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NONLINEAR ANALYSIS OF LANDSLIDE PROCESSES UNDER SEISMIC EFFECTS

NELINEARNA ANALIZA KLIZIŠNIH PROCESA POD DJELOVANJEM SEIZMIČKIH UČINKA

Grishin A.V., Siplivets A.A.

Prethodno priopćenje

Sažetak: *U članku se razmatra stanje naprezanja-deformacija padine prouzročenih seizmičkom aktivnošću uzimajući u obzir geometrijsku i fizičku nelinearnost promatrane sredine.*

Ključne riječi: *padina, metoda konačnih elemenata, oscilacije, pomak, dinamičko opterećenje*

Preliminary notes

Abstract: *This paper investigates the stress-strain state of the slope caused by seismic activity, and taking into account the geometrical and physical nonlinearity.*

Key words: *slope, finite element method, oscillations, displacement, dynamic loading*

1. INTRODUCTION

It is known that soil slopes are nonlinear complex environment, which stress-strain state depends essentially on the way, time and type of loading. Therefore, it is desirable to take into calculation model from infinite number of factors that characterize this complex system only a finite number of them, reflecting its most important properties. Thus, they must be really determined from experimental data, and subject to further numerical implementation using computer programs.

Therefore, the calculation model is not exactly the same as the real system, but should reflect its main properties. Although the model is "poorer" than the material object, but, according to the rules, it is accessible, informative and user friendly. It also allows better understanding of basic properties of the object, to predict the consequences of changing the material properties and various influences on object.

Currently, there is a rapid development of mathematical modeling methods which led to a development of large number of different types of models. The most detailed description and classification is stated, for example, in [5, 6, 7]. It should be noted that such approach in scientific research is today most applicable and productive.

2. FORMULATION OF MATHEMATICAL MODELING

As shown in [6], the formulation of mathematical modeling problem can be divided into three stages: a model - an algorithm - a program (software). Let us briefly consider these stages in relation to the slopes which are under the influence of its own weight and previously applied static, followed by seismic loads.

A complex model is constructed in the first stage, taking into consideration a complex system, which in mathematical form displays the most important of its properties, formulated in the form of the fundamental laws of nature. Work by authors [3] shows that soil slopes even under the effect of their own weight are in the elastic-plastic state, so their model must consider this factor.

Further, soils slopes are subject to complex loading, which means that deformation theory of plasticity is not applicable. Therefore, a more sophisticated theory of plastic flow with hardening [3] must be used. Definition of physico-mechanical properties of soils characterizing its properties that are necessary for this theory are described in [2].

Large deformation and displacement may arise in the slopes under the influence of the static and dynamic loads, which must also be taken into account in the model and defined from calculations. In this case, using Lagrangian (material) approach, the relationship between Cauchy-Green strain tensor C and displacement vector u will take the form [3].

$$\boldsymbol{C} = \frac{1}{2}(\overset{\circ}{\nabla} \boldsymbol{u} + \overset{\circ}{\nabla} \boldsymbol{u}^{\mathrm{T}} + \overset{\circ}{\nabla} \boldsymbol{u} \cdot \overset{\circ}{\nabla} \boldsymbol{u}^{\mathrm{T}}),$$
$$\gamma_{\kappa s} = \frac{1}{2}(u_{\kappa,s} + u_{s,\kappa} + u_{t,\kappa} u_{t,s}). \qquad (1)$$

Difficulties occur when solving nonlinear problems that are associated with the lack of information on actual configuration $\boldsymbol{V}_t$, which is determined by the

Cauchy stress tensor $\boldsymbol{\sigma}$. Therefore, when solving nonlinear problems it is more convenient to find state of stress using the reference configuration $\boldsymbol{V_0}$, described by the source data.

With regard to this, it is necessary to operate with arising stress tensor determined by this configuration. Here we will use a symmetric Kirchhoff stress tensor K, which is defined through Cauchy stress tensor, as follows:

$$\mathbf{K} = \sqrt{D}\nabla \boldsymbol{x}^{\mathrm{T}} \cdot \boldsymbol{\sigma} \cdot \nabla \boldsymbol{x}. \qquad \boldsymbol{\sigma} = \frac{1}{\sqrt{D}} \overset{\circ}{\nabla} \boldsymbol{X}^{\mathrm{T}} \cdot \mathbf{K} \cdot \overset{\circ}{\nabla} \boldsymbol{X}. \tag{2}$$

The equation of motion, balance and static boundary conditions using Kirchhoff tensor $\boldsymbol{K}$ is nonlinear, depending on the strain state of a continuous medium. Ratio of virtual work principle with total stresses and with finite increments using Kirchhoff tensor defines component form as follows:

$$\int_{\overset{\circ}{V}} \left[\kappa_{\mathbf{sm}} \delta\gamma_{\mathbf{sm}} + \left(\rho \ddot{\mathbf{u}}_{\mathbf{m}} + \mathbf{c_m}\dot{\mathbf{u}}_{\mathbf{m}} - \overset{\circ}{\mathbf{F}}_{\mathbf{m}} \right) \boldsymbol{\delta}\mathbf{u_m} \right] \mathbf{d}\overset{\circ}{V} - \\ - \int_{\overset{\circ}{S}_1} \overset{\circ}{\mathbf{q}}_{\mathbf{m}} \boldsymbol{\delta}\mathbf{u_m} \mathbf{d}\overset{\circ}{\mathbf{S}} - \int_{\overset{\circ}{S}_p} \overset{\circ}{\mathbf{p}}_{\mathbf{m}} \boldsymbol{\delta}\mathbf{u_m} \mathbf{d}\overset{\circ}{\mathbf{S}} = 0, \quad \mathbf{t} \in (0, \infty),$$

$$\int_{\overset{\circ}{V}} \left\{ \begin{array}{l} \Delta\kappa_{sm}\, \delta\, \Delta\gamma^{\Lambda}_{sm} + \underline{(\rho\, \Delta \ddot{u}_s + c_s\, \Delta \dot{u}_s)\delta\, u_s} + \\ \frac{1}{2}\kappa_{sm}\, \delta(\Delta u_{n,s}\, \Delta u_{n,m}) - \Delta \overset{\circ}{F}_s\, \delta\, \Delta u_s + \\ \left[\kappa_{sm}\, \delta\, \Delta\gamma^{\Lambda}_{sm} + \left(\rho\, \ddot{u}_s + c_s\, \dot{u}_s - \overset{\circ}{F}_s \right) \delta\, \Delta u_s \right] \end{array} \right\} d\overset{\circ}{V} - \\ - \int_{\overset{\circ}{S}_1} \left(\Delta \overset{\circ}{q}_s + \overset{\circ}{q}_s \right) \delta\, \Delta u_s\, d\overset{\circ}{S} - \int_{\overset{\circ}{S}_p} \left(\Delta \overset{\circ}{p}_s + \overset{\circ}{p}_s \right) \delta\, \Delta u_s\, d\overset{\circ}{S} = 0, \\ t \in (0, \infty). \tag{3}$$

The equation of state in increments can be written as follows [3]

$$\Delta\kappa_{\boldsymbol{ij}} = \overline{D}_{ijnm}\, \Delta\gamma^{\Lambda}_{nm}, \\ \overline{D}_{ijnm} = D^{(N)} D_{\alpha\beta\xi\eta} x_{i,\alpha} x_{j,\beta} x_{n,\xi} x_{m,\eta}, \ \frac{dV}{d\overset{\circ}{V}} = \sqrt{D} = \left| X_{\kappa,s} \right|. \tag{4}$$

$$\Delta\kappa_{\boldsymbol{ij}} = \overline{D}_{ijnm}\, \Delta\gamma^{\Lambda}_{nm}, \\ \overline{D}_{ijnm} = D^{(N)} D_{\alpha\beta\xi\eta} x_{i,\alpha} x_{j,\beta} x_{n,\xi} x_{m,\eta}, \ \frac{dV}{d\overset{\circ}{V}} = \sqrt{D} = \left| X_{\kappa,s} \right|. \tag{4}$$

In the study of problems with small extensions and translations is assumed that the current body configuration $\boldsymbol{V_t}$ coincide with its reference configuration $\boldsymbol{V_0}$. This significantly simplifies the solution. However, under deformation with over 10% of displacement gradient, this approach can lead to both qualitatively and quantitatively wrong results.

Therefore, the process of solving problems with large deformations creates additional difficulties associated, firstly, with the geometric nonlinearity of the original equations and, secondly, the lack of information about the current configuration of the body $\boldsymbol{V_t}$.

The first problem leads to the introduction of various stress and strain tensors and the two main approaches in the study of problems in mechanics: Lagrange's and Euler's. The second problem generates widely used methods for solving the problems of incremental (high-speed) type.

Such approach allows replacement of a full load with equivalent number of small sequential steps and then to determine the body configuration, as well as the stresses and strains at a subsequent step from the previous where they are known. In this case, the reference configuration of a subsequent loading is adopted from previous current configuration. This enables the results determination process for each load to be written in the form of operation of the same type using the algorithm for solving the geometrically linear problems.

