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SCALE-FREE NETWORK THEORY IN STUDYING THE STRUCTURE OF THE ROAD NETWORK IN POLAND

ABSTRACT

This paper discusses the issue of statistical analysis of traffic flow in different regions of Poland. Such analysis allows us to identify "valuable (sensitive) areas" whose damage or blockage may provoke considerable disturbances or even stoppage of traffic flow in the examined road network. The results of the studies indicate that the road network in Poland has the properties of a scale-free network. The distribution of the examined variables does not have a normal character, whereas the relationship between the number of nodes and the number of connections is a power-law feature.

KEY WORDS

transport; traffic; critical points

1. INTRODUCTION

Networks are omnipresent structures. Examples of networks include the Internet, roads, railways, air transport systems, energy transmission systems and social networks. Networks and network models provide a means for demonstrating phenomena, objects and their interrelations. A network model allows reflecting objects which are spatially diverse and have a distinct centre, and/or varied connections between such objects.

Studies of network structure are a very interesting topic for scientists all over the world. Since the explosion of the complex network science, many real-world networks have been examined. Examples include the structure of the Internet [1], criminal groups [2, 3], cyber-terrorism [1], social groups [4-6], trade [7-9], railroads and gas/oil pipeline systems [10], energy networks [11-12], scientific cooperation networks [13], citation networks [14] and many others.

The spatial structure of the transport network, its distinctive roles of nodes and connection with actual traffic flows are the primary interests of geographic inquiry [15].

Jiang [16] studied the hierarchies of urban streets from the perspective of both geometry and topology and from the point of view of traffic flow in reality. Bono and Gutiérre [17] employed spatial analysis to analyse how urban space accessibility decreases when the road network is damaged. Zhang and Li [18] proved that the fractal behaviour observed in the structure of road networks is of particular interest because it indicates that road networks are self-organized at several scales. They scaled the size of the network to the geodesic distance, and this universal behaviour, as well as the small-world and scale-free properties at several scales, represent an optimal organization that ensures maximal capacity at minimal cost.

The relationship between street centrality measures and land use intensity, when street centrality is calibrated in terms of a node's closeness, betweenness and straightness on the road network was examined by Wang et al. [19]. Delling et al. [20] studied the problem of computing all Pareto-optimal journeys in a dynamic public transit network for two criteria: arrival time and number of transfers. Kalapala et al. [21] examined how the recursive nature of model networks generates a scale-invariant journey structure and suggests a simple relationship between the scaling exponent of the dual degree distribution α and the fractal dimensions governing the placement of roads and intersections.

The public transport systems in Polish cities were analysed by Sienkiewicz and Hołyst [22], who concluded that the distribution degree can follow a power law or be described by an exponential function, depending on the assumed definition of network topology. All of the above studies concern free-scale network.

This article analysed the road network structure in Poland and identified critical "valuable (sensitive) areas” in this structure using general traffic measurements.

Numerous publications on the geography of road networks in Poland have mainly dealt with their topology. This paper discusses the issue of statistical analysis of traffic flow in different regions of Poland. Such analysis allows us to identify “valuable (sensitive) areas” whose damage or blockage may provoke considerable disturbances or even stoppage of traffic flow in the examined road network. The knowledge about the location of the most important nodes or areas in the communication network plays a vital role as far as safety improvements and limitation of transport negative influence on the environment and living conditions are concerned. Road network is a part of national critical infrastructure; however, some of its corridors are included in the Trans-European Road Networks. Their main roles are: continuity, action integrity and proper functioning. Providing those elements is possible owing to inter alia the knowledge about the characteristics, dynamics and the evolution of the analysed roads.

2. STUDY AREA AND DATA COLLECTION

Poland is situated in central-eastern Europe. Since 2004, Poland has been a member of the European Union and, together with Finland, Estonia, Latvia and Lithuania, it forms its eastern border. Poland is one of the largest European countries: it covers an area of 312,679 km^2 and has a population of 38.1 million [23]. In Poland, the structure of transportation is mainly based on car transport. In 2010, this type of transport constituted as much as 70.4% of all transportation operations and is steadily growing. Other forms represent a minor part of the structure of transportation: railway transport – 15.4%, air transport – 0.1%, pipeline transport – 7.6%, inland shipping – 0.3%, and marine shipping – 6.2% [23]. In 2010, the total length of roads in Poland amounted to 406,122.1 km, including 66,343.2 km in urban areas and 339,778.9 km in non-urban areas [23]. Public roads in Poland are divided into national, voivodeship, district and communal roads [24, 25].

