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# Value of Monitoring in Asset Management: A Social Cost-benefit Analysis Approach

Mariem Zouch*[,a], Wim Courage[a], Oswaldo Morales Nápoles[a,b]

[a]TNO, Van Mourik Broekmanweg 6, 2628 XE Delft, The Netherlands
[b]Delft University of Technology, delft Institute of Applied Mathematics, Mekelweg 4, 2628 CD Delft, The Netherlands
mariem.zouch@tno.nl

We present a framework to investigate new monitoring techniques for infrastructures and assess their potential value for the network management. This framework is based on a social cost benefit analysis tool that aims to (i) assist decision makers in selecting and developing cost-effective new monitoring techniques and (ii) provide managers with socially optimal maintenance and rehabilitation strategies that take into account output from these monitoring systems. Potential value of monitoring consists mainly in enabling condition-based strategies and providing more accurate and relevant information that should result in more cost-effective strategies. Monitoring provides information about either the structure degradation level or its environment. The condition of the structure is represented by a set of technical performance indicators that reflect its degradation level and are linked to a set of end-user service levels. Finally, the end-user service levels are valuated to optimize the cost and benefits of maintenance and rehabilitation strategies. Main feature of the tool we develop is to enable optimal, dynamic and reliability-based decisions that are reviewed and updated every time a new relevant information is available. Transition probabilities to predict future deterioration levels are estimated and updated using monitoring data to assess risks and optimize its expected cost. Moreover, the derived strategies are socially optimal and take into account indirect impacts of degradations and M&R strategies on the society and the environment. This is done by consideration and valuation of end-user service levels. We use Markov decision processes which are an appropriate framework for decision-making under uncertainty to incorporate reliability and risk measures within the optimization problem.

## 1. Introduction

Road and bridge networks represent a huge public investment that is specially significant for citizens and contribute directly to social and economic activities. Unacceptable levels of pavement deterioration may reduce economic efficiency, lead to user safety and comfort issues, and incur extra maintenance and societal costs. Such problems could be mitigated by defining and implementing efficient strategies which, together with adequate material selection and construction practices, minimize total costs and maximize the availability of the networks. Moreover, with the climate change and sustainability issues, road owners and managers are more and more interested in promoting sustainable strategies that are based not only on the technical and structural condition of the pavements but also on societal and environmental considerations. Such considerations can be taken into account by defining and ensuring acceptable performance levels, also called end-user service levels (EUSLs). Introducing EUSLs will also enforce the need to enable different perspectives related to different stakeholders. Eventually, the EUSLs are directly related to the technical condition of the structures (i.e., the network performance decreases as the pavements deteriorate), but they also change as a result of the change in societal, political or economic targets, e.g., change in traffic intensities or change in objectives like availability or emissions thresholds. Defining such asset management policies is a complex task. First, there is a high level of uncertainty in the degradation process of the structure that makes it difficult to assess its degradation level as well as to predict its future evolution, especially because most of the existing infrastructures are ageing and their lifetimes are approaching the design service life. Second, considering societal and environmental factors

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into the decision-making process implies that indirect impacts of maintenance decisions should be evaluated, quantified and monetized. In order to reduce uncertainties and their impacts on the robustness of the decisions, adequate monitoring systems and decision-making frameworks that handle both uncertainties and information updating should be promoted.

In this work, we develop a reliability-based social cost benefit analysis framework for road networks management. It aims to (i) demonstrate the value of monitoring road networks and compare different monitoring techniques, and (ii) assist road owners in managing and maintaining efficiently road networks. The model derives optimal maintenance strategies for a given input data from monitoring systems and associated costs and benefits. It is based on three different type of variables: monitoring variables, condition indicators (CIs) and performance indicators (EUSLs). These variables are linked through relationships that can be determined by experts or estimated from available data. The framework also requires models to predict the pavement degradation processes after different maintenance actions. These degradation models, and consequently maintenance strategies, are dynamically updated using monitoring data. A Markov decision process model is used to derive optimal maintenance decision rules. Different monitoring techniques can be then compared. The main motivation of this work is to highlight the impact of monitoring output on network management policies. On one hand, a maintenance decision-making tool can help to identify drivers for monitoring systems for supplying specific, more reliable data on technical conditions and their evolution in time. On the other hand, the efficiency of stochastic dynamic decision-making techniques such as Markov decision processes that have been increasingly used during the last decades in different fields such as maintenance planning (Zouch et al., 2011), finance (Schal, 2002) and robotics, depends closely on the quality of available data. Hence, investing in monitoring techniques for infrastructure networks is expected to improve the efficiency of maintenance strategies. A special feature of our framework is that it can be used to investigate and possibly promote (i) the monitoring non-technical parameters, e.g., influence factors such as environmental loads, traffic or even directly the EUSLs, and (ii) operational actions such as speed limitations or maximum vehicle loads (to mitigate risk or further degradation) or even substantial redesigns like increasing the number of lanes. The paper is organized as follows. The problem of asset management is described and formulated in Section 2 where the approach for maintenance strategies based on monitoring techniques is presented. The value of monitoring is shown in Section 3. A numerical example is shown and analyzed in Section 4. Finally, conclusions and perspectives are presented in Section 5.

