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Location problems in transport network

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ABSTRACT

This paper presents the importance of site selection for certain logistic infrastructure facilities as a key element for the quality of logistic processes. Different approaches to solving location problems in transport and logistics are presented. Basic procedures for solving discrete location problems are shown on concrete numerical examples in order to analyze necessary data for specific methods and possibilities of applying certain mathematical algorithms and models for the processes of logistic chains optimization and therefore optimization of logistic systems as bearers of logistic chains realization.

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1. Introduction

Location planning for facilities such as warehouses, logistic distribution centers, intermodal terminals, landfills, airports and similar comes under strategic planning. Location of the facility affects numerous operational and logistic processes. High costs of facility construction often prevent relocation of facilities. Projects of this type require long-term investments where facilities must perform certain functions for a long period of time. Therefore, decision-makers must select locations which will not perform their function well only under the current state of the system, but also in accordance with predictions of future changes in the system.

Quality of transport and logistic services as well as total costs of logistic system depend significantly upon the position of certain facilities in the transport network. Location of the facility in the network where some serving is performed, depends upon the type of the service. Number and location of certain facilities in transport network where some services are provided, are in the function of specific criteria and requirements posed by logistic systems. Over the past four decades a large number of papers that address the question of facility location in the transport network have been published. Warehouses, distribution centers, intermodal terminals, landfills, airports, hubs, garages and other represent some of the facilities with locations that can be determined by applying location analysis method.

Operational research has developed a series of mathematical models for the location problem. Unfortunately, some models do not give good results when applied to logistic systems because they do not take into consideration the specificities of logistic processes that affect reaching a good solution [1]. In approaching this problem, for this reason it is necessary to include logistic experts who are familiar with the technology of logistic processes as well as with specificities of individual processes.

Location theory tries to answer the following questions:

- What is the optimal number of facilities in the network on whose nodes a certain service is provided?
- In which nodes should facilities be located?
- How to perform the allocation of clients who need a service in one of the facilities?

Facilities on a transport and logistic network are usually located in nodes while in special cases facilities may be located at any point of observed space and then we talk about continuous location problems.

The study of location theory began in 1929 when Alfred Weber discussed how to position a warehouse in order to minimize the total distance between the warehouse and several clients [2]. Location theory is examined by Hakimi in several papers [3, 4, 5].

Location theory often examines questions of where to locate a new facility and how many new facilities should be located in the area where other facilities already exist and where in between the facilities there are certain relations that also have an important role in decision-making. Discrete space can be modelled using weighted graphs or networks. In numerous tasks of modelling location problems, nodes represent existing and/or potential (new) locations for the facility (facilities) that we want to locate and/or user spots and graph branches correspond to their relations (roads, communication lines, powerlines, etc.).

Quality of the service and total costs of the system depend essentially upon the location of certain facilities on the network. Nodes that represent facilities refer to those real objects where an activity or service is performed or to objects where clients are searching for a specific service. Sometimes the same facility can simultaneously provide and request a service. Weight assigned to the node represents the number of clients or demand for services in that node.

In practice, for the purpose of setting and solving location tasks, oriented i.e. asymmetric networks where $c_{ij} \neq c_{ji}$ are especially interesting. Network is oriented if the branches are determined in it by arranged pair of nodes [*i*, *j*], where *i* is the beginning and *j* is the end of the branch. For solving location problems on a network which can be presented in a plane, a variety of methods have been developed.

2. A simple mathematical model for the location problem

We examine an unoriented network *M (X, L, D)* Figure 1, where *X = {A, B, ...}* set of *n* nodes, L is a set of arcs (branches) and D a set of the distances between the nodes. Next to each node a number of request for services z_j, j = 1,2, ... *n* is written out. It is necessary to determine *p* nodes where the facilities for performance of certain services would be located. The shortest path between the nodes *i* and *j* is indicated as d_{ij}. We introduce into consideration binomial variables x_{ij} which are defined as following:

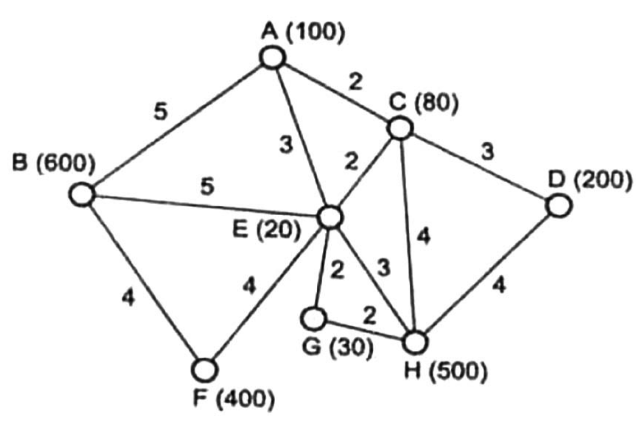

Fig. 1 Network for the location of the facility (Teodorović, 2007, p. 403) [6]

x_{ij} = 1, if the object is located on i^{th} node and serves clients from j^{th} node 0, if the object is not located.

