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ORTHONORMAL FUNCTIONS BASED MODEL PREDICTIVE CONTROL OF PH NEUTRALIZATION PROCESS

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Original scientific paper

This paper presents use of Legendre and Laguerre orthonormal functions for representation of the control trajectory in discrete model predictive control, on a pH neutralization process, which is a process with static non-linearity. Performance is tested with plant-to-model mismatch present. Orthonormal functions are used for efficient parameterisation of the difference of control signal as in the case of linear process. This approach has better computational efficiency compared to the classical predictive control algorithms.

Keywords: *model predictive control, non-linear process, orthonormal functions, pH neutralization*

Ortonormalne funkcije u prediktivnom upravljanju modelom kod pH procesa neutralizacije

Izvorni znanstveni članak

U radu se opisuje primjena Legendre i Laguerre ortonormalnih funkcija za predstavljanje upravljačke putanje u prediktivnom upravljanju diskretnim modelom, kod pH procesa neutralizacije, procesa sa statičnom nelinearnošću. Provedba je testirana uz postojanje nepodešenosti između postrojenja i modela. Ortonormalne funkcije su rabljene za učinkovitu parametrizaciju razlike upravljačkog signala kao u slučaju linearnog procesa. U ovom je pristupu računalna učinkovitost bolja u usporedbi s klasičnim algoritmima prediktivnog upravljanja.

Ključne riječi: *nelinearni proces, ortonormalne funkcije, pH neutralizacija, prediktivno upravljanje modelom*

1 Introduction

Recently proposed approaches of model predictive control suggest use of different model descriptions: hybrid fuzzy model [13, 15], Wiener-model based on PWL [14], hybrid MPC based on genetic algorithms [16], probabilistic neural-network [17]. Approach used in this article proposes use of orthonormal functions as in [2] tested on a non-linear pH neutralization process.

As described in [1] the technique used in the design of discrete model predictive controller is based on modeling the control trajectory, control signal $u(k)$ or the difference of control signal $\Delta u(k)$ by forward shift operators, which can lead to possible large number of forward shift operators used for the description of control trajectory if complicated dynamics of the process, fast sampling or high demands on closed-loop performance are present. Fast changes of control signal are also possible as there is no structural constraint on the future control signal. Performance of the model predictive controller, based on orthonormal Laguerre and Legendre functions, is tested on a pH neutralization process, having static non-linearity. It can be concluded that plant-to-model mismatch is present, so robustness of this approach is tested.

As described in [1, 2] advantage of such approach is reduction in number of parameters used for description of the control trajectory compared to the classical approach. The change of control trajectory is managed through adjustment of scaling factor present in orthonormal function.

Paper consists of Section 1, being Introduction, Sections 2 and 3, present algorithm for model predictive controller design using Laguerre and Legendre functions, respectively. Section 4 presents pH neutralization process. In Section 5 results of simulation examples are presented. After simulation section, follow conclusions.

2 Laguerre orthonormal functions

The z-transforms of the discrete-time Laguerre networks are written as

$$\begin{aligned}
&\mathrm{L}_1(z)=\frac{\sqrt{1-a^2}}{1-az^{-1}},\\
&\mathrm{L}_2(z)=\frac{\sqrt{1-a^2}}{1-az^{-1}}\frac{z^{-1}-a}{1-az^{-1}},\\
&\vdots\\
&\mathrm{L}_N(z)=\frac{\sqrt{1-a^2}}{1-az^{-1}}\left(\frac{z^{-1}-a}{1-az^{-1}}\right)^{N-1},
\end{aligned}\tag{1}$$

$l_i(k)$ denotes the inverse z-transform of $\mathrm{L}_i(z, a)$. In Fig. 1 Laguerre network is presented. The set of discrete-time Laguerre functions is expressed in a vector form as

$$\mathrm{L}(k)=\left[l_1(k)\ l_2(k)\ \dots\ l_N(k)\right]^{\mathrm{T}}.\tag{2}$$

Set of discrete Laguerre functions satisfies the following difference equation:

$$\mathrm{L}(k+1)=A_l\mathrm{L}(k).\tag{3}$$

A_l is a quadratic matrix of dimension ($N \times N$) and is function of parameters a and $\beta = 1 - a^2$. Initial state is given by

$$\mathrm{L}(0)^{\mathrm{T}}=\sqrt{\beta}\left[1\quad -a\quad a^2\quad -a^3\quad \dots\quad (-1)^{N-1}a^{N-1}\right].\tag{4}$$

Example for $N = 5$:

$$A_l=\begin{bmatrix} a & 0 & 0 & 0 & 0 \\ \beta & a & 0 & 0 & 0 \\ -a\beta & \beta & a & 0 & 0 \\ a^2\beta & -a\beta & \beta & a & 0 \\ -a^3\beta & a^2\beta & -a\beta & \beta & a \end{bmatrix}.$$

Design of discrete MPC using the pulse operator corresponds to the case where the parameter $a = 0$ in the Laguerre polynomial.

At time k_i, the control trajectory $\Delta u(k_i)$, $\Delta u(k_i+1)$, $\Delta u(k_i+2)$, …, $\Delta u(k_i+k)$, …, is regarded as the impulse response of a stable dynamic system. A set of Laguerre functions, $l_1(k)$, $l_2(k)$, …, $l_N(k)$ is used to capture the dynamic response with a set of Laguerre coefficients that are to be determined from the design process. Based on this,

$$\Delta u(k_i+k)=\sum_{j=1}^{N} c_j(k_i)l_j(k), \tag{5}$$

where k_i is the initial time of the moving horizon window and k is the future sampling instant, N is the number of terms used in the expansion, c_j, j = 1, 2, ..., N, are the coefficients being functions of the initial time of the moving horizon window, k_i.

Eq. (5) can also be expressed in a vector form:

$$\Delta u(k_i+k)=\mathrm{L}(k)^{\mathrm{T}}\eta, \tag{5a}$$

where η has N Laguerre coefficients:

$$\eta=\begin{bmatrix} c_1 & c_2 & \dots & c_N \end{bmatrix}^{\mathrm{T}}. \tag{5b}$$

So, the coefficient vector η is optimized and computed in the design.

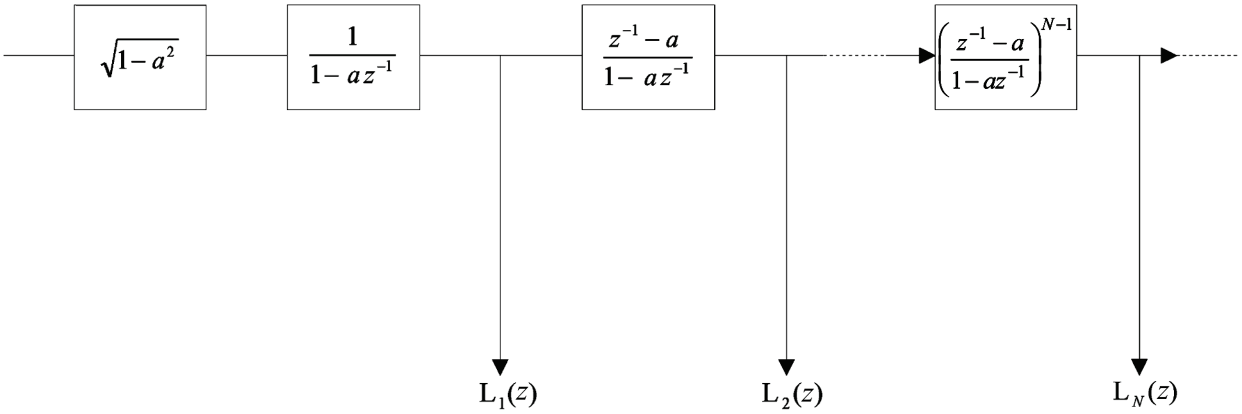

Figure 1 Laguerre network

3 Legendre orthonormal functions

The z-transforms of the discrete-time Legendre networks are written as

$$B_n(z)=z^d\left(\frac{\sqrt{1-|\xi_n|^2}}{z-\xi_n}\right)\prod_{k=0}^{n-1}\left(\frac{1-\overline{\xi}_k z}{z-\xi_k}\right);\ d=0 \text{ or } 1,$$

$$\xi_k=\frac{2-\alpha(2k+1)}{2+\alpha(2k+1)}, \tag{6}$$

$p_i(k)$ denotes the inverse z-transform of $B_i(z,\xi_1\dots\xi_i)$. The set of discrete-time Legendre functions is expressed in a vector form as

$$\mathrm{P}(k)=\begin{bmatrix} p_1(k) & p_2(k) & \dots & p_N(k) \end{bmatrix}^{\mathrm{T}}. \tag{7}$$

Set of discrete Legendre functions satisfies the following difference equation

$$\mathrm{P}(k+1)=\varOmega\mathrm{P}(k), \tag{7a}$$

similar to the Laguerre case.

By analysing Legendre network given in Fig. 2, or by analyzing construction given by Eq. (6) one obtains

$$\varOmega=\begin{bmatrix} \xi_1 & 0 & 0 & \dots & 0 \\ 0 & \xi_2 & 0 & \dots & 0 \\ 0 & 0 & \xi_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & \xi_n \end{bmatrix}, \tag{7b}$$

$$\mathrm{P}(0)=\begin{bmatrix} \sqrt{\beta_1} \\ -\xi_1\sqrt{\beta_2} \\ \xi_1\xi_2\sqrt{\beta_3} \\ -\xi_1\xi_2\xi_3\sqrt{\beta_4} \\ \xi_1\xi_2\xi_3\xi_4\sqrt{\beta_5} \\ \vdots \end{bmatrix}, \tag{7c}$$

β_i being $\beta_i=1-\xi_i^2$.

At time k_i, the control trajectory $\Delta u(k_i)$, $\Delta u(k_i+1)$, $\Delta u(k_i+2)$, …, $\Delta u(k_i+k)$, is regarded as the impulse response of a stable dynamic system. A set of

Legendre functions, $p_1(k)$, $p_2(k)$, …, $p_N(k)$ is used to capture the dynamic response with a set of Legendre coefficients that are to be determined from the design process. Based on this,

$$\Delta u(k_i + k) = \sum_{j=1}^{N} c_j(k_i) p_j(k)\,, \tag{8}$$

where k_i is the initial time of the moving horizon window and k is the future sampling instant, N is the number of terms used in the expansion, c_j, j = 1, 2, …, N, are the coefficients being functions of the initial time of the moving horizon window, k_i.

Eq. (8) can also be expressed in a vector form:

$$\Delta u(k_i + k) = \mathrm{P}(k)^{\mathrm{T}} \eta, \tag{9}$$

where η has N Legendre coefficients:

$$\eta = \begin{bmatrix} c_1 & c_2 & \dots & c_N \end{bmatrix}^{\mathrm{T}}. \tag{10}$$

So, the coefficient vector η is optimized and computed in the design.

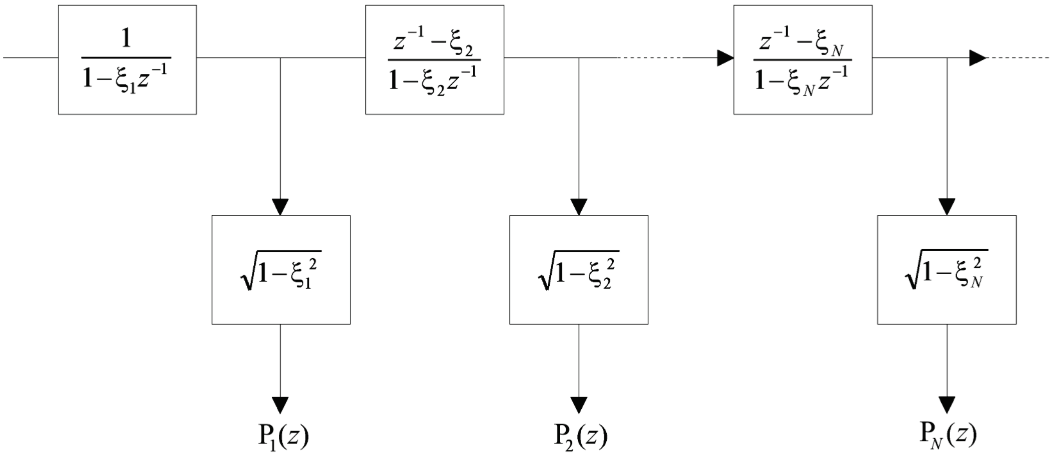

Figure 2 Legendre network

As thoroughly explained in [1], an alternative formulation of the cost function is used and formulated, for both, Laguerre functions presented in Section 2, and Legendre functions presented in this section. The task is finding the coefficient vector η to minimize the cost function:

$$J = \sum_{m=1}^{N_p} x(k_i + m|k_i)^{\mathrm{T}} \boldsymbol{Q} x(k_i + m|k_i) + \eta^{\mathrm{T}} \boldsymbol{R}_L \eta, \tag{11}$$

$\boldsymbol{Q} \geq 0$ and $\boldsymbol{R}_L > 0$ being weighting matrices.

Having optimal parameter vector η, the receding horizon control law is realized as

$$\Delta u(k_i) = \mathrm{P}(0)^{\mathrm{T}} \eta. \tag{11a}$$

Design parameter N is the number of terms used in capturing the control signal, in both, Laguerre and Legendre case.

Parameter ξ_i defined in Eq. (6) is like parameter a in the Laguerre case, except it has progression.

Stability of the closed loop system can be guaranteed under certain circumstances. An approach that uses terminal constraints on the state variables, which forces the terminal state variables to be zero is thoroughly explained in [1] for Laguerre case, and can be used in the Legendre case.

4 pH neutralization process

As presented in [3] the pH neutralization process is explained as follows. An acid stream (flow Q_1), a buffer stream (flow Q_2) and a base stream (flow Q_3) are mixed in a tank T_1. The acid and base streams are equipped with flow control valves. Before mixing, the acid stream passes through a tank T_2, which introduces additional flow dynamics. The liquid levels h_1 and h_2 and effluent pH are measured variables. The single-input single-output control scheme considers the effluent pH to be the controlled variable y and the base stream flow Q_3 to be the manipulated variable u.

A more detailed description of this process can be found in [4].

Fig. 3 shows block diagram of the process described above.

Model of the process described with cascade of a discrete linear block $G_0\left(z^{-1}\right) = \dfrac{0{,}0563 z^{-1} + 0{,}1659 z^{-2}}{1 - 0{,}9508 z^{-1} + 0{,}173 z^{-2}}$ and static nonlinearity f is estimated and presented in [3]. Both Laguerre and Legendre based predictive controllers were obtained based on the discrete linear block and gain equaling one. So, in different working points, there is plant-to-model mismatch present, as controllers were obtained for gain equaling one, and true gain is anywhere between approximately 0,2 and 2 in different working points, as can be seen in [3].

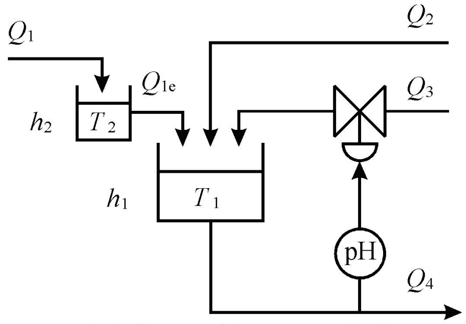

Figure 3 Block diagram of the pH neutralization process

4 Simulation

Fig. 4 presents block scheme used for simulation of pH neutralization process.

Fig. 5 shows reference trajectory. Figs. 6 and 7 show output of the pH neutralization process obtained using MPC based on Laguerre and Legendre orthonormal functions.

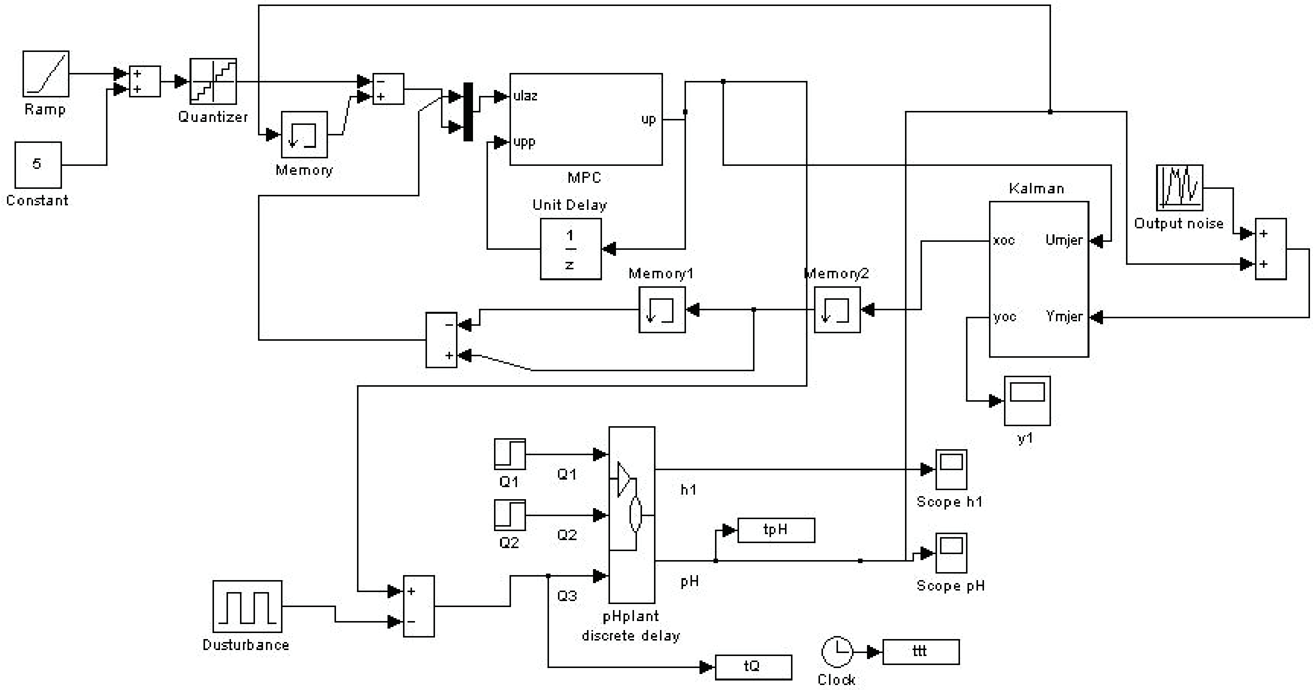

Figure 4 Block scheme used for simulation of process

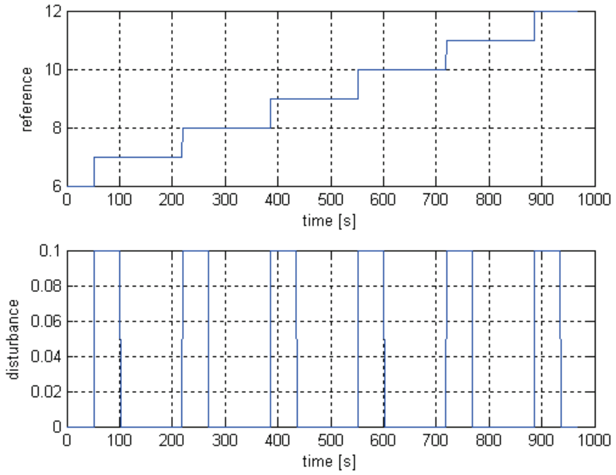

Figure 5 Reference and disturbance trajectories

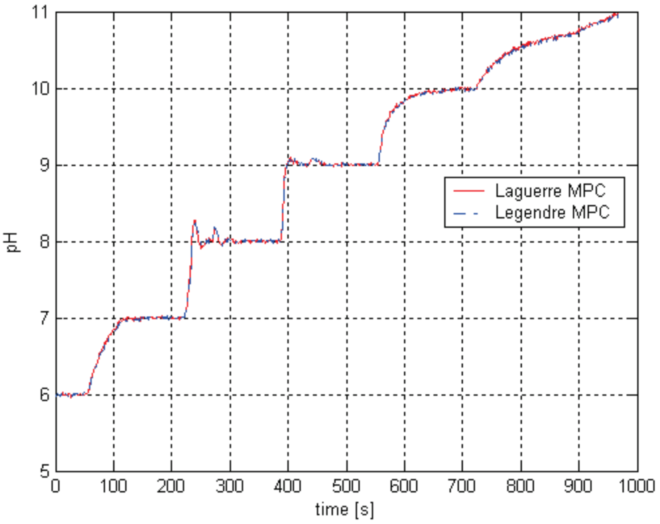

Figure 6 Output of the process obtained using Laguerre based MPC and Legendre based MPC

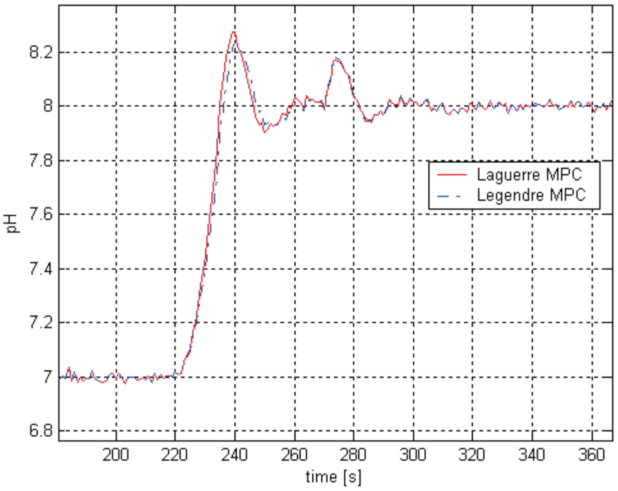

Figure 7 Zoomed portion of previous figure showing output of the process for Legendre and Laguerre based MPCs

5 Conclusion

As can be seen from results presented, proposed use of Laguerre and Legendre functions in model predictive control in the case of pH neutralization process that has static non-linearity, is justified. As working point moves away from the one used for obtaining the MPC, output of the process becomes slower, in both cases, Laguerre and Legendre MPCs. In Fig. 6 one can see that there is a slight difference in outputs, Legendre based MPC produces output with slightly smaller overshoot than the Laguerre one. Both MPCs were obtained for equal number of functions $N = 5$, and equal prediction horizon $N_p = 15$. In Laguerre case, $a = 0{,}998$; in Legendre case middle ξ is $\xi_{sr} = 0{,}998$. Also, disturbance is present with amplitude of 0,1, whose pulse width is 30 % of its period, so after each reference step, follows step of disturbance as can be seen in Figs. 6 and 7. Disturbance rejection can be noticed.

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