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# Calculation of the Mean Output Power of Base Transceiver Station in GSM

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Original scientific paper

In this paper we calculate the distribution of output power of traffic channels of base station in GSM network depending on the traffic load. The principle of the calculation is to find the distribution of the output power of one traffic channel, and then to combine this distribution with the distribution of the number of busy traffic channels. Numerical example refers to the simplest distributions, but the principle can be implemented on the more complicate distributions.

**Key words:** GSM network, BTS, Output power, Telephone traffic, Probability density function

**Proračun funkcije raspodjele srednje izlazne snage bazne stanice u GSM-u.** U radu se izračunava raspodjela izlazne snage bazne stanice u GSM mreži u ovisnosti o prometu. Princip računanja je nalaženje raspodjele izlazne snage jednog prometnog kanala, a zatim kombiniranje ove raspodjele s raspodjelom broja zauzetih prometnih kanala. Brojčani se primjeri odnose na najjednostavnije raspodjele, ali se princip može primijeniti i na složenije.

**Ključne riječi:** GSM mreža, BTS, izlazna snaga, telefonski promet, funkcija gustoće vjerojatnosti

## 1 INTRODUCTION

The usage of radio connections and mobile communications increases every day. That's why it is concluded about five years ago that energy must be saved on this field, considering at least two reasons. The first one is that energy is more and more expensive, and the second one is environment protection, because increased energy consumption has detrimental effect on the nature. Energy saving is requested more and more in the mobile network construction, [1]. The program of energy saving, called *GREEN Radio*, is well known, [2, 3]. This program has few aspects, but for us in this paper is interesting the one, which uses the characteristics of human communication, i.e. traffic. The methods, which consider traffic, are often designated by the term TANGO (*Traffic-Aware Network planning and Green Operation*). The dependence of energy consumption on the traffic in one base station (BTS) is presented in [4], and the results are proved by the measured results. The influence of micro-cells and the network architecture on energy savings is considered in [5], while in [6] is analyzed the possibility to save energy by adjusting the function of BTS according to traffic load.

Telecommunication traffic is mathematically expressed human need for communication. It is obvious that energy

consumption in the network of mobile users depends on the telecommunication traffic. In this paper we investigate dependence of output power of mobile GSM network on the traffic. We can consider this dependence as the dependence of mean power, but, also, as the dependence of power distribution on traffic values.

The considered model is presented in Section 2. Then, we evaluate the power of one traffic channel in Section 3 and the total output power of one BTS in Section 4. Numerical examples and the results of simulation are presented in Section 5 and the method of simulation is briefly described in Section 6.

## 2 MODEL, ASSUMPTIONS AND DESIGNATIONS

Let us consider one BTS with dynamic power control in GSM network, [7, 8]. The network uses FDMA (Frequency Division Multiple Access) and TDMA (Time Division Multiple Access), implemented within each frequency. The number of frequencies used in one BTS is  $N_f$ , and the number of time slots used on each frequency is  $N_s (= 8)$ . The number of traffic channels,  $N_t$ , which are used for telephone connection realization, is slightly lower than the total number of channels, ( $N_t < N_c = N_f N_s$ ), because some of the channels are used for signalling. It

is supposed that each connection occupies one channel. The offered traffic to all traffic channels in BTS is  $A$ . It is assumed that the number of users, i.e. mobile stations (MS)  $N_{ms}$  in one cell, which is overlaid by the considered BTS, is much higher than the number of traffic channels, i.e.  $N_{ms} \gg N_t$ . This assumption allows us to use the well-known Erlang model for calculation of the number of busy channels ( $j$ ), [8], Section 2.4.4.

The call loss probability, caused by the lack of free radio channels, is  $B$ , and the served traffic is  $Y = (1 - B)A$ .

