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APPLICATION OF FRACTAL GEOMETRY IN URBAN PATTERN DESIGN

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Preliminary notes
Fractal geometry studies the structures characterized by the repetition of the same principles of element distribution on multiple levels of observation, which is recognized in the case of cities. Since the holistic approach to planning involves the introduction of new processes and procedures regarding the consideration of all relevant factors that can influence the choice of urban pattern solutions, we point to the awareness of the importance of fractality in natural and built environment for human well-being. In this paper, we have reminded of the fractality of urban environment as a quality of an urban area and pointed on the disadvantages of conventional planning concepts, which do not consider pattern fractality as criterion for environmental improvement. After a brief review of the basis of fractal theory and the review of previous studies, we have presented previously known and suggested new possibilities for the application of fractal geometry in urban pattern design.

Keywords: *fractal geometry, fractals, holistic approach, urban pattern design*

Primjena fraktalne geometrije u projektiranju gradskog obrasca

Prethodno priopćenje
Fraktalna geometrija proučava objekte koje karakterizira ponavljanje istih principa raspodjele elemenata promatranjem na više nivoa, a to se prepoznaje u slučaju gradova. Budući da holistički pristup planiranju uključuje uvođenje novih procesa i postupaka u odnosu na razmatranje svih relevantnih faktora koji mogu utjecati na izbor rješenja gradskog obrasca, naglašavamo koliko je za ljudsko blagostanje potrebna svijest o važnosti fraktalnosti u prirodnom i izgrađenom okolišu. U ovom radu podsjećamo na fraktalnost gradskog okoliša kao kvalitete urbanog područja i ističemo nedostatke uobičajenih koncepata planiranja, koji ne uzimaju u obzir fraktalnost obrasca kao kriterij za poboljšanje okoliša. Nakon kratkog pregleda osnova fraktalne teorije i prethodnih istraživanja, predstavljamo ranije poznate mogućnosti i sugeriramo nove za primjenu fraktalne geometrije u projektiranju gradskog obrasca.

Ključne riječi: *fraktalna geometrija, fraktali, holistički pristup, projektiranje urbanog obrasca*

1 Introduction

Since urban environment reflects the social order, economic and political power, cultural level of the community, as well as the ability of individuals to make decisions on behalf of the community, it is not to wonder why it is said that the construction is "the most important testimony of the history of a nation or a country" [1]. But, under what conditions modern generations are going to leave the testimony of their existence to further generations? Undoubtedly, under conditions of limited space at their disposal. The over-population of urban areas not only threatens primary biological and psychological human needs, but also causes irrational management of the space which often results in higher population density. This not only denies the human needs for space, but also what are supposed to be the natural human rights: clean air, land and water. The alienation of modern cities and the conflict of humans with natural laws and ecological phenomena are present. But knowing that a city form affects directly pollution, waste waters, toxic gas emissions, green areas and energy consumption, it is expected the physical structures of the cities could preserve to a large extent the essential relationship between humans and their environment. So, in view of demographic trends and excessive consumption of space as the most valuable resource, the profession has an obligation to provide its protection.

New scientific achievements and modern techniques must not harm natural and built environment, but rather on the contrary, they should be the tools of their ecological integration. For this reason, we consider justified to shed new light on urban pattern design in accordance with new findings from other scientific disciplines. Fractal geometry is a relatively new scientific discipline which can provide new ways of visualization of the influence of decision-making on cities as well as new possibilities for a better city form through its application in urban pattern design. Various studies have shown that urban patterns have fractal characteristics in their structure, and that fractal geometry provides more appropriate models for the description of such space forms, rather than Euclidean geometry [1 ÷ 7]. The characteristics of urban patterns, such as heterogeneity, self-similarity and hierarchy, are essential features of fractal structures, too [8]. On the other hand, fractals have the ability to summarize complexity, density and heterogeneity of space distribution in a single value - fractal dimension, which is independent of the scale [3, 5, 9].

The effects of the modern era, such as increasing car traffic, adversely affect the hierarchical principles typical for such structures, i.e. agglomerations become more uniformly shaped. The loss of physical complexity of built environment adversely affects the human mind [1], as well as the lack of structures in human scale [10]. In this regard, the studies [1] have given emphasis to the necessity of relationships and structures at all scales of urban pattern, from the agglomeration level to the neighbourhood level and beyond, to the architectural details.

