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ANALYSIS OF MATERIAL RELIABILITY OF AlSi17Cu5 ALLOY USING STATISTICAL WEIBULL DISTRIBUTION

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Analysis of reliability of AlSi17Cu5 alloy was carried out using the so-called „permanence measure". Two-parameter Weibull distribution, defining stability of material features, is its numerical equivalent. Mechanical properties (HB hardness HB, tensile strength R_m, and yield strength $R_{0,2}$) were selected as parameters for statistical evaluation, whereas inoculation with phosphorus (CuP10) and overheating to 920 °C (separately and jointly) together with intense cooling of the alloy were the factors shaping the influence of technology type. Statistical calculations, preceded by verification of hypotheses on conformity of distribution of the investigated resultant characteristics with normal distribution, were supported with variance estimators, and correlation and regression analyses.

Key words: cast Al-Si alloy, material and technological reliability, Weibull distribution, reliability theory

INTRODUCTION

Casts used under variable operation conditions, are often subjected to factors with stochastic character, and the time of failure-free operation of the whole system depends on components with significant dispersion of variability features [1, 2]. Therefore, the majority of problems of reliability theory is based on mathematical methods, mostly those of statistics and calculus of probability [2, 3]. The reliability itself consists in determination of the influence of factors other than random ones, recognition of mechanisms initiating a defect, selection of proper materials with known history of their manufacturing and type of technology used, increasing the "warranty" of correct operation for a given time [4, 5].

The reliability is also related with the notion of material and technological stability or construction durability, that is a set of features determining the failure-free time of utilization of a mechanical system. The analysis of reliability is carried out using a so-called reliability measure, which, being a function of reliability time, may be written down with the following expression [5]:

$$R(t) = p\{T>t\}$$

where: $R(t)$ – reliability measure in time t, T – time of operation before the first failure, or probability (p) of failure-free operation of the object for a time longer than t.

Assuming $Q(t)$ as a measure of the object unreliability equal to 1 – $R(t)$, and existence of a time derivative of the distribution function t, probability density $f(t)$ equals to:

$$f(t) = \frac{dQ(t)}{dt}$$

Hence, the unreliability measure is defined by the following relation:

$$Q(t) = \int_0^t f(t)dt$$

while the reliability amount to:

$$R(t) = 1 - \int_0^t f(t)dt$$

Many technical objects is characterized by such distribution of operational reliability, particularly mechanical devices with character close to normal distribution, log-normal distribution or Weibull distribution [4]. Form of these courses as a graphical formulation of probability density of a random variable, depends on damage physics of the studied object. In the case of material reliability, not only in cast alloys, or operational reliability, the main causes of unserviceability of mechanical equipment are: fatigue, surface and volumetric defects, corrosion, abrasive wear, creep, and particularly exceeding a given resistance. These features are an adequate measure for estimation of material stability of casts subjected to mechanical tests [6-8].

AIM AND SCOPE OF STUDIES

The aim of the paper was to estimate the material reliability as a component of operational reliability of AlSi17Cu5 cast alloy using statistics of two-parameter Weibull distribution. In order to achieve this goal, scope of the studies includes:

- list of results of mechanical properties (HB, R_m, $R_{0,2}$);
- verification of hypotheses on conformity of distribution of the studied characteristics with z normal distribution;

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- determination of basic point and range estimators;
- determination of two-parameter Weibull distribution and characteristics of descriptive statistics using correlation, regression analyses and variance;
- graphical interpretation of distribution of the estimated material reliability.

TEST MATERIALS AND METHODS

An AlSi17Cu5 alloy close to A3XX.X series for casting in sand moulds and permanent moulds was the subject of the study. The alloy is intended for casts of combustion engine pistons, cylinder blocks and frames for compressors, pumps and brakes. The alloy was melted in a Balzers VSG induction furnace, using pure Al of A00 grade (99,98 %Al), technical silicon (99,96 %Si) and copper (99,93 %Cu) in a SiC crucible. Inoculation with phosphorus was carried out using CuP10 preliminary alloy. Refining was carried out using "Rafglin-3" preparation in the amount of 0,3 % by wt. The chosen alloy was casted into a copper permanent mould, obtaining 3 samples simultaneously. 10 casts (30 samples) for each of the following technologies were done:

1. The alloy after inoculation with 0,05 % by wt. P (CuP10), (designation SM).
2. The alloy overheated at 920 ºC for 30 min. in the furnace (SP).
3. The alloy overheated and inoculated with the parameters as above (SPM).
4. The alloy overheated and inoculated, casted into a permanent mould placed in liquid nitrogen (SPMA).

The alloys obtained using the aforementioned technologies were compared with the alloy in its initial state (SW).

Measurement of HB hardness by Brinell method was carried out according to PN-EN ISO 6506-1 standard using a Zwick ZHF hardness tester, under load of 187,5 kg, using a steel ball with diameter of 2,5 mm and measurement time of 35 s. Static tensile test was carried out according to PN-EN ISO 6892-1 standard using an Instron 3382 machine, at the ratio of 20:1 and constant tension rate of 5 mm/min. Tensile strength (R_m) and yield strength ($R_{0,2}$) were determined for these samples. Results of these measurements constituted the material for statistical studies.

RESULTS AND DISCUSSION

Results of analysis of chemical composition of the studied alloy are shown in Table 1.

Table 1 **Chemical analysis of the examined silumin /wt.%**

Alloy	Content of constituents						
	Si	Cu	Ni	Mg	Fe	Ti	Al
AlSi17Cu5	16,36	3,48	0,01	0,03	0,11	0,01	rest

Verification of statistical hypotheses on conformity with normal distribution was carried out first. Significance level of $p(\alpha)\leq 0{,}05$ was assumed. Using Statistica 7.1 software, a standard normality plot was prepared, and proper ranks were attributed to material features (HB, R_m, $R_{0,2}$) for selected technologies [6]. Standardized values were calculated for them and normality histograms were prepared. Using Kolmogorov-Smirnov test of goodness of fit, supported by Lilliefors test and Shapiro-Wilk test, the hypothesis on conformity of the studied characteristics with normal distribution was confirmed in each case.

Then, using the "Data analysis" package of MS Excel, basic parameters of descriptive statistics of regression and correlation, and estimators of single-factor variance analysis were determined.

Weibull modulus (parameter "*m*") determined by the degree of fit using Pearson R coefficient (correlation coefficient) and R^2 determination coefficient serves statistical method of strength determination, thus an evaluation of material reliability. Survival probability $P_S(V_0)$ for identical samples with unitary volume V_0 was defined as:

$$P_S(V_0) = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

where: σ_0 – selected material characteristics of the alloy (e.g. R_m), σ_0 – value, for which 37 % exceeds the given load considering the studied property (e.g. R_m), m – numerical value of Weibull modulus.

Numerical comparison of selected point estimators, regression analyses and Weibull modulus "*m*" indices calculated based on results of measurements of HB hardness, tensile strength R_m and yield strength $R_{0,2}$ for all technologies is shown in Table 2.

Weibull modulus "*m*" may be determined by curve matching, however the graphical method is easier, using double logarithming of dependence (5). The following formula is obtained then:

$$\ln\left[\ln\left(\frac{1}{P_S(V_0)}\right)\right] = m \cdot \ln\left(\frac{\sigma}{\sigma_0}\right)$$

where $\ln\left[\ln\left(\frac{1}{P_S(V_0)}\right)\right]$ as $\ln\left(\frac{\sigma}{\sigma_0}\right)$ function is a straight line with slope "*m*".

Using a model according to dependence (6), the course of distribution of survival probability "*p*" as a function of HB hardness of the AlSi17Cu5 alloy for all technologies together with logarithmic trend line, shown in Figure 1a, while the logarithmic form of Weibull distribution function is shown in Figure 1b.

Similar dependencies (for all technologies) were determined for tensile strength R_m and yield strength $R_{0,2}$. Graphical interpretation of these dependencies is shown in Figures 2 and 3.

CONCLUSIONS

In engineering practices, Weibull analysis serves modeling of a set of data that contain values pertaining

Table 2 **Selected statistical parameters determined based on HB, R_m and $R_{0,2}$ results for all applied technologies**

Type of technology	SW	SM	SP	SPM	SPMA
Statistics	Hardness / HB				
Average, x̄	95,43	119,50	127,50	132,63	147,70
Standard deviation, s	16,28	7,78	4,68	4,82	4,94
Variability index, z	0,170	0,065	0,036	0,036	0,033
Correlation coefficient, R	0,967	0,984	0,983	0,988	0,978
Determination coefficient R^2	0,935	0,968	0,984	0,988	0,987
"*m*" modulus	6,53	18,55	32,35	32,75	34,26
	R_m / MPa				
Average, x̄	180,41	209,87	216,93	210,11	231,12
Standard deviation, s	35,76	19,18	12,57	10,29	10,77
Variability index, z	0,198	0,091	0,058	0,049	0,046
Correlation coefficient, R	0,953	0,979	0,988	0,980	0,982
Determination coefficient R^2	0,909	0,959	0,973	0,957	0,958
"*m*" modulus	5,689	13,401	20,064	24,850	26,967
	$R_{0,2}$ / MPa				
Average, x̄	135,00	157,33	161,00	161,22	167,73
Standard deviation, s	33,50	15,65	8,54	7,85	4,89
Variability index, z	0,248	0,099	0,053	0,048	0,029
Correlation coefficient, R	0,978	0,989	0,967	0,973	0,975
Determination coefficient R^2	0,953	0,977	0,928	0,941	0,946
"m" modulus	4,759	11,376	21,232	23,899	33,708

where: z – variability index as a relative measure of differentiation of distribution of the studied resultant characteristics.

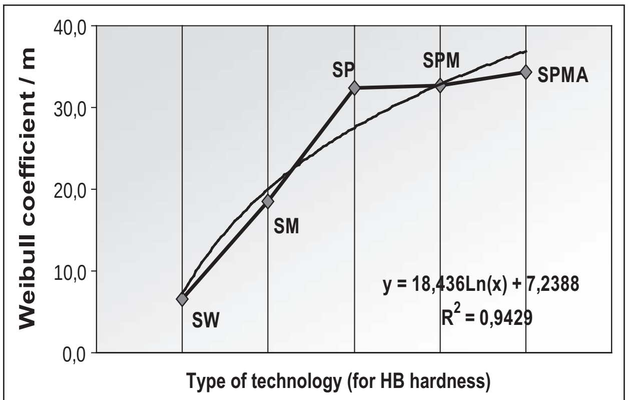

Figure 1 Weibull coefficient of AlSi17Cu5 cast alloy for HB hardness as a function of applied technologies

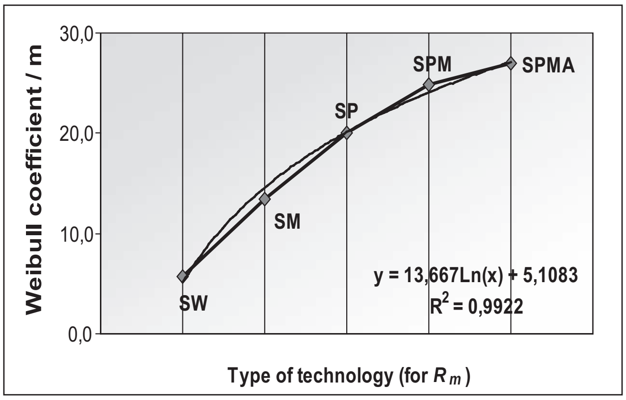

Figure 2 Weibull coefficient of AlSi17Cu5 cast alloy for tensile strength R_m as a function of applied technologies

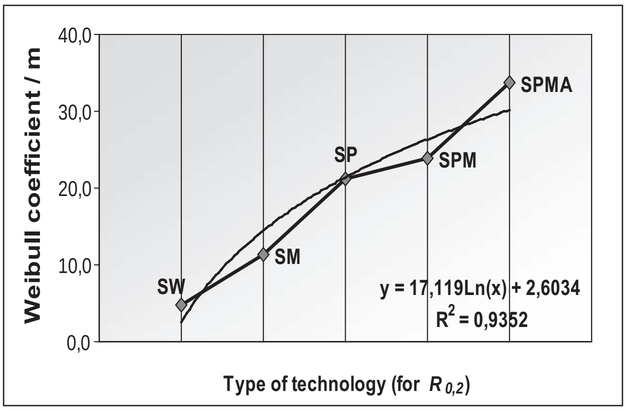

Figure 3 Weibull coefficient of AlSi17Cu5 cast alloy for yield strength $R_{0,2}$ as a function of applied technologies

to defects. It is used in many spheres of engineering, such as estimation of reliability of ceramic materials or strength of engineering materials. The evaluation enables prediction of permanence of products, comparisons of reliability of competing product projects, statistical determination of warranty policies or proactive management of spare part resources.

Evaluation of the influence of the applied technologies (inoculation, overheating and intense cooling) on technological permanence of the AlSi17Cu5 alloy was carried out based on a population of 30 samples (large test), casted according to the assumed research method. Were carried standard calculations of parameters of descriptive statistics and of the hypothesis on conformity of the studied resultant characteristics with normal distribution.

Using Kolmogorov-Smirnov, Lilliefors and Shapiro-Wilk tests of goodness of fit, the hypothesis on conformity of the studied characteristics (HB, R_m and $R_{0,2}$) with normal distribution was confirmed in each case.

Using MS Excel's "Data Analysis – Regression" tool, parameters of correlation and variance statistics were determined. The determined estimators (Table 2) show that the smallest dispersion of HB hardness (measured by standard deviation and variance index) characterizes the population of AlSi17Cu5 alloy samples subjected to overheating (to 920 ºC) and to inoculation with subsequent overheating. The smallest dispersion of tensile strength R_m and yield strength $R_{0,2}$ results was found for combined process of inoculation and overheating, intensified additionally with fast cooling (SPMA). These results reflected material reliability determined by statistical two-parameter Weibull "m" distribution. This parameter, defined by survival probability, indicates that:

- for HB hardness, the highest values of Weibull modulus, thus that of material reliability, characterizes the alloy overheated to 920 ºC (Figure 1). Inoculation with phosphorus and combined process of inoculation and significant overheating above $T_{liq.}$ with intense cooling do not contribute significant changes into the value of survival probability;
- for tensile strength, depending on the applied technologies, the Weibull coefficient increases non-line-

arly, reaching a maximum value of approx. 27 for the combined process of inoculation with phosphorus, overheating to 920 ºC and fast cooling of the AlSi17Cu5 alloy (Figure 2). The course of its function may be described with the logarithmic dependence y=13,667·ln(x)+5,108, at R^2 equal to 0,99;

- for yield strength, depending on the applied technology, the Weibull coefficient increases, but the increase is gradual (Figure 3);
- evaluating Weibull modulus determined for tensile strength and yield strength, the highest material reliability characterizes AlSi17Cu5 alloy subjected to inoculation, overheating and intense cooling.

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Note: The responsible translator for English language is Dr Janusz Mrzigod, Katowice, Poland