The network of national roads includes highways and express roads, international roads and other communication pathways that ensure the integrity of national road system, access roads to public border crossings, roads that constitute ring-roads of large urban agglomerations and roads with military functions. The network of voivodeship roads consists of roads that connect cities and roads that are important for the given voivodeship and military roads (not included in the national road system) [24]. According to their functions within the road network, the length of national roads amounted to 18,607.9 km, voivodeship roads 28,461.1 km [23]. Poland has a limited network of highways: in 2010 their length was 854.7 km, which constituted app. 2 km per 1,000 km^2 and 2 km per 100,000 citizens while the mean values for 27 EU member-countries were 13 km and 15 km, respectively [23].

The density of roads (national and voivodeship) in the individual Polish voivodeships is variable. The highest density of roads is found in the voivodeship of Śląskie (20.70 km/100 km^2) and it is higher by 38% than the mean road density in Poland (15.05 km/100 km^2). The density of roads is also higher than the average value in Poland in such voivodeships as Dolnośląskie, Opolskie, Lubuskie, Małopolskie, Kujawsko-Pomorskie, and Świętokrzyskie. Except for Kujawsko-Pomorskie, these voivodeships are located in the south-west and western part of Poland. Industry is the main branch of economy in this region. The voivodeships where the average density of roads approximates the mean value for Poland include: Wielkopolskie, Pomorskie, Zachodniopomorskie and Mazowieckie. The first three are situated in the north and north-west of Poland. The fourth, Mazowieckie, is located in the central region and includes Warsaw, the capital of Poland. In other voivodeships (Łódzkie, Podkarpackie, Wamia and Mazury, Lubelskie, Podlaskie) the density of road network is below the national mean value.

For research purposes, the results were acquired of direct measurements taken under the General Traffic Measurement performed in 2010 for the General Directorate for National Roads and Motorways in Warsaw, Poland. The basic parameter (calculated based on the General Traffic Measurement for all sections of the national and voivodeship road network) is the average daily traffic in a given year. It is defined as the average number of vehicles passing through a given section of a road within 24 consecutive hours during one year.

4. METHODS

It was assumed for research purposes that where there are two points and an idea of how to connect them, a network may be formed for the purpose of these studies, it was assumed that vehicles moving from point A to point B passed a number of points. If points with characteristic features (for example, intersections with other roads, characteristic points with high traffic volume, etc.) are marked, and the frequency of vehicle passages is studied, a network is obtained consisting of nodes (measuring points) and edges/links (passage route of a given vehicle). The number of connections per node k (vertex degree) is expressed by the measure of the number of passages through a given node x, within specified time t.

Scale-free networks are distinguished by the power law distribution of the totality of nodes

$$P(k) = \frac{C}{k^{\alpha}} \qquad (1)$$

also known as the Pareto principle, which describes the scale-free networks when the exponent α belongs to a narrow range of values between 2.0 and 3.0. The scale-free property is of fundamental importance here and applies to the entire distribution. This is because the power law is invariant in relation to the operation of scaling the degrees of the nodes.

If the investigated correlation $P(k)$ is expressed with a double logarithmic scale, it was found that the power distribution of probability is a straight line [26].

$$\ln P(k) = -\alpha \ln k + \ln C \qquad (2)$$

The power distribution expressed with novel coordinates, i.e. $X = \ln k$, $Y = \ln P(k)$, becomes equivalent to a linear correlation:

$$Y = aX + b \qquad (3)$$

where $a = -\alpha$ and $b = \ln C$ [26].

Another feature of scale-free networks is that, in general, a randomly-selected subnet of a scale-free network also maintains the power law of degree distribution and is, therefore, itself scale-free [26]. This indicates the self-similarity of those networks, when considered on various scales of distance; therefore, they have fractal properties [21].

The topology of a scale-free network, conditioned by the power law, is usually characterised by the presence of a small number of high-degree nodes (hubs) and a relatively large number of low-degree nodes. Therefore, such a network is generally resistant to random failures [27] while being susceptible to precisely targeted attacks [28].

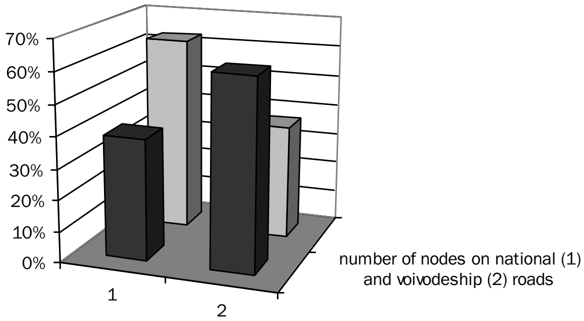
This is due to the fact that a large number of nodes connect to the centres; therefore, a virus introduced into such a network structure will be instantly transferred to an enormous number of other nodes. It will attack one of the centres and subsequently the others and will take control of the entire system. Such a topology may be rather dangerous, although it may become an "ally" when used properly.

5. RESULTS

The research was examined on the latest general traffic measurements performed in 2010 at 4,637 points (nodes), where 33,200,289 passages of vehicles (links) were counted to assess the road network traffic in Poland. The minimum node degree was 25 (on voivodeship roads – the measuring station No. 14242, situated near Garwolin and Parysów, to the south-east of Warsaw, the crossroads of voivodeship roads No. 805 and 800), while the maximum node degree amounted to 104,339 (on national road – the measuring station situated in the voivodeship of Śląskie, the area of Katowice, both to the north towards Sosnowiec and to the south-west towards the national border).

On national roads, 1,792 nodes with a total number of 20,998,478 connections were examined, while on voivodeship roads 2,845 nodes with a total number of 12,201,811 connections were examined.

As shown in chart No. 1 (*Figure 1*), 39% of all nodes are located on national roads and represent 63% of the total traffic, while 61% of nodes are located on regional roads and represent 37% of the total traffic.



	1	2
number of nodes on national (1) and voivodeship (2) roads	39%	61%
number of connections on national (1) and voivodeship (2) roads	63%	37%

Figure 1 - The number of nodes on national and voivodeship roads along with the number of connections provided by those nodes

5.1 Statistical distribution of variables

The distribution of nodes in the investigated road network was examined with a Kołmogorov-Smirnov test. This statistical method tests the null hypothesis which assumes that the distribution of variables in a sample is normal. If the K-S statistic is statistically significant ($p < 0.05$), the null hypothesis assuming the normality of variable distribution is rejected. In the examined road network, the null hypothesis was rejected and it was thus assumed that the distribution of the sample was not normal. In this paper, we used the least squares method to estimate the characteristic exponent α. According to Fronczak and Fronczak [26] „application of linear regression requires a lot of caution". Usually this method does not deal well with power-law distributions. Both estimated parameters and their uncertainty can be biased. This means the above mentioned methods can be used only when the accuracy of the result is not important. However, when the accuracy of results is important more reliable tools are required. Such a method of precise study is presented in Clauset et al. [29], in which maximum-likelihood fitting methods with goodness-of-fit tests based on the Kolmogorov-Smirnov and likelihood ratios are used. The correlation between the number of nodes and the number of connections is shown in *Figure 2*.

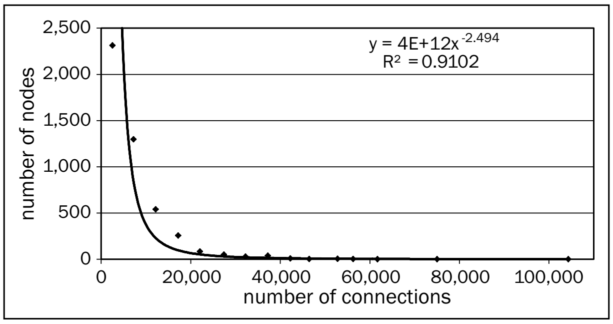


Figure 2 - Distribution of degrees of vertices and the number of connections

This correlation was also investigated with a double logarithmic scale. In this scale, the power distribution of the nodes is a straight line, while the scale coefficient does not change (see: *Figure 3*).