Through their experience of space, people are connecting to the environment and to the world, in one of the fundamental ways. The human relationship to his living environment is two-way: a man forms, shapes his environment, while built and created areas in turn affect his material and spiritual world [7]. Therefore, fractality as an important feature of the human environment, should not be neglected in urban pattern design.

In order to be able to understand the relationship between complex systems, such as cities, and fractals, in this paper we have firstly presented the basic concepts of complexity theory and fractal geometry. Then, we have given some theoretical background for their application in urban pattern design and, as a synthesis of the above-presented, we have revealed some suggestions for that. In conclusion, in this paper we have provided recommendations for future studies in order to get better criteria selection for environmental improvement, design and reconstruction of urban areas using the methods of fractal geometry as geometry of "organized complexity".

2 About the complexity and fractals

Chaos is a key term in understanding the complexity theory. Chaotic systems are characterized by the irreversibility of time in them, and their status at any time is very sensitive to initial conditions. The complex system is composed of many chaotic systems and it becomes more complex as time goes on [4]. The chaotic complex systems are also characterized by fractals, i.e. fractal is the resulting image of chaotic behaviour of the systems within a complex system [11].

The term fractal derives from the Latin adjective "fractus" meaning broken, and describes the objects which are not geometrically regular at first sight. Euclidean geometry describes them as shapeless or amorphous, and they mostly occur in nature. Fractals are found in the microstructure of DNA, crystals, plants, certain organs in humans and animals, clouds, mountains, river streams, lightning, coasts, galaxy, in heartbeat fluctuations, stock trends, weather forecast, earthquakes...

The basic properties of fractals are: generation through iterations, fractal dimension and self-similarity.

All types of fractals are created through iteration, such as:

- Iterated function systems (linear fractals), where the transformation of the initiator (input) into the generator (output) determines the functional system, the rules within the system and the output of each iteration [12]. Two of those fractals, Koch's Curve and Koch's Snowflake, are presented in Fig. 1.

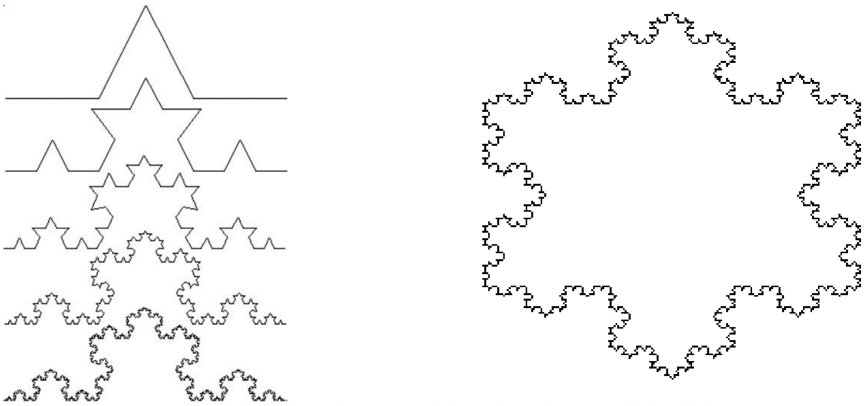

Figure 1 Generation of Koch's Curve (left) and Koch's Snowflake (right)

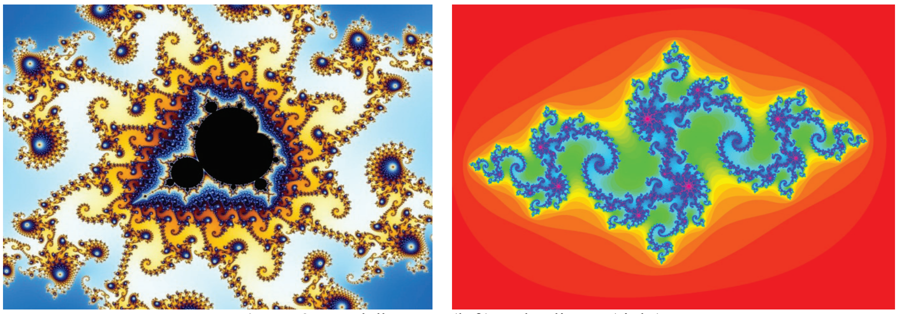

Figure 2 Mandelbrot set (left) and Julia set (right)