Distribution function of random variables will be designated by  $F$ . For example: let us consider the random variable  $r$  and one of its values  $x$ . As it is well known, the (cumulative) probability distribution function of random variable  $r$ ,  $F_r(x)$ , represents the probability that random variable  $r$  is less or equal to  $x$ :

$$F_r(x) = P\{r \leq x\}. \quad (1)$$

The probability density function of random variable  $r$  is designated by  $f_r(x)$  and it is, for continuous random variables, equal to the derivative of the distribution function  $F_r(x)$ .

The output power of one traffic channel  $w$  is defined as mean power during the useful part of GSM burst. The output power of one traffic radio channel,  $w$ , is random variable; its mean value is  $w_m$ , and maximal value  $w_{max}$ .

The part of one channel power in total power,  $\omega$ , is one eighth of channel power  $w$  ( $\omega = w/8$ ). The mean value of random variable  $\omega$  is  $\omega_m$ . The BTS output power is designated by  $w_B$ , probability density function (PDF) is  $f_{w_B}(w)$ , cumulative distribution function (CDF) is  $F_{w_B}(x) = P\{w_B \leq x\}$ , the mean value is  $w_{Bm}$  and the greatest value is  $w_{Bmax}$ .

The mean output BTS power,  $w_{Bm}$ , is the sum of mean power of all traffic,  $w_{Bm} = \sum \omega_{im}, i = 1, 2, \dots, N_t$ , Fig. 1.

We shall make some assumptions, which facilitate the calculation, but do not reduce the method quality.

- A1. The first assumption is that output power of BTS is adjusted for all active traffic channels on all frequencies only according to the user's distance.
- A2. The second assumption is that MSs are uniformly distributed in the cell area. The cell area is the circle with radius  $R$ .
- A3. The third assumption is that output power of one channel ( $w$ ) as the random variable depends on the random distance, ( $d$ ), between MS and BTS.

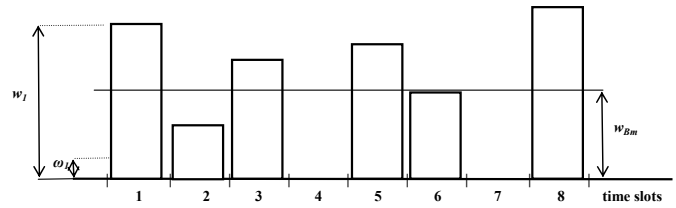


Fig. 1: Symbolic presentation of output power for one BTS with 8 traffic channels:  $w_1$  output power of first traffic channel,  $\omega_1$  output power of first channel as part of mean BTS output power,  $\omega_1 = w_1/8$ ,  $w_{Bm}$  mean output power of BTS,  $w_{Bm} = \sum \omega_{im}, i = 1, 2, \dots, 8$

Let one channel output power,  $w$ , depends only on the distance  $d$ . It is dependent random variable. This dependence is expressed by well-known equation, [8]:

$$w = g(d) = ad^\gamma, \quad (2)$$

where  $a$  represents constant (factor) of proportionality, and the value  $\gamma$  is between 2 and 5, [8]. It is obvious that  $w_{max} = aR^\gamma$ .

The distance between BTS and MS,  $d$ , is independent random variable and its distribution function

$$F_d(x) = P\{d \leq x\}, \quad (3)$$

represents probability that the distance is less or equal to some value  $d$ . This assumption means that output power of one traffic channel is continuous random variable (as also  $d$ ), which is in practice not completely true, because the output power changes in steps of 2 dB. The distribution density of random variable  $d$  is  $f_d(x)$ .

### 3 THE POWER OF ONE CHANNEL

According to the assumption A.2, the distribution of users in the cell is uniform. Then CDF of independent random variable  $d$  is

$$F_d(x) = \frac{\pi x^2}{\pi R^2} = \left(\frac{x}{R}\right)^2, \quad (4)$$

because probability  $P\{d \leq x\}$  is equal to the ratio of the area of the circle with radius  $x$  and the area of the cell. It is obvious that PDF of random variable  $d$  is

$$f_d(x) = \frac{2x}{R^2}. \quad (5)$$

One channel output power,  $w$ , depends only on the distance  $d$  and it is obvious that  $w$  is dependent random variable.

PDF of dependent random variable  $w$  can be expressed by PDF of independent random variable  $d$  and (2), [9]:

$$f_w(w) = \left[ \frac{1}{|g'(d)|} \right] f_d(d) = \frac{2}{\gamma a R^2} \left( \frac{w}{a} \right)^{-\frac{\gamma-2}{\gamma}} \quad (6)$$

and CDF of one channel output power is, obviously

$$F_w(x) = \int_0^x f_w(w)dw = \frac{a^{-\frac{2}{\gamma}}}{R^2} x^{\frac{2}{\gamma}}. \quad (7)$$

Mean output power of one channel is

$$\begin{aligned} w_m &= \int_{w_{\min}}^{w_{\max}} w f_w(w)dw \\ &= \frac{2a}{(2+\gamma)R^2} R^{2+\gamma} = \frac{2}{2+\gamma} w_{\max}. \end{aligned} \quad (8)$$

It is obvious that  $w_m = 0.5w_{\max}$  for  $\gamma = 2$  and  $w_m < 0.5w_{\max}$  for  $\gamma > 2$ .

#### 4 TOTAL OUTPUT POWER OF THE BASE STATION

The total output power of the BTS,  $w_B$ , can be expressed by its mean value,  $w_{Bm}$ , by its PDF,  $f_{wB}(w)$ , and by CDF,  $F_{wB}(x)$ .

The mean value of the BTS output power can be calculated as the sum of mean values of powers,  $\omega_m$ , caused by each active channel:

$$w_{Bm} = \sum_{k=1}^{N_t} k \omega_m P(k, A, N_t). \quad (9)$$

In (9), the variable  $P(k, A, N_t)$  is the probability that  $k$  channels are busy in the group of  $N_t$  channels with the offered traffic  $A$ .

According to the assumptions about the number of traffic sources and about the number of channels,  $N_{ms} \gg N_t$ , we can use Erlang model, i.e. truncated Poisson distribution, [10], Section 4.3.1.:

$$\begin{aligned} P(k, A, N_t) &= ERL(k, A, N_t) \\ &= \frac{\frac{A^k}{k!}}{\sum_{i=0}^{N_t} \frac{A^i}{i!}}, k = 0, 1, 2, \dots, N_t. \end{aligned} \quad (10)$$

As it is

$$\sum_{k=1}^{N_t} k \cdot ERL(k, A, N_t) = Y = (1 - B)A \quad (11)$$

the served traffic in the group of  $N_t$  channels with the offered traffic  $A$ , [10–12], the mean BTS output power is obtained using the equations (8) - (11):

$$w_{Bm} = \omega_m A(1 - B). \quad (12)$$

#### 5 NUMERICAL EXAMPLES

Let us consider one GSM cell and the BTS, which functions in the class 4:  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$ . One frequency is used for 8 traffic channels, i.e.  $\omega = w/8$ . Figure 2 presents CDF of random variables  $w$  and  $\omega$ , i.e. CDF of one channel output power,  $w$ , for the duration of channel  $F_w(x) = P\{w \leq x\}$  and CDF of one channel output power,  $\omega$ , which is the part of total power  $F_\omega(y) = P\{\omega \leq y\}$ .

Figure 3 presents calculated values of the mean output power of one BTS for the example with the same numerical values as in Fig. 2:  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$ ,  $\omega = w/8$  and for the groups of  $N_t = 6, 14$  and 30 channels. The values from Fig. 3 are proved by computer simulation, as explained in Section 6. Verification of the values of mean BTS output power for the group of  $N_t = 6$  channels is performed in three points ( $A = 3$  Erl, 6 Erl and 9 Erl), for the group of  $N_t = 14$  channels in four points ( $A = 9$  Erl, 12 Erl, 15 Erl and 18 Erl) and for  $N_t = 30$  channels also in four points ( $A = 21$  Erl, 27 Erl, 33 Erl and 39 Erl). We performed five simulations for each point. The mean value of output BTS power, obtained by simulation, does not differ from the calculated value more than 2% in neither case.

Figures 4 and 5 present CDF of BTS output power, obtained by the simulation for two cases. The first one is when traffic load is such that there is nearly no loss, and the second one is for traffic overload. Figure 4 presents CDF of the output power of one BTS functioning in the class 4, obtained by the simulation in the same conditions as the ones stated for the example from Fig. 2 and Fig. 3 ( $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$ ,  $\omega = w/8$ ). The simulation is performed for the group of  $N_t = 14$  channels and for two values of traffic:  $A = 5$  Erl and  $A = 13$  Erl. As can be seen from [13], or more precisely from [14], when it is  $N_t = 14$  and traffic loss is negligible ( $B = 0.05\%$ ), the offered traffic can be  $A = 5$  Erl at maximum. If traffic loss is increased on 15%, the allowed traffic can raise up to  $A = 13$  Erl. When it is  $N_t = 30$  channels, the offered traffic  $A = 15.8$  Erl correspond to small loss ( $B = 0.05\%$ ), and offered traffic  $A = 31$  Erl correspond to great loss ( $B = 15\%$ ), [13, 14]. As it can be seen, the increased traffic causes the increased BTS output power, besides the increased traffic loss.

The influence of traffic changing on the BTS output power can be seen in Fig. 4 and Fig. 5. Let us consider CDF if  $N_t = 14$ , Fig. 4. It is obvious that  $F_w(x) = P\{w \leq x\} = 0.95$  is satisfied by the power  $x = 18$  W if traffic is  $A = 5$  Erl, and by the power of about  $x = 30$  W (or 66% more) if traffic is  $A = 13$  Erl.

In the case of the group with  $N_t = 30$  channels, during 95% of time ( $F_w(x) = P\{w \leq x\} = 0.95$ ) the power of

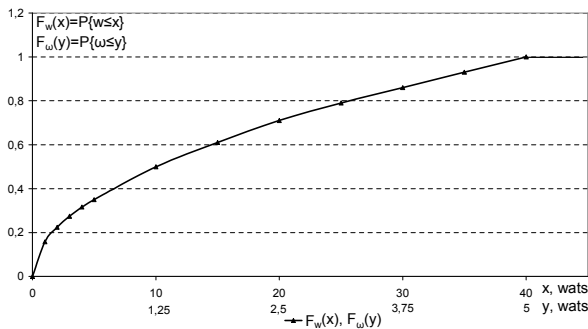


Fig. 2: Cumulative distribution of the probability  $F_w(x)$  of one channel output power  $w$  and cumulative distribution of the probability  $F_\omega(y)$  of one channel output power, that is the part of total power  $\omega$ ,  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$

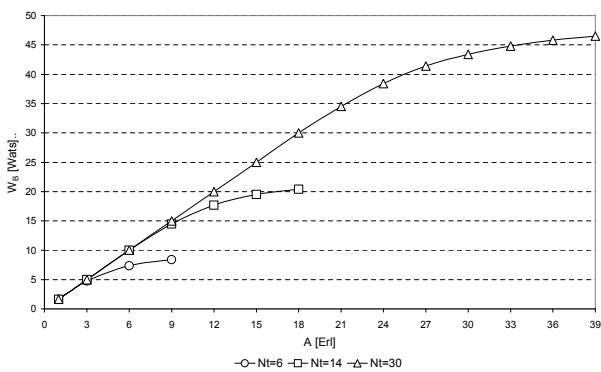


Fig. 3: Mean output power of BTS,  $w_{Bm}$ , as a function of offered traffic  $A$  for  $N_t = 6, 14$  and  $30$  channels,  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$