Problem is posed in such a manner that we aim to minimize the sum of travelled distances *F* between the service facilities and clients so the objective function can be formulated in the following manner:

To minimize

$$min\ F = \sum_{i=1}^{n}\sum_{j=1}^{n} z_j d_{ij} x_i, \qquad (1)$$

Each client can be served in only one facility so this constraint can be expressed as:

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \dots, n \qquad (2)$$

Constraint of locating *p* facilities on the network can be expressed as:

$$\sum_{j=1}^{n} x_{jj} = p \qquad (3)$$

Each client located in the node where the facility is also located receives a service from that facility which is ensured by the constraint:

$$x_{jj} \geq x_{ij},\ i,j = 1, 2,..., n;\ i \neq j \qquad (4)$$

As stated earlier x_{ij} binominal variable:

$$x_{ij} \in \{0, 1\},\ i,j = 1, 2,..., n \qquad (5)$$

In Figure 1 it is possible to determine the shortest distance between the nodes and those can be shown by a matrix $[d_{ij}]$.

In the next step we shall calculate the expression $z_j d_{ij}$ by multiplying each matrix column of shortest distances with the number of requests for services in node *j*, first column is multiplied with 100, second with 600 and so on, this way we get a matrix $[z_j d_{ij}]$.

$$[d_{ij}] = \begin{bmatrix} 0 & 5 & 2 & 5 & 3 & 7 & 5 & 6 \\ 5 & 0 & 7 & 10 & 5 & 4 & 7 & 8 \\ 2 & 7 & 0 & 3 & 2 & 6 & 4 & 4 \\ 5 & 10 & 3 & 0 & 5 & 9 & 6 & 4 \\ 3 & 5 & 2 & 5 & 0 & 4 & 2 & 3 \\ 7 & 4 & 6 & 9 & 4 & 0 & 6 & 7 \\ 5 & 7 & 4 & 6 & 2 & 6 & 0 & 2 \\ 6 & 8 & 4 & 4 & 3 & 7 & 2 & 0 \end{bmatrix} [z_j\, d_{ij}] =$$

=

	A	B	C	D	E	F	G	H	Σ
A	0	3000	160	1000	60	2800	150	3000	10170
B	500	0	560	2000	100	1600	210	4000	8970
C	200	4200	0	600	40	2400	120	2000	9760
D	500	6000	240	0	100	3600	180	2000	12620
E	300	3000	160	1000	0	1600	60	1500	7620
F	700	2400	480	1800	80	0	180	3500	9140
G	500	4200	320	1200	40	2400	0	1000	9660
H	600	4800	320	800	60	2800	60	0	9440

This problem can be solved by using completed software packages such as *WinQSB.* This location problem has 64 variables and for the purpose of simplifying the usage of *WinQSB*+ software we may introduce new variables $x_1 = x_{11}, x_2 = x_{12}, x_3 = x_{13} \dots x_{63} = x_{87}, x_{64} = x_{88}$.

Objective function (1) is min($0\, x_1 + 3000\, x_2 + 160\, x_3 + \dots + 60\times63 + 0\times64$).

We have 65 constraints. Due to constraints (2) only one variable in each column is 1, others are zero, so in new terms we have: $x_1 + x_9 + x_{17} + \dots + x_{57} = 1$, for 8 columns we have 8 constraints. Constraint (3) for $p = 1$ is $x_1 + x_{10} + x_{19} + x_{28} + x_{37} + x_{46} + x_{55} + x_{64} = 1$.

Constraint (4) for the presented problem is: $x_1 \geq x_2$ $(x_1 - x_2 \geq 0)$, $x_1 \geq x_3$ $(x_1 - x_3 \geq 0)$, ... $x_{10} \geq x_9$ $(x_{10} - x_9 \geq 0)$ ... $x_{64} \geq x_{63}$ $(x_{64} - x_{63})$ and we have $7\times 8 = 56$ such constraints.

By using QSB+ software we get a solution:
$x_{33} = x_{34} = x_{35} = x_{36} = x_{37} = x_{38} = x_{39} = x_{40} = 1$, objective function is $minF = 7620$ which in matrix $[z_j d_{ij}]$ matches Σ of line *E* and this means that facility should be located in node *E*. As it can be seen, clients from node *E* use the services in *E*, $x_{37}=1$.

If two facilities should be placed on the transport network shown in Figure 1, then in constraint (3) $p = 2$ i.e. $x_1 + x_{10} + x_{19} + x_{28} + x_{37} + x_{46} + x_{55} + x_{64} = 2$. By using QSB+ two solutions appear: $x_{10} = 1$, $x_{33} = 1$, $x_{35} = 1$, $x_{36} = 1$, $x_{37} = 1$, $x_{38} = 1$, $x_{39} = 1$, $x_{40} = 1$ or $x_{10} = 1$, $x_{14} = 1$, $x_{33} = 1$, $x_{35} = 1$, $x_{36} = 1$, $x_{37} = 1$, $x_{39} = 1$, $x_{40} = 1$. For both solutions objective function is 4620. In Figure 2, second solution is presented where the clients from nodes B and F are supplied in B, and clients from A, C, D, E, G, H are supplied in E. Facilities are located in nodes E and B, clients from the node B are served in B, which means that locating 2 facilities on a network will reduce the total travel for 7620 – 4620 = 3000.

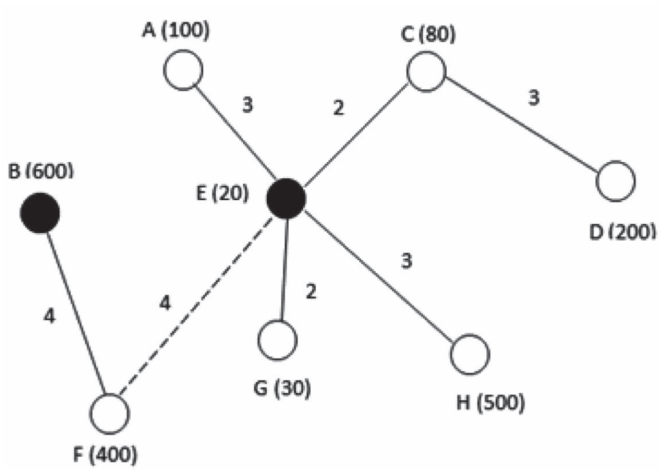

Fig. 2 Network with two facilities

Source: made by the authors

3. Median

Median is a type of facility determined by minimizing average distance (average travel time, average transport costs) between the facility and the service user. Median problems have a large influence on logistics because this group of problems is met during planning different distribution systems.

Algorithm used to locate one median on a network is defined by Hakimi and algorithm belongs to the group of algorithms for determining a set of acceptable solutions and they are used for determining one median on an unoriented network. It consists of the following steps:

Step 1 To calculate the shortest travel distances d_{ij} between all pairs of nodes (*i,j*) on the network G and to present these in the minimum distance matrix $[d_{ij}]$ (nodes *i* represent possible locations for a median while nodes *j* represent locations of clients who request services)

Step 2 To multiply the j^{th} column of the minimum distance matrix by the service demand weight z_j from the node *j*. Element $z_j\, d_{ij}$ of the matrix $[a_j\, d_{ij}]$ represents the distance travelled by clients from the node *j* to node *i* where they are served.

Step 3 To compute the sum for each row *i* of the $[z_j\, d_{ij}]$ matrix. Term $\sum_{j=1}^{n} a_j\, d_{ij}$ represents total distance travelled by users in case when the facility is located in the node *i.*

Step 4 Node with a row corresponding to the minimum total distance travelled by users, represents the location of the median.

In the previous paragraph, shortest distance matrix $[d_{ij}]$ for the network in Figure 1 is defined as well as a matrix $[z_i d_{ij}]$. In the matrix $[z_i d_{ij}]$ below the line Σ, sums of rows are written out i.e. the average distance travelled by the clients from j^{th} nodes to node *i* in which the facility is located. The minimum sum is found for node E so the facility should be located in node E and then the average distance travelled will be 7620. The same result is given by using linear programming.

The presented algorithm for determining one median in case of unoriented network can be fully applied to determine the location of the input or output median on an oriented network. It is necessary only to take account of the orientation of the network, i.e. the length of the shortest paths between certain pairs of nodes. When the network is oriented and when clients go to the facility in order to be served, we have an input median and in case where the means of transport leave from the facility located in the i^{th} node to the j^{th} node, we have an output median.

In case when two facilities are located on the network, the first facility as shown should be located in E while the second can be placed in one of the nodes {A, B, C, D, F, G, H}. Table 1 shows the sums of the shortest paths for different location combinations of two nodes.

According to Table 1, the best solution is to locate the second facility in node B, if each client node is connected to the closest node where the facility is, the result is shown in Figure 2. When two facilities are located on a network, sum of distances is reduced for 7620-4620 = 3000. According to the data in Table 1, node E has the least distance sum and the next node with the smallest sum of distances is node B. This model is largely simplified be-

Table 1 Sum of the shortest paths for different location combinations of two facilities

Combination	Sum of shortest paths
AE	7320
BE	4620
CE	6960
DE	6629
FE	5020
GE	7020
HE	5320

Source: made by the authors

cause when the costs were analyzed, the cost of building the facility in a particular node and other expenses were not taken into account.

