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A FEM-Formulation for Virtual Reality Applications

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Virtual reality (VR), as a novel technology, represents one of the most powerful tools to assist or even play the major role in many areas, such as development of new designs, training medical practitioners or assembly operators, entertaining industry, etc. On the other hand, the finite element method (FEM) imposed itself as an essential technical support for the needs of computing flexible bodies' deformational behavior. FEM together with CAD are important ingredients of VR. In the VR applications that imply interactive simulations with flexible bodies included, the efficiency of FEM formulations is of crucial importance. The paper presents a co-rotational FEM-formulation developed to meet the needs of simulating geometrically nonlinear deformational behavior at interactive frame rates. It is presented here in combination with a rather simple linear tetrahedral element. The formulation is enriched with a coupled-mesh technique to enable the usage of rougher FEM models to compute deformational behavior of complex geometries. The advantages of an iterative solver and the solution procedure for both static and dynamic analyses are discussed.

Keywords

Virtual reality
Finite element method
Co-rotational FEM formulation
Tetrahedral element

MKE-formulacija za aplikacije virtualne stvarnosti

Izvornoznanstveni članak

Virtualna stvarnost (VR), kao nova tehnologija, predstavlja jedan od najmoćnijih alata koje podržavaju rad ili čak igraju glavnu ulogu u mnogim područjima, kao što su razvoj novih dizajna, trening liječnika ili montažera, industrija zabave, itd. S druge strane, metoda konačnih elemenata (MKE) se nametnula kao osnovna tehnička podrška za potrebe proračunavanja deformacijskog ponašanja elastičnih tijela. MKE je zajedno s CAD-om, važan dio VR-a. U VR aplikacijama koje podrazumijevaju interaktivnu simulaciju s elastičnim tijelima, efikasnost MKE formulacije je od presudne važnosti. Rad predstavlja korotacijsku MKE formulaciju razvijenu s ciljem simuliranja geometrijski nelinearnog ponašanja u interaktivnoj domeni. Formulacija je predstavljena u kombinaciji s vrlo jednostavnim linearnim elementom tipa tetraedra. Formulacija je proširena tehnikom spregnutih mreža kako bi se omogućilo korištenje grubljih MKE modela za određivanje deformacijskog ponašanja složenih geometrija. Razmotrene su prednosti iterativnog solvera kao i procedura rješavanja statičke i dinamičke analize.

Ključne riječi

Virtualna stvarnost
Metoda konačnih elemenata
Korotacijska FEM formulacija
Tetraedar element

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1. Introduction

In the last several decades, classical computer aided engineering (CAE) software packages have offered invaluable assistance to engineers. For years, they have represented the maximum in terms of performance and reliability in a variety of engineering tasks. The initial development of virtual reality (VR) technology aimed primarily at displaying models previously made in other CAE applications, thus facilitating the review process of developed design. However, the strong development of advanced visualization systems, followed by further development of already existing components (CAD, FEM, etc.) and necessary hardware, have enabled the integration of VR technology into the very core of engineering tasks. And furthermore, the possibility of application of VR concept in a number of other fields has also been recognized [1], thus being even integrated into daily life and activity.

As Franković [2] has noticed, a multidisciplinary approach is nowadays an imperative in many research areas. This is also the case with the field of VR, which involves mechanics, numerical computation, computer graphics – to name but a few.

With the VR concept, the user is offered the possibility of manipulating and analyzing a 3D virtual world, as if the objects were right in front of him. In certain applications, it suffices to incorporate rigid-body behavior in the VR environment. But in many other areas of application, VR requires simulation of deformable object's behavior at interactive frame rates, quite often involving large or moderately large deformations. The finite element method (FEM), as dominant one in the field of structural mechanics, is widely used to compute deformational behavior. But whereas typical engineering "off-line" computations pose accuracy as the primary simulation objective, it is often the efficiency of computation, i.e. the