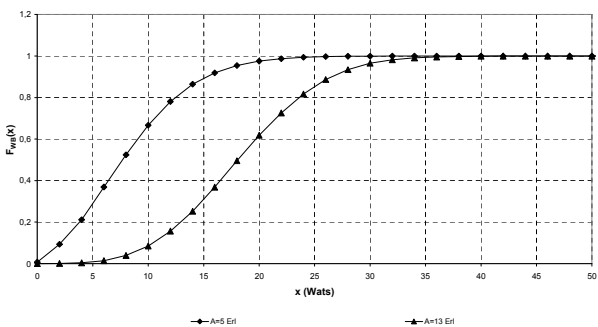


Fig. 4: CDF of BTS output power ( $N_t = 14$ ,  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$ ,  $\omega = w/8$ ) for two values of traffic  $A = 5$  Erl and  $A = 13$  Erl,  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$

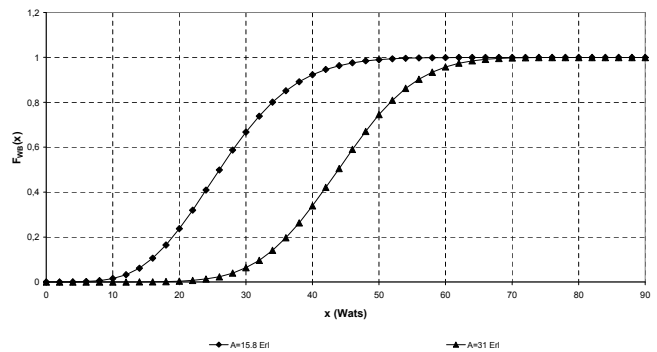


Fig. 5: CDF of BTS output power ( $N_t = 30$ ,  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$ ,  $\omega = w/8$ ) for two values of traffic  $A = 15.8$  Erl and  $A = 31$  Erl,  $R = 10$  km,  $w_{max} = 40$  W,  $\gamma = 4$

42 W is satisfactory if offered traffic is  $A = 15.8$  Erl, while it is necessary 60 W (or 42% more) if the offered traffic is  $A = 31$  Erl.

## 6 SIMULATION

In this paper computer simulation is used to verify the calculated values of mean BTS output power and to estimate CDF of BTS output power. Simulation program is upgraded program for telephone traffic simulation, developed 40 years ago, [15–17], and called Roulette or Monte Carlo method. This method is simple: new connection, termination of the existing connection, or empty event is generated on the basis of generated random number. We used this method, expanded by the second random number generator, which determines random user distance. In this way output power estimation is the result of traffic process randomness, but also the result of position randomness of mobile user. Simulation process for the verification of values from Fig. 3 is performed until at least  $5000N_t$  connections, i.e. at least 5000 connections per one channel, are realized. Each mean value of BTS output power, presented in Fig. 3, for the defined conditions ( $N_t, A, R, \gamma, w_{max}$ ) is verified by five simulation trials.

## 7 CONCLUSION

The mean output power of BTS depends on the distribution of the distance between BTS and MS and on the distribution of the number of busy channels. As the traffic is always calculated for the busy hour, we can conclude that the distribution of the power varies during the day. The numerical example refers to the uniform distribution of users in the cell area and Erlang traffic model with loss, as the simplest examples. It is clear that this principle of calculating the mean output power of one BTS can be implemented also when the distribution of MSs in cell area

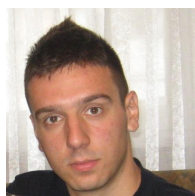
is not uniform, or when the traffic distribution is closer to Engset distribution.

The upgraded model of telephone traffic simulation in one GSM cell enables the verification of the results obtained by the calculation. This model can be also adjusted to other conditions in the cell (e. g. non-uniform distribution of MSs, or Engset model of traffic).

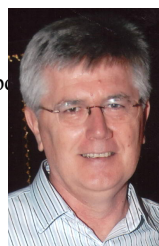
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