

JOINT PRODUCTION AND LABOUR VALUES

E. ZALAI

This paper investigates the problem of labour value determination in the presence of joint production. The purpose is to clarify the main problems involved, and critically review some recent proposals. In the case of joint production the inputs can not be divided between products in a natural way. As a result, none of the three alternative definitions given by *Marx* for labour values can generally be applied in this case. This means that the definition of labour values has to be generalized. The author points out that any such generalization will contradict some original Marxian proposals; therefore, there is no Marxian solution to the problem. One could still attempt to find a generalization Marxian enough in its spirit. The author lists some criteria that such a generalization could be tested against. He points out that the solution suggested by *Morishima* fails to meet most of the above criteria. It will be argued that a solution which relies on a price dependent division of inputs between products is closer to the general spirit of Marxian analysis.

The renewed interest in Marx's economics has contributed a lot to a better understanding of the content and scope of his economic concepts. Formal models have proved to be especially instrumental in this process, as exemplified by the pioneering works of *Bródy* [1] and *Morishima* [2]. There remained, however, quite a few problems not satisfactorily solved. In connection with the labour values heterogeneous labour and joint production have traditionally been sorted out as two major problems posing serious challenge to their general conceptual validity. In a related paper, I have addressed the first issue (*Zalai* [3]), in this paper I will deal with the second.

Joint production is a common phenomenon in our modern times, for in most production processes there are some inputs which can not be directly divided between the produced goods. The theoretical importance of joint production is also underlined by the fact that the proper treatment of durable capital goods itself leads to a model of joint production. This point was already emphasized by *Marx* but it was not until the works of *von Neumann* [4] and *Sraffa* [5] that this issue started to be seriously studied. It can thus rightly be expected that a value or price theory can accommodate the case of joint production as well.

Marx himself paid little attention to the problem of joint production. The alternative definitions of labour values given by him only apply directly to such a situation, in which the commodity and labour inputs necessary for the production of various commodities are separately given for every single commodity. Later analysis revealed that these definitions would not, in general, give meaningful results, if joint production is present. Thus, in short, there is no Marxian solution to the problem.

Dealing with the problem of value determination in the case of joint production Bródy [6] left the question basically open and only noted the possible arbitrariness in assigning values to joint products. Láng [7] took this issue further and proposed to consider labour values as a set rather than a single point. He defined this set by all the solutions one could get from an input-output model based on arbitrary divisions of joint inputs between products. Morishima [2] and later Morishima and Catephores [8] proposed to generalize the definition of labour values in a linear programming framework, partly in response to Steedman [9], who produced negative "Marxian" labour values as an evidence of their fallacy.

In this paper we will first review Marx's alternative definitions of labour values and point out the related problems caused by joint production. Then, the various proposals, especially those of Morishima, will be critically investigated. It will be argued that his suggested solutions do not meet some important Marxian criteria and are, therefore, not acceptable. Finally, a possible 'second best' solution will be discussed.

Joint production: a test of Marx's triple definition of values

Three alternative definitions of labour values can be distilled from Marx's *Capital*: labour content, labour required for the final (net) output and total (direct and indirect) labour input. These three concepts, which also appear in input-output analysis, are known to produce the same result in the case of single production. In the case of joint production, however, they do not always provide operational concepts. It will be also interesting to see how different these concepts in fact are.

Let us define an output (Z) and input (Q) matrix containing m rows and n columns each, where the rows refer to commodities and the columns to production processes, respectively. Similarly, let a row vector $w = (w_j), j = 1, 2, \dots, m$ contain the homogeneous labour inputs and denote labour values by vector $p = (p_i), i = 1, 2, \dots, n$.

The *first*, the labour content definition of value given by Marx rests upon his assumption that if a commodity is produced under socially average conditions, then the labour entering into its production is conserved by the value of the product. Suppose that every process considered operates under socially average conditions—an assumption to which we will come back later. To find these labour conserving values then means to solve the following system of equations (labour inflow = labour outflow):

$$pQ + w = pZ, \quad (1)$$

where, of course, we expect p to be positive (or at least nonnegative).

The *second* definition is based on Marx's assumption that in a given period the value of the net product of a society is equal to the amount of (live) labour that created

it. Let us assume that the input and output coefficients remain unaffected by the change in the general intensity (level) of production in every process. Based on this assumption the value of any commodity could be determined as the labour required to produce one unit of final (net) output of the given commodity. Let us denote by vector t_i the level of the various processes, at which one unit net output of commodity i will be produced. Thus, the value of the i^{th} commodity is $p_i = \mathbf{w}t_i$, where

$$\mathbf{Z}t_i - \mathbf{Q}t_i = \mathbf{e}_i, \quad (2)$$

and \mathbf{e}_i stands for the i^{th} unit vector. Again, the solution of (2) is expected to be nonnegative.