Symbols/Oznake

$\mathbf{B_L}$	- element strain-displacement matrix - matrica deformacija-pomicanja elementa	$\{u_s\}$	- nodal displacements of entire FE model - pomaci u čvorovima cijelog FE modela
$\mathbf{C_s}$	- structural damping matrix - matrica strukturnog prigušenja	V_e	- volume of the element - volumen elementa
$\{F_e\}$	- element internal elastic forces - unutarnje elastične sile elementa	$\mathbf{X}$	- element matrix containing global nodal coordinates - matrica elementa koja sadrži globalne koordinate čvorova
$\{F_{ext}\}$	- structural external forces - vanjske strukturalne sile	$\{x_{0e}\}$	- initial element configuration vector - vektor početne konfiguracije elementa
$\{F_{int}\}$	- structural internal forces - unutarnje strukturalne sile	$\{x_e\}$	- current element configuration vector - vektor trenutne konfiguracije elementa
$\mathbf{H}$	- material Hooke's matrix - Hooke-ova matrica materijala	x, y, z	- global coordinate system's coordinates - koordinate globalnog koordinatnog sustava
$\mathbf{J}$	- element Jacobian matrix - Jakobijeva matrica elementa		**Greek letters/Grčka slova**
$\overline{\mathbf{J}}$	- sub-matrix of the Jacobian matrix - podmatrica Jacobijeve matrice	Δ	- increment of a quantity - prirast veličine
$\mathbf{K_e}$	- linear element stiffness matrix - linearna matrica krutosti elementa	ξ	- natural coordinate of finite element - prirodne koordinate konačnog elementa
$\mathbf{K_e^R}$	- rotated element stiffness matrix - rotirana matrica krutosti elementa		- **Subscripts/Indeksi**
$\mathbf{M_s}$	- structural mass matrix - strukturalna inercijalna matrica	1, 2, 3, 4	- element's local nodal numbers - lokalni brojevi čvorova elementa
N_L	- element shape functions - funkcije oblika elementa	e	- the quantity defined for an element - veličina definirana za jedan element
$\mathbf{R_e}$	- element rotational matrix - rotacijska matrica elementa	t	- moment in time at which the quantity is given - vremenski trenutak u kojem je definirana veličina

CPU time needed for the computation, that plays the major role in VR applications. Therefore, FEM-formulations that offer high efficiency and, furthermore, a possibility of trade-off between the accuracy and numerical effort are of great interest for VR applications.

Due to their simplicity, mass-spring systems, as a possible approach to meet the requirements of VR simulations, have been in the researchers' focus in the last couple of decades ([3], [4], [5]). But numerous issues that this approach raises as well as rapid hardware development led to computationally more demanding continuum-based FEM approaches. Zhuang [6] has applied FEM in combination with quadratic strain terms to account for geometric nonlinearities and graded mesh technique to reduce the complexity of models. Capell et al. [7] have used an approach that incorporates a division of objects into sub-domains, large local rigid-body rotation of which has been included in the formulation. Mueller at al. [8] have developed a similar approach with local rotations accounted for at nodal level. Etzmuss at al. [9] have improved this approach by including local rotations on element level and they have used it to model cloth by means of triangular 2D elements. In recent years, the solutions have also been sought in meshless FEM formulations [10], [11].

This paper describes a simplified geometrically nonlinear co-rotational FEM-formulation for 3D structures in combination with a linear tetrahedral element. An interested reader may find more about the rigorous co-rotational framework in [12]. Features of the present formulation meet the requirements of VR applications.

2. 4-node tetrahedral element and simplified co-rotational FEM formulation

Numerical efficiency of tetrahedral element with linear shape functions (4 nodes) combined with its high meshing ability were the decisive factors to choose this 3D finite element as a workhorse element within the first

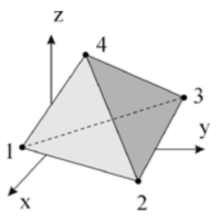

Figure 1. 4-node linear tetrahedral element

Slika 1. Linearni tetraedar element s 4 čvora

steps of developing and testing the proposed formulation. Fig. 1 depicts the element with local nodal numbers.