3. PROBLEM SOLUTION AND EQUATION SET-UP

Solution to the first problem allows us to represent our problem in algebraic form ie move from an infinite number of degrees of freedom of the slope to a finite number. It can be implemented using various projection methods. We will use finite element method. Solution to the second problem enables to linearize the original non-linear equations. In this case, the adjustment of their coefficients can be performed at each iteration solution or through their given number.

Discretization of equations using finite element method and static calculation methods for calculating the slopes are in detail considered in the monographs [3, 4], so we'll move to the solution of dynamic problems. In matrix form the equation of body motion at the time tn is transformed in the following form:

$$\left(\boldsymbol{M} + \frac{1}{\rho} [\boldsymbol{h}]^{\mathrm{T}} [\boldsymbol{H}]^{-1} \boldsymbol{h} \right) \ddot{\boldsymbol{\delta}}_{\boldsymbol{n}} + \boldsymbol{C}\dot{\boldsymbol{\delta}}_{\boldsymbol{n}} + \mathbf{K}(\boldsymbol{\delta}_{\boldsymbol{n}})\boldsymbol{\delta}_{\boldsymbol{n}} = \boldsymbol{Q}_{\boldsymbol{n}}. \tag{5}$$

The second term in brackets is called the matrix of associated masses and it takes into account the impact from the water environment on the slope with its oscillations [3]. Let us denote sum in parentheses with and call it the reduced mass.

The second term in (5) is the matrix of damping which for each finite element equals:

$$C^{ij}_{(e)} = \int_{-1}^{1}\int_{-1}^{1} \left[N^{i}_{(e)} \right]^{m} c_{(e)} \left[\overline{N}^{j}_{(e)} \right] h_{(e)} \det\mathbf{J}_{(e)}\, d\eta_1\, d\eta_2 = \\ = \int_{-1}^{1}\int_{-1}^{1} c^{ij}_{(e)}\, d\eta_1\, d\eta_2. \tag{6}$$

$\mathbf{K}(\delta)$ is called the system matrix of stiffness and it depends through a matrix $\boldsymbol{D}$ of the global vector of nodal displacements.

To solve the equation (5) we use an explicit or implicit methods of integration. Solutions shows that the second type methods are most acceptable regarding considered problems, so the Newmark method and its possible modifications will be used.

4. METHOD APPLICATION ON AN EXAMPLE

For determination of displacement in time we use the following equation:

Consider the example of the slope design scheme shown in Fig. 1.

We take that the ground slope located in the vertical plane with coordinate = –60 m receives displacement impulse of 10 cm from right side, which caused slope oscillation. At the slope boundaries wave oscillations are not reflected and they freely passing through.

Fig. 2 shows diagrams of horizontal and vertical oscillation displacement of slope point A.