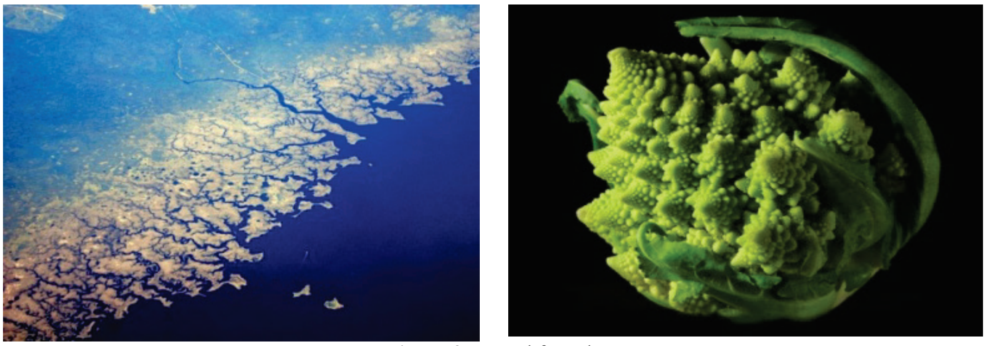

Figure 3 Natural fractals

This type of fractals has been produced by humankind as part of traditional artefacts and buildings for millennia [13].

- Mathematically or computer-generated fractals are generated by recursive algorithms, which create substructure on a frame on increasingly smaller scales, or build up a complex whole by progressively adding contributions that create the whole out of smaller components [14]. Mandelbrot set and Julia set are the best known examples of these fractals, Fig. 2.
- Natural fractals (e.g. ferns, broccoli, relief...), shown in Fig. 3, usually occur in nature as a result of natural processes and they are characterized by the limited number of iterations and by statistical self-similarity.

The basic elements of Euclidean geometry - point, line and plane - have well known topological dimension: point 0, line 1, plane 2. However, the "father of fractals" Mandelbrot says: "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth". Thus, neither Euclidean geometry, with its regular geometric shapes and integer dimensions, can describe in the most appropriate way the forms found in nature [15]. Therefore, fractal dimension (D_f) is introduced to "measure" how well a structure fills the certain space. The examples from nature, such as the root of the tree, blood vessels, brain, lungs, etc. fill properly the space of their location. In a similar way, the street network in the city tends to reach every part of the city and each facility.

A fractal dimension does not have to be an integer, as the Euclidean dimension is. The characteristic of the fractal dimension is that it remains constant for the specific fractal, regardless of the enlargement/scale of its observation. It measures the increasing of structurality (or appearance of the details) by zooming; it is the measure of the complexity of a given structure.

To date, various studies have proposed several definitions of fractal dimension. For scientific purpose, Hausdorff's dimension is the most used and the most present in computer tools for fractal analysis and thus can be easily and automatically calculated.

One or more methods for fractal dimension estimation can be appropriate for the analysis of urban pattern. The methods are based on the same principle of logarithmic equation and tend to define a relationship between the object size and different scales of fractal dimension estimation. If the subject of the analysis is the ground plan of an inhabited area, the calculation of its fractal dimension is based on the separation of the inhabited from the uninhabited areas (occupied and free pixels - regarding the image of the inhabited area) according to binary logic - assigning one colour to occupied pixels and other colour to free pixels. The distribution of built-up areas can be described in this manner.

The fractal dimension of the urban pattern describes the distribution of structures in the plane of observation (ground plane or elevation) and can have values between 0 and 2. If $D_f = 2$, it means that the pattern is uniform, while 0 corresponds to marginal case where the pattern is made of one point i.e. an isolated object; $D_f < 1$ corresponds to a pattern consisting of separate unrelated elements; $D_f > 1$ indicates groups of elements related by fractal law, where isolated elements can also appear. The more D_f is close to 2, the more elements are gathered in a large cluster [8].

3 The theoretical background for the application of fractals in urban pattern design

3.1 The inadequacy of conventional approach

The changes in scientific paradigms and consequential development of new scientific disciplines such as fractal geometry and complexity theory, inevitably lead to the review of existing methods in various fields. As a result, the inadequacy in the application of existing methods based on concepts such as rationalism, reductionism, comprehensive long-term planning, etc. for the management of built environment has been more evident.