In order to use this element in combination with the simplified co-rotational formulation described later in this section, it is necessary to have its linear stiffness matrix. Although the development of the element is widely known in the FEM community, for the sake of completeness the main steps are to be given here. For more details, an interested reader may address to reference [13]. Derivation of the linear stiffness matrix is done with respect to the element's natural coordinates. The natural coordinates are defined so as to have the value 1 at the corresponding node and are equal to zero over the opposite surface of the tetrahedron, thus:

$$\xi_L \in [0,1] \quad L = 1,...,4 . \tag{1}$$

Of course, only three coordinates are independent from each other and the fourth is determined using the condition:

$$\xi_1 + \xi_2 + \xi_3 + \xi_4 = 1 . \tag{2}$$

And the shape functions are simply given as:

$$N_L = \xi_L \quad L = 1,...,4 , \tag{3}$$

thus already fulfilling the condition that the sum of all shape functions at any point within the element equals 1. The shape functions may be obtained using the relation between the Cartesian and local tetrahedral coordinates given below:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} = \mathbf{X} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} . \tag{4}$$

Expressing the local coordinates from Eq. (4), one also obtains the shape functions. Another simple approach leading to the same result can be used as well. Namely, any point within the tetrahedral element subdivides the element into 4 smaller tetrahedrons formed between the point and faces of the element. The local coordinates of the point are given by ratios between the volume of sub-tetrahedrons and the volume of tetrahedral element itself. And, as already emphasized by Eq. (3), this yields at the same time the shape functions.

The isoparametric approach is used, which implies that the same shape functions are used for description of the displacement field and for the geometry. Partial derivatives with respect to the global, Cartesian coordinates are related to partial derivatives with respect to the local, tetrahedral coordinates by means of Jacobian matrix:

$$\begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{bmatrix} = \overline{\mathbf{J}}^{\mathrm{T}} \begin{bmatrix} \dfrac{\partial}{\partial \xi_1} \\ \dfrac{\partial}{\partial \xi_2} \\ \dfrac{\partial}{\partial \xi_3} \end{bmatrix} . \tag{5}$$

Since the entries of matrix $\mathbf{X}$ are constant for an element, its inversion directly yields the Jacobian matrix, whose entries are also constant over the element's domain and depend only on the element's geometry:

$$\mathbf{J} = \mathbf{X}^{-1} = \left[\begin{array}{ccc:c} & & & J_{14} \\ & \overline{\mathbf{J}} & & J_{24} \\ & & & J_{34} \\ \hdashline J_{41} & J_{42} & J_{43} & J_{44} \end{array} \right] . \tag{6}$$

Since shape functions are defined in natural coordinates, Eqs. (5) and (6) are used to obtain the partial derivatives of shape functions with respect to the global coordinates. Finally, the element linear stiffness matrix is given in a quite general manner by the following integration:

$$\mathbf{K}_{\mathrm{e}} = \int_{V_{\mathrm{e}}} \mathbf{B}_{\mathrm{L}}^{\mathrm{T}} \, \mathbf{H} \, \mathbf{B}_{\mathrm{L}} \, dV_{\mathrm{e}} , \tag{7}$$

where the linear strain-displacement matrix, $\mathbf{B_L}$, reads:

$$\mathbf{B_L} = \begin{bmatrix} N_{1,x} & 0 & 0 & N_{2,x} & ... & 0 \\ 0 & N_{1,x} & 0 & 0 & ... & 0 \\ 0 & 0 & N_{1,x} & 0 & ... & N_{4,z} \\ N_{1,y} & N_{1,x} & 0 & N_{2,y} & ... & 0 \\ 0 & N_{1,z} & N_{1,y} & 0 & ... & N_{4,y} \\ N_{1,z} & 0 & N_{1,x} & N_{2,z} & ... & N_{4,x} \end{bmatrix} , \tag{8}$$

and partial derivatives of shape functions N_L with respect to the global coordinates in matrix $\mathbf{B_L}$ are determined as discussed above.

A few characteristics of the 4-node tetrahedral element are to be mentioned here. The linear shape functions imply that the element is capable of representing constant strain and stress fields at best. The entries of all above mentioned matrices are constant over the element's domain, which further means that the integration in Eq. (7) does not require multiple evaluations at different integration points. This simplicity of the element accounts for its advantage. On the other hand, the disadvantage is that the FEM models exploiting this finite element typically yield a stiffer behavior compared to actual structural behavior. The effect is even more pronounced when the element is used with incompressible materials due to its susceptibility to volumetric locking. Those are, however, mostly acceptable drawbacks from the point of VR simulations where a compromise between accuracy and numerical efficiency is a necessity with realistic physical behavior as the primary objective.