One can immediately see that neither of the two equation systems (1) and (2) will always have a solution, and if it does this may have negative elements, and the solution may not be unique. To guaranty in general the solvability of (1) one should assume that the rank of the matrix $(\mathbf{Z} - \mathbf{Q})$ is equal to n (i.e. $m \geq n$), whereas in the second case its rank should be equal to m (i.e. $m \leq n$). Thus, except for the specific case when $(\mathbf{Z} - \mathbf{Q})$ is a quadratic, nonsingular matrix, one can not expect in general that the two solutions will coincide. Even in such a case, the solution may be meaningless from the economic point of view, because some of its elements may be negative.

It is also clear that the *third* definition, which is based on the phase-by-phase calculation of the labour input, can not be used if there are joint products, since one can not tell how much labour was used for the production of the single commodities.

Thus, none of the three definitions given by Marx proves to be universally and meaningfully applicable to the case of joint production. Based on this observation one can rightly conclude that there is no Marxian solution to the problem of labour values in the case of joint production. In what follows we will turn our attention to various interpretations of this conclusion.

Steedmans example: negative labour values?

Steedman [9], challenging the basic Marxian thesis that the source of profit is the surplus value, produced a simple numerical example, which contained two processes and two commodities. The data in his example were as follows:

$$\mathbf{Z} = \begin{pmatrix} 30 & 3 \\ 5 & 12 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 25 & 0 \\ 0 & 10 \end{pmatrix} \quad \mathbf{w} = (5 \quad 1)$$

To determine the labour values he mechanically applied the above discussed Marxian definitions (it could be interpreted as either of the first two, since in this case they coincide). As a result he got $p_1 = -1$ and $p_2 = 2$. On the basis of these "labour values" he then showed that the surplus value was negative.

Morishima [10] and Wolfstetter [11] pointed out that his claim that these numbers are the labour values is unfounded, because he applied Marx's definition to a

case different from what Marx considered. From our previous discussion it should also be clear, that for the joint production case one has to use a generalized notion of labour values, in order to get an operational concept. Wolfstetter has also pointed out that the negative value in Steedman's example may be attributed to the fact that the first process is not efficient in the net sense, i.e. the second process yields larger net output using the same amount of labour. (Applying one unit of labour the net output of the second process is 3 and 2, whereas that of the first is only 1 and 1 from the two commodities, respectively.)

One can easily generalize Wolfstetter's point for the case of any invertible $(Z - Q)$ matrix. Let us assume that the "labour value" of some commodity, say the i^{th} , is negative, i.e. $p_i = w(Z - Q)^{-1}e_i < 0$. Define $x = (Z - Q)^{-1}e_i$ and $x_1, x_2 \geq 0$ such that $x_1 - x_2 = x$. Clearly, we have $w x_1 < w x_2$ and $(Z - Q)x_1 \geq (Z - Q)x_2$, which means that the composite process of x_1 is more efficient than that of x_2 . Conversely, let us assume that there is an inefficient collection of processes in the technology, i.e. we may find $x_1, x_2 \geq 0$, such that $(Z - Q)x_1 \geq (Z - Q)x_2$ and $w x_1 < w x_2$. From this it follows that $w(x_1 - x_2) = w(Z - Q)^{-1}(Z - Q)(x_1 - x_2) < 0$. Because $(Z - Q)(x_1 - x_2) \geq 0$, $w(Z - Q)^{-1}$ must contain at least one negative element.

There is an important consequence of the above phenomenon in the context of labour values. We have emphasized that the labour content definition is based on the assumption that the various processes operate under socially average conditions. This assumption, however, cannot be maintained if a process (or a group of processes) is clearly less efficient than another one. In such a situation, according to Marx, negative and positive extra-surpluses emerge in the various processes, thus one could not use equation (1), which does not contain such extra-surpluses. Steedman's example, therefore, violates the Marxian concept of labour values in this respect, too.

Based on this observation one may even wonder whether or not Steedman's example can picture a market economy in equilibrium at all. The answer to this question is affirmative. Based on the Sraffian concept of producer's prices Steedman showed that his example can be regarded as an equilibrium state of an economy. Suppose, workers get 3 and 5 units out of the 8 and 7 units net output of the two commodities, respectively. Then $p_1 = 1/3, p_2 = 1$ prices, $p_0 = 1$ wage rate and $r = 0.2$ profit rate satisfy the following equilibrium price condition:

$$pZ = (1 + r)pQ + p_0w$$

and wages are equal to the cost of consumption. Thus the question remains how one could define labour values in a more meaningful way for such a case.

Morishima: optimal and true values

Morishima followed the second definition of labour values, that is, the one based on the labour requirement of net output. In accordance with this definition the labour value of any bundle of commodities (\mathbf{b}) is determined in two steps. First, such activity levels have to be determined which give rise to a net output vector \mathbf{b} . As indicated earlier, this problem may not have meaningful (nonnegative) solution and/or it may have several solutions. Thus, in this form, it will not provide a universal method for the determination of labour values.

