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The Teaching of Geometry under the new Syllabus.

A PAPER

READ BY

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The Teaching of Geometry under the New System.

Here at the beginning of the 20th century. we are witnessing the birth of the New Education. What is this New Education? The principal feature is new methods, new methods of teaching the old subjects, and there is also the addition of some new subjects, such as manual training and nature study. And why these new methods and these new subjects? Because the teacher has come to look at his pupil from a new point of view; he has a new ideal. In the old education the *amount* of knowledge was considered of primary importance; the *power* of getting it and consequently the *means* used were of secondary importance. In the new the order is reversed; the amount of knowledge gained is not the one thing needful; the master seeks to arouse the boy's *interest* in the knowledge he is acquiring; to make his knowledge real by making him handle and see and make and measure; to widen his interests so as to bridge the chasm between school life and his duties as a citizen; in short, to stimulate and direct his innate powers and energies. In the old the criterion of success was the extent to which the pupil's power of *receptivity* had become developed; the master was a setter and hearer of tasks: in the new the criterion of success is the growth of the pupil's power of *initiative*; the master is the educator and director of activity. Between the old teacher and his pupil the conflict raged, the former compelling, the latter resisting and moved forward only by an appeal to his sense of duty; between the new master and his pupil the struggle largely disappears and the fond hope is entertained that the strife is o'er, the battle won. So much in praise of the new. But let us not forget that in all ages there have been teachers, more indeed than we wot of, who have had no other ideal than this, and who have discovered the new methods for themselves. To-day although there are still some apologists for the old, most teachers are inquiring for the new way. The Reform has come from within, from the ranks of teachers themselves; and the reformer has laid his hand on every subject in the school curriculum and on some not in it and has proclaimed the methods by which the end is to be reached. So we have the new Mathematics. What is it? Not new truths surely, for here at any rate what is new is not true. But the teacher is allowed a wider discretion in the choice of methods; further he is told to co-ordinate the branches of his subject and not keep them in water-tight compartments labelled "Arithmetic" "Geometry" etc. but to use whatever is most servicable in any part of his mathematical equipment; for example, wherever praticable, to use Geometry to illustrate Algebra and Arithmetic, and so on; and further he is asked to make large use of the concrete, to preface the study of abstract geometry by an experimental course and in all cases to make an extensive use of graphs to visualize results; finally, as far as possible to see that the rules which the pupil uses are generalisations from his own experience. It may not be amiss to note here that Mr. Hall, while frankly admitting the value of graphs as a corrective, as a means of ready reckoning, and as throwing light on and giving a meaning to certain solutions, sounds a warning note against their excessive use and bids the teacher beware of graphomania.

To come now more particularly to the subject of the New Geometry. In 1902 the Mathematical Association, at the instance of the British Association, appointed a committee to make recommendations with regard to the teaching of Elementary Mathematics. The Committee was representative of all the great Schools of England. We find the names of Godfrey, Siddons (who was Secretary), Barnard and Baker, all of whom have written books on Geometry. In view of the report of this Committee the Cambridge University published a detailed syllabus of work required in Geometry. This syllabus the University of Melbourne has adopted in toto. Roughly, it demands:—

(*a*) A preliminary course of simple practical work with instruments, and

(*b*) The knowledge of certain problems and theorems, the enunciations of which are set forth, and the application of them to riders and deductions.

For the primary examination these cover Euclid I. 1-34, but the number of propositions is reduced to 26, or about three-fourths of Euclid's number. For the Junior Examinations, extending into Euclid's IV th book, we have 67 propositions against Euclid's 98, a reduction to two-thirds. The Senior Examinations go on to Euclid XI. 21, but with large omissions from Book VI. and the removal of incommensurables from elementary geometry. The first Primary Examination Paper appeared in June of this year. It is on very good lines, but appears to be pitched in too high a key for children of 13, while the length of it, for two hours' work, and in face of the announcement that hand-writing, etc., would be taken into account, is excessive. The first Junior paper will appear in December, 1906, and although the Mathematical Committee of the Public Examinations Board in vain did their utmost to have specimen papers prepared and circulated, I hazard the conjecture that the new examination will not exhibit any violent change from the old. There will be some practical exercises, but we shall have the well known theorems and problems, while the unfamiliar ones, little known because seldom if ever used, will have disappeared. *Riders will doubtless form a very important part of the paper.*

Let me offer a few remarks on each of the parts of the Geometry syllabus, and first let us take the course of experimental geometry. Here the pupil is taught to draw with the greatest possible accuracy angles, triangles, quadrilaterals, circles and various lines connected with them, bisectors, perpendiculars etc. from given data, and so he gains a clear definite idea of the terminology of geometry. By the way, it is amazing how inaccurately much of this mechanical work is done by boys. Further he is taught to observe and to generalise results ; he finds that the three angles of every triangle he draws make two right angles, and so he reaches out to geometrical truth. This is all that is demanded, but the teacher will seek to elicit some reason or proof, however imperfect and distinct from the severe logical methods of deductive geometry. For instance a simple application of the idea of an angle as the "amount of rotation" will satisfy the pupil as to the sum of the three angles of a triangle. So too a simple reason for many exercises is found by folding the paper along the line of symmetry. Boys are very much attracted by this experimental course, which goes a long way to overcome their idea of geometry as deadly dull and dry. But it is necessary to sound a warning, not to spend too much time here : most books greatly overdo the business. Bright lads who begin their secondary course at the age of 14 can spend a term or even a term and a half at it with undoubted profit ; the ordinary boy may begin geometry a year earlier than he has hitherto done ; say at 12, and take a lesson a week throughout the year. He will look forward with keen pleasure to Geometry day. If the School has a drawing master, who is not merely an artist, the two subjects, Drawing and Geometry, may be co-ordinated, but this will rarely be found a happy arrangement. The work of correcting and marking a set of papers will be greatly facilitated if the master draw the figure accurately for himself on tracing paper ; by placing it over the pupil's work, instead of applying the instruments to each, he will at once detect the inaccuracies.