FEM formulations for VR simulation are supposed to be stable and computationally as inexpensive as possible. Linear FEM models are suitable regarding both requirements, but are not suitable for deformations involving large rotations. One of the ideas how to cope with this problem arises from the principles of incorporating flexible bodies in Multi-Body-System (MBS) dynamics. In MBS dynamics, the overall motion of flexible bodies is described as a superposition of large rigid-body motion and small deformation with respect to a body-fixed reference frame, which performs the same rigid-body motion as the body. The idea can be extended to FEM to yield a geometrically nonlinear co-rotational FEM formulation. Since such an approach decouples rigid-body motion from deformational motion, it allows the usage of engineering strain and stress measures in the formulation and, furthermore, decoupling geometrical from material nonlinearities. Theoretically, a co-rotational reference frame may be assigned to each material particle. In co-rotational FEM formulations, a local reference frame is typically defined at each element integration point as those points are used in the evaluation of element matrices and vectors. However, in the simplified co-rotational formulation, the essence of which is presented in this section, the idea is to account for rigid-body rotation on a somewhat rougher level. Actually, an average rigid-body rotation is determined for each finite element (Fig. 2) and further used in the computation. It should be noticed that the applied 4-node tetrahedral element already requires only one rotation per element, which is due to the aforementioned properties of the element.

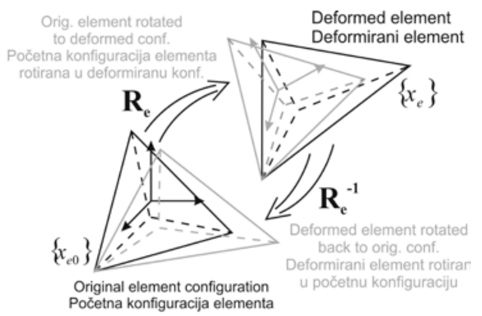

Figure 2. Decomposition of element's motion into rotation and deformation

Slika 2. Dekompozicija kretanja elementa na rotaciju i deformaciju

The spatial resolution of accounting for material rigid-body rotation is not the only simplification of this formulation compared to rigorous geometrically nonlinear FEM formulations. The more important simplification resides in the fact that the behavior of finite elements remains purely linear with respect to the co-rotational reference frame. This actually means that both the change in the element geometry over the course of deformation and the stress stiffening effects are neglected in this formulation. Hence, the only geometrically nonlinear effect taken into account is the averaged rigid-body rotation of single finite elements. This offers significant improvement in efficiency compared to rigorous formulations, a very stable simulation and an acceptable accuracy with the plausibility of deformational behavior as a minimum requirement being certainly met.

The very core of the formulation is given by the following equation which defines internal elastic forces on the element level:

$$\{F_e\} = \mathbf{R_e K_e}\left(\mathbf{R_e}^{-1}\{x_e\} - \{x_{0e}\}\right). \tag{9}$$

It would be worthwhile to give the interpretation of this equation. The first term in the parenthesis on the right-hand side of the equation yields the current element configuration rotated back to the original element orientation. Observing Fig. 2, that configuration is obtained when the deformed element is rotated backwards through $\mathbf{R_e^{-1}}$. Thus, the entire term in the parenthesis gives the rotation-free nodal displacements. Multiplied with the linear element stiffness matrix, they yield the element internal (elastic) forces with respect to the original element configuration. Finally, what remains is to rotate the internal forces to the current element configuration, which is achieved by multiplication with the element rotational matrix.

Very simple algebra transforms Eq. (9) into the following form:

$$\{F_e\} = \mathbf{R_e}\,\mathbf{K_e}\,\mathbf{R_e}^{-1}\{x_e\} - \mathbf{R_e}\,\mathbf{K_e}\,\{x_{0e}\}, \tag{10}$$

which reveals how the element stiffness matrix is obtained for the current configuration by simply using the element rotational matrix and the element linear stiffness matrix computed for the original configuration:

$$\mathbf{K_e^R} = \mathbf{R_e}\,\mathbf{K_e}\,\mathbf{R_e}^{-1}. \tag{11}$$

It should be now more obvious where the efficiency of this formulation resides. In a pre-step prior to interactive simulation, the linear stiffness matrix of each element is computed. During the simulation, the information about the current and initial element configurations is used to extract the rotational matrix describing the part of motion which is rigid-body rotation. The matrix is further used to obtain the rotated element stiffness matrix (Eq. (11)), re-assemble the current structural stiffness matrix and compute the internal elastic forces (Eq. (10)). With those quantities, the solver may proceed to solve the FE system of equations.

A rigorous geometrically nonlinear FEM approach would require update of each element stiffness matrix, which would not only take into account the change in the element configuration, but also the current stress state of the element (geometric stiffness matrix). Those operations are much more time-consuming compared to the above presented simplified approach that only demands extraction of the rotational part of motion and rotation of the pre-computed element stiffness matrix.

3. Solving the FE system of equations

Solving the FE system of equations is another key-element where significant time savings can be gained.

Direct solvers have been standard in commercially available FEM software packages for years. They are very robust and have gone through several optimization phases. For those reasons, direct solvers are often the users' first choice. In the last decade, however, iterative solvers have gained in importance and proven to be more than a decent alternative for direct solvers, in many cases even a primary choice.

Having in mind the demands of VR simulations, the authors have decided to use an iterative solution procedure – the preconditioned conjugate gradient method [14] for solving the FE system of equations. The basic idea of an iterative solver is that it starts out with an initial guess, i.e. initial solution vector, and goes through an iterative process to update the solution vector in every iteration using the system matrix and a pre-conditioner matrix to converge faster to the solution. A convergence criterion is used to determine whether the accuracy of the solution is acceptable or more iteration steps are needed to improve the accuracy.

The method benefits from the system matrix in the sparse form and is very convenient for real-time simulations. First of all, the preconditioned conjugate gradient method involves three matrix-vector products, three vector updates and four inner products per iteration. The number of operations is somewhat greater compared to the conjugate gradient method without preconditioning, but the "pre-conditioned version" is more effective provided the pre-conditioner is well chosen and reduces the system condition number, thus improving the convergence rate of the method enough to make up for the additional cost caused by the conditioning. An additional idea on how to get a well-conditioned FE system will be addressed later in the paper.

Furthermore, the method provides a very easy way of performing a trade-off between the solution accuracy and computational effort by limiting the number of performed iterations. Finally, the efficiency of the iterative solver can be noticeably improved by a reasonable choice of starting vector of the iterative process. This is particularly interesting in dynamics with the system of equations solved for nodal velocities. Namely, the nodal velocities typically do not change dramatically within a time-step, especially not in VR applications where low-frequency behavior is of primary interest. Hence, a good choice would be to take the velocities from the previous time-step as a starting vector for the iterative solver in the next time-step. This improves numerical efficiency of simulation as fewer iteration steps are needed to arrive to the solution.

Regarding dynamics, the question of solver type is actually closely related to the time integration scheme. Although the chosen type of solver already reveals a great deal of information on this subject, it will be addressed once again later in the paper.

4. Implementation for static cases, example

The presented FE-formulation is primarily developed for dynamics within VR simulations. However, it can also be used for efficient computations of static cases. This may also serve to get a very first impression about the level of accuracy offered by the formulation compared to rigorous geometrically nonlinear FEM formulations. Solution to the geometrically nonlinear FE equations in static cases requires that internal elastic forces balance the external forces:

$$\{F_{int}\} = \{F_{ext}\}. \tag{12}$$

In order to obtain the solution, the FE system of equations is linearized for the current structure configuration, so that:

$$\mathbf{K}_{\mathrm{st}}\,\{\Delta u_s\}_{\mathrm{t}} = \{F_{ext}\}_{\mathrm{t}} - \{F_{int}\}_{\mathrm{t}}\ . \tag{13}$$

The nonlinear problem is solved by means of Newton-Raphson, or modified Newton-Raphson iterative procedure. This iterative procedure should be distinguished from the iterative method used within the solver to get the solution of the linearized FE system of equations. As can be seen from Eq. (13), within the rigorous geometrically nonlinear FEM-formulations, the external loads are typically applied incrementally (step by step). Time is only an auxiliary variable to denote incrementally increasing external loads. Within each incremental step, the above mentioned Newton-Raphson iterative procedure is used to reach the solution that provides the balance between the external and internal forces. One of the reasons for incrementally applied external loads is the stability issue. Namely, if the full external load is applied at once, then the linearized system often results in an unacceptably large structural deformation and, upon the update of all quantities, the system of equations might not converge any more. The presented simplified formulation offers a better computational stability, thus allowing much larger increments and, in many cases, even the application of full load in one step. This is due to the fact that the behavior of single elements remains linear with respect to the co-rotational reference frame.

