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ONTO-SEMIOTIC ANALYSIS OF VISUALIZATION AND DIAGRAMMATIC REASONING TASKS

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The explicit recognition of the objects and processes implied in mathematic activity is a competence that teachers should develop. This competence allows teachers to understand, design, and manage the processes of mathematical learning, and assess them with suitability standards previously set. Consequently, formative processes to develop this competence should be designed. This paper describes a training design aimed at developing the teachers' competence of epistemic and cognitive analysis of mathematics instruction processes, highlighting the role of visual and analytical languages in the constitution of mathematical objects.

INTRODUCTION

The Didactic-Mathematical Knowledge model (DMK) proposed by Godino (2009) includes as categories of teachers' knowledge the epistemic (institutional knowledge) and cognitive (personal knowledge) aspects, and they are considered as components of specialized knowledge of mathematical content. Teachers should be able to provide possible solutions to a mathematical task recognizing the sequence of operative and discursive practices that the resolver should implement. They should also be able to identify the network of ostensive (languages and artefacts) and non - ostensive objects (concepts, propositions, procedures and arguments) that intervene in the mathematical activity, the synergic relations between them, and the conflicting relationships between different types of languages brought into play and mathematical processes involved.

THEORETICAL FRAMEWORK, METHOD, AND BACKGROUND

In this research, the approach of the training activity is supported by the model of mathematic teacher's knowledge, described by Godino (2009) as "didactic-mathematical knowledge" (DMK) using theoretical and methodological tools from the Onto-Semiotic Approach (OSA) of cognition and instruction (Godino, Batanero, & Font, 2007; Font, Godino, & Gallardo, 2013).

The methodology applied is related to the design based – research approach (Kelly, Lesh, & Baek, 2008) or didactic engineering in the generalized sense proposed by Godino, Batanero, Contreras, Estepa, Lacasta, and Wilhelmi (2013), according to which the design is developed in four phases: preliminary study, design, implementation, and retrospective analysis.

Competence of epistemic and cognitive analysis

Teachers need competences to analyse the mathematics teaching and learning processes to achieve a suitable teaching, as well as to synthesize the existing didactical knowledge on the design, implementation and evaluation of teaching practice (Godino, Rivas, Castro, & Konic, 2012). This analysis competence allows teachers to comprehend and evaluate the students' mathematical activity, anticipate conflicting meanings and possibilities of institutionalization of mathematical knowledge (Godino et al., 2007), and assess their effectiveness and cost. The type of task analysis we propose to trainee teachers in the instructional design is an evolution of the onto-semiotic analysis technique described in Godino et al. (2012).

Visualisation and diagrammatic reasoning

Arcavi (2003) considers that mathematics, as a human and cultural creation that deals with objects and entities which are very different from any physical phenomena, strongly supports visualization in its different forms and levels, not only in the field of geometry. Duval (2006) attributes an essential role to the treatment of the signs within each system of semiotic representation and the conversion between different systems. Dörfler (2003, p. 41) argues that a widespread "inventory" of diagrams, supports and favours the creative and inventive use of diagrams.

We believe that mathematics teachers should be aware of the relationship between visual or diagrammatic representations, as well as the non - ostensive mathematical objects necessarily involved. They also should know the uses and limitations of different languages, recognizing the epistemic and cognitive possibilities of visual means of expression. This working hypothesis conditions the following planned instructional design.

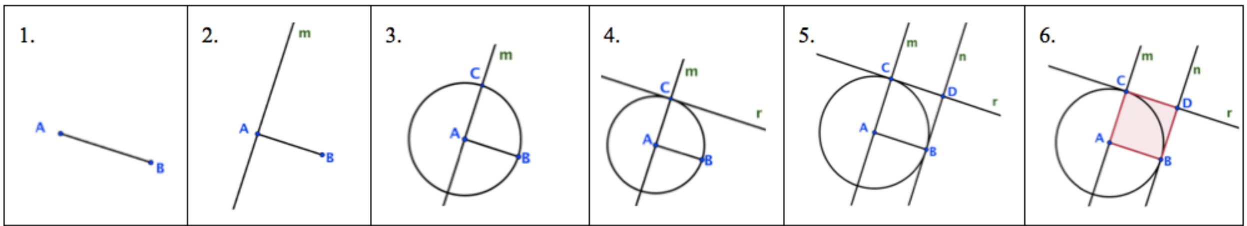
INSTRUCTIONAL DESIGN

The instructional design aims to reflect about the characteristics of the visualization and diagrammatic reasoning and its role in the teaching and learning of mathematics; it is also about recognizing the diversity of objects and processes involved in mathematical tasks performed by applying visualizations and diagrammatic reasoning.

Below we show the onto-semiotic analysis of a task proposed by an epistemic subject and it is, therefore, an institutional analysis.

A priori task analysis

The sequence shown below is the procedure followed by a student to build a square with Geogebra.



1.	2.	3.	4.	5.	6.
a) I represent a segment AB.	b) I build a straight line *m* perpendicular to segment AB through point A.	c) I build a circumference of centre A and radius AB. d) I call C to the point of intersection between the circle and the straight line *m*.	e) I build a straight line *r* parallel to the segment AB through point C.	f) I build a straight line *n* perpendicular to the segment AB through point B. g) I call D to the point of intersection between the straight lines *n* and *r*.	h) The quadrilateral ABCD is a square.

I: Justify that, indeed, the quadrilateral ABCD is a square.
II: Identify the knowledge involved in the construction and justification of the square with Geogebra.

Figure 1. Building a square with Geogebra

Expected answer to question I

1) We designate with 'square' a quadrilateral with four congruent sides and four right interior angles. (You can substitute other alternative definitions of square).
2) The A angle is right because the AC straight line is drawn perpendicular to AB.
3) The AC side is congruent to AB because it is the circle radius with center A and radius AB.
4) *r* and *m* are perpendicular because *r* is parallels to AB and *m* is perpendicular to AB. Therefore, the C angle is straight.
5) D is straight angle because *r* and *n* are perpendicular.
6) The CD side is congruent to AB because *r* and AB are parallel (including parallel segments between parallel lines are equal)
7) DB is congruent with AC because *m* and *n* are parallel.
8) Then, the four sides of ABCD are congruent and the four corners are straight.

Figure 2. Expected answer question I

Expected answer to question II

The table 1 shows the configuration of objects and processes involved in the construction of the square and in its justification.

	OSTENSIVE OBJECTS (Means of expressions)	**NON - OSTENSIVE OBJECTS (MEANINGS)** (Concepts, propositions, procedures, arguments)
	Construction of the square	
a)	I represent a segment AB.	*Concepts*: segment (general); segment endpoint *Procedure*: drawing of a generic segment with Geogebra. *Particularization*: fixed segment defined by points A and B.
b)	I build a straight line *m* perpendicular to segment AB through point A.	*Concepts*: straight line; point of a segment; perpendicular to a segment by a point; right angle. *Procedure*: construction with Geogebra for generic straight line perpendicular to a segment by one end. *Proposition*: two perpendiculars lines determine four right angles. *Particularization*: figures given.
	...	...
h)	The quadrilateral ABCD is a square.	*Proposition*: ABCD is a square
	Justification	
1)	The term square we designate a quadrilateral …	*Concepts*: square; quadrilateral; congruent sides; interior angles of a polygon; right angle.
2)	The A angle is right because the AC straight line ...	*Proposition*: The angle A is right. *Argumentation*: based on the perpendicular lines definition. *Particular – general*: the diagram figures refer to any figure that meets the conditions given.
	...	...
8)	Then, the four sides of ABCD ...	*Proposition*: thesis *Justification*: steps 1 to 7

Table 1: Configuration of objects and processes that intervene in mathematical practices

The table 1 shows that there is a narrow overlapping between the objects that intervene in the mathematics activity, specifically between: the diagrammatic – visual and sequential languages, the ostensive (material) and non-ostensive objects (immaterial), and the extensive (particular) and intensive objects (general). The use of diagrams in the mathematical practice should be accompanied by other means of non-visual expressions to communicate, justify or explain the development of the operative and discursive practices implied.

CONCLUSSION

The type of analysis we described in this paper should be an instrumental competence of the mathematics teacher because it enables him/her to recognize the complexity of objects and meanings at stake in mathematical activities, foresee potential conflicts, and adapt to the students' abilities and learning objectives. It is necessary to design and implement didactical situations for teacher training whose main objective is to develop teacher's competence to carry out the meta-analysis (Jaworski, 2005) of a key component of teaching: the mathematical activity understood both from the institutional and personal point of view.

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