Morishima, therefore, proposed to reformulate the above problem into an inequality system in the fashion common in modern mathematical economics, i.e. allowing for excess supply, as follows

$$(\mathbf{Z} - \mathbf{Q})\mathbf{t} \geq \mathbf{b}. \quad (3)$$

If the production system examined is productive, i.e. there exists a $\mathbf{t}^* \geq \mathbf{0}$ such that $(\mathbf{Z} - \mathbf{Q})\mathbf{t}^* > \mathbf{0}$, then the above inequality system will always have nonnegative solutions. The only problem that remains to be solved, according to Morishima, is to select an appropriate solution, because the inequality system (3) will, as a rule, have many feasible solutions. He proposes to choose such a solution that minimizes the amount of labour. This minimal labour required to produce at least as much net output as in vector \mathbf{b} is what Morishima calls the *true* labour value of the commodity bundle \mathbf{b} . Let us denote it by $p(\mathbf{b})$, which is thus defined as follows:

$$p(\mathbf{b}) = \min \{ \mathbf{w}\mathbf{t} : (\mathbf{Z} - \mathbf{Q})\mathbf{t} \geq \mathbf{b}, \mathbf{t} \geq \mathbf{0} \} \quad (4)$$

In his earlier work [2] he proposed to consider the dual solution of problem (4), that he called *optimal* labour values, as the generalization of labour values. The optimal value of a commodity bundle is equal to its true value. However, the optimal values are not always uniquely determined, unlike the true values. This is why he switched to the latter concept. Morishima viewed thus his true labour values as the proper generalization of the Marxian labour values for the case of joint production. To justify his claim he referred to a passage in the "Poverty of philosophy", which reads as follows: "It is important to emphasize the point that what determines value is not the time taken to produce a thing but the minimum time it could possibly be produced, and this minimum is ascertained by competition." (Marx [12], p. 66.)

It should be emphasized that the context of the above quotation suggests that Marx was concerned with market values, that is, with the centre of prices rather than with values defined as socially average labour contents. This double interpretation of values, i.e. natural center of prices *versus* average labour content, is a source of confusion and debates among Marxist economists, since these two concepts seem to be qualitatively different. Nevertheless, in this paper we will stick to the second interpretation of values and show that Morishima's true values might fit in with the above concept of market values but not with that of average labour content which appears in most places of *Capital*.

In the next section we will define some criteria which we believe this latter concept of labour values should fulfill. To prepare the ground for the assessment of Morishima's true values it should be noted that they are not additive, that is, in general

$$p(\mathbf{b}) \neq \sum p(\mathbf{e}_i)b_i.$$

Also, if \mathbf{y} is the observed net output, the true value of \mathbf{y} is in general smaller than the labour actually used for its production. Thus, for example, even if all the net output is consumed by the workers, Morishima's true values might indicate exploitation (surplus labour). On the other hand, it can occur that with the true values one is not able to identify surplus labour even if there is capitalist consumption. These features seem to contradict the Marxian analysis.

Morishima's true values are based on a marginal rather than average concept of valuation. They can in fact be interpreted as marginal values only if the technology exhibits constant returns to scale. Such an assumption, however, is not needed in order to determine labour values in the case of single production. It is easy to show that one does not even need to know the input coefficients. In order to determine labour values one can solve an equation system based on the actual amount of inputs and outputs. Let us denote the output vector by \mathbf{q} in this latter case. The labour values are simply defined by the following equation:

$$p\langle \mathbf{q} \rangle = p\mathbf{Q} + w$$

where $\langle \cdot \rangle$ denoted a diagonal matrix.

To see the essential difference between Marx's and Morishima's labour value concept clearly let us consider an economy with alternative, but *single* product technologies. The same commodity is thus produced with different input requirements. In such a situation one can easily tell (*ex post*) what the socially average input requirement of various products was in a given period. Thus, the determination of labour values can be reduced to the familiar input-output scheme. Once the values have been determined one can evaluate the different processes. The individual processes will, as a rule, exhibit positive and negative extra-surpluses. It is, in fact, these extra-surpluses that allow us to judge which processes are operated under better or worse conditions than the social average.

We think that in the above situation the outlined approach should be followed in the spirit of Marx's related analysis. Following Morishima's definition of true labour values one would arrive at a completely different result. His true values should be determined by solving a linear programming model based on a von Neumann-Leontief technology, which is a problem familiar from the "nonsubstitution theorem" (see, for example, Gale [13]). In that solution one would only find negative extra-surplus, i.e. only the most labour efficient processes would 'conserve' the labour input. This is in sharp contrast with the Marxian average concept outlined above.