4. Application of game theory for solving location problems

The case considered here is one where two competing trading companies A and B sell identical goods in four villages which are shown in Figure 3 [7]. Out of the total population in these villages, 20% lives in the first, 40% in the second and 20% in the third and fourth village respectively. The distances between neighboring villages are indicated in Figure 3.

Both companies decided to open a warehouse in different or the same villages out of the four potential sites so they examine an optimal location. Statistical research show that stock turnover depends upon the distance of the warehouse from the clients in the following manner. Company A will have 80% of turnover in every village that is closer to its warehouse, 60% of turnover in villages equidistant from both warehouses and 40% of turnover in the village which is closer to the warehouse of the company B.

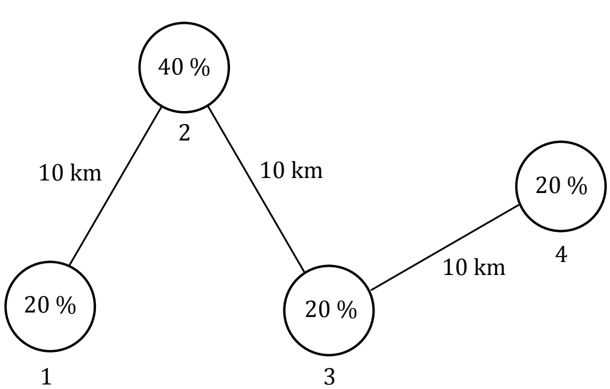

Fig. 3 Selection of the warehouse location

Source: Gren J. (1972). Gry statystyczne i ish zastosowania, Panstwowe Wydawnictwo Ekonomiczne, Warszawa.

A conflict situation between companies can be described with a matrix game. Company A will take the role of the first player and company B of the second player. Each player has four strategies – to open a warehouse in villages 1, 2, 3 or 4. Profit of the first player is a share of turnover expressed in a percentage achieved by the company A. If company A has a *k* percent of turnover, then company B will have a turnover deducted by that percentage. So, it is a zero sum game. It only remains to determine the payoff matrix. If A and B open a warehouse in the same location we get strategy $a_{11} = a_{22} = a_{33} = a_{44} = 0.60$ or 60%. If A opens a warehouse in the first and B in the second village then the profit of A company equals

$a_{12} = 0.80\times0.20 + 0.40\times0.40 + 0.40\times0.20 + 0.40\times0.20 = 0.48$

For several strategies (1,3) we have a matrix element $a_{13} = 0.80\times0.20 + 0.60\times0.40 + 0.40\times0.20 + 0.40\times0.20 = 0.56$ and so on. Payoff matrix is

Table 2 Payoff matrix

	B				Min
A	0,60	0,48	0,56	0,64	0,48
	0,72	0,60	0,64	0,68	**0,60**
	0,64	0,56	0,60	0,72	0,56
	0,56	0,52	0,48	0,60	0,48
max	0,72	**0,60**	0,64	0,72	

In each line on the right side of the table, a minimum element is written out and below each column there is its maximum element. Saddle point is (2, 2) and the value of the game $a_{22} = 0.60$ or 60 %. If A does not choose a strategy $i = 2$, one cannot expect a higher profit. The same goes for B. Therefore, strategies $i = 2$, $j = 2$ are called optimal and arranged triplet (2, 2, 0.60) is the solution of the game. The value v = 0.60 is a *compromised gain* (*loss*), and strategies $i = 2$, $j = 2$ ensure to the players *equilibrium in the game.* This is why a saddle point is called *equilibrium point*. If there is a saddle, players do not need to hide their optimal strategies. If B knows that A has chosen an optimal strategy, it cannot get any advantage because one cannot reduce the profit of the opponent. This works the other way around. Each matrix game with a saddle has such characteristics.

This way the game theory can be used to solve specific location problems and to assist in decision making.

5. Conclusion

Location problems draw attention of numerous researchers in different fields. Depending on the set up objective function or multiple objective functions, constraint structure, dimensions of the problem, a variety of methods and algorithms is being used.

Algorithm for generating a set of permissible solutions involves an examination of all possible solutions of *p*-median locations, calculation of the corresponding values of defined function criteria and determination of the optimal solution. This approach which is presented in this paper can be applied only in case of networks with a smaller number of nodes where a small number of facilities should be located.

When determining the location, flow of goods in asymmetric networks, i.e. oriented networks has an important influence on the decision.

Due to the increasing quality of logistic services, new and modern development strategies for transport and logistic systems, many research papers and studies have been devoted to this area. Despite the rich and diverse literature, location problems face theoretical and practical challenges because every location problem requires a research approach, appropriate model and methods suitable for solving. This paper is an attempt to provide insight and inspiration for solving practical problems by presenting several basic methods for solving location problems.

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