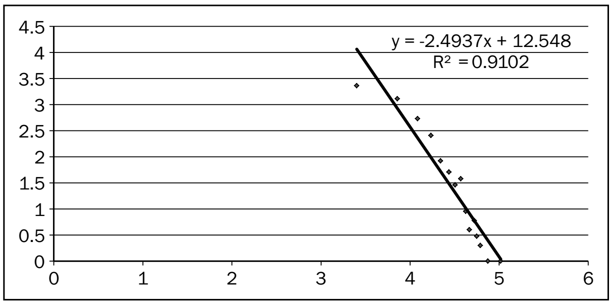


Figure 3 - Distribution of degrees of vertices and the number of connections in log-log scale

The examined road network, consisting of national and voivodeship roads, is based on traffic flow and shows the features of scale-free networks. In a mathematical arrangement, this correlation is invariable in relation to the scale, whereas the characteristic exponent α = 2.494 after scaling did not change and the slope coefficient stayed at the same level as the characteristic exponent α.

5.2 Distribution of node degrees in the sub-networks

This part of the studies focused on investigating the sub-networks in a spatial arrangement. To this end, the examined road network was spatially divided into sub-networks at the voivodeship level. Sixteen sub-networks were established which corresponded to the administrative division of Poland into voivodeships. The results are summarized in *Table 2*.

Table 2 presents the results of estimation of the characteristic exponent of power function and the slope coefficient in a double logarithmic scale. These findings indicate that the distribution of nodes in the sub-networks is not always scale-free in its nature. In one case, in the voivodeship of Łódzkie, the sub-network did not show such properties. All national and voivodeship roads in this voivodeship are burdened with traffic flow. No predominant centre that could meet the examined criterion was detected.

5.3 Hierarchical analysis of the nodes at the national level and determination of "valuable (sensitive) areas"

The structure of the investigated road network is arranged in a sort of hierarchy. If we organize them at the regional and national level, a clear hierarchical

Table 2 - The subnets along with the fitted power law function

Name of voivodeship	Characteristic exponent α	Characteristic exponent α after log-log function
Śląskie	2.472	2.472
Dolnośląskie	2.404	2.404
Opolskie	2.412	2.412
Lubuskie	2.336	2.336
Małopolskie	2.355	2.355
Kujawsko-Pomorskie	2.493	2.493
Świętokrzyskie	2.244	2.244
Mazowieckie	2.241	2.241
Wielkopolskie	2.414	2.414
Pomorskie	2.059	2.059
Zachodniopomorskie	2.281	2.281
Łódzkie	3.485	0.579
Podkarpackie	2.811	2.811
Warmińsko-Mazurskie	2.385	2.385
Lubelskie	2.783	2.783
Podlaskie	2.312	2.312