This means that, with the presented formulation, Eq. (13) can typically be solved with only one incremental step used (full load applied) by means of the Newton-Raphson iterative procedure, whereby the system stiffness matrix and internal forces are updated in each iterative step.

The below given example considers large deformation of a part of sliding door hinge made of steel. It is the part of a door hinge that runs over the guide rail fixed onto the car chassis. The main purpose of the example is to gain an insight about the achievable accuracy with the presented FE formulation. The FEM model (courtesy of Volkswagen AG) of the considered part is depicted in Fig. 3 from two perspectives. It consists of 3342 tetrahedral elements and 1288 nodes. The boundary conditions, which involve fixed nodes over the surface of the holes for the hinge pin and the applied external force, are chosen for the purpose of testing the formulation. It can be noticed that the chosen boundary conditions actually resemble the case when a car door opens quickly and hits against the bump stops. This is an interesting case from the point of view of car industry because strong competitiveness pushes the limits each and every day. It should be emphasized that such a case also involves materially nonlinear and even plastic behavior, which is however neglected within the framework of this paper.

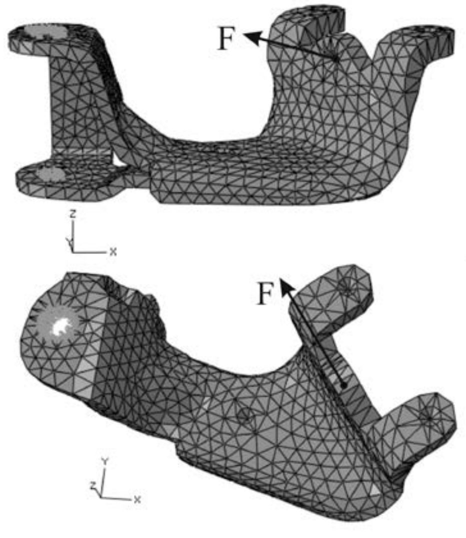

Figure 3. FEM model of the sliding door hinge

Slika 3. FEM model šarke kliznih vrata

The magnitude of the force is set so that large geometrically nonlinear deformations are induced. The amount of induced deformation can be visually explored in Fig. 4 from different perspectives.

The results for displacements of the point at which the force acts in all three global directions yielded by: 1) linear, 2) 'rigorous' geometrically nonlinear (result from ABAQUS) and 3) proposed formulation, are depicted in Fig. 5 against the external force. The results talk in favor of a very good agreement between the rigorous geometrically nonlinear analysis and the simplified co-rotational formulation.