Accordingly, environment management should not only consider the elements of economic development and the society demands in administrative sense, but also the criteria such as nature protection, application of renewable resources, economic development, revitalization of urban areas, prevention of urban sprawl, etc. The problem becomes more complex if the descriptive criteria are considered, such as the quality of life in an urban environment, spirit of the place, feeling of satisfaction in a pleasant environment, safety, etc. which are very difficult to quantify. Some studies speak in favour of the observation that these qualities are associated with the complexity of built environment [1, 2]. So far it has not been possible to express the quantity of these properties, thus the cities have not been generally treated as complex systems. "Architects, planners, urbanists and settlement builders treated them as simple predictive systems that needed to be arranged and reduced to their components for easier urban modelling and urban problem solving". The conventional planning methods, based on the application of Euclidean geometry, have shown many disadvantages [2].

New findings about complex systems have contributed the city not to be considered as simple system arranged by linear geometry, but a complex organism that demonstrates a certain order through its form, at various scales of observation. The city is characterized by the complex organization as a result of the interactions between its elements. It is an open system and its form and function continually change and affect the environment, as the environment affects them. Therefore, cities are perceived, more than any other human creation, as self-organized living systems, which cannot be described by conventional linear and mechanical principles [2]. As a result, the studies have claimed that "cities are ideal candidates for the application of fractal analysis". This statement implies the fact that cities are created through gradual action of local factors, which generate highly organized global patterns, according to the "bottom-up" principle, and a small change of local factors (initial conditions) may lead to unexpected pattern changes. Therefore, detection of small changes on a local level, which can be considerable on a global scale, is very important for urban pattern design.

3. 2 Reasons for the application of fractal theory in urban pattern design

Along with the development of science and changes of scientific paradigms, the human thinking about space and the perception of the city and urban morphology has changed. The period of Cartesian paradigm and causal determinism as well as Newtonian physics and the mechanistic perception of the universe (XVII, XVIII and XIX century) are characterized by the consistent application of Euclidean geometry in urban design. In the first half of the twentieth century, quantum theory, uncertainty principle and chaos theory reveal the uncertainty and unpredictability even about the events that were previously considered fully predictable. Also, they reveal the order on a deeper level in seemingly accidental events. Based on the works of David Bohm and Karl Pribram, in the second half of the twentieth century, quantum-holographic paradigm appears as an assumption of quantum-holographic nature of the universe and the human brain, as well as an assumption of oneness at a deeper level, which recognizes physical systems as wholes, whose parts contain the whole again. The geometric (mathematical) interpretation of such order in chaos, i.e. fractals, was presented by Benoit Mandelbrot, who thus founded fractal geometry.

In the examples from the nature, fractal theory is reflected in the fact that elements are always at least a little different one from another, but the configuration of elements contributes to the impression of naturalness, wholeness and uniformity. The traditional builders have built urban areas in similar manner: forms and elements of the system have been defined, but their configurations have been chosen in order to provide perfect adaptation to existing conditions. [7] Although until the eighties of the last century, the connection between self-similarity and fractals was unknown, there were many examples of the applied self-similarity in the past [16], mostly for creation of hierarchical structure of elements imitating the forms from the nature.

Complexity theory and fractals can be applied to many natural and artificial systems which dynamics, i.e. interaction between local factors, generates in ordered patterns. Fractal theory provides an explanation of complexity which arises in the form and structure of such systems. Many properties of complex systems (for example, a number of factors in interaction, existence of the loop, sensitivity of the system to initial conditions, self-organization, adaptation, hierarchy of relationships and self-similarity) are noticed within urban systems, but have been considered too complex to be analyzed for a for long, therefore often neglected [11]. It was in accordance with the principles of reductionism and simplification of the issues of urban systems.

Along with the development of new sciences, the planning has been developed going from classical thought (positivism, reductionism, linear and static systems, Euclidean geometry) to complex thought (alternative epistemology, holism, non-linear and dynamic systems, fractal geometry, which are all characteristics of complexity theory). If cities show features of complexity, they can be studied as complex systems [4].