## 2. Problem description and formulation: from monitoring to optimal maintenance strategies

We consider a road sub-network that is comprised of a main route and two alternatives. We assume that maintenance decisions will be taken only for the main road, whereas impacts of those actions will be assessed on the totality of the sub-network, i.e., main road and alternatives. This can be later used to define efficient strategies on bigger networks where maintenance decisions should be taken simultaneously on many roads. In order to determine its level of deterioration, the main road is monitored either periodically (e.g., visual inspections) or continuously (e.g., sensor-based monitoring systems). $\mathbf{R}$ parameters, denoted $\boldsymbol{O_1, \ldots, O_R}$, are directly measured to evaluate $\mathbf{K}$ key condition indicators (CIs) denoted $\boldsymbol{X_1, \ldots, X_K}$. The CIs can be related to different types of defect, e.g., deformation, loss of material, discontinuity. Levels of $\boldsymbol{X_1, \ldots, X_K}$ are obtained from monitoring $\mathbf{R}$ variables $\boldsymbol{O_1, \ldots, O_R}$ using a set of transfer functions $\boldsymbol{f_k(O_1, \ldots, O_R), k = 1, \ldots, K}$. Note that levels of $\boldsymbol{X_1, \ldots, X_K}$ can be expressed as real values (e.g., percentage) or rates (e.g., very good, very bad). Note that although the general notation $\boldsymbol{f_k(O_1, \ldots, O_R)}$ is used here, all the monitoring variables are not necessarily related to each technical indicator $\boldsymbol{X_k}$.The performance of the sub-network, i.e., the main road and alternatives, is assessed using $\boldsymbol{E}$ EUSL variables, denoted $\boldsymbol{Z_1, \ldots, Z_E}$. An example of EUSLs is network availability. The EUSLs can correlated to one or more condition indicators. Depending on the application, current EUSLs cab be known either through direct monitoring or assessed using information about the current CIs.

The decision horizon is discretized into decision periods (e.g., one year) denoted $\mathbf{t}$. In the beginning of each period, based on available information about the main road pavement condition (through monitoring) and the sub-network performance, the decision-maker decides whether to do nothing for the current period and wait until the next one or to maintain the pavement. A panel of maintenance actions varying from minimal repairs such as patching, to milling and resurfacing with different thicknesses, to complete renewal is available to the decision-maker. Let $\boldsymbol{\pi_1, \ldots, \pi_A}$ denote the set of available maintenance actions and $\boldsymbol{\pi_0}$ denote the do-nothing action. Maintenance decisions have different impacts on both the pavement condition and the sub-network performance. We assume that the condition and the performance levels of the pavement after $\pi_0$ is selected are the same as just before the decision, and that any maintenance action $\pi_i, i = 1, \dots, A$ has two types of impacts on the main road CIs as well as the sub-network EUSLs: (i) immediate impact on the degradation and the performance levels, and (ii) future impact on their evolution processes. Immediate impacts on both the CIs and the EUSLs are modelled using the general impact functions denoted $\varphi(X_1, \dots, X_K, \pi_i)$and $\gamma(Z_1, \dots, Z_E, \pi_i)$, $i = 1, \dots, A$, respectively. We assume that $\varphi(X_1, \dots, X_K, \pi_0) = X_1, \dots, X_K$ and $\gamma(Z_1, \dots, Z_E, \pi_0) = Z_1, \dots, Z_E$. Future impact on the CIs will be modelled through the change of degradation processes of the impacted CIs, i.e., the transition probabilities from one level to another. Similarly, future impacts on the EUSL is modelled through impacts on their future evolution estimations. Let $P^{\pi_i}$ denote the (joint) transition probability matrices of $X_1, \dots, X_K$ and $Z_1, \dots, Z_E$ , respectively, after action $\pi_i$ is performed. Explanation on how $P^{\pi_i}$ could be obtained and updated with monitoring data will be given in the next section.