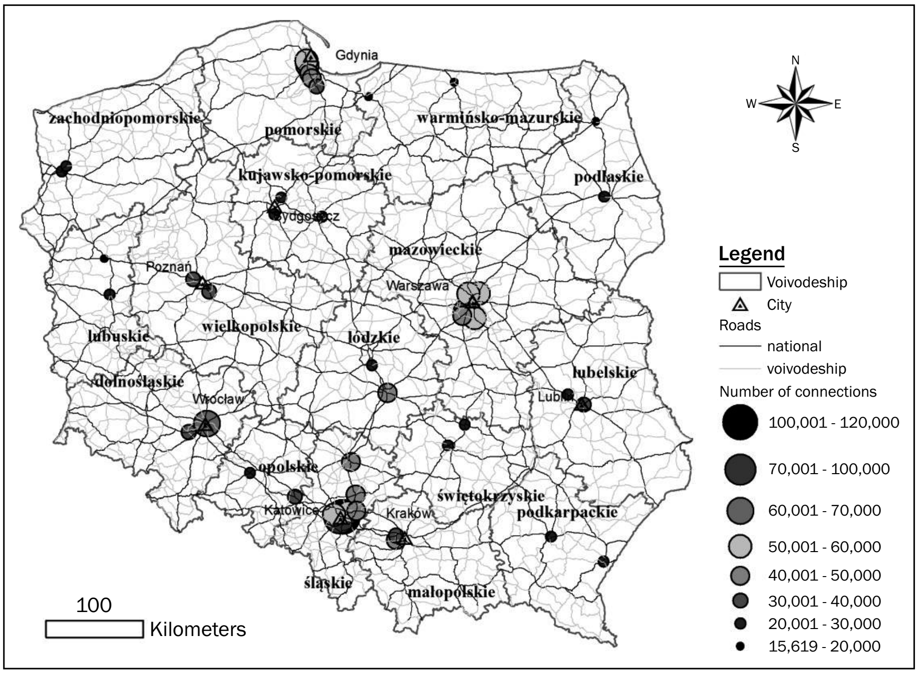


Figure 4 - The spatial distribution of the most important nodes in the road network in Poland

structure of roads will be generated. Not all networks with a power distribution of top degrees have a hierarchical structure. In the study area there are twenty-five such points.

Spatial analysis of the nodes with the highest number of connections reveals that they do not occur as single items (see *Figure 4*). Specific node regions may be selected. In this way, "valuable (sensitive) areas" may be identified based on the most important nodes found in a given region. These regions are critical to the functioning of the whole network. Identification of such nodes is important in terms of the features of investigated network. This paper focuses mainly on the attributes related to susceptibility to targeted attacks and resistance to accidental damage. Knowledge of the most important spots in the road network constituting regions ("valuable (sensitive) areas") creates the possibility for emergency management of these areas, particularly for supervision and protection. *Figure 4* shows the spatial distribution of the most important nodes in the road network in Poland.

The most significant regions in terms of communications include the area of Katowice (the geographical region named the Upper Silesia), Warsaw (the Warsaw Region) and Gdynia (the geographical region called the Northern Region). These regions overlap with the geographical territories and industrial zones in Poland.

Upper Silesia (Górny Śląsk) is a node region located in the voivodeship of Śląskie with the highest density of roads in Poland (20.7 km/100 km^2). They are concentrated around such cities as Katowice, Sosnowiec and Chorzów. The Warsaw Region is situated in the voivodeship of Mazowieckie. The density of roads approximates the national average (15.04 km/100 km^2) and they are concentrated around the capital city. The Northern Region is situated in the voivodeship of Pomorskie (Pomeranian). The density of roads is slightly below the Polish average (14.85 km/100 km^2) and they are concentrated around Trójmiasto (Tricity), which includes Gdańsk, Sopot and Gdynia.

5.4 Hierarchical analysis of the nodes at the voivodeship level and determination of "valuable (sensitive) areas"

The presence and importance (hierarchy) of node regions is also seen at the regional level. Within each voivodeship, there are so-called "valuable (sensitive) areas". This structure has been identified based on the nodes with the highest number of connections in the investigated voivodeship provided they constitute a cohesive area instead of dispersed nodes.

At the voivodeship level, hierarchical analysis has revealed a tendency towards the formation of local node sub-regions which are located in the following voivodeships: Dolnośląskie, Małopolskie, Kujawskopomorskie, Łódzkie and Lubelskie (see: *Figure 5*). Traffic flow is concentrated around five cities: Wrocław, Kraków, Bydgoszcz, Poznań, and Lublin.

In Dolnośląskie, this zone is centred around Wrocław. The roads from the south-east towards Opole and from the west towards Legnica are the most loaded with traffic flow. Wrocław, together with the neighbouring cities, is located within the Sudetes Industrial Area.

In Małopolskie, this zone centres around Kraków. The roads from the south towards Zakopane and from

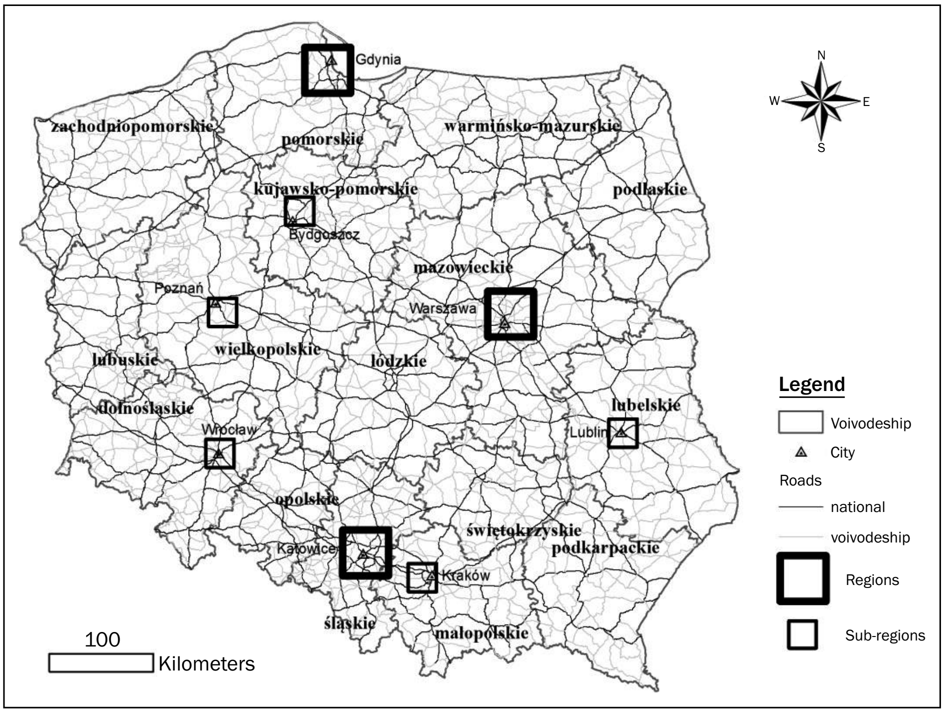


Figure 5 - The spatial distribution of the most important regions and sub-regions in the road network in Poland

the west towards Katowice are the most loaded with traffic flow. The city, together with the neighbouring towns, forms the Kraków Industrial Area.

In Kujawskopomorskie, there is a zone around Bydgoszcz, the roads from the north towards Grudziądz and from the east towards Toruń are the most loaded with traffic flow.

Within the voivodeship of Wielkopolskie, there is a zone around Poznań. The roads from the south towards Luboń and from the west towards Września and Swarzędz have the heaviest traffic flow.

In Lubelskie, this zone centres around Lublin. The roads from the north towards Puławy and from the south-east towards Zamość and Dorohusk (the border crossing with Ukraine) are the most loaded with traffic flow.

The spatial analysis did not reveal any node subregions in the following voivodeships: Opolskie, Lubuskie, Świętokrzyskie, Zachodniopomorskie, Łódzkie, Podkarpackie, Warmińsko-Mazurskie and Podlaskie. In these voivodeships, the most important nodes were separated from each other and they did not form a cohesive structure, i.e. a local sub-region.

6. DISCUSSION

The spatial structure of the transport network, its distinctive roles of nodes and connection with actual traffic flows are the primary interests of geographic inquiry. Such correlation was investigated for roads in Poland. The relationship between the number of nodes and the number of connections is power-law in its nature. A few nodes are characterized with a strong position, while the rest of them create a "long tail". Such networks also include the so-called "centres". The distribution of top degrees in such networks is a power-type distribution and has been called "scale-free" because of their stability in relation to scaling.

Derived characteristic exponent "α" is stable in reference to scale, so the network is self-similar. The dependency being investigated can be described in the following way: if the traffic flow of 50 cars/day happens eight times more frequently than the traffic flow of 100 cars/day, then the same ratio of frequency should take effect for the traffic flow of 100 and 200 cars/day as for 500 and 1,000 cars/day.

The process of checking the characteristic distribution of nodes in the network was important since this feature induces the sequence of further features. In this case, a feature related to the resistance of scale-free networks to accidental damage and susceptibility to targeted attacks becomes a critical element. The road network is a part of the critical infrastructure and is important for the economy since it enables the transfer of commodities and people [30]. Acts of targeted terrorism may disturb the functioning of a given region or even a country if they are directed at the points which are critical for network functionality.

Having information on the nature of the network and, in this case, on its invariability in relation to the scale, it also becomes possible to investigate other features (e.g. the preferential attachment of new nodes to the existing nodes) and resistance to accidental damage. The first (1) of the features is explained by real-world situations. While making decisions concerning new road investments, the decision-makers are most frequently guided by the number of connections per node. New national or voivodeship roads will take traffic from the roads most heavily burdened by the vehicle fleet and will be connected with them. As for the second (2) feature, which describes the interferences in a random node, it may be concluded that it does not disturb the smoothness of the flow throughout the entire network. Generally, such occurrences take place all the time – examples include road renovation, car accidents, stopping in prohibited locations, etc. The role of a damaged node is assumed for a period of time by side roads or additional roads, i.e. detours organised for the period of a temporary failure while the entire system is functioning properly.

The power distribution which is attributed to the investigated road network does not have a natural scale, as opposed to Gauss, and is thus scale-free [26]. The traditional methods of analysing traffic are based on the assumption that a road network is random. In such case the amount of links in nodes (measurement points) that are beyond the average appear scarcely. Hubs (nodes that have more than the average number of links) cannot exist in such a network. Meanwhile, the presented research shows that the nature of a road network may have a specific feature of a scale-free network. Nodes in such a network develop not in an accidental way (drivers will more often choose well-known roads and those indicated by other drivers who have already used them). Consequently, hubs, i.e. nodes whose number of links exceeds the average, are created frequently. Theoretically, the network may develop endlessly; however, considering transport network, there are physical limitations to the process (i.e. the number of roads). That is why the analysis of road network is so essential and discussing the mean values of traffic flow in a voivodeship or a selected region is at least contraindicated and, in many cases, using the idea of average degree leads to serious errors.

7. CONCLUSION

This article analysed the road network structure in Poland and identified critical “valuable (sensitive) areas” in this structure using general traffic measurements and scale-free network analysis. The studies of the road network in Poland were based on the traffic flow measurements that were taken on the national and voivodeship roads. The analysis allows us to identify “valuable (sensitive) areas” based on the most important nodes found in a given region.

The investigated network meets the basic criterion of a scale-free network. It has a power-law distribution of top degrees and the characteristic exponent is within a narrow range <2; 3>. After scaling, the function graph becomes a straight line and the slope coefficient remains at the same level as the characteristic exponent α. This confirms the stability of distribution in relation to the scale. An attempt was made to determine whether, after spatial division of the examined network into sub-networks (at the voivodeship level in this case), they would also be in conformity with the power rule.

The study carried out on the road network in Poland indicated that there are eight important node regions, the so-called “valuable (sensitive) areas”: three regions at the national level and five regions at the voivodeship level. The national regions include Górny Śląsk (Upper Silesia), the Warsaw Region and the Northern Region, while at the voivodeship level the group includes the sub-regions of Wrocław, Kraków, Bydgoszcz, Poznań and Lublin. Studies have shown the existence of several communication centres in Poland. With the information on their position, it is possible to provide special supervision for public security or impact on the natural environment.

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STRESZCZENIE

ZASTOSOWANIE TEORII SIECI BEZSKALOWYCH DO BADANIA SIECI DROGOWEJ POLSKI

W artykule omówiono statystyczną analizę natężenia ruchu w różnych regionach Polski. Analiza ta pozawala na zidentyfikowanie „wrażliwych miejsc” w strukturze sieci, których zniszczenie lub blokada może wywołać poważne zakłócenia lub nawet zatrzymanie ruchu w badanej sieci drogowej. Badania wykazały, iż badana sieć w Polsce nie ma charakteru normalnego, a związek pomiędzy liczbą węzłów oraz liczbą połączeń posiada charakter bezskalowy.

SŁOWA KLUCZOWE

transport; ruch pojazdów; punkty krytyczne

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