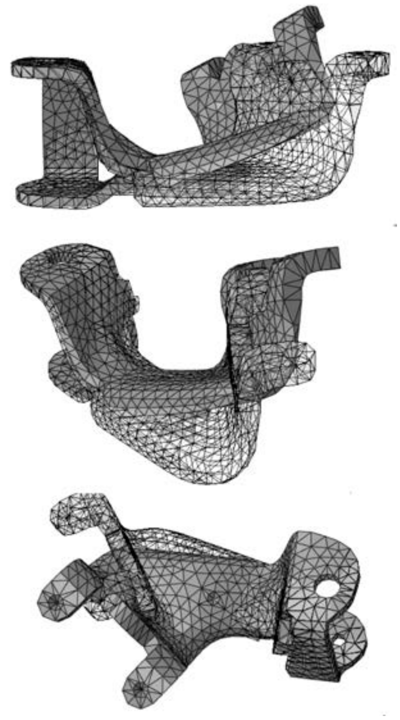

Figure 4. Deformed and initial configuration of the hinge

Slika 4. Deformirana i početna konfiguracija šarke

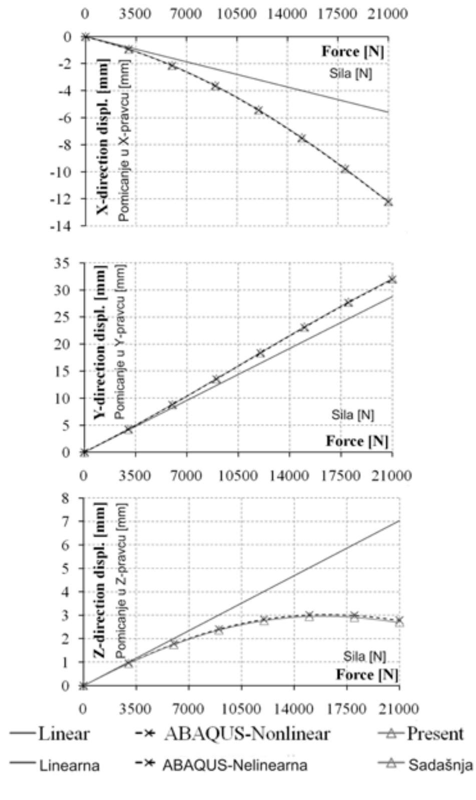

Figure 5. Displacements in global directions

Slika 5. Pomaci u globalnim smjerovima

5. Dynamics in VR applications, coupled-mesh technique and plausibility of deformational behavior

In geometrically nonlinear analysis, the dynamic FE equations are given in the following form:

$$\mathbf{M}_{\mathbf{s}}\left\{\ddot{u}_s\right\}_t+\mathbf{C}_{\mathbf{s}}\left\{\dot{u}_s\right\}_t=\left\{F_{ext}\right\}_t-\left\{F_{int}\right\}_t. \quad (14)$$

The very important question of time integration scheme arises at this point. This question has already been implicitly tackled in discussion about the solver. Time integration can be done based on an explicit or an implicit scheme. An explicit time integration procedure avoids the need to build complete structural matrices and to perform matrix inversions. The equations are decoupled, which is a big advantage of the approach, but the time-step size is severely limited by the stability requirements of the method and depends on the highest eigenfrequency of the system. Since a nonlinear system of equations is solved, this implies that the critical time-step also varies over the course of simulation. At this point, it should be emphasized that high-frequency behavior is usually (but not always) of less interest within VR applications. With soft materials an explicit time integration scheme could be advantageous.

However, the discussion about the applied solver type reveals that the authors actually keep the system of equations coupled. Hence, already after that discussion, it was apparent that the authors' choice is an implicit time-integration scheme. Since it is the human perception that plays the major role in VR applications, the low-frequency behavior is put in the focus. Regarding the computational effort for one time step, an implicit time integration scheme is indeed more expensive than an explicit time-integration scheme, but, on the other hand, it allows significantly larger time-steps. In this manner, it also effectively filters out high-frequency behavior. An implicit time integration scheme keeps the system of equations coupled and this is the reason why an efficient solver was needed, which can also perform a trade-off between the accuracy and numerical efficiency. The authors use Newmark time-integration scheme [14].

With the currently available average pc configurations, this formulation allows FE models of up to several thousand elements at interactive frame rates. But an FE mesh with such a number of elements usually cannot offer an appealing representation of surface when objects of rather complex geometry are modeled. On the other hand, it should be kept in mind that a plausible behavior suffices in most cases. This is where the idea of coupled mesh technique is brought into play. It consists in introducing two meshes – an FE mesh, which is used to calculate the deformation and a detailed triangulated surface mesh, which is used to represent the actual, complex geometry of the object. As a pre-step of interactive simulation, the two meshes are coupled to each other. For each vertex of the surface mesh a corresponding finite element is found and the surface vertex is assigned to this element. The criterion to find the corresponding element is based on the local coordinates of vertex with respect to the elements. Upon the computed deformation, the current nodal coordinates of the FEM model are used to re-compute the current coordinates of each surface vertex. The coupled-mesh technique is demonstrated in Fig. 6. The dog model (courtesy of Matthias Mueller, NVidia Switzerland), consists of 314 nodes and 700 tetrahedral elements, coupled to 12520 surface vertices, interconnected into 24835 triangular faces.

Figure 6. Coupled mesh technique

Slika 6. Tehnika spregnutih mreža

Fig. 7 gives snapshots from an interactive simulation in order to present achievable results. Both the complex geometry and the FEM model, coupled to each other, are shown. The figure depicts a plausible deformational behavior of the complex model.

