., Murty, M. N., Shevade, S. K.: *Multiclass Core Vector Machine*, Proc. of ICML'07, 2007, 41-48.
- [34] Tsang, I. W., Kwok, J. T., Zurada, J. M.: *Generalized Core Vector Machines*, IEEE Transactions on Neural Networks, 17 (2006), 5, 1126-1140.
- [35] Wang, Y. Q., Li, Y., Chang, L.: *Approximate Minimum Enclosing Ball Algorithm with Smaller Core Sets for Binary Support Vector Machine*, Proc. CCDC'2010, 2010, 3404-3408.
- [36] Wang, Y. Q., Niu, X. T., Chang, L.: *Multiclass Core Vector Machine with Smaller Core Sets*, Proc. CCDC'2010, 2010, 525-530.