In addition to direct costs of performing maintenance, decisions incur indirect costs on the whole sub-network, i.e., user, safety and environmental costs. User costs are comprised of vehicle operating costs and travel time loss whereas environmental costs consist mainly in air pollution, noise and global warming costs. Note that user, safety and environmental impacts do not have a monetized values, hence reference values from valuation techniques such as *willingness-to-pay* could be used. Indirect costs are comprised of action-related costs that are incurred by maintenance and construction works, and condition-related costs that result from the main road condition during the decision period after the maintenance decision (including do-nothing) has been implemented. Details of these costs are given below. The objective is to define, given the outcome of monitoring systems, an optimal maintenance decision rule that minimizes the total expected discounted costs (i.e., agency, user, safety and environment costs) over the infinite horizon. An optimal maintenance decision rule is a map that associates to each possible state of the sub-network, i.e., condition and performance levels, the optimal action to perform. The benefits of a monitoring system can therefore be obtained by comparing the total expected cost of the derived maintenance strategy to strategies that are based on the pavement age only or to reference strategies.

**2.1 Action-dependent degradation and performance evolution processes**

Several models for degradation evolution are available in the literature for both discrete or continuous variables. One can refer to Brownian motion process (Myers et al. 1998, Whitmore et al. 1998), Gamma process (Abdel-Hameed 1975) or Markov chains. Although widely applied in a degradation evolution context, these models are not very suitable when monitoring systems are used and degradation evolution updating is frequently required. In this work, we propose a general framework based on a graphical model (discrete Bayesian Network) (Pearl 1988) to predict and update the degradation evolution processes with new available data. This framework is illustrated in Figure 1 where monitoring observations (green), CIs (blue) and EUSLs (red) are represented. In addition to CI variables $X_1, \dots, X_K$ and EUSL variables $Z_1, \dots, Z_E$, we also consider the variables representing increments these variables, i.e., $\Delta X_1, \dots, \Delta X_K$ and $\Delta Z_1, \dots, \Delta Z_E$ to predict their evolution within each decision period. Bold green arcs represent deterministic relationships, i.e., transfer functions of monitoring data to CIs whereas dashed green arcs represent eventual deterministic relationships that can be additionally taken into account such as transfer functions from monitoring data or from actual CI levels into actual EUSL levels. Black arcs represent probabilistic dependence between different variables.

**2.2 Costs and benefits**

Performing any maintenance action $\pi$ incurs both direct and indirect costs and results consequently in indirect benefits (comparing with the do-nothing option). Direct costs are the action costs comprised of fixed costs $c_F(\pi)$ and a variable cost $c_V(\pi, l)$ where $l$ is the total maintained length of a road section.

Indirect costs of actions are classified into action-related and condition-related costs. Action-related costs are caused by maintenance construction works and concern the totality of the sub-network. They are a function of the resulting EUSLs during the maintenance performance period. Action-related indirect costs are comprised of user, safety and environment costs denoted $c_{a.user}(\pi(Z_1, \dots, Z_E))$, $c_{a.safety}(\pi(Z_1, \dots, Z_E))$ and $c_{a.env}(\pi(Z_1, \dots, Z_E))$, respectively, where $\pi(Z_1, \dots, Z_E)$ denotes the resulting EUSLs while action $\pi$ is performed. User costs are comprised of vehicle operating costs and time loss values. Environment costs are comprised of air pollution, global warming and noise costs. Condition-related costs are incurred during the remaining decision period after maintenance decision are implemented and are a function of both resulting condition levels of the main road pavement after the maintenance decision is implemented and the resulting EUSLs of the sub-network. Similarly to action-related costs, condition-related costs are comprised of user, safety and environment costs that we assume functions of the CIs and the EUSLs immediately after performing a maintenance decision $\pi$, denoted $X^{\pi} = \varphi(X_1, \dots, X_K, \pi)$ and $Z^{\pi} = \gamma(Z_1, \dots, Z_E, \pi)$respectively as well as the CIs and the EUSLs at the end of the decision period denoted $X'$ and $Z'$, respectively. The condition-related user, safety and environment costs after action $\pi$ is performed are denoted $c_{c,user}(X^{\pi}, Z^{\pi}, X', Z')$, $c_{c,safety}(X^{\pi}, Z^{\pi}, X', Z')$ and $c_{c,env}(X^{\pi}, Z^{\pi}, X', Z')$. Moreover, we assume that an expected penalty cost called quality cost and denoted $c_{Q}(X^{\pi}, Z^{\pi}, X', Z')$ could be paid at the end of each decision period if the pavement reaches unacceptable degradation levels or the sub-network witnesses unacceptable EUSLs. In addition to its practical significance, the cost of quality reflects a risk-avoidance attitude of the decision-maker in his decisions by avoiding to frequently select the do-nothing option.

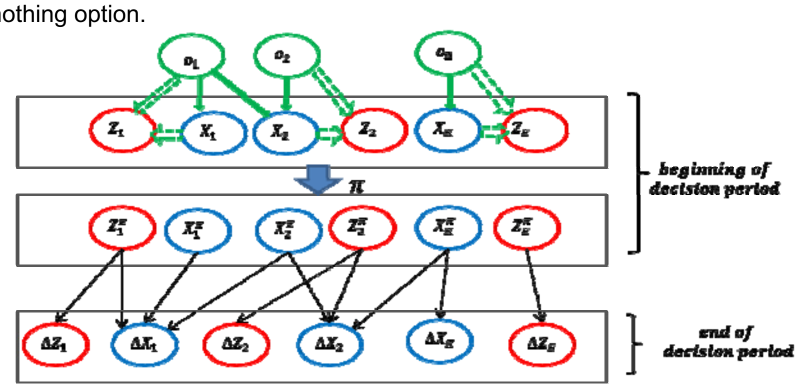


*Figure 1: Graphical model used to predict evolution of the CIs $(X_1, \ldots, X_K)$ and the EUSLs $(Z_1, \ldots, Z_E)$ with monitoring input $(o_1, \ldots, o_R)$.*

The total cost associated with a monitoring system is the sum of the fixed and variable costs of monitoring and the total expected cost of the derived optimal maintenance. The benefits can be calculated by comparing the incurred costs and the costs of an age-based strategy. Hence, the net present value of the monitoring technique can be calculated as the difference between total expected costs and benefits

Given the initial/current condition of the main road pavement $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_K)$ and the current EUSLs $\mathbf{Z} = (\mathbf{Z}_1, \ldots, \mathbf{Z}_E)$ at the period $\mathbf{t} = \mathbf{1}$ , the maintenance optimization problem can be formulated as a stochastic dynamic program (SDP) with decision parameters ($\mathbf{X}, \mathbf{Z}$ ). Moreover by including the type of the last performed maintenance denoted $\hat{\pi}$ , $\hat{\pi} = \pi_1, \ldots, \pi_k$ (to determine the type the transition probabilities) as a decision parameter, the SPD is equivalent to a Markov decision process (MDP) (Puterman 1994). Let $\pi^*$ denote the optimal action to perform in the state ($\mathbf{X}, \mathbf{Z}, \hat{\pi}$); $\pi(\mathbf{Z})$ the EUSLs during the performance of action $\pi$ ; $\hat{\pi}' = \hat{\pi}$ if $\pi_0$ is selected and $\hat{\pi}' = \pi$ otherwise; $\lambda$ is discount factor. We also introduce the following notations: $c_{action}(\pi, l) = c_F(\pi) + c_V(\pi, l)$; $c_{ind_{action}}(\pi(Z)) = c_{a,user}(\pi(Z)) + c_{a,safety}(\pi(Z)) + c_{a,env}(\pi(Z))$; $c_{ind_{cond}}(\pi, X, Z, X', Z') = c_{c,user}(X^{\pi}, Z^{\pi}, X', Z')$ + $c_{c,safety}(X^{\pi}, Z^{\pi}, X', Z') + c_{c,env}(X^{\pi}, Z^{\pi}, X', Z')$.
The MDP formulation is then given by the following:

$$X', Z', \hat{\pi}^{t+1} \qquad (1)$$

## 3. Value of monitoring

The maintenance optimization model described in the previous section is used as a tool to investigate different monitor systems and compare them in terms of costs and benefits. We propose to derive monitoring benefits by comparing the derived optimal strategies to age-based strategies where the decision is based only on the network age. Different monitoring systems can therefore be compared based on their value. Such a comparison can ultimately be useful in the monitoring system design phase to select monitoring parameters that are more relevant (in terms of total costs) to the network management.