Criteria for testing the generalizations

There should be no doubt: both the optimal and true labour values of Morishima generalize the Marxian values in the general sense. If joint production and technological alternatives are absent, both of them result in the same valuation system as the Marxian labour values. This is a necessary but not sufficient condition that a generalization should meet. The crux of the matter is that we are dealing here with a situation in which we want to find not only a more general labour value concept, but—to paraphrase Morishima—a “true Marxian” labour value concept.

Since Marx did not give a general definition and he is not alive any more, this problem will remain unsolvable or a matter of subjective judgement. Nevertheless, one might still try to set up a few fundamental criteria, based on the original Marxian concept and surrounding analysis, against which various generalizations could be tested. A partial list of criteria is provided below as an example.

1. The values are uniquely determined, once the socially necessary input and output data are known (*unique existence*).

2. The values are nonnegative, but positive if labour is indispensable for the production of the given commodity (*positivity*).

3. The joint value of a bundle of commodities is the quantity-weighted sum of the unit values (*additivity*).

4. The total value of outputs is equal to the sum of the value of commodity inputs plus live labour input, if and only if a production process is operated under socially average conditions (*average property*).

5. If some processes do not operate under socially average conditions, then both positive and negative extra-surpluses arise (*symmetry of average property*).

6. The value of the actual net output of the economy is equal to the amount of labour actually used (*net product identity*).

7. If there is (no) surplus product, the surplus value is positive (zero) (*surplus identity*)*.

8. The values are independent of prices (*price independence*).

The readers of Marx's *Capital* will most probably agree on that the above principles are fundamental characteristics, almost axioms of the Marxian value concept. It is therefore not necessary to comment on them in more detail here. One could probably add a few more items to the list or question the reasonability of some criteria listed. But this is not the real issue here. Our aim was to collect those Marxian statements, the validity of which have been questioned in one or another way in the foregoing analysis. The only exception to this rule is the criterion of price independence, which will only later enter our discussion.

* The surplus product is a vectoral magnitude. If there are both negative and positive elements in it, one can not tell whether or not surplus is produced. This is probably why the concepts of surplus value and surplus labour were introduced by the classical economists.

Let us now briefly summarize Steedman's and Morishima's understanding of labour values in the light of the above criteria. Steedman, as we have seen it, extended mechanically Marx's original definition to the case of joint production. This approach would result in a concept that would violate several of the above criteria. Neither the existence, uniqueness nor the nonnegativity of values is guaranteed by such a concept. It would not bear the average property either, as we have already shown (see the basis of negative values). The surplus identity would not be fulfilled either. Steedman himself has shown with his example that there may be negative surplus value associated with positive surplus product. If the values can be determined at all, they will thus only satisfy the additivity, net product identity and the price independence criteria.

Unfortunately, Morishima's concept does not score much better at all in the above test either, as we have already seen. Although the true values will always be uniquely determined and nonnegative, but only for a bundle of commodities (not additively). They do not fulfill, in general, the average requirements, the net product and surplus identities. Apart from these, Morishima's concept rests upon the assumption of constant input and output coefficients, and on the choice of techniques which is rather alien to the Marxian analysis, too. It is much closer to a neoclassical price theory and to the notion of opportunity cost. It can certainly not explain the source of the social net product (the actual labour input—according to Marx), which is distributed among the classes of capitalist society through rents, profits and wages.

What other solution is left?

On the above ground we may thus conclude that the challenge posed by joint production to the Marxian labour value concept, articulated forcefully by Steedman, was not successfully met by Morishima's otherwise interesting generalized notion. It seems to be almost certain, too, that any revision or extension of Marx's original definition will lead to conclusions, which will not be in full conformity with those of Marx. The question is therefore how one could find such a solution that is reasonably close to the spirit of the Marxian analysis and way of reasoning.

This is admittedly a matter of taste as well, because it rests upon subjective judgement. At present I can not think of any better solution than to resort to Marx's original definition. We have emphasized at the beginning that Marx's original definitions can only be used if the commodity and labour inputs are divided among the produced commodities. This division takes place in real life through the cost and price calculations according to socially agreed principles. One could, therefore, rely on this social mechanism in defining labour values, in a similar vein as Marx included the social, historical elements into the determination of the necessary consumption of labour.*

* This solution was first proposed by the author in an unpublished dissertation in 1980 (Zalai [14]). *Flaschel* [15] has independently also argued in favour of such a solution as against Morishima's true values.

Such a solution could be objected to on the basis that it brings in the prices into the definition of values, i.e. values are no more price-independent. But how could one deny at all that prices indirectly influence the size of values? The decisions about what to produce, what technologies to use and so on depend to a large extent on prevailing and expected prices, and these decisions form the necessary costs of production, which in turn determine values. The logical priority of values to prices in Marