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DISCRETE-TIME VARIABLE STRUCTURE CONTROLLER FOR AIRCRAFT FLIGHT ANGLE TRACKING

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Summary

The paper presents the longitudinal, short-period aircraft dynamics and its application on the climb angle tracking. For the aircraft flight angle tracking the stable system centre technique is developed for controlling the output in a discrete-time non-minimum phase causal system using the sliding mode control. The developed discrete-time stable system centre technique transforms the output tracking problem to a corresponding state variable tracking problem by asymptotically identifying the ideal internal dynamics for the unstable internal states of a discrete-time system. A numerical simulation example is given to show the effectiveness of the method.

Key words: *discrete-time non-minimum phase system, stable system centre, sliding mode control*

1. Introduction

The sliding mode control technique was introduced into control engineering five decades ago. The nature of this technique is known to be suitable to control nonlinear systems with parameter uncertainties and bounded input disturbances. This advantage is an obvious reason why the sliding mode has been developed for flight control systems in the past 20 years.

Many real-life objects, including aircraft [1-7] have unstable zero (internal) dynamics, i.e. they have a non-minimum phase (NMPH) nature. A dynamic system is of a NMPH if its dynamics is unstable [8]. Devasia, et al. [9] have presented a stable solution of unstable zero dynamics with an application to an NMPH object by employing iterative algorithms for a solution of unstable differential equations. Output regulation in non-minimum phase systems is studied by Byrnes and Isidori in [10] and a path-following problem that is less general than the output reference profile tracking is studied in [11].

Shtessel [12] and Shkolnikov and Shtessel [13] used the sliding mode control with a dynamic sliding hypersurface for the NMPH output tracking with matched or unmatched disturbances. Also, in papers by Shtessel and Shkolnikov [14-16], the non-minimum phase output tracking in disturbed, uncertain, and causal systems is reduced to a state tracking problem. Bounded state tracking profiles are generated by equations of the stable system centre and followed by the variable structure sliding mode control, whose utilization is very beneficial due to its robustness to the matched disturbances and uncertainties. There are several papers dedicated to the output tracking in NMPH discrete-time (DT) linear systems, among them being [17] and [18]. The output set point regulation is studied in [17]. The optimal control that drives a NMPH DT linear system from a known constant initial to a known constant final position is studied in [18].

However, the output tracking in causal NMPH DT linear systems is still a challenge. Also, there were no attempts in the referenced papers to analyse the possibilities of implementing sliding modes in a causal NMPH discrete-time variable structure system (DTVSS). The obvious advantage of using the sliding mode control technique is in its robustness to the matched disturbances. In [19], the method of stable system centre, that is developed in [14-16] for controlling the output in causal NMPH continuous systems, is extended to the output tracking in NMPH DTVSSs. The developed DT stable system centre technique is illustrated on a variable structure system design for the flight path angle tracking.

2. Flight path angle tracking

An approximate model of an F-16 aircraft in the pitch plane is considered [16]:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 4 & -1.2 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ -0.2 \\ -20 \end{bmatrix} \delta_e \tag{1}$$

where: θ is the pitch angle, α is the angle of attack, q is the pitch rate and δ_e is the elevator deflection. The commanded output of the aircraft (1) is the flight path angle γ (Fig.1), that is the angle of the velocity vector in the vertical plane: $\gamma = \theta - \alpha$.

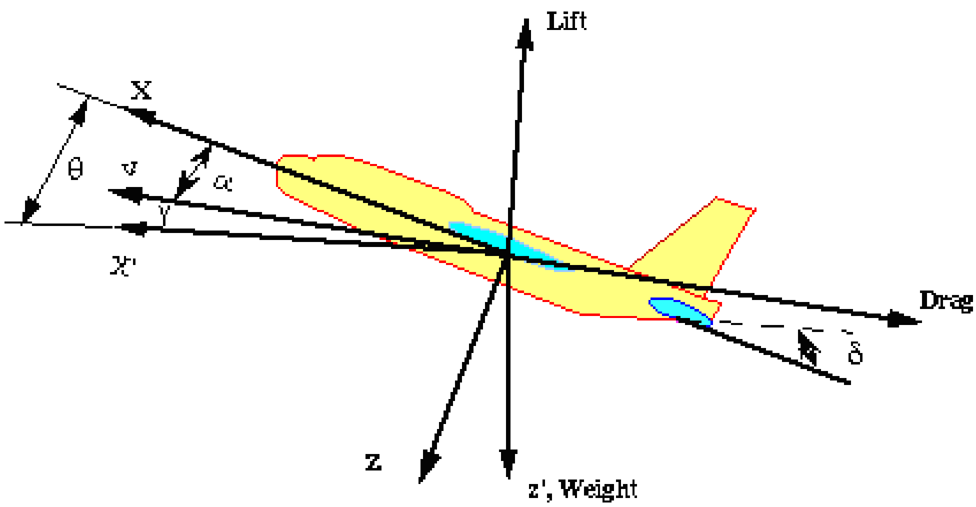

Fig. 1 The pitch plane

The control input is the elevator deflection δ_e. The transfer function of the input δ_e to the output γ for the aircraft F-16 is identified as

$$W(s) = \frac{\gamma(s)}{\delta_e(s)} = \frac{0.2(s+10.816)(s-9.616)}{s(s+3.102)(s-0.902)} \tag{2}$$

The system (1) is of an NMPH nature, because its transfer function (2) has one zero and one pole in the right half of the complex plane. The dynamics of this system is unstable. Following the approach in [8], for the system (1), the following can be identified:

– the internal dynamics (ID)

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -1.5 \end{bmatrix} \begin{bmatrix} \theta \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (-4\gamma - 20\delta_e) \tag{3}$$

– the input/output (I/O) dynamics

$$\dot{\gamma} = -\gamma + \theta + 2\delta_e .$$

The output stabilization problem of the flight path angle γ can be solved by using the sliding mode control method. Let us assume the sliding hypersurface $g = \gamma = 0$. The equivalent control providing the system motion in the sliding hypersurface g is:

$$\delta_e = u_{eq} = -5\theta . \tag{4}$$

Substituting (4) in (3), the reduced-order system dynamics g is obtained:

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 104 & -1.2 \end{bmatrix} \begin{bmatrix} \theta \\ q \end{bmatrix} . \tag{5}$$

The dynamics (5) has a set of eigenvalues $\{-10.816, 9.616\}$ and it is obvious unstable. This instability causes the equivalent control (4) to grow unbounded. Thus, no bounded sliding mode control, switching on the sliding hypresurface $g = 0$ can keep $\gamma = 0$ for a long time.

The unstable dynamics (5) can be stabilized by a traditional PD controller. Shtessel and Shkolnikov [16] chose the PD controller in the form:

$$\delta_e(s) = -2(s+0.7)e(s), \quad e = \gamma_{ref} - \gamma . \tag{6}$$

The disadvantages of this method are:

– the method is not developed for the problem of tracking an arbitrary profile.

– the regulation problem solution is not robust to the object parameter variations and external disturbances.

It has been shown that the standard application of the sliding mode control method does not solve the NMPH tracking problem. Because of that Shtessel and Sholnikov [16] have developed a stable solution to the unstable inverse (internal) dynamics by employing the stable system centre method**.** They have designed a new sliding mode controller to provide tracking in a continuous-time NMPH causal system with matched disturbances and uncertainties.

3. Synthesis of DTVSS for flight path angle tracking

The transfer function (2) can be given in the form with parameter uncertainties:

$$W(s) = \frac{d_3 s^2 + d_2\, s + d_1}{s^3 + a_3\, s^2 + a_2\, s + a_1}, \tag{7}$$

$$a_1 = 0, \quad -3.0778 \le a_2 \le -2.5182, \quad 1.98 \le a_3 \le 2.42,$$

$$-23.6 \le d_1 \le -18, \quad 0.216 \le d_2 \le 0.264, \quad 0.1 \le d_3 \le 0.3 .$$

Disturbance is acting on the object as follows:

$f(t) = 2\,h(t-8)\sin \pi t$, where $h(t-8)$ is the Heaviside step function.

The continuous-time state-space model of object (7) for the nominal values of parameters $a_{2n} = -2.798004,\ a_{3n} = 2.2,\ d_{1n} = -20.8013312,\ d_{2n} = 0.24$ and $d_{3n} = 0.2$ is:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2.798004 & -2.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t),$$

$$y(t) = \begin{bmatrix} -20.8013312 & 0.24 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}. \tag{8}$$

The sampling period $T = 0.4 ms$ was chosen so that the theorem of sampling is fulfilled. Using the δ transform

$$\delta\, x(kT) \overset{\Delta}{=} \frac{x((k+1)T) - x(kT)}{T}, \qquad T > 0 .$$

for the selected sampling period, the DT model of (8) is:

$$\begin{aligned} \delta\, \mathbf{x}(kT) &= \begin{bmatrix} 0 & 1 & 0.002 \\ 0 & 0 & 0.9996 \\ 0 & 2.7972 & -2.192 \end{bmatrix} \mathbf{x}(kT) + \begin{bmatrix} 0 \\ 0.0002 \\ 0.9996 \end{bmatrix} u(kT), \\ y(kT) &= \begin{bmatrix} -20.8013312 & 0.24 & 0.2 \end{bmatrix} \mathbf{x}(kT), \quad k = 0,1,2,... \end{aligned} \tag{9}$$

We wish to design a DTVS controller to achieve output error dynamics in the sliding mode for system (2), such that a control is (in the next explanation k stands for kT)

$$u(k) = u_{eq}(k) + u_{sw}(k), \tag{10}$$

where: $u_{eq}(k)$ is the equivalent control and $u_{sw}(k)$ is the switching control. $u(k)$ is a function accomplishing the following general goals:

- the output $y(k)$ of the NMPH system (9) must follow the desired signal (the output tracking profile) $y_d(k)$ given in a current time, that is $\lim_{k\to\infty} \left| y_d(k) - y(k) \right| = 0$.
- the existence of the sliding mode on the predefined hypersurface should be provided.

Applying the transformation matrix $\mathbf{P}$ with the condition that $\mathbf{x}=\mathbf{P}\,\mathbf{z}$:

$$\mathbf{P}=\begin{bmatrix} 45.7836 & -5.7709 & -0.0004 \\ -50.6927 & -4.6903 & 0.0002 \\ -20.0813 & 0.24 & 0.2 \end{bmatrix},$$

DT system (9) is transformed into a normal canonical form:

$$\begin{aligned} &\delta\,\mathbf{z}(k)=\begin{bmatrix} -10.8514 & 0 \\ 0 & 9.6514 \end{bmatrix}\mathbf{z}(k)+\begin{bmatrix} -23.8059 \\ -23.4278 \end{bmatrix} q(k) \\ &\delta\,q(k)=\begin{bmatrix} -2.0099 & 2.2256 \end{bmatrix}\mathbf{z}(k)-0.9975\,q(k)+0.2\,u(k)+0.2f(k) \\ &y(k)=q(k) \end{aligned} \tag{11}$$

The pair of controllable matrices of the internal dynamics is

$$\mathbf{A}_{11}=\begin{bmatrix} -10.8514 & 0 \\ 0 & 9.6154 \end{bmatrix},\quad \mathbf{A}_{12}=\begin{bmatrix} -23.8059 \\ -23.4278 \end{bmatrix}.$$

The problem is in designing the sliding mode control (10) for the NMPH DT linear system (11) to provide for the asymptotic output tracking $\lim_{k\to\infty}\left|y_d(k)-y(k)\right|=0$ in the presence of the bounded disturbances $f_1(k), f_2(k)$.

4. DT state generator design

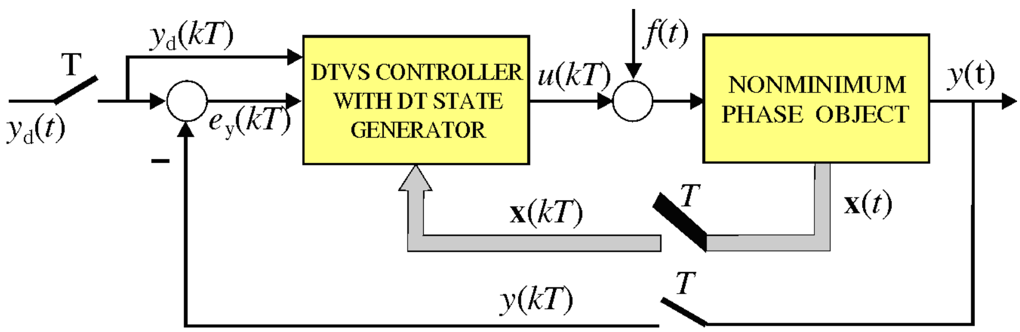

Fig. 2 Variable structure system block diagram

A functional diagram of the sliding mode control system that is supposed to provide $y(k)\to y_d(k)$ as $k\to\infty$ is presented in Fig.2. Firstly, a DT state reference profile generator (DT state generator) is to be designed to generate the bounded state reference profile $x_d(k)$ (stable system centre) so that $x(k)\to x_d(k)$ implies $y(k)\to y_d(k)$. Secondly, the DTVS control (10) is to be designed to provide $x(k)\to x_d(k)$ in the presence of the bounded disturbances $f_1(k), f_2(k)$.

Following this approach, one has to use a dynamic inverse of the object to build the DT state generator. For the non-minimum phase output this inverse is unstable.

For a system (11) with the known ID and the desired signal, $y_d(k)$, the problem to find the bounded internal state $\mathbf{z_r}(k)$ can be reduced to determining the stable system centre [14-

16]. The bounded internal state $\mathbf{z_r}(k)$ is called the ideal internal dynamics (IID) of the system (11).

The desired signal $y_d(k)$ and the disturbance $f(k)$ can be piecewise presented by the known linear exosystem. The model of the desired signal $y_d(k) = const$ can be defined by the characteristic polynomial

$$P_1(z) = z\ . \tag{12}$$

Using the characteristic polynomial (12), we can define a stable system centre for the system (11) by theorem 1 [19].

The desired asymptotic behaviour of the internal state dynamics is of the first order. We define it to be

$$\delta\, \mathbf{z_c}(k) + c_0\, \mathbf{z_c}(k) = 0 \quad \text{with } c_0 = 1\text{, i.e. } z + p_0 \text{ with } p_0 = 0\,.$$

We define the input to the stable system centre

$$\mathbf{s_c}(k) = \mathbf{A}_{12}\, y_d(k)$$

and the equation of the internal state generator (13) is

$$\delta\, \mathbf{z_c}(k) + c_0\, \mathbf{z_c}(k) = -\mathbf{P}_0\, \mathbf{s_c}(k)$$

Matrix $\mathbf{P}_0$ is computed to be

$$\mathbf{P}_0 = c_0\, \mathbf{A}_{11}^{-1} = \begin{bmatrix} -0.092467 & 0 \\ 0 & 0.1039998 \end{bmatrix}\text{, then is } \mathbf{P}_0\, \mathbf{A}_{12} = \begin{bmatrix} 2.2 \\ -2.44 \end{bmatrix}.$$

The internal states are generated by

$$z_{1c}(k) = \frac{-2.2}{z+1}\, y_d(k), \quad z_{2c}(k) = \frac{2.44}{z+0.3}\, y_d(k)\,.$$

5. DTVS controller design

The stable system centre design transforms the output error problem into the state error stabilization problem. Now, the state error will be stabilized to zero in the presence of disturbances via sliding mode control [20].

Let us assume

$$e_y(k) = y_d(k) - y(k),$$
$$e_{z_1}(k) = z_{1c}(k) - z_1(k),$$
$$e_{z_2}(k) = z_{2c}(k) - z_2(k).$$

The internal state error dynamics can be written as

$$\delta\, \mathbf{e_z}(k) = \mathbf{A}_{11}\, \mathbf{e}_z(k) + \mathbf{A}_{12}\, e_y(k), \tag{13}$$

We define the sliding hypersurface $g(k)$ as

$$g(k) = e_y(k) + \mathbf{M}\ \mathbf{e_z}(k), \qquad \mathbf{M} \in \Re^{1\times(n-1)},\ \mathbf{M}\text{ - constant matrix,} \tag{14}$$

and consider $e_y(k)$ as a virtual control in the sliding mode on the sliding hypersurface (14)

$$g(k)=0 \quad \Leftrightarrow \quad e_y(k)=-\mathbf{M}\ \mathbf{e_z}(k)\,. \tag{15}$$

The relation (14) can be rewritten in the closed loop as

$$\delta\,\mathbf{e_z}(k)=(\mathbf{A}_{11}-\mathbf{A}_{12}\,\mathbf{M})\,\mathbf{e}_z(k)\,. \tag{16}$$

The internal state error dynamics (13) will be substituted by one order selecting the sliding hypersurface as (14), where, in the sliding mode, the compensated error dynamics is (16). Let us select the eigenvalues of the compensated internal error dynamics at $\lambda_{1,2}=0.998$, then the design gain is $\mathbf{M}=\begin{bmatrix}0.1734 & -1.8323\end{bmatrix}$. The selecting sliding hypersurface (15) is

$$g(k)=e_y(k)+0.1734\,e_{z_1}(k)-1.8323\,e_{z_2}(k)\,. \tag{17}$$

Using the sliding hypersurface (17), we can define a sliding node control by theorem 2 [19]:

$$u(k)=-9.3769\,e_{z_1}(k)-8.8091\,e_{z_2}(k)+19.3994\,e_y(k)+ \\ \min(2500\left|g(k)\right|,35+0.1\left|g(k)\right|)\,\mathrm{sgn}(g(k))\,.$$

The simulation results are shown in the form of step responses of the system (Fig.3 to Fig.10) for the object parameter variation in the above given boundaries and matched disturbance

$$f(t)=2\,h(t-8)\sin\pi\,t\,.$$

Based on the results obtained, it can be concluded that the system is stable and robust to the object parameter variation as well as to the external matched disturbance action. The control signal is smooth and the switching function dynamics is with reduced chattering. If we want the dynamics of the system to stay the same (the balanced state has no overshoot) for the object parameter variation, then we must estimate the object parameter variation exactly. Using a similar approach to incorporate the minimal polynomial of a linear exosystem into a servo compensator [21], we can estimate the object parameter variation. The DTVS controller robustness with respect to the disturbance and object parameter variation is implemented.

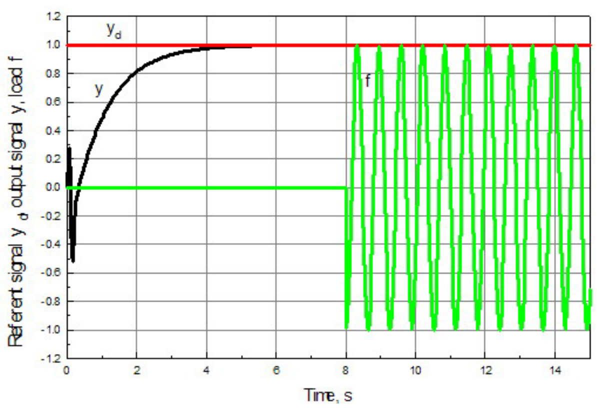

Fig. 3 Reference and output signal of the nominal system

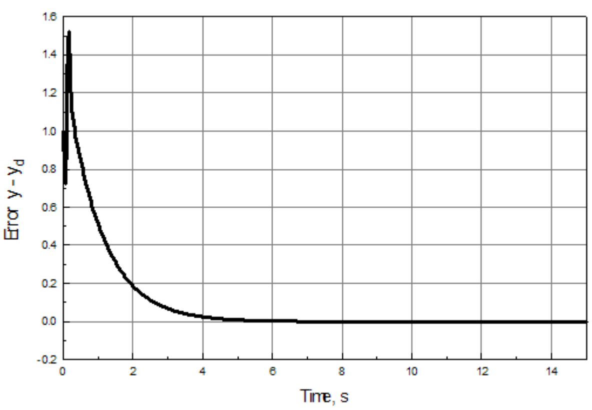

Fig. 4 Tracking error of the nominal system

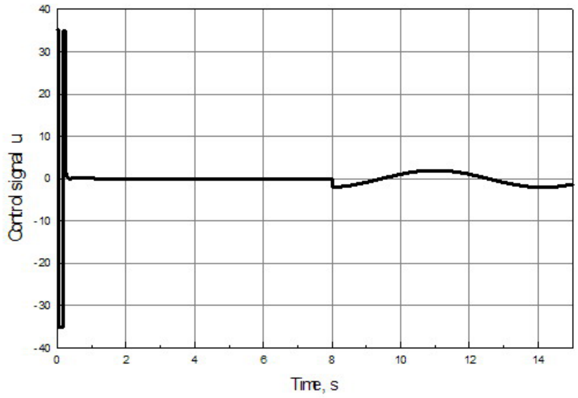

Fig. 5 Control signal of the nominal system

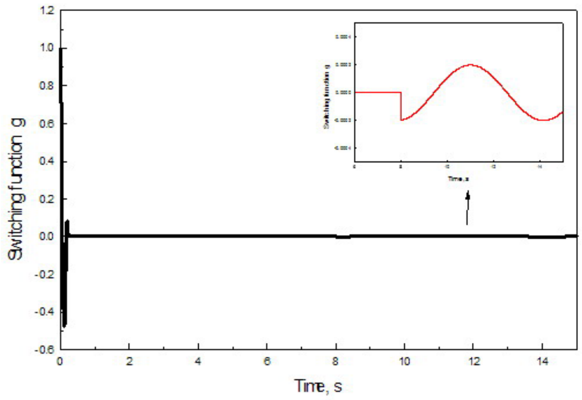

Fig. 6 Switching function of the nominal system

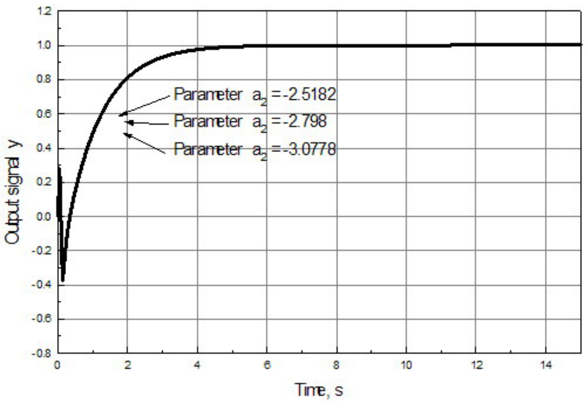

Fig. 7 Output signal with parameter a_2 variation

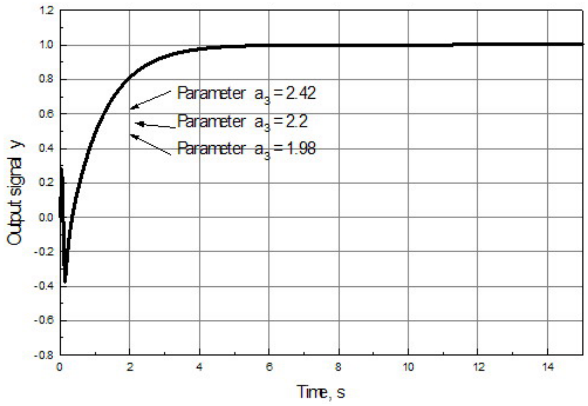

Fig. 8 Output signal with parameter a_3 variation

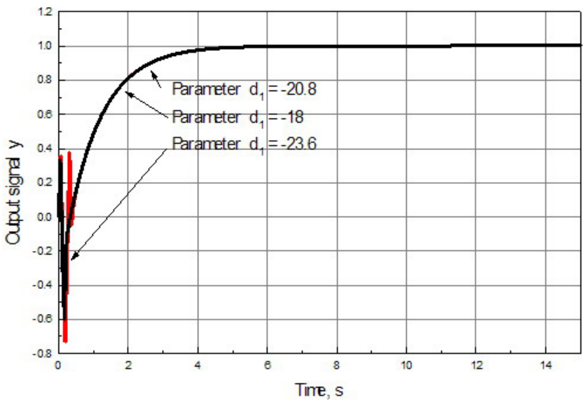

Fig. 9 Output signal with parameter d_1 variation

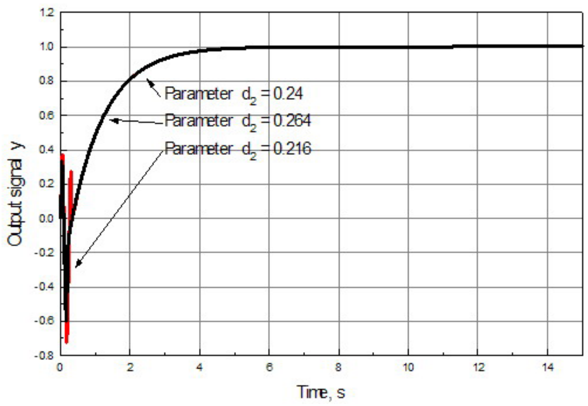

Fig.10 Output signal with parameter d_2 variation

6. Conclusion

The stable system centre method that was originally developed for controlling the output in a continuous-time non-minimum phase causal system using the sliding mode control is extended to discrete-time systems. The developed discrete-time stable system centre technique transforms an output tracking problem in causal discrete-time systems to a corresponding state variable tracking problem by asymptotically identifying the ideal internal dynamics for unstable internal states of a discrete-time system. The theoretical results are confirmed by the robust discrete variable structure control design for the aircraft flight angle tracking. Simulations demonstrate stability and high accuracy tracking performance.

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