In *Fractal Cities*, inevitable book regarding this topic, fractal geometry has been described as "geometry of the system at many scales, geometry of organized complexity". The authors conclude that cities, like most real systems, show an infinite complexity and this fact must change the existing concepts of urban planning, which so far have been simplified and unrealistic [2]. Elementary pure solids express reductionist design while fractals express ordered complexity that is a result of mixing a hierarchy of linked scales [14].

Most cities contain planned and unplanned parts, thus, their form is never regular and strictly geometrical. The growth of the city can follow to some extent the adopted plans, but its form experiences constant changes under the influence of many factors, i.e. shows deviations from the planned. "Even planned cities adapt to the context in natural way when it comes to implementation" [2]. Euclidean geometry has not managed to describe the complexity that exists within the urban fabric. It describes irregular organic forms, very frequent in urban centres, only as amorphous and unorganized. Therefore, it is necessary to define the fractality of urban environment and take it into consideration for criteria selection for environmental improvement.

3.3 Previous studies

Fractal structures occur in many natural systems. If the fractal principle is the principle by which nature is organized [15], then human beings are expected to perceive their environment in the same manner. "Our mind somehow has intrinsically fractal structure, and therefore more readily accepts fractal information. As a consequence of this anatomical trait - and this point is crucial to architecture and design - we tend to imagine fractal forms as the most natural. There appears to be a certain resonance between our cognitive apparatus and environments that possess fractal properties. The brain is constantly computing characteristics of our environment, evaluating features that are essential for our survival, so this resonance has deep meaning [14]."

Fractality is an inherent property of human environment because human beings have been surrounded by fractals for millions of years, so, a large part of the structure of human mind comes from this ancient relation. The environment, i.e. what human eyes can see, influences humans, thus the human mind is fractal. Only recently humans are surrounded by structures that are not fractal [1]. "The architecture that is adapted to human physiology is nourishing because it generates positive feelings through positive cognitive response to symmetries and fractal structures [17]." People react to the level of complexity of what they see [11]. Natural structures are fractal, in contrast to built structures, where only in rare examples, mostly in traditional architecture, fractality occurs. Traditional concepts used the ideas imposed by the nature and by the moment of technological (and their entire civilizational) development, while the modern concepts are limited to linear geometric processes arising from Euclidean geometry.

Modern cities are based on a large-scale regular geometric pattern created to adapt the cities to the needs of traffic and rapid population growth, so they are anti-fractal. As a consequence, human beings are physically and psychologically separated from the surrounding areas, while the environment becomes inappropriate for living. Human beings are losing support from the environment, which was used by their mind to create its mental structure. Unlike them, the traditional cities are characterized by great variety of properties; they are picturesque and proportional to human needs. Some researchers attribute it to the fractal properties of these cities [1] and state that an artificial environment with those measurable qualities provides a better quality of life. By contrast, stressful environments with the opposite characteristics induce anxiety and depressive behaviour, and ultimately pathology in their users and residents. [14]

The possibilities of the application of fractal geometry in urban pattern design were examined at the examples of the cities: Milan [18], Thessaloniki [19], Basel [20] and Wallonia [3], as well as the possibilities of its application in morphology comparison between cities [21]. Some researchers have measured the fractality of urban networks [5], street vistas [6] and rail networks [22].

4 The possibilities of application of fractal geometry in urban pattern design

In regard to planning from the spatial/physical aspect of space, some of its manifestations such as configuration, accessibility, plot size, size and distribution of free areas, dimensions and distribution of facilities can be designed and improved by the application of fractal geometry in one of the following ways.

1) Fractal simulation of urban systems, aiming to obtain predictions of urban growth, is related to the application of methods which include cellular automata and agent based systems. It serves for the development of possible scenarios of specific urban area growth [9]. This field is not yet applicable in practice.
2) The determination of fractal dimension (observing only ground plan of urban pattern, for now) can be a useful tool in the process of urban pattern design, as the following:

- The fractal analysis of urban pattern can mathematically identify the level of complexity of its structure. Densely populated and urbanized cities are more homogeneous (D_f close to 2), as well as those more dispersed areas where the same pattern is repeated several times. The heterogeneous patterns occur as a consequence of particular planning concepts or less controlled development [21].
- The determination of D_f enables the estimation of the changes in physical complexity caused by particular project demands or interventions, which have a direct morphological influence on urban patterns, by comparing D_f estimated before and after the "marked" change [11]. Based on this, it is possible to gain insight into the obtained level of integration of the considered new building with the existing urban space.
- Fractal classification, i.e. zoning according to the value of D_f, can help planners to identify more precisely the boundaries of the zones where some interventions can be implemented to maintain complexity of the urban area [11].
- The information about D_f of the urban boundaries and D_f of the urban area can be used as a parameter for criteria selection for spatial development. For example, when planning new residential areas, additional free areas are often used for further spatial development, which is not in accordance with the principles of rational space use. Analyzing these two D_f values of the observed area, planner can draw conclusions about the fragmentation or the compactness of the pattern. The increase of the area D_f and the reduction of the urban boundaries D_f in the peripheral areas of the city, indicate more dense, homogeneous development with tendency to inhabit the existing free spaces. The opposite trend indicates the process of urban sprawl characterized by fragmented development [21].

3) Fractality as a concept in urban pattern design refers to detecting of the desired characteristic of urban pattern and its repetition with lesser or greater distortion for better adaptation to existing environment. It was used in construction in the past mostly intuitively or as a synthesis of different skills possessed by the communities. The implementation of fractal concept must be developed at more scales: regional scale, city scale, neighbourhood scale and architectural scale. What contributes to the feeling of pleasure in the ambience is the recognition of the order of things, from the lowest to the highest scale of observation, with obligatory existence of human scale.
4) Fractals in the process of criteria selection for environmental improvement have sense and purpose within the context of application, for example: consideration of the urban structure development or distribution of building blocks, greenery, open spaces, walking paths, road crossings. In this regard, the application of fractal geometry in the initial phase of urban planning would provide better results in the space design in terms of identity/authenticity of the inhabited area. It refers to the creation or recognition of fractals that will be used for pattern design at all scales of observation, aiming to connect "small to big", "part to a whole". That could also provide a base for further research and studies, and consequently for planning and investment activities. Here are some suggestions for fractal application in urban pattern design. The reference models serve only to illustrate the basic concepts, and can be further developed at any scale of observation.

- The demand for easier accessibility to the green areas and protection of certain level of compactness of the urban pattern reminds of Dragon Curve or Koch Island, Fig. 4. Due to knowing fractal dimension of these fractals, a desirable range of D_f values of urban boundaries and urban areas can be formed, allowing the control of the penetration of green areas into urban areas and vice versa - the control of urban sprawl. Or, varying fractal dimension of urban boundaries and fractal dimension of the observed area can locally increase the population density without

significant loss of life quality, by planning focus on increase of urban boundaries D_f.
- The preservation of green areas and the connection of residential areas with open spaces are of great importance for every community. In the presented fractal structures the penetration of free zones has several scales of observation, Fig. 5. The possibility to leave free spaces of different size for different uses, including those left to be used (and planned) by future generations, is in accordance with the principles of sustainable development.