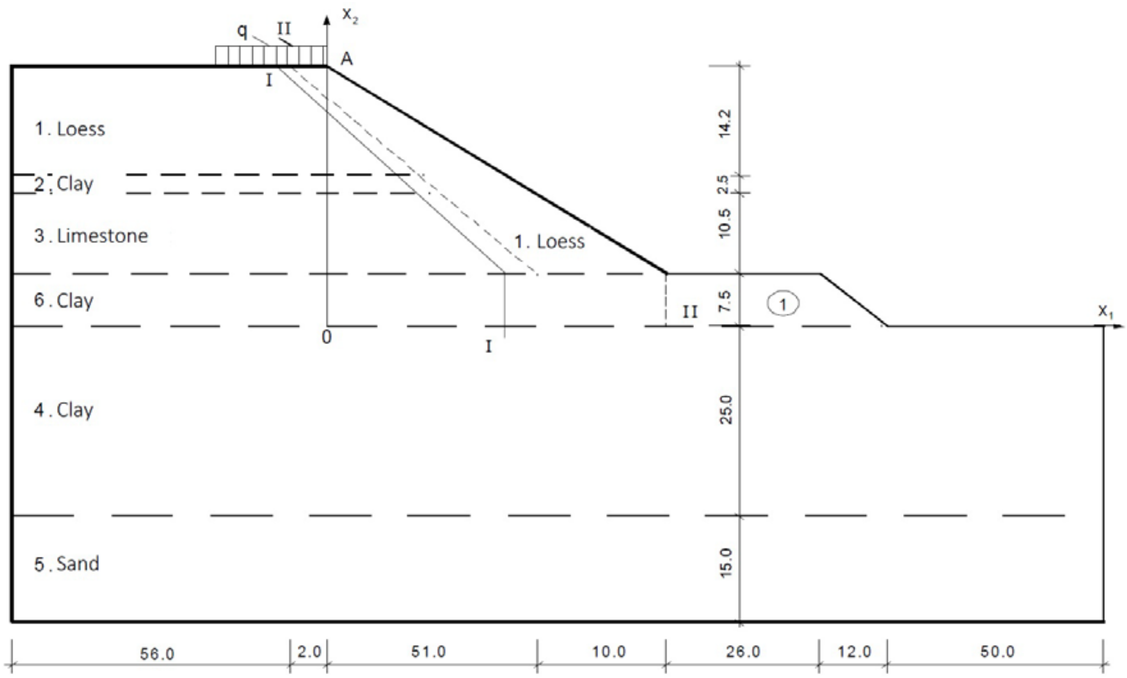
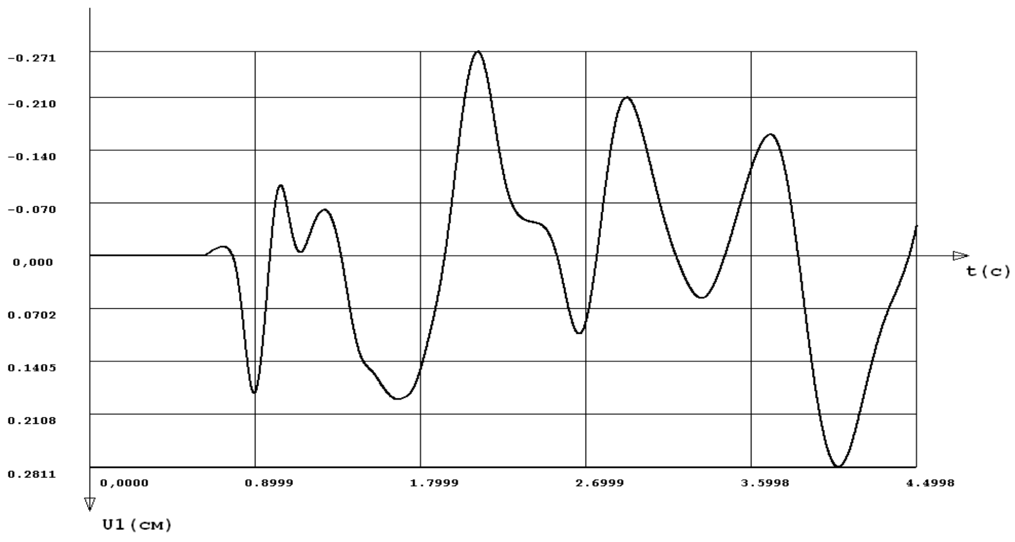


Figure 1. Design scheme of a planned slope



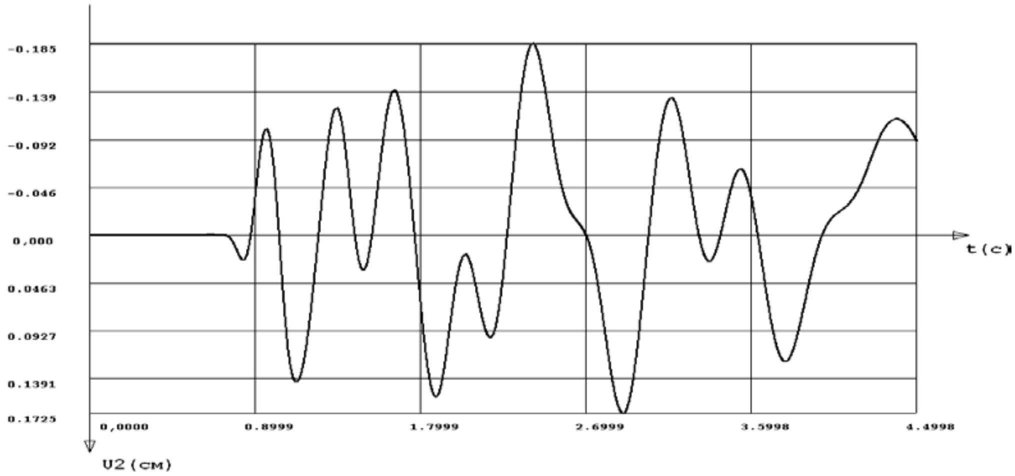


Figure 2. Diagrams of horizontal and vertical oscillation displacement of point A

Fig. 3 shows the diagrams of the normal and tangential stress oscillation at slope point B.

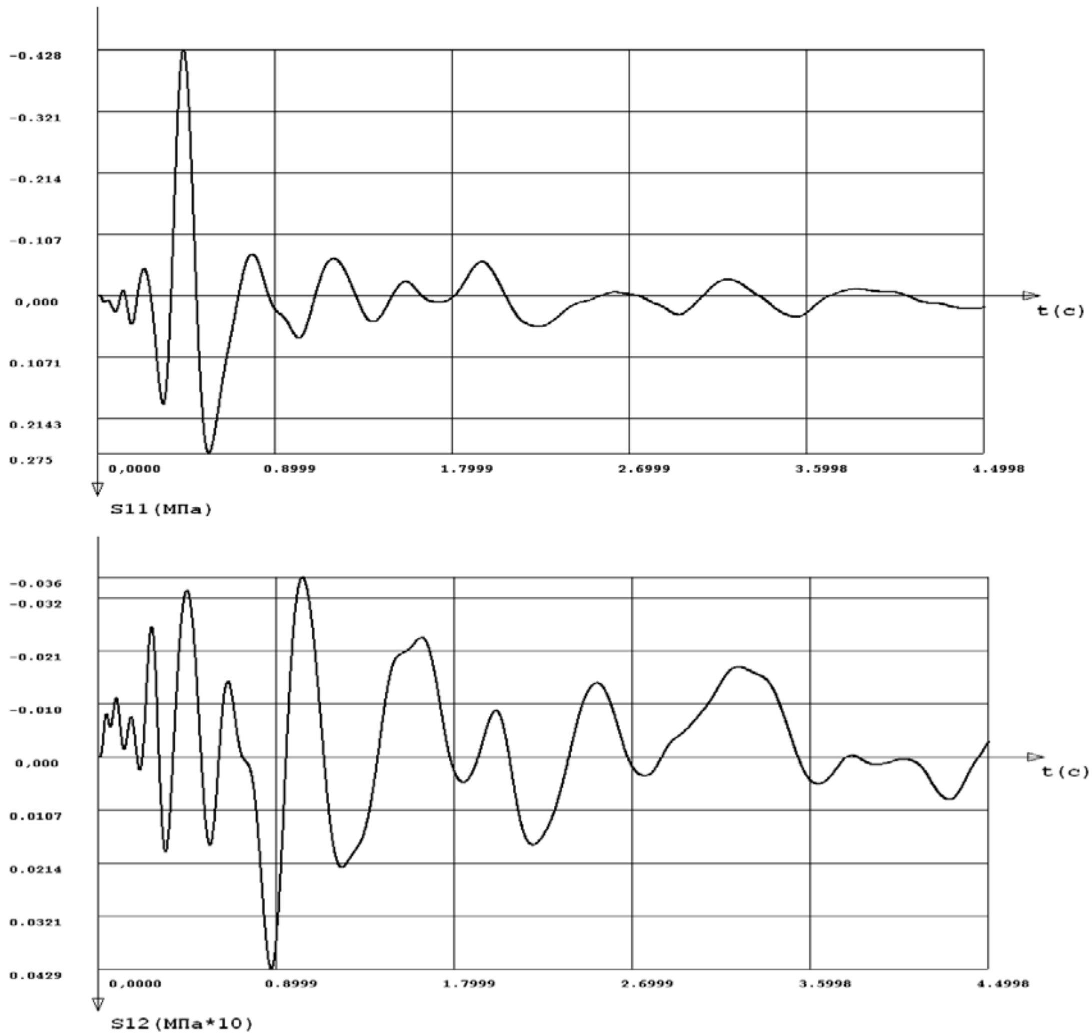


Figure 3. Diagrams of normal and tangential oscillation of stresses at slope point B.

5. CONCLUSION

Thus, using the proposed software we can examine complex oscillation processes occurring at any point in the slope caused by static and seismic loadings. It enables the results determination process for each load to be written in the form of operation of the same type using the algorithm for solving the geometrically linear problems. The possibilities of complex program are not limited only to the solution of problems given by initial displacement of the soil slope: accelerograms of earthquakes can also be defined.

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Kontakt autora:

Professor Oleg Popov

A.V.Grishin Ph.D., Professor,
Head of Department

A.A.Siplivets post graduate

Prorector for international relationship
Odessa State Academy of Civil Engineering and Architecture
65029 Odessa, Didrihsona 4, Ukraine
isi@ogasa.org.ua