### 3.1 Condition-based policies versus age-based policies

In order to show the value of monitoring the sub-network, we compare condition-based policies obtained with the model in Equation (1) to age-based maintenance policies. Age-based maintenance policies can be obtained using the same model in Equation (1) where the condition variables are replaced by the pavement age, i.e., the time interval since the last complete renewal. Eventually, all the relations such as transfer matrices and degradation matrices should be changed and estimated from time-based data only.

### 3.2 Condition-based policies: Quality of information

In a monitoring and management context, the main key issues are selecting the relevant parameters to monitor as well as decision rules updating. Moreover, one can also be interested in finding out (i) whether or not it is beneficial to invest in a time-continuous monitoring techniques or is the periodical visual inspection enough for an efficient network management, and (ii) whether or not more accurate monitoring systems will have more value. For this the following aspects are relevant: **Choice of variables and indicators:** can be achieved by comparing the net present values of different strategies obtained using different monitoring and condition variables. **Decision updating frequency:** BN updating can be used to detect "significant" change in the deterioration process (e.g., change in the deterioration speed). The MDP model can be run again to update the maintenance decision rule every time a critical change in the pavement behavior is detected. **Quality of information**: Some monitoring systems are less accurate than others. However the impact of higher accuracy on the management policies is not obvious. A sensitivity analysis can be done to assess the impact of data accuracy on the total expected costs of maintenance. Another way to evaluate the quality of information is to compare the strategies obtained by the MDP model and their associated costs to the strategies derived using a partially-observed Markov decision process where the deterministic observations are replaced by, possibly, a subjective probability distribution over all possible observations.

## 4. Numerical example

We consider the case of a sub-network that is periodically inspected to measure 2 parameters $\mathrm{O}_1$ and $\mathrm{O}_2$. Collected measurements are used to calculate current values of two condition indicators $\mathrm{X}_1$ and $\mathrm{X}_2$ representing cracking and raveling indicators, respectively. Values of $\mathrm{X}_1$ and $\mathrm{X}_2$ are then classified into 5 categories (*very good=1*, *good=2*, *fair=3,bad=4, very bad=5*). The performance of the sub-network is assessed through two EUSLs $\mathrm{Z}_1$ and $\mathrm{Z}_2$ that can be in one of 5 levels (*very good=1,…, very bad=5*).
In addition to the do-nothing option $\pi_0$, 4 actions $\pi_1, \dots, \pi_4$ are available to the decision-maker. $\pi_1$ is an operational action that has no impact on the condition indicators $\mathrm{X}_1$and $\mathrm{X}_2$ but only on the EUSLs $\mathrm{Z}_1$ and $\mathrm{Z}_2$. Actions $\pi_2, \dots, \pi_5$ are maintenance actions that have direct impacts on $\mathrm{X}_1$and $\mathrm{X}_2$, and consequently impacts on $\mathrm{Z}_1$ and $\mathrm{Z}_2$. More specifically, $\pi_2$ impacts $\mathrm{X}_1, Z_1$ and $Z_2$, $\pi_3$ impacts $\mathrm{X}_2$ and $Z_2$and $\pi_4$is the complete renewal action that reset the pavement to its best state. We consider a pavement of length 1000m. The joint transition probability matrices of $(\mathrm{X}_1, \mathrm{X}_2, \mathrm{Z}_1, \mathrm{Z}_2)$ (after different actions) are obtained and updated using the model described in Section 2.1. More specifically, these probabilities are calculated from the conditional distribution of $(D\mathrm{X}_1, \mathrm{DX}_2, \mathrm{DZ}_1, \mathrm{DZ}_2)$ given $(\mathrm{X}_1, \mathrm{X}_2, \mathrm{Z}_1, \mathrm{Z}_2)$ as illustrated in Figure 2. using the graphical model described in Section 2.1 and simulated data. In Figure 2, we only show one example of one action and one state $(\mathrm{X}_1 = 1, \mathrm{X}_2 = 2, \mathrm{Z}_1 = 1, \mathrm{Z}_2 = 2)$.

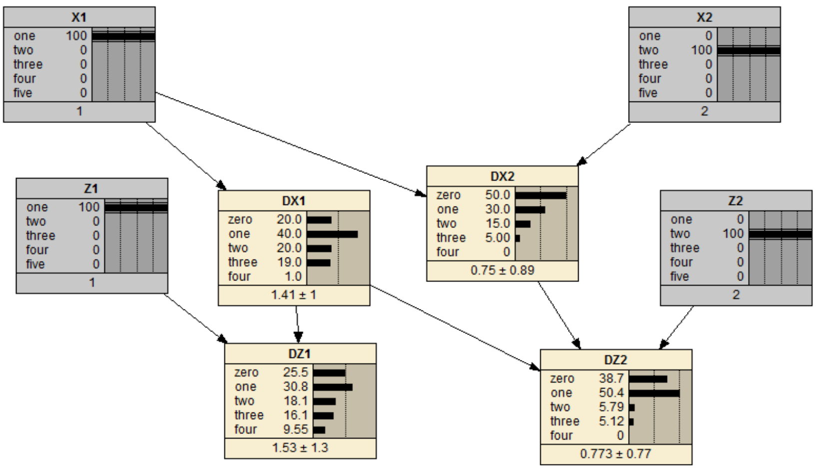


*Figure 2: Graphical model to predict the joint transition probabilities of the four variables* $(\boldsymbol{X_1}, \boldsymbol{X_2}, \boldsymbol{Z_1}, \boldsymbol{Z_2})$ .

Solving the MDP model given by Equation (1) gives the optimal decision rule and the associated costs (K euro). Given the large number of states, we only show the results for some states in Table 1 where detailed costs are presented. The total immediate cost is the expected cost for the current decision period and it is the sum of the maintenance decision direct costs and the incurred indirect costs including the expected penalty cost. The total cost is the expected cost of the decision on the infinite horizon which is the sum of the immediate expected action costs and its future costs. Because of space limitations, we only present the method to derive optimal strategies given monitoring inputs and do not detail here the calculation of costs and benefits for the cost-benefit analysis.

## 5. Conclusions and perspectives

In this paper we propose a framework for a reliability-based, societal cost benefit analysis to assess and compare different monitoring systems from a management perspective. The framework combines monitoring data, condition indicators and performance indicators and is based on two main components. The first one is a graphical model to predict both condition and performance indicators based on monitoring data. The second component is a Markov decision model that uses the prediction model to derive optimal maintenance decision rules that minimize the total expected discounted costs, i.e., agency, user, safety and environment costs. The total cost associated with a given monitoring system is then given by the sum of the monitoring cost and the expected cost of the derived maintenance strategy. The benefits are obtained by comparing the total expected cost of the derived maintenance strategy and the total expected cost of an age-based strategy. Although the graphical model is used to update the evolution process of the network indicators (and possibly detect critical changes in their evolution), it is a data-driven model and depends on priors that are either estimated from data analysis or by experts. Future works concern mainly (i) the use of a partially observed Markov decision process to derive the benefits of monitoring systems (instead of age-based strategies), and (ii) the application of the framework to a business case for results analysis (iii) the possible use of a dynamic BN for modelling the evolution of network indicators in time.

*Table 1: Optimal strategy and associated costs*

| $(X_1, X_2, Z_1, Z_2)$ | Optimal action | Total cost | Direct action cost | Action-related cost | Condition - related cost | Expected Penalty | Total immediate cost | Future cost |
|---|---|---|---|---|---|---|---|---|
| (1,1,1,1) | $\pi_0$ | 2649,70 | 0 | 0 | 39,85028 | 28,05 | 67,89 | 2581,81 |
| (2,1,2,3) | $\pi_0$ | 2676,46 | 0 | 0 | 53,05964 | 36,81 | 89,87 | 2567,71 |
| (2,2,5,4) | $\pi_1$ | 2753,11 | 65 | 12,708 | 58,57533 | 49,11 | 185,39 | 2587,60 |
| (4,3,4,4) | $\pi_2$ | 2799,75 | 114 | 21,776 | 38,40553 | 43,91 | 218,09 | 2478,47 |
| (3,4,4,1) | $\pi_2$ | 2680,38 | 114 | 5,444 | 38,09742 | 44,36 | 201,91 | 2581,65 |
| (5,5,1,4) | $\pi_3$ | 2843,29 | 181 | 10,388 | 22,18004 | 41,25 | 254,82 | 2588,47 |
| (5,5,1,4) | $\pi_3$ | 2843,29 | 181 | 10,388 | 22,18004 | 41,25 | 254,82 | 2575,09 |
| (5,5,2,2) | $\pi_4$ | 2899,46 | 268,5 | 15,128 | 9,834349 | 30,90 | 324,36 | 2575,09 |
| (5,5,4,1) | $\pi_4$ | 2899,46 | 268,5 | 15,128 | 9,834349 | 30,90 | 324,36 | 2575,09 |
| (5,5,5,5) | $\pi_4$ | 2978,88 | 268,5 | 94,55 | 9,834349 | 30,90 | 403,79 | 2575,09 |

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