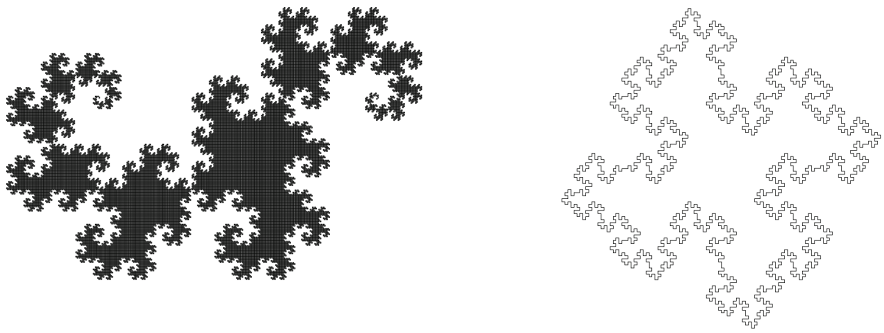

Figure 4 Fractals: Dragon Curve and Koch Island

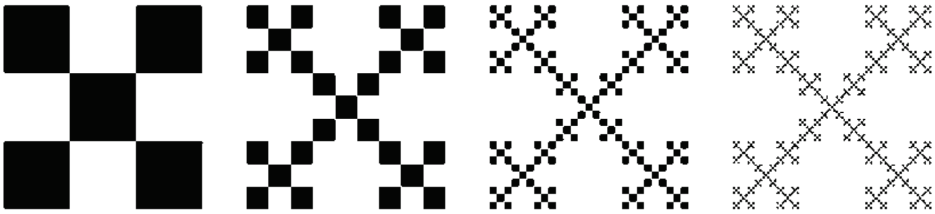

Figure 5 Sierpinski Carpet fractal: generation through iteration

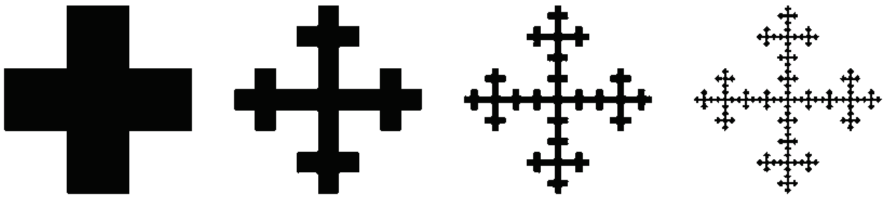

Figure 6 Vicsek fractal: generation through iteration

- The analysis and further application of fractals in urban pattern design can refer to configuration, connection between certain facilities or their availability. Figures 5 and 6 present the permeating of urban and free space at all scales. It can significantly reduce the density of traffic that depends on the distribution of different facilities within the urban pattern.
- The urban infrastructure design process aims to create infrastructure system that will operate effectively along with the population growth in the city. For this purpose, the hierarchy of the roads could be analyzed, as well as their ability to "cover the area", i.e. to achieve the maximum coverage of the area or availability of the facilities, with the minimum space consumption. Since fractal dimension of a line is the measure of its ability to fill the space, as well as the measure of its hierarchy, it is reasonable to analyze urban networks by the application of fractal geometry.

5 Conclusion

In order to make better, more meaningful and valuable decisions in the process of urban pattern design, it is necessary to take into account all relevant factors that may affect that process. The above presented studies have revealed that urban environment with fractal properties like self-similarity, hierarchy and complexity is desirable or even necessary for human well-being. Fractality of inhabited area in so-called primitive communities proves that such structurality is deeply encoded or essential for human beings and their architectural heritage. Therefore, it is important to define direction of development in order to maintain, protect or improve the existing fractal characteristics of areas.

The application of fractal concept is preferable having in mind that the same pattern can be repeated at several scales, always with a certain level of transformation of the initial form, and at the same time can be adapted to environmental conditions, natural or urban. It provides a special quality of the final form, leaves a good aesthetic and overall impression to observers i.e. users of the space.

New plans must consider cities as complex systems. It includes the introduction of new principles of pattern design, in the direction of holistic and sustainable development, which will lead to:
- obtaining the predictions of population distribution, space use or urban sprawl;

- estimation of the changes in physical complexity of urban pattern due to larger or smaller interventions or project requirements;
- prevention or limitation of the fragmentation of urban pattern to preserve high-quality landscapes and/or agriculture areas in the periphery;
- enabling the penetration of green areas into urban fabric, as a significant factor for good ventilation of the central areas;
- good accessibility to places for recreation and contact with nature;
- good accessibility to central facilities.

The properties of some well-known fractals can be applied usefully in urban pattern design regarding the distribution of free areas/green areas and their size, urban sprawl or hierarchy of traffic networks.

Further research can be conducted in different directions, according to the requirements of each branch or its purpose. Some general guidelines are:

- It is not sufficient to analyze fractality of built environment only in one projection. It is also necessary to consider the other projection, i.e. the elevation of built environment, for clearer determination of the fractality of human environment and all its effects.
- Further study on psychological effect of fractal environment on humans would give a clear answer to the question of whether there is a connection between the urban quality of life and the level of fractality of urban morphology. In particular, it is necessary to obtain concrete data of how lower or higher environment fractality affects the quality of life in it, which can be transformed to criteria for environmental improvement.
- Fractal dimension provides information on the manner in which a particular area is filled, but there are indications that fractal dimension is not a sufficient parameter to provide unique categorization of urban patterns. It is necessary to investigate which parameters or pattern characteristics would be best to complement fractal dimension for this purpose.

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