Come now to the course of theorems and problems prescribed. In most text books the theorems appear in one group, numbered in their order, not Euclid's it need hardly be said, and the problems in another part, sometimes numbered, sometimes unnumbered. This separation, making the course of theorems independent of the methods of construction, is rendered possible by the liberty now granted to use hypothetical constructions. For example, in a theorem, a bisector may be used *before* the student has reached the theorem necessary for proving the soundness of the construction for the bisection of an angle. The pons asinorum may be solved by bisecting the vertical angle and discovering two congruent triangles. A revolution truly, and one that the Euclidean vehemently protests against. But after all it is quite rational ; it is as if one were to say—there is such a thing as a bisector ; if I imagine it or draw something to represent it bisecting, say, the vertical angle of this isosceles triangle I have straightway two triangles with two sides and the contained angle equal, and the required result follows. The Mathematical Committee, mentioned above, recommends that the prescribed theorems and problems should be studied *pari passu*, as two parallel courses, and not all the theorems first and then all the problems. As soon as the pupil has mastered the congruence of angles with three sides given, he will tackle the problem "to bisect a given angle" and be able to satisfy himself of the validity of his construction.

A word or two, more in detail, about the theorems. Our syllabus gives them in the following order (I shall use Euclid's numbers for reference) 13, 14, 15, 27, 28, 29, 30, 32, 4, 26, 5, 6, 8, etc. Noteworthy omissions are 7, 16, 17, 20, 21, 24, 25, 33, etc. But it is distinctly provided that the order in which the theorems are here mentioned is not *imposed* as the sequence of their treatment but that any proof which appears to the examiners to form part of a systematic treatment of the subject will be accepted. Latitude enough surely ! Euclid himself has still a place. However there is a concensus of opinion that the Cambridge order is the best yet suggested, and I should strongly recommend the teacher to select as his text-book of Geometry one built on this plan and to stick to it. Another point is this—In the multitude of text-books with theorems in varying order, what reasons are to be given for the different steps of the proof ? The old landmarks Euc. 1.8 1.32 etc. have disappeared. Unless proof is to become vague and uncertain some reason must be supplied for the various steps and there are only two ways open, to refer to the theorem that substantiates the step either by the *number* given to it in the text-book used or by *summarising its enunciation.* The latter, although clumsy, is really the more satisfying and will frequently be resorted to in teaching: angle A = angle B (being alternate); $\triangle$ ABC $\equiv$ $\triangle$ DEF (two sides and contained angle). The former is shorter, and emphasises the place of the prop. in the system adopted, and will, I expect, be much used in written work. Pity the examiner! On the whole, I think that reference to a prop. by its number alone should now be regarded as insufficient.

The Problems do not call for much further remark. They are regarded as a practical course, but the syllabus states that in cases where the validity of a construction is not obvious the reasoning by which it is justified may be required. Clearly it is necessary to be armed with a theoretical proof of all the problems enumerated and that is provided for in all the text-books I have seen. There need not be any confusion between the mode of treating an experimental exercise and a problem so called. The experimental exercise is a particular case: "Draw an angle of 92°, and show the construction for bisecting it." Here no proof is required; marks are given for the accuracy of the figure as tested by measurement, and for the sufficiency of the written explanation, if any. The problem so called is the general case:—"Bisect a given angle:" Marks are given for the general accuracy of the figure and for the theoretical sufficiency of the construction and of the proof.

A word about text-books. There are several points to consider. First and foremost, how far does the book coincide with the syllabus? Is there a preliminary course of practical work? Are the riders representative and sufficient in number and variety? In the next place, what is the mode of treatment? What definitions are given of an angle, of parallel straight lines, of a tangent etc.? Are they the old or the new? Do we prefer the old or the new? In passing, I cannot help feeling that it is a pity that so many of the books give the newer definition of an angle as "rotation" in place of the older one of "inclination," but make little or no use of the newer idea when they deal with such a theorem as Euclid I, 13 or 15. Then, too, there is the question of cost and whether the book is published in parts.

I have looked through five of the new books, those by Barnard and Child, Godfrey and Siddons, Hall and Stevens, Baker and Bourne, and Alcock. The last two are on conservative lines, while Hall and Stevens's is a compromise between the old and the new, and none of the three follows the Cambridge order. The two first named do follow this order. Let me repeat, however, that the *order* is not an essential point of the Cambridge syllabus. Still, I think Godfrey and Siddons's text-book will be found the most suitable for the Primary and Junior exams. while the scholarly and thorough treatment of the subject by Barnard and Child will best meet the requirements of the Senior exams. The Teacher will receive many valuable hints from the perodical "School World" published by Messrs. Macmillan, or from "School" published by John Murray, and if he wishes to keep abreast of fresh developments in secondary education he should subscribe to one or other of them.

In conclusion, while many are still disposed to think that the boy with mathematical ability will receive a sound geometrical training more rapidly by continuing to study Euclid's elements, there can be little doubt that the average boy, to whom Euclid is often difficult and uncongenial and who seldom sees the wood for trees, will find the new way more attractive, more easy and more valuable, while the boy who never could or never would see anything in Euclid will gain something of worth in the practical exercise and even he may assimilate some geometrical truth.

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