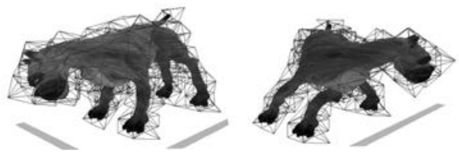

Figure 7. Plausible deformational behavior

Slika 7. Realistično deformacijsko ponašanje

The coupled-mesh technique does not only allow usage of rougher FE-models in combination with fine geometries, but it also has a positive influence on the efficiency of the solver. Namely, the FE mesh can be formed of elements, which are quite similar in size and shape. This has a positive influence on the system condition number, thus speeding up the iterative solver.

Another rather simple model is considered to tackle the aspect of plausibility of deformational behavior by visually comparing the results given by linear formulation and the presented co-rotational formulation. A rather soft beam is exposed to gravity load. The beam model consists of 8 cell-cubes stacked in the length direction of the beam, with each cell-cube discretized by 5 tetrahedral elements. The authors are aware of the fact that it is a rather rough model, but it suits the intended objective. A wire model of the beam is shown in Fig. 8. Fig. 8a gives the undeformed configuration of the beam, whereby the left end of the beam is clamped. Figs 8b

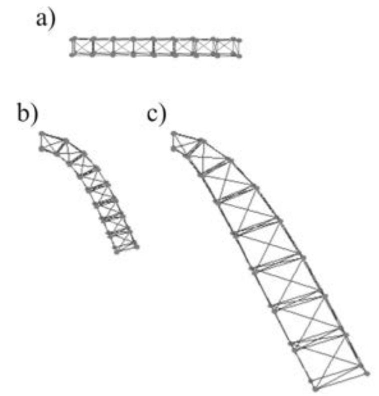

Figure 8. Beam model under gravity load: a) initial configuration, b) co-rotational formulation; c) linear formulation

Slika 8. Model grede pod utjecajem gravitacije: a) početna konfiguracija; b) ko-rotacijska formulacija; c) linearna formulacija

and 8c represent snapshots from an interactive simulation, where Fig. 8b gives the result by the co-rotational formulation, while the result in Fig. 8c is obtained by linear formulation. It should be noticed that elements toward the free end of the beam exhibit large rotations compared to the original configuration. As Fig. 8c reveals, the linear formulation cannot handle such large rotations successfully. An artificial enlargement of the structure volume is obvious. It should be emphasized that the deformed configurations (Figs. 8b and c) are given in the same scale. The easiest way to notice this is by comparing the depicted beam thickness at the clamped end of the beam (not influenced by deformation).

6. Conclusions

Virtual reality gains in importance in many areas of application. As a demanding multi-disciplinary field, it requires efficient innovative approaches to provide real-time or nearly real-time solutions. Mechanics of deformable objects is only one segment of it, but in many applications the very important one. The paper puts focus to the simplified co-rotational FE formulation, which offers high numerical efficiency, reasonable accuracy and good stability of simulation. Both static and dynamic cases have been considered in the paper. Static cases are of less interest for VR applications. Nevertheless, application of the proposed formulation to resolve geometrically nonlinear static deformations has been demonstrated with the primary objective to point out the achievable accuracy. Actually, the accuracy of the formulation strongly depends on the nature of deformation. If deformation is dominated by large local rotations, with relatively small strains, then the formulation represents a very decent alternative for rigorous geometrically nonlinear formulations. This was to be expected since the formulation accounts for arbitrarily large local rotations. The considered static case has confirmed it. However, the main advantages of the formulation and applied solver come to the fore in VR applications involving interactive dynamics. Dynamics at interactive frame rates with the presented FE formulation offers good plausibility of deformational behavior, which is the minimum requirement in many areas of VR application. This renders the formulation applicable for various VR simulators, such as laparoscopic surgery simulators.

In the future work, it is planed to extend the functionality of the developed software in order to include materially nonlinear deformational and plastic behavior as well as modeling material tearing and contact. Contact detection, as a first step in handling contact, has been already tackled by the authors [15]. A very important step in further work is extension of the formulation to include other finite element types, particularly those involving rotational degrees of freedom.

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