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Examining Prospective Middle School Mathematics Teachers' Modelling Skills of Multiplication and Division in Fractions

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Abstract

Modelling can be considered as an effective way of ensuring permanent learning in the 'Fractions' topic, as well as the other topics of mathematics. When the effect of teacher's knowledge on students' learning is considered, it is important to investigate modelling skills of tomorrow's teachers regarding fractions. Within this context in the present study, prospective middle school mathematics teachers' modelling skills of multiplication and division in fractions were examined. The study was carried out with a total of 104 prospective middle school mathematics teachers who study at the Faculties of Education at two different universities in Turkey. The Modelling Fractions Test (MFT) with six items, three of which were on multiplication and three of which were on division in fractions, was used as a data collection tool. In analyzing the data, descriptive statistics was used and items were evaluated one by one. The findings indicated that the participants showed better performance on modelling with operations in multiplication than with the ones in division, created better models regarding operations requiring multiplication and division of a whole number and unit fraction, and also half of the participants answered questions in MFT as completely correct.

Key words: *fractions; mathematics teaching; modelling multiplication and division in fractions; prospective middle school mathematics teachers.*

Introduction

A fraction is defined as an equal part or several equal parts of a whole (Baykul, 2005). Like whole numbers, fractions also signify an amount, but in fractions, it is the number of parts that is important instead of wholes (Altun, 2008). Sizes, which are represented with the same fraction, can be different. For instance, one part of the triangular, rectangular and circular areas, which are divided into two equal parts shown in Figure 1, can be expressed with the same fraction ($\frac{1}{2}$). However, it has been found that students fall prey to the mistake that different sizes, which are defined with the same fraction, are considered equal (Erdem, 2015). It is stated that the underlying reasons for this mistake can be as follows: students do not have adequate experience regarding the fact that the amount represented with fraction is related with the whole that is taken as reference, and they fail to improve in this important reasoning (Alacaci, 2009). In parallel to this, Lamon (1996) points out that considering referenced whole plays a significant role in learning the fraction concepts in a meaningful way.

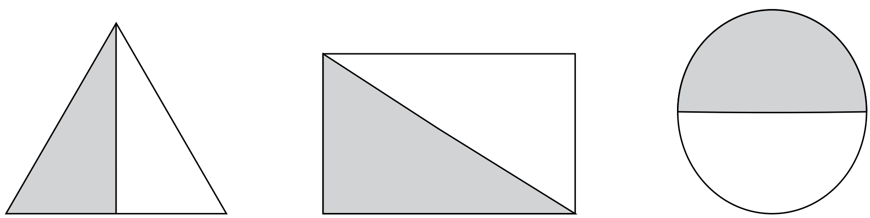


Figure 1. Different sizes that can be expressed with the same fraction ($\frac{1}{2}$)

Fractions can sometimes be confused with rational numbers and perceived as a number set. In the broadest sense of the term, fractions are the expressions that are used to represent rational numbers and that can exist in infinite numbers (Altun, 2008). As it is known, the expression $\frac{1}{2}$ can be taken as both a rational number and a fraction. However, expressions such as $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots$, which are equivalent to this fraction, are fractions, but they are not rational numbers since they do not fit the term rational number (*the fact that the numerator and denominator are relatively prime*). This provides us with the conclusion that a rational number can be represented by more than one fraction. Accordingly, it is understood that fractions cannot be a number set since this contradicts with the fact that the set is well defined.

It is stated by a great deal of studies that fractions, which can cause different perceptions and confusions, is one of the topics in mathematics that the students experience the most difficulty with in learning (Aksu, 1997; Behr, Lesh, Post, & Silver, 1983; Bezuk & Bieck, 1993; de Castro, 2008; Erdem, 2015; Ersoy & Ardahan, 2003; Gökkurt, Şahin, Soylu & Soylu, 2013; Işık & Kar, 2012; Moss & Case, 1999; Olkun & Toluk-Uçar, 2012; Pesen, 2008; Soylu & Soylu, 2005; Şiap & Duru, 2004; Stafylidou & Vosniadou, 2004; Tirosh, 2000; Toluk-Uçar, 2009; Ünlü & Ertekin, 2012). Many reasons are given in the literature for experiencing difficulty in learning fractions:

- ❑ Fractions are presented predominantly by means of rules rather than conceptual instruction (Aksu, 1997; Bezuk & Bieck, 1993; de Castro, 2008; Gökkurt et al., 2013; Moss & Case, 1999; Pesen, 2008; Soylu & Soylu, 2005; Tirosh, 2000; Toluk-Uçar, 2009)
- ❑ Fraction concept is a considerably abstract one (Olkun & Toluk-Uçar, 2012)
- ❑ Fractions and their properties do not exhibit much accord with natural numbers and their properties with which the students are familiar (Stafylidou & Vosniadou, 2004; Tirosh, 2000)
- ❑ Unit fraction concept is not fully understood (Ersoy & Ardahan, 2003)
- ❑ The numerator and denominator of the fraction are perceived as two unrelated whole numbers (Doğan & Yeniterzi, 2011; Şiap & Duru, 2004)
- ❑ Teachers or prospective teachers have had incomplete or memorized knowledge about fractions in their previous experiences (Toluk-Uçar, 2009)
- ❑ An expression like $\frac{a}{b}$ can have different meanings (Ünlü & Ertekin, 2012). These different meanings are explained in the literature as follows: "*a) **part-whole comparison** – it signifies the relationship between a whole and a part, b) **ratio** – it signifies the relationship between two quantities, c) **quotient** – it signifies the division operation, d) **operator** – it signifies the equivalence of the fractions and multiplication operation, e) **measurement** – it signifies how much amount the fraction represents*" (Behr et al., 1983).
- ❑ Thinking of fractions as only a certain section of the whole or its amount by ignoring other meanings of the fractions (Işık & Kar, 2012).

As for other reasons, the fact that teachers do not have adequate content knowledge and pedagogical content knowledge about fractions can render learning about fractions difficult. University years, in which teachers season these two types of knowledge, play a significant role at this point. Literature shows that the education obtained by prospective teachers, especially at university, is important in performing instruction activities in their professional lives in an effective way (Arslan & Özpınar, 2008; Erdem & Soylu, 2013; Erdem, 2015; Gürbüz, Erdem, & Gülburnu, 2013; Hill, Rowan, & Ball, 2005; Peker, 2009; Smith, 2000; Ubuz, 2002). However, when the conducted studies are examined, it is observed that mathematics teachers and prospective mathematics teachers do not have an adequate level of knowledge about fractions (Ball, 1990; Gökkurt et al., 2013; Işıksal, 2006; Kılcan, 2006; Li & Kulm, 2008; Ma, 1999; Newton, 2008; Simon, 1993). It can be stated that the fact that teachers and prospective teachers learn fractions predominantly by using rules rather than conceptual learning *(i.e. in multiplication operation, numerators are multiplied and the result of this multiplication is written as the numerator; denominators are multiplied and the result of this multiplication is written as the denominator. In division operation, the first fraction is left as it is; the second fraction is inverted according to multiplication, and it is multiplied with the first fraction)* is one of the most important factors that play a part in the emergence of this condition.

The conducted studies show that middle school students experience more difficulty in the multiplication operation and particularly division operation in fractions compared to other operations (Birgin & Gürbüz, 2009; Durmuş, 2005; Parmar, 2003; Toluk, 2002; Ünlü & Ertekin, 2012). It is even stated that the division operation in fractions is one of the most difficult topics of mathematics at this level (Ma, 1999). The fact that the fraction concept is a considerably abstract one (Olkun & Toluk-Uçar, 2012) and the fact that it was determined by many conducted studies that the students experience difficulty in understanding this concept and other operations about this concept makes it necessary to use different and effective methods in teaching this concept (Erdem, 2015). When we think of the middle school students, it is considered that mathematical modelling, which is a concretization tool, can be an effective method in teaching fractions conceptually. In many conducted studies, it is emphasized that models have to be used in teaching fractions where difficulties are experienced in learning and teaching (Ball, 1993; Behr et al., 1983; de Castro, 2008; Erdem, 2015; Lamon, 1996; Parmar, 2003; Toluk-Uçar, 2009).

Generally, three different models are put forth in teaching fractions to middle school students (Parmar, 2003). These models, which are expressed as an area or region models (a), length model (b) and set model (c)-(d), are shown in Figure 2. In view of the conducted literature review, it was found that the area or region model is the most widely used model (de Castro, 2008; Forrester & Chinnappan, 2010; Parmar, 2003; Toluk-Uçar, 2009). On the other hand, models, in which proper geometric shapes are used, are recommended in teaching the operations that are performed with fractions (Kieren, 1988; Pesen, 2008; Vergnaud, 1988). For this reason, if the area or region model is to be used, it is stated that it can be difficult to divide triangular or circular region models into equal parts, and accordingly, rectangular region models must be used instead (Doğan-Temur, 2011). Apart from this, when the literature is examined, it is observed that volume models are also used in teaching fractions (Pesen, 2008).

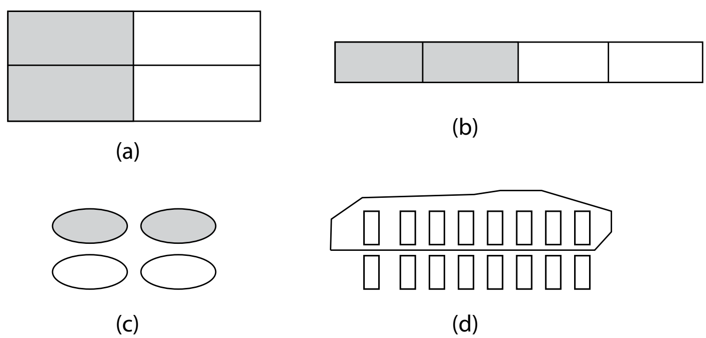


Figure 2. Different models used in teaching fractions

The fact that selected models are appropriate for the levels of the students is one of the important points that must be taken into account in an instruction that is performed by using models. The fact that the teacher has an adequate level of content knowledge and pedagogical content knowledge is important in performing such an instruction. In other words, the teacher must possess adequate conceptual and operational knowledge and must transfer this knowledge to the students in an effective way. As a matter of fact, many conducted studies emphasize that the teacher's content knowledge and pedagogical content knowledge is important in order to perform effective instruction (Ball, 1988; 1990; Cankoy, 2010; Davis & Simmt, 2006; Erdem & Soylu, 2013; Gökkurt et al., 2013; Gürbüz et al., 2013; Hill et al., 2005; Rowan, Chiang, & Miller, 1997; Shulman, 1986; 1987; Tchoshanov, 2011). In this respect, the education acquired by the teachers in university years plays a significant role in their reaching adequate levels in both these types of knowledge.

As it was mentioned before, when we consider the fact that middle school students experience more difficulty in multiplication operation and particularly division operation in fractions compared to other operations and the underlined importance of modelling, especially in teaching fractions (Ball, 1993; Behr et al., 1983; de Castro, 2008; Lamon, 1996; Parmar, 2003; Toluk-Uçar, 2009), it is important to examine prospective middle school mathematics teachers' skills of forming models regarding multiplication and division operations in fractions. In this sense, the aim of this study is to examine prospective middle school mathematics teachers' skills of modelling multiplication and division operations in fractions.

Methods

Since this study aims to determine an existing condition, it has a descriptive research quality. In such studies, the studied situations and conditions are researched in detail, their relationship with previous situations and conditions are examined, and an attempt is made to describe "what" they are (Tanrıöğen, 2011).

Participants

The study group is composed of a total of 104 fourth-year students who are studying at the Department of Middle School Mathematics Teaching at the Faculty of Education in two state universities located in Turkey. The reason for selecting fourth-year students for the research is that these students took *Special Methods of Instruction I-II* courses that are important in terms of students' content and pedagogical content knowledge. As a matter of fact, when the course contents of middle school mathematics teaching undergraduate programme of the Council of Higher Education (CHE/YÖK) in Turkey are examined, it can be observed in these courses that the general objectives of field teaching are stated and methods, techniques, tools and materials which are to be used in teaching field-specific concepts are introduced.

Data Collection

The "Modelling Fractions Test (MFT)", which was composed of six items (three of which were about the multiplication operation in fractions and three of which were about the division operation in fractions), was used as the data collection tool.

Literature, Curriculum for 6^{th}-8^{th} Grades Middle School Mathematics Course (MNE/MEB [Ministry of National Education], 2009) and the opinions of three middle school mathematics teachers who are experienced in their fields (*9-13-15 years of experience*) were utilized in order to determine the appropriateness of the items in MFT to the middle school students. In view of the pilot study, which was conducted with 35 prospective teachers, it was decided that a duration of 50 minutes would be given to the prospective teachers in the actual implementation. Furthermore, Cronbach's Alpha coefficient was calculated as .819 as a result of the analyses that were conducted in order to determine the reliability of the test. Apart from this, in order to get more accurate and detailed information about model forming performance, the prospective teachers were requested to explain how they modelled the operations in the test.

Data Analysis

Descriptive statistics was used in analyzing the data. Each question was studied and evaluated separately. Frequency and percentage tables were given regarding each item in MFT. Moreover, since it was believed that direct quotations would be effective in reflecting the individuals' thoughts (Yin, 1994), some answers, which could serve as examples to each item, were quoted. The rubric in Table 1, which was developed with the help of literature (Forrester & Chinnappan, 2010; Toluk-Uçar, 2009), was used in assessing the answers given by the participants to the items in MFT.

Table 1
The rubric developed for assessing items in MFT

Level	Score	Content
Completely Correct Answer	4	Correct model–correct explanation
Partially Correct Answer–A	3	Correct model–partially correct explanation
Partially Correct Answer–B	2	Correct model–wrong explanation or no explanation
Wrong Answer	1	Wrong model–wrong explanation or no explanation
Unanswered	0	No model–no explanation

Results

Descriptive statistics results of the answers given by the prospective teachers to the items in MFT are featured in this section. Some sample answers of the students regarding these items are presented.

As it is seen in Table 2, it was found that approximately half of the participants (50.5%) gave completely correct answers to the operations in MFT. It was observed that the participants generally showed better performance on modelling in multiplication operations than in division operations. It was also observed that the participants exhibited the best performance in the operation $2 \times \frac{1}{3}$ (73.1%). On the other hand, it was determined that the participants experienced the most difficulty in the item $\frac{2}{3} \div \frac{1}{2}$ (28.8%), and they gave the least statements for the models formed in this item. Furthermore, it was found that the prospective teachers formed better models and gave more true statements in the operations that required the multiplication and division of a whole number and a unit fraction.

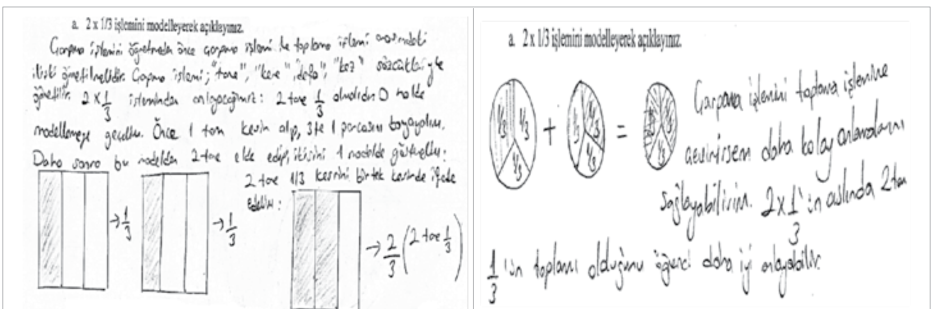
Table 2

Percentages and frequencies of answers to items in MFT

Item	Completely Correct		Partially Correct-A		Partially Correct-B		Wrong		Unanswered	
	f	%	f	%	f	%	f	%	f	%
$2 \times \frac{1}{3}$	76	**73.1**	11	10.6	3	2.9	13	12.5	1	1
$\frac{1}{2} \times \frac{2}{3}$	55	52.9	7	6.7	13	12.5	24	23.1	5	4.8
$\frac{2}{5} \times \frac{1}{3}$	51	49.0	6	5.8	16	15.4	19	18.3	12	11.5
$2 \div \frac{1}{2}$	61	58.7	8	7.7	5	4.8	21	20.2	9	8.7
$\frac{1}{3} \div \frac{1}{6}$	42	40.4	12	11.5	8	7.7	25	24.0	17	16.3
$\frac{2}{3} \div \frac{1}{2}$	30	**28.8**	11	10.6	10	9.6	24	23.1	29	27.9
Mean	53	50.5	9	8.8	9	8.8	21	20.2	12	11.7

Explain the operation $2 \times \frac{1}{3}$ by modelling.

In this item, the prospective teachers were expected to realize that the multiplication is actually a repetitive addition operation, and to show one third of two equal wholes separately and combine them in a whole. In view of the conducted analyses, it was found that the participants gave answers which fit in the following categories: 73.1% *completely correct*; 10.6% *partially correct-A*, 2.9% *partially correct-B*, 12.5% *wrong* and 1% *unanswered*. Among all the operations, the operation $2 \times \frac{1}{3}$ was found to be the question in which correct models were mostly formed and correct statements were mostly provided. It was observed from the given answers that many of the prospective teachers had the knowledge that $2 \times \frac{1}{3}$ operation was the addition of 2 units of $\frac{1}{3}$ and that they were able to form appropriate models using this knowledge (Figure 3a-3b)

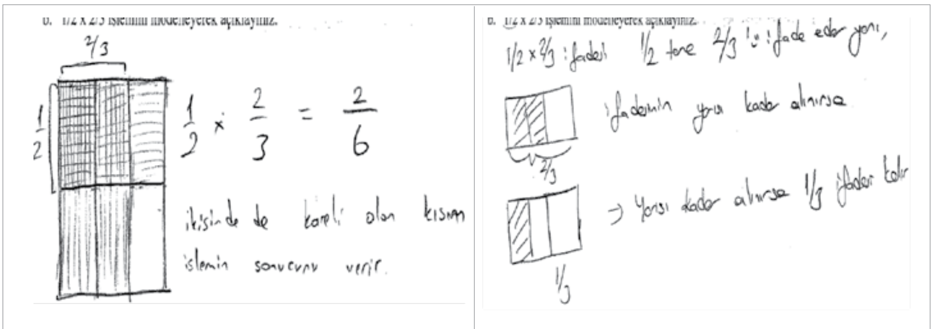


(a)	(b)

Figure 3. Some sample answers to item $2 \times \frac{1}{3}$

Explain the operation $\frac{1}{2} \times \frac{2}{3}$ by modelling.

In this item, the prospective teachers were expected to determine two thirds of a whole in a certain direction (*horizontal or vertical*), then determine half of the same whole in a different direction, and lastly show where these two regions intersect. From the analyses, it was found that the participants gave answers which fit in the following categories: 52.9% *completely correct*; 6.7% *partially correct-A*, 12.5% *partially correct-B*, 23.1% *wrong* and 4.8% *unanswered*. The prospective teachers were able to show the intersection of these two regions as the model of this operation after showing one of the multipliers horizontally on the geometric figure and the other multiplier vertically on the geometric figure (Figure 4a). On the other hand, it was observed that some prospective teachers regarded this operation as a half of the fraction $\frac{2}{3}$ and formed a model accordingly (Figure 4b).

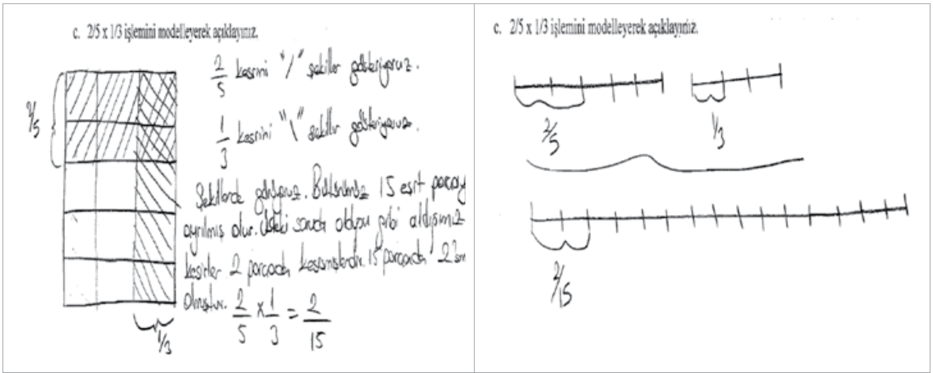


(a)	(b)

Figure 4. Some sample answers to item $\frac{1}{2} \times \frac{2}{3}$

Explain the operation $\frac{2}{5} \times \frac{1}{3}$ by modelling.

In this item, the prospective teachers were expected to determine two fifths of a whole in a certain direction, then one third of the same whole in a different direction, and lastly show where these two regions intersect. In view of the analyses, it was found that the participants gave answers which fit in the following categories: 49% *completely correct*; 5.8% *partially correct-A*, 15.4% *partially correct-B*, 18.3% *wrong* and 11.5% *unanswered*. It was determined from the given answers that this operation is the multiplication operation in which the prospective teachers experienced the most difficulty in forming a model. It was observed that some prospective teachers gave answers at the expected level although this was the operation in which they experienced the most difficulty (Figure 5a). On the other hand, it was observed that many of the participants used region or area model for modelling the operations whereas some prospective teachers incorrectly used the length model (*modelling its components and result instead of modelling the operation itself*) in this operation as in the other ones (Figure 5b).

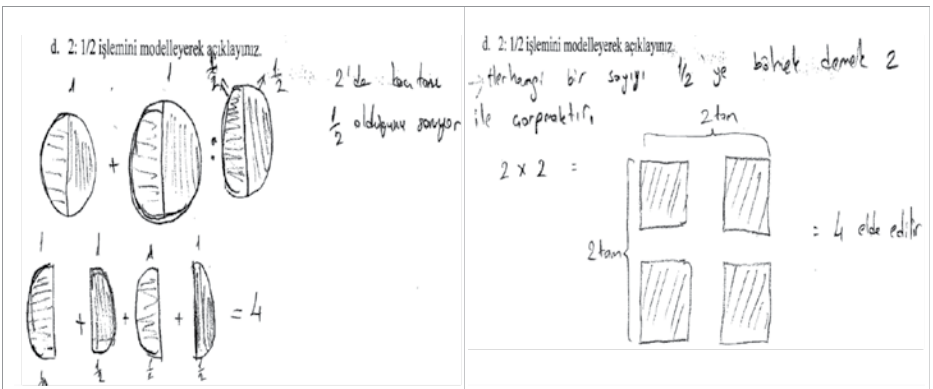


(a) (b)

Figure 5. Some sample answers to item $\frac{2}{5} \times \frac{1}{3}$

Explain the operation $2 \div \frac{1}{2}$ *by modelling.*

In this item, the prospective teachers were expected to divide equally each of the two equal wholes into two and show the number of half figures. From the analyses, it was found that the participants gave answers which fit in the following categories: 58.7% *completely correct*; 7.7% *partially correct-A*, 4.8% *partially correct-B*, 20.2% *wrong* and 8.7% *unanswered.* It was found from the given answers that this operation was the division operation in which the prospective teachers exhibited the best performance in forming a model. It was observed that some prospective teachers had the knowledge that the number of halves ($\frac{1}{2}$) in 2 wholes were expressed by the operation $2 \div \frac{1}{2}$ (Figure 6a) whereas some prospective teachers did not form a correct model regarding this operation and they just implemented the rule of division in fractions (*The first fraction is left as it is; the second fraction is inverted and it is multiplied with the first fraction*) (Figure 6b).

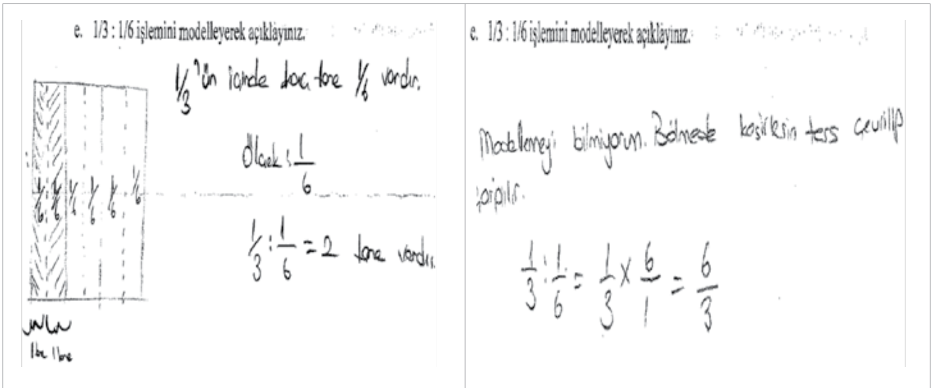


(a) (b)

Figure 6. Some sample answers to item $2 \div \frac{1}{2}$

Explain the operation $\frac{1}{3} \div \frac{1}{6}$ by modelling.

In this item, the prospective teachers were expected to determine one third of a whole and show the number of areas, which are represented as one sixth of the same whole, in the region signed with $\frac{1}{3}$. It was found that the participants gave answers which fit in the following categories: 40.4% *completely correct*; 11.5% *partially correct-A*, 7.7% *partially correct-B*, 24% *wrong* and 16.3% *unanswered*. When the given answers were examined, it was observed that some prospective teachers were able to think that the fraction $\frac{1}{6}$ was as a scale and the operation $\frac{1}{3} \div \frac{1}{6}$ could be solved by finding the number of magnitudes signified by the fraction $\frac{1}{6}$ in the magnitude signified by the fraction $\frac{1}{3}$ (Figure 7a). On the other hand, it was observed that some prospective teachers did not form a model for this operation and they only implemented the rule of division in fractions as in the previous division operation (Figure 7b).



(a)	(b)

Figure 7. Some sample answers to item $\frac{1}{3} \div \frac{1}{6}$

Explain the operation $\frac{2}{3} \div \frac{1}{2}$ by modelling.

In this item, the prospective teachers were expected to determine two thirds of a whole and show the number of areas, which are represented as the halves of the same whole, in the region signed with $\frac{2}{3}$. From the analyses, it was found that the participants gave answers which fit in the following categories: 28.8% *completely correct*; 10.6% *partially correct-A*, 9.6% *partially correct-B*, 23.1% *wrong* and 27.9% *unanswered*. When the given answers were examined, it was found that this operation is the operation in which prospective teachers experienced the most difficulty in forming a model and gave the least correct statements. It was determined that in this operation the majority of the participants was not able to grasp that the operation $\frac{2}{3} \div \frac{1}{2}$ could be solved by finding the number of magnitudes signified by the fraction $\frac{1}{2}$ in the magnitude signified by $\frac{2}{3}$. For this reason, many participants preferred modelling the numerator ($\frac{2}{3}$), denominator ($\frac{1}{2}$) and the result ($\frac{4}{3}$) they found instead of modelling the division operation itself (Figure 8b).

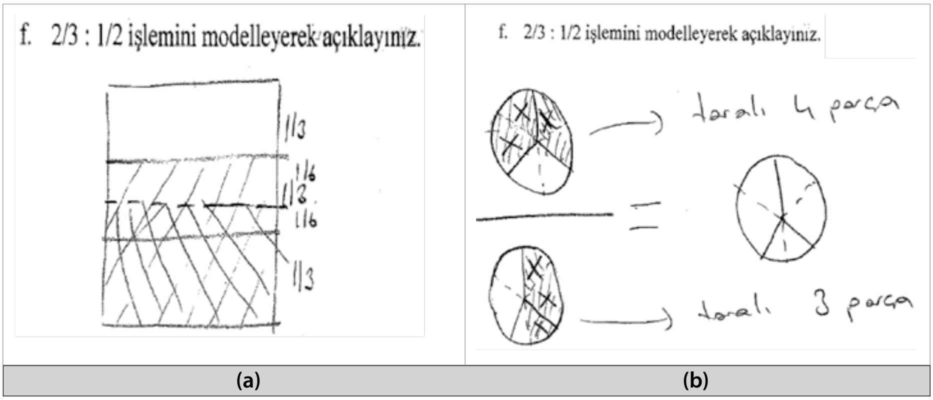


Figure 8. Some sample answers to item $\frac{2}{3} \div \frac{1}{2}$

To sum up, the following was found in this study: (a) approximately half of the prospective teachers fell under the *completely correct answer* category; (b) they exhibited better performance in modelling multiplication operations in fractions than in the division operations; (c) they formed better models in the operations that required the multiplication and division of a whole number and the unit fraction; and (d) many prospective teachers utilized memorized knowledge (*In the division operations, the first fraction is left as it is; the second fraction is inverted and it is multiplied with the first fraction*) especially in modelling the division operations.

Discussion and Conclusions

The aim of this study was to examine prospective middle school mathematics teachers' skills of modelling multiplication and division operations in fractions. In view of the conducted analyses, it was found that half of the prospective teachers gave completely correct answers to the operations in MFT. It can be stated that prospective teachers' skills of modelling multiplication and division operations in fractions are at medium level. However, the effect of the answers given to the multiplication and division operations on this result was at a different level. On the average, the ratio of giving completely correct answers to multiplication operations in fractions was 58% whereas the ratio of giving completely correct answers to division operations was 42%. This result showed that the prospective teachers formed better models in multiplication operations than in division operations. This result supports Ma's (1999) opinion that the division operation in fractions is one of the most difficult topics of middle school mathematics. Similarly, a number of research studies showed that middle school students experienced more difficulty in multiplication operation and especially in division operation in fractions (Birgin & Gürbüz, 2009; Durmuş, 2005; Parmar, 2003; Toluk, 2002; Ünlü & Ertekin, 2012).

It can be stated that the fact that the prospective teachers did not have an adequate level of conceptual knowledge was effective on their exhibiting lower performance generally in division operations. This conclusion resulted from the fact that many prospective teachers performed memorization-based algebraic operation instead of forming a model regarding the division operations. This result is parallel with the results of some studies (Ball, 1990; Işıksal, 2006; Li & Kulm, 2008; Newton, 2008; Simon, 1993) in the literature. On the other hand, it was observed that the prospective teachers formed better models in the operations that required the multiplication and division of a whole number and a unit fraction. In this sense, the results of the conducted analysis showed that the prospective teachers exhibited the best performance in the operation $2 \times \frac{1}{3}$ among the multiplication operations whereas they exhibited the best performance in the operation $2 \div \frac{1}{2}$ among the division operations. Apart from this, it was observed that the prospective teachers experienced more difficulty in forming models as the numbers in the numerator and denominator of the fractional expressions increased. For instance, the prospective teachers exhibited lower performance in the operation $\frac{2}{5} \times \frac{1}{3}$ than in the operation $\frac{1}{2} \times \frac{2}{3}$ although unit fractions were used in both of these operations (See Table 2). In view of this, it can be argued that it may be more difficult to model the operations in which big fractions are used as also stated by de Castro (2008). The important thing is to make students realize that a similar reasoning can be proposed for big fractions after teaching them how to model the operations that are performed with small fractions.

It was also found that the prospective teachers were able to correctly model the components in multiplication or division operations whereas they incompletely or incorrectly modelled the operations themselves or they were not able to model them at all. For instance, some prospective teachers correctly modelled the fractions $\frac{1}{2}$ and $\frac{2}{3}$ in the operation $\frac{1}{2} \times \frac{2}{3}$ while they experienced difficulty in modelling the multiplication operation itself. Similar findings were encountered in the literature (de Castro, 2008; Redmond, 2009; Tirosh, 2000; Ünlü & Ertekin, 2012) regarding prospective teachers. In view of this, it can be argued that modelling fractions and modelling the operations in fractions are different skills. It can be stated that the fact that teachers and prospective teachers learn fractions predominantly by means of applying rules *(i.e. in multiplication operation, numerators are multiplied and the result of this multiplication is written as the numerator; denominators are multiplied and the result of this multiplication is written as the denominator. In division operation, the first fraction is left as it is; the second fraction is inverted according to multiplication, and it is multiplied with the first fraction)* was effective in the emergence of this result. Similarly, it was stated in many conducted studies (Aksu, 1997; Bezuk & Bieck, 1993; de Castro, 2008; Gökkurt et al., 2013; Moss & Case, 1999; Pesen, 2008; Soylu & Soylu, 2005; Tirosh, 2000; Toluk-Uçar, 2009) that teaching rules is influential in experiencing difficulties in learning about fractions.

When the given answers were examined in detail, it was observed that some prospective teachers formed correct models regarding the operations, but they did not make any statements or made incomplete statements (see Figure 8a). In other words, it was found that some prospective teachers were not able to express the processes, which were utilized in modelling the operations mathematically at an adequate level. It can be argued that the fact that the prospective teachers were not able to convey their thoughts in mathematical language at an adequate level was influential in the emergence of this condition. Educators have important responsibilities at this point. As a matter of fact, Çalıkoğlu-Bali (2003), Hiebert, Morris and Glass (2003), and Straker (1993) mentioned the importance of the educators' effective usage of the mathematical language in the learning environments for the understanding of mathematical concepts. Therefore, it is important for the educators to use mathematical language frequently and in a correct manner while teaching mathematical concepts.

Using the modelling method effectively in the learning environments, beginning with early childhood, will provide great contribution to learning the fraction concepts and the operations in fractions in a meaningful way. When it is considered that teacher's knowledge plays a key role in this regard, it is important to use the modelling method effectively in teaching fractions at university level. It is believed that, with this method, teachers will gain understanding of their students' learning related to the operation process rather than just observe whether the result is correct. On the other hand, teachers must keep in mind that modelling the fractions and modelling the operations with fractions are different skills in teaching fractions. Moreover, prospective teachers' possession of an adequate level of content knowledge for mathematics instruction may not be enough to perform effective instruction. Prospective teachers must also be enabled to have an adequate level of pedagogical content knowledge. In this regard, it is important to form environments in which prospective teachers learn mathematical concepts by structuring them into the Special Methods of Instruction courses at university level.

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Ispitivanje vještina modeliranja budućih nastavnika matematike u višim razredima osnovnih škola na zadatcima množenja i dijeljenja razlomaka

Sažetak

Modeliranjem se učinkovito osigurava trajno učenje matematičkoga sadržaja razlomaka i ostalih matematičkih sadržaja. Kako bi se utvrdilo znanje koje nastavnici imaju o učeničkom učenju, važno je ispitati vještine modeliranja budućih nastavnika u rješavanju računskih operacija s razlomcima. U ovome se radu u navedenome kontekstu istražuju vještine modeliranja koje posjeduju budući nastavnici matematike u višim razredima osnovnih škola u rješavanju računskih operacija množenja i dijeljenja razlomaka. Istraživanje je provedeno na uzorku od 104 buduća nastavnika matematike u višim razredima osnovnih škola koji studiraju na učiteljskim fakultetima na dva različita turska sveučilišta. Za prikupljanje podataka upotrijebljen je Test modeliranja računskih operacija s razlomcima (engl. Modeling Fractions Test – MFT) koji se sastoji od tri pitanja o množenju razlomaka i tri o dijeljenju razlomaka. U analizi podataka upotrijebljena je deskriptivna statistika te je svako pitanje analizirano zasebno. Ispitanici su pokazali bolje vještine modeliranja u svojim odgovorima na pitanja s računskim operacijama množenja, nego dijeljenja, izradili su bolje modele u slučaju operacija množenja i dijeljenja cijelih brojeva i jediničnih razlomaka te je polovica ispitanika potpuno točno odgovorila na pitanja u TMR-u.

Ključne riječi: *budući nastavnici viših razreda osnovnih škola; modeliranje računskih operacija množenja i dijeljenja razlomaka; podučavanje matematike; razlomci.*

Uvod

Razlomak se definira kao jednak dio ili više jednakih dijelova cjeline (Baykul, 2005). Poput cijelih brojeva, razlomci također označavaju količinu, ali u razlomcima je važan broj dijelova umjesto cjelina (Altun, 2008). Istim se razlomkom mogu predstaviti različite veličine. Primjerice, jedan dio površina trokuta, četverokuta i kruga podijeljenih u dva jednaka dijela (Slika 1) može se izraziti istim razlomkom ($\frac{1}{2}$). Međutim, učenici znaju griješiti misleći da se različite veličine, koje su definirane istim razlomkom, smatraju jednakima (Erdem, 2015). Smatra se da su razlozi za tu grešku sljedeći: učenici nemaju odgovarajuće iskustvo vezano uz činjenicu da se količina izražena razlomkom odnosi na referentnu cjelinu te se ne usavršavaju u ovom važnom rezoniranju (Alacaci, 2009). U skladu s navedenim, Lamon (1996) ističe da promišljanje o referentnoj cjelini ima važnu ulogu u smislenom učenju o konceptu koji predstavlja razlomak.

Slika 1.

Učenici ponekad razlomke miješaju s racionalnim brojevima i doživljavaju ih kao brojevne skupove. U najširem smislu termina, razlomci su izrazi koji se koriste u predstavljanju racionalnih brojeva i koji mogu postojati u beskonačnim brojevima (Altun, 2008). Kao što je poznato, izraz $\frac{1}{2}$ može se shvatiti kao racionalan broj i kao razlomak. Međutim, izrazi kao što su to $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots$, koji su jednaki ovome razlomku, jesu razlomci, ali nisu racionalni brojevi jer ne odgovaraju terminu racionalan broj (*činjenica da su brojnik i nazivnik relativno prosti brojevi*). Time dolazimo da zaključka da se racionalan broj može predstaviti s više nego jednim razlomkom. Također se podrazumijeva da razlomci ne mogu biti brojevni skup jer je to u protuslovlju s činjenicom da je skup dobro definiran.

Velik broj istraživanja utvrrđuje da razlomci kao nastavna tema mogu izazvati različita shvaćanja te tako zbuni učenike. Stoga predstavljaju jednu od matematičkih tema s kojima učenici imaju najviše poteškoća (Aksu, 1997; Behr, Lesh, Post, i Silver, 1983; Bezuk, i Bieck, 1993; de Castro, 2008; Erdem, 2015; Ersoy, i Ardahan, 2003; Gökkurt, Şahin, Soylu i Soylu, 2013; Işık, i Kar, 2012; Moss, i Case, 1999; Olkun i Toluk-Uçar, 2012; Pesen, 2008; Soylu, i Soylu, 2005; Şiap, i Duru, 2004; Stafylidou, i Vosniadou, 2004; Tirosh, 2000; Toluk-Uçar, 2009; Ünlü, i Ertekin, 2012). U literaturi nalazimo mnoge razloge zbog kojih učenici nailaze na pteškoće u učenju o razlomcima:

- ❑ Razlomci se pretežno podučavaju s s pomoću pravila umjesto konceptualno (Aksu, 1997; Bezuk, i Bieck, 1993; de Castro, 2008; Gökkurt i sur., 2013; Moss, i Case, 1999; Pesen, 2008; Soylu, i Soylu, 2005; Tirosh, 2000; Toluk-Uçar, 2009)
- ❑ Koncept razlomka prilično je apstraktan (Olkun, i Toluk-Uçar, 2012)
- ❑ Razlomci i njihove karakteristike često nisu u skladu s prirodnim brojevima i njihovim karakteristikama koje učenici poznaju (Stafylidou, i Vosniadou, 2004; Tirosh, 2000)
- ❑ Koncept jediničnoga razlomka nije u potpunosti jasan (Ersoy, i Ardahan, 2003)
- ❑ Brojnik i nazivnik se u razlomku percipiraju kao dva nepovezana cijela (Doğan, i Yeniterzi, 2011; Şiap, i Duru, 2004)
- ❑ Učitelji ili budući učitelji ne vladaju potpuno znanjem o razlomcima (Toluk-Uçar, 2009)
- ❑ Izraz poput $\frac{a}{b}$ može imati različita značenja (Ünlü i Ertekin, 2012). Ta različita značenja obješnjena su u literaturi na sljedeći način: "*a)* ***usporedba dijela i cjeline*** *– označava odnos između dijela i cjeline, b)* ***omjer*** *– označava odnos među dvjema*

količinama, c) ***količnik*** *– označava operaciju dijeljenja, d)* ***operator*** *– označava jednakost razlomaka i računsku operaciju množenja, e)* ***mjera*** *– označava koliku količinu predstavlja razlomak*" (Behr i sur., 1983).

- ❑ Razlomci se doživljavaju samo kao određen dio cjeline ili njezine količine, a u isto se vrijeme ignoriraju ostala značenja razlomaka (Işık, i Kar, 2012).

Činjenica da nastavnici ne posjeduju odgovarajuće znanje o predmetu i o pedagogiji podučavanja razlomaka može otežati učenje o razlomcima. Vrijeme visokog obrazovanja tijekom kojega nastavnici stječu ove dvije vrste znanja ima značajnu ulogu. Iz literature saznajemo da je obrazovanje koje budući nastavnici stječu na fakultetu osobito važno u učinkovitom izvođenju podučavanja u njihovim profesionalnim životima (Arslan, i Özpınar, 2008; Erdem, i Soylu, 2013; Gürbüz, Erdem, i Gülburnu, 2013; Hill, Rowan, i Ball, 2005; Peker, 2009; Smith, 2000; Ubuz, 2002). Međutim, provedena istraživanja potvrđuju da nastavnici matematike ne posjeduju odgovarajuće znanje o razlomcima (Ball, 1990; Gökkurt i sur., 2013; Işıksal, 2006; Kılcan, 2006; Li, i Kulm, 2008; Ma, 1999; Newton, 2008; Simon, 1993). Činjenica da nastavnici i budući nastavnici uče razlomke pretežno koristeći se pravilima umjesto konceptualnim metodama učenja (*U računskim operacijama množenja množe se brojnici pa se rezultat te operacije upisuje kao brojnik; nazivnici se zbrajaju pa se rezultat te operacije upisuje kao nazivnik. U računskim operacijama dijeljenja prvi se razlomak ostavlja nepromijenjen; drugi se razlomak invertira u odnosu na množenje te se množi s prvim razlomkom.*) jedan je od najvažnijih čimbenika koji uzrokuju takvo stanje.

Provedena istraživanja pokazuju da učenici viših razreda osnovnih škola imaju više poteškoća s računskim operacijama množenja, a osobito s računskim operacijama dijeljenja u razlomcima u odnosu na ostale računske operacije (Birgin, i Gürbüz, 2009; Durmuş, 2005; Parmar, 2003; Toluk, 2002; Ünlü, i Ertekin, 2012). Autori čak navode da je računska operacija dijeljenja najteži element matematike kao školskoga predmeta na toj razini (Ma, 1999). Činjenica da je koncept razlomka relativno apstraktan (Olkun, i Toluk-Uçar, 2012) i da su mnoga istraživanja utvrdila da učenici imaju problema s razumijevanjem toga koncepta i operacija vezanih uz taj koncept upućuju na prijeko potrebnu upotrebu različitih i učinkovitih metoda u podučavanju toga koncepta (Erdem, 2015). Smatra se da je modeliranje računskih operacija s razlomcima, kao sredstvo konkretizacije, učinkovita metoda u konceptualnom podučavanju razlomaka učenicima viših razreda osnovnih škola. U mnogim se provedenim studijama naglašava da se modeliranje treba koristiti u podučavanju razlomaka gdje nailazimo na poteškoće u učenju i podučavanju (Ball, 1993; Behr i sur., 1983; de Castro, 2008; Erdem, 2015; Lamon, 1996; Parmar, 2003; Toluk-Uçar, 2009).

Općenito, ističu se tri različita modela podučavanja razlomaka učenicima viših razreda osnovnih škola (Parmar, 2003). Ti su modeli prikazani na slici 2. Oni su izraženi kao model područja ili polja (a), model duljine (b), i model skupa (c) i (d). Pregledom literature pronađeno je da je model područja ili polja najčešće korišten model (de Castro, 2008; Forrester, i Chinnappan, 2010; Parmar, 2003; Toluk-Uçar, 2009). S druge strane, modeli u kojima se koriste geometrijski oblici preporučuju se u podučavanju računskih operacija s razlomcima (Kieren, 1988; Pesen, 2008; Vergnaud, 1988). Stoga, ako se koristi model polja, može doći do poteškoća pri dijeljenju trokutnihih ili kružnih modela polja u jednake dijelove te se preporuča korištenje pravokutnih modela (Doğan-Temur, 2011). Osim toga, pregled literature pokazuje da se modeli voumena također koriste u podučavanju razlomaka (Pesen, 2008).

Slika 2.

U podučavanju računskih operacija s razlomcima s pomoću modeliranja važno je znati koji su modeli primjereni učeničkoj razini. Kako bi se to postiglo nastavnik mora imati primjerenu razinu predmetnoga kao i pedagoškoga znanja. Drugim riječima, nastavnik mora posjedovati primjereno konceptualno i operativno znanje te mora na učinkovit način prenijeti to svoje znanje učenicima. Mnoga istraživanja naglašavaju važnost nastavnikova predmetnoga i pedagoškoga znanja u svrhu postizanja učinkovite poduke (Ball, 1988; 1990; Cankoy, 2010; Davis, i Simmt, 2006; Erdem, i Soylu, 2013; Gökkurt i sur., 2013; Gürbüz i sur., 2013; Hill i sur., 2005; Rowan, Chiang, i Miller, 1997; Shulman, 1986; 1987; Tchoshanov, 2011). Stoga obrazovanje koje učitelji stječu tijekom svojega visokoga obrazovanja ima važnu ulogu u njihovu učinkovitom podučavanju materije vezane uz obje navedene vrste znanja.

Kako je prije spomenuto, kad uzmemo u obzir činjenicu da učenici viših razreda osnovnih škola imaju više poteškoća s računskim operacijama množenja, a osobito dijeljenja u razlomcima nego s ostalim računskim operacijama i kad se uzme u obzir važnost modeliranja u podučavanju računskih operacija s razlomcima (Ball, 1993; Behr i sur., 1983; de Castro, 2008; Lamon, 1996; Parmar, 2003; Toluk-Uçar, 2009), važno je istražiti vještine modeliranja računskih operacija množenja i dijeljenja razlomaka koje posjeduju budući učitelji matematike učenicima viših razreda osnovnih škola. Stoga je cilj ovoga rada istražiti vještine modeliranja računskih operacija množenja i dijeljenja razlomaka u budućih učitelja matematike u višim razredima osnovnih škola.

Metode

S obzirom na to da je cilj ovoga rada utvrditi postojeće stanje, radi se o deskriptivnom istraživanju. U takvim se istraživanjima proučavane situacije i uvjeti detaljno studiraju, istražuje se njihov odnos s ostalim situacijama i uvjetima te se nastoji opisati „što“one jesu (Tanrıöğen, 2011).

Sudionici

Testna se skupina sastoji od ukupno 104 studenta četvrtih godina koji pohađaju odsjeke za podučavanje matematike u višim razredima osnovnih škola na Učiteljskim fakultetima na dva državna sveučilišta u Turskoj. Za istraživanje su odabrani studenti četvrtih godina zbog toga što su oni pohađali kolegije *Posebne metode podučavanja I-II*. Ti su kolegiji važni za formiranje predmetnog i pedagoškog znanja u studenata.

Zapravo, kad se prouče sadržaji kolegija dodiplomskog programa poučavanja matematike u višim razredima osnovnih škola koje je donijelo Vijeće za visoko školstvo (CHE/YÖK) u Turskoj, vidi se da su su uvedeni opći ciljevi podučavanja u praksi postavljeni kao metode, tehnike, sredstva i materijali koji se trebaju koristiti u podučavanju predmetno-specifičnih koncepata.

Prikupljanje podataka

Test modeliranja računskih operacija s razlomcima (engl. Modeling Fractions Test – MFT), koji se sastojao od četiri zadatka (tri zadatka računskih operacija množenja u razlomcima i tri zadtka računskih operacija dijeljenja u razlomcima) upotrijebljen je kao sredstvo prikupljanja podataka. Literatura, Kurikulum za matematiku u 6. i 8. razredima osnovnih škola (MNE/MEB [Ministarstvo nacionalnog obrazovanja], 2009) i mišljenja iskusnih nastavnika matematike (*9-13-15 godina radnog iskustva*) u višim razredima osnovnih škola koristili su se kako bi se utvrdila primjerenost zadtaka ispitivanih u MFT na obrazovanje učenika viših razreda osnovnih škola. U svjetlu pilot istraživanja koje je provedeno na uzorku od 35 budućih učitelja odlučeno je da će testiranje budućih učitelja u glavnom dijelu istraživanja trajati 50 minuta. Zatim je izračunat Cronbach Alpha koeficijent (0,819) kao rezultat analiza koje su provedene kako bi se utvrdila valjanost testa. Osim toga, kako bi se prikupile točnije i detaljnije informacije o izradi modela, buduće se učitelje zamolilo da objasne kako su modelirali operacije u testu.

Analiza

U analizi je upotrijebljena deskriptivna statistika. Svako je pitanje zasebno proučeno i ocijenjeno. Prikazane su tablice frekvencija i postotaka za svaki zadatak iz MFT-a. S obzirom na to da se smatralo kako će izravni citati biti učinkoviti za odražavanje misli pojedinaca (Yin, 1994), neki su primjeri o zadatcima navedeni točno tako kako su izneseni. Rubrika prikazana u Tablici 1, koja je razvijena uz pomoć literature (Forrester, & Chinnappan, 2010; Toluk-Uçar, 2009), upotrijebljena je pri ocjenjivanju odgovorima sudionika na zadatke u MFT-u.

Tablica 1.

Rezultati

U ovome su dijelu rada prikazani deskriptivni rezultati odgovora budućih učitelja na zadatke u MFT-u. Prikazani su primjeri nekih odgovora studenata na te zadatke.

Tablica 2.

Kao što se vidi iz Tablice 2, otprilike polovina sudionika (50,5%) dala je potpuno točne odgovore na operacija u MFT. Sudionici su općenito bolje modelirali računske operacije množenja razlomaka nego računske operacije dijeljenja razlomaka. Sudionici su najbolje riješili zadatak $2 \times \frac{1}{3}$ (*73,1%*), a najviše su poteškoća imali sa zadatkom

$\frac{2}{3} \div \frac{1}{2}$ (*28,8%*), zbog čega su dali najmanje izjava o modeliranju te računske operacije. Budući su učitelji izradili najbolje modele i dali su najtočnije izjave o operacijama množenja i dijeljenja cijeloga broja i jediničnoga razlomka.

Objasni operaciju $2 \times \frac{1}{3}$ **s pomoću modeliranja.**

U ovome su zadatku budući učitelji trebali razumjeti da je množenje zapravo repetitivno zbrajanje te prikazati jednu trećinu dvaju jednakih cjelina posebno te ih spojiti u jednu cjelinu. Analizom je utvrđeno da su sudionici dali odgovore koji spadaju u sljedeće kategorije: 73,1% *potpuno točno*, 10,6% *djelomično točno-A*, 2,9% *djelomično točno-B*, 12,5% *netočno* i 1% *neodgovoreno*. Među svim operacijama, na zadatku $2 \times \frac{1}{3}$ izrađen je najveći broj točnih modela i dane su najčešće točne izjave. Mnogi budući učitelji znali su da operacija $2 \times \frac{1}{3}$ predstavlja zbrajanje dviju jedinica od $\frac{1}{3}$ te su znali izraditi primjerene modele koristeći se tim znanjem (Slike 3a-3b).

Slika 3.

Objasni operaciju $\frac{1}{2} \times \frac{2}{3}$ s pomoću modeliranja.

U ovome su zadatku budući učitelji trebali odrediti dvije trećine jedne cjeline u određenome smjeru (*horizontalnom ili vertikalnom*) te potom odrediti polovicu iste cjeline u drugome smjeru. Na kraju su trebali pokazati područje gdje se ta dva polja sijeku. Analizom je utvrđeno da su sudionici dali odgovore koji se ubrajaju u kategorije: 52,9% *potpuno točno*, 6,7% *djelomično točno-A*, 12,5% *djelomično točno-B*, 32,1% *netočno* i 4,8% *neodgovoreno*. Budući su učitelji znali pokazati sjecište dvaju polja kao model te operacije nakon što su pokazali jedan množitelj vodoravno na geometrijskome liku, a drugi okomito na geometrijskome liku (Slika 4a). S druge strane, utvrđeno je da su neki budući učitelji vidjeli tu operaciju kao pola razlomka $\frac{2}{3}$ te su tako izradili i model (Slika 4b).

Slika 4.

Objasni operaciju $\frac{2}{5} \times \frac{1}{3}$ s pomoću modeliranja.

U ovome su zadatku budući nastavnici trebali odrediti dvije petine jedne cjeline u određenome smjeru te potom odrediti jednu trećinu iste cjeline u drugome smjeru. Na kraju su trebali pokazati područja gdje se ta dva polja sijeku. Analizom je utvrđeno da su sudionici dali odgovore koji se ubrajaju u sljedeće kategorije: 49% *potpuno točno*, 5,8% *djelomično točno-A*, 15,4% *djelomično točno-B*, 18,3% *netočno* i 11,5% *neodgovoreno*. Budući su učitelji imali najviše poteškoća s izradom modela za tu operaciju množenja. Neki odgovori budućih učitelja na ovo pitanje bili su na očekivanoj razini iako su s ovom operacijom imali najviše poteškoća (Slika 5a). S druge strane, uočeno je da su mnogi sudionici upotrijebili model područja ili polja za modeliranje ove računske operacije, dok su neki budući učitelji netočno upotrijebili model duljine (modeliranje komponenata i rezultata umjesto modeliranja same operacije) u ovoj i ostalim operacijama (Slika 5b).

Slika 5.

Objasni operaciju $2 \div \frac{1}{2}$ ***s pomoću modeliranja.***

U ovome su zadatku budući nastavnici trebali podijeliti na dva jednaka dijela svaku od dviju jednakih cjelina te prikazati broj polovina. Analizom je utvrđeno da su sudionici dali odgovore koji se ubrajaju u sljedeće kategorije: 58,7% *potpuno točno*, 7,7% *djelomično točno-A*, 4,8% *djelomično točno-B*, 20,2% *netočno* i 8,7% *neodgovoreno*. Na ovoj su operaciji dijeljenja budući nastavnici bili najuspješniji u izradi modela. Neki su budući nastavnici znali da se broj polovina ($\frac{1}{2}$) u dvije cjeline izražava operacijom $2 \div \frac{1}{2}$ (Slika 6a) dok neki nisu izradili točan model ove operacije te su samo primijenili pravilo dijeljenja u razlomcima (*Prvi se razlomak ne mijenja, drugi se invertira te se množi s prvim razlomkom*) (Slika 6b).

Slika 6.

Objasni operaciju $\frac{1}{3} \div \frac{1}{6}$ ***s pomoću modeliranja.***

U ovome su zadatku budući nastavnici trebali odrediti jednu trećinu cjeline te u danome polju pokazati broj područja koja su prikazana kao jedna šestina iste cjeline. Analizom je utvrđeno da su sudionici dali odgovore koji se ubrajaju u sljedeće kategorije: 40,4% *potpuno točno*, 11,5% *djelomično točno-A*, 7,7% *djelomično točno-B*, 24% *netočno* i 16.3% *neodgovoreno*. Neki su budući nastavnici znali da razlomak $\frac{1}{6}$ predstavlja skalu i da bi se operacija $\frac{1}{3} \div \frac{1}{6}$ mogla riješiti pronalaženjem broja veličina koje predstavljaju razlomak $\frac{1}{6}$ u veličini koju predstavlja razlomak $\frac{1}{3}$ (Slika 7a). S druge strane, uočeno je da budući nastavnici nisu izradili model te operacije i da su samo primijenili pravilo dijeljenja razlomaka kao što je to bio slučaj u prethodnoj operaciji dijeljenja (Slika 7b).

Slika 7.

Objasni operaciju $\frac{2}{3} \div \frac{1}{2}$ ***s pomoću modeliranja.***

U ovome se zadatku od budućih nastavnika očekivalo da odrede dvije trećine cjeline te u dobivenom polju pokažu broj područja koja se prikazuju kao polovine iste cjeline. Analizom je utvrđeno da su sudionici dali odgovore koji se ubrajaju u sljedeće kategorije: 28,8% *potpuno točno*, 10,6% *djelomično točno-A*, 9,6% *djelomično točno-B*, 23,1% *netočno* i 27,9% *neodgovoreno*. S modeliranjem te operacije budući su nastavnici imali najviše problema pa su dali najmanje točnih izjava. U tome zadatku većina sudionika nije mogla shvatiti da se operacija $\frac{2}{3} \div \frac{1}{2}$ mogla riješiti pronalaženjem broja veličina koje označava razlomak $\frac{1}{2}$ u veličini označenoj s $\frac{2}{3}$. Zbog toga su mnogi sudionici radije modelirali brojnik ($\frac{2}{3}$), nazivnik ($\frac{1}{2}$) i rezultat ($\frac{4}{3}$) umjesto same operacije dijeljenja (Slika 8b).

Slika 8.

Zaključno gledano, istraživanjem se pokazalo sljedeće: (a) odgovori otprilike polovine budućih nastavnika spadaju u kategoriju *potpuno točno*; (b) budući su nastavnici bolje rješavali zadatke modeliranja računskih operacija množenja razlomaka nego zadatke modeliranja računskih operacija dijeljenja razlomaka; (c) izrađivali su bolje modele u operacijama u kojima se zahtijevalo množenje i dijeljenje cijeloga broja i jedinični razlomak i (d) mnogi budući nastavnici koristili su se memoriziranim znanjem (*u operacijama dijeljenja prvi razlomak ostaje nepromijenjen, drugi je razlomak invertiran i množi se s prvim razlomkom*) osobito u modeliranju operacija dijeljenja razlomaka.

Rasprava i zaključci

Cilj je ovoga rada bio istražiti vještine modeliranja računskih operacija množenja i dijeljenja razlomaka budućih učitelja matematike u višim razredima osnovnih škola. Rezultati analize pokazali su da je polovina budućih nastavnika dala potpuno točne odgovore na zadatke operacija u MFT. Vještine modeliranja računskih operacija množenja i dijeljenja razlomaka budućih nastavnika na srednjoj su razini. Međutim, učinak odgovora koje su ispitanici dali na operacije množenja i dijeljenja razlomaka bio je na različitoj razini. U prosjeku je omjer davanja potpuno točnih odgovora na operacije množenja razlomaka bio 58%, a omjer davanja potpuno točnih odgovora na operacije dijeljenja bio je 42%. Taj je rezultat pokazao da su nastavnici izradili bolje modele u operacijama množenja nego u operacijama dijeljenja. Takav rezultat je u skladu s mišljenjem koje zastupa Ma (1999) o tome da operacije dijeljenja u razlomcima predstavljaju jedno od najtežih gradiva u matematici za više razrede osnovnih škola. Također se u mnogim istraživanjima pokazalo da su učenici viših razreda osnovnih škola imali više poteškoća s operacijama množenja, a osobito s operacijama dijeljenja razlomaka (Birgin, i Gürbüz, 2009; Durmuş, 2005; Parmar, 2003; Toluk, 2002; Ünlü, i Ertekin, 2012).

Rezultati su pokazali da budući nastavnici nisu imali primjerenu razinu konceptualnoga znanja i da je to negativno utjecalo na njihove rezultate u operacijama dijeljenja. Taj je zaključak proizišao iz činjenice da je puno budućih nastavnika pribjeglo algebarskoj operaciji utemeljenoj na memoriziranju umjesto da su izradili model u skladu s operacijama dijeljenja. Takav je rezultat u skladu s rezultatima nekih ranijih studija (Ball, 1990; Işıksal, 2006; Li, i Kulm, 2008; Newton, 2008; Simon, 1993). S druge strane vidjeli smo da su budući nastavnici izradili bolje modele u operacijama u kojima se zahtijevalo množenje i dijeljenje cijeloga broja i jediničnoga razlomka. U tom smislu rezultati analize pokazali su da su budući nastavnici najbolje riješili operaciju $2 \times \frac{1}{3}$ među operacijama množenja, a imali su najbolje rezultate u operaciji $2 \div \frac{1}{2}$ među operacijama dijeljenja. Osim toga, vidjeli smo da su budući nastavnici imali tim više poteškoća u izradi modela što se vrijednost brojnika i nazivnika u razlomcima povećavala. Primjerice, budući su nastavnici imali više s operacijom $\frac{2}{5} \times \frac{1}{3}$ nego s operacijom $\frac{1}{2} \times \frac{2}{3}$, iako su jedinični razlomci upotrijebljeni u obje operacije (v. Tablicu 2). S obzirom na to može se reći da je teže modelirati operacije u kojima se koriste veliki razlomci, što tvrdi i Castro (2008). Važno je da učenici shvate da se slično rezoniranje može predložiti u velikim razlomcima nakon što ih se poduči modeliranju operacija koje se provode s malim razlomcima.

Također je pokazano da su budući nastavnici znali točno modelirati komponente u operacijama množenja i dijeljenja dok su nepotpuno ili netočno modelirali same operacije ili ih uopće nisu bili u stanju modelirati. Primjerice, neki su budući nastavnici točno modelirali razlomke $\frac{1}{2}$ i $\frac{2}{3}$ u operaciji $\frac{1}{2} \times \frac{2}{3}$, a imali su poteškoća s modeliranjem same operacije množenja. Slični se rezultati budućih nastavnika nalaze i u literaturi (de Castro, 2008; Redmond, 2009; Tirosh, 2000; Ünlü, i Ertekin, 2012; Zembat, 2004). S obzirom na to može se reći da su vještine modeliranja razlomaka i modeliranje operacija u razlomcima različite vještine. Može se reći da nastavnici i budući nastavnici uče razlomke pretežno primjenjujući pravila (*U operacijama množenja brojnici se množe te se rezultat ove operacije bilježi kao brojnik; nazivnici se množe te se rezultat ove operacije bilježi kao nazivnik. U operaciji dijeljenja prvi se razlomak ostavlja takvim kakav jest, drugi se invertira s obzirom na operaciju množenja te se potom množi s prvim razlomkom*.) Takvo je njihovo znanje učinkovito u generiranju rezultata. Također se u mnogim istraživanjima navodi da su pravila podučavanja utjecajna pri suočavanju s poteškoćama u učenju o razlomcima (Aksu, 1997; Bezuk, i Bieck, 1993; de Castro, 2008; Gökkurt i sur., 2013; Moss, i Case, 1999; Pesen, 2008; Soylu, i Soylu, 2005; Tirosh, 2000; Toluk-Uçar, 2009).

Kad su se rezultati temeljito analizirali, uočeno je da su neki budući nastavnici izradili točne modele što se tiče operacija, ali nisu dali ni jednu izjavu, ili su dali nepune izjave o tome kako su ih radili (v. Sliku 8a). Drugim riječima, neki budući nastavnici nisu bili u stanju opisati procese koji su upotrijebljeni u operacijama na matematički primjerenome stupnju. Može se reći da je činjenica da budući učitelji nisu bili u stanju prenijeti svoje misli na matematičkom jeziku i na primjerenoj razini bila presudna u pojavi tog uvjeta. Edukatori tu imaju važne odgovornosti. Çalıkoğlu-Bali (2003), Hiebert, Morris i Glass (2003) i Straker (1993) spominju važnost učinkovite upotrebe matematičkoga vokabulara u okolinama u kojima se uči s ciljem postizanja razumijevanja matematičkih koncepata. Stoga je važno da se edukatori često i točno koriste matematičkim vokabularom kad podučavaju matematičke koncepte. Učinkovitom upotrebom takve metode učenja u okolini u kojoj se uči, a počevši od ranoga djetinjstva, uvelike će se pridonijeti učenju koncepata razlomaka i operacija u razlomcima na smislen način. Uzme li se u obzir to da znanje nastavnika ima ključnu ulogu u ovom aspektu nastave, važno je učinkovito upotrijebiti metodu modeliranja u podučavanju razlomaka na sveučilišnoj razini. Vjeruje se da će s pomoću te metode nastavnici steći spoznaje o učenju njihovih učenika koje se odnosi na proces operacije radije nego da samo provjeravaju točnost rezultata. S druge strane, nastavnici moraju imati na umu to da su modeliranje razlomaka i modeliranje operacija s razlomcima različite vještine u podučavanju razlomaka. Osim toga, posjedovanje primjerene razine predmetnoga znanja u budućih nastavnika možda nije dovoljno za učinkovito podučavanje. Budući nastavnici također moraju posjedovati primjerenu razinu znanja pedagoškoga sadržaja. S obzirom na to važno je oblikovati okoline u kojima će budući nastavnici učiti matematičke koncepte tako što će ih se uvrstiti u sveučilišne kolegije Posebnih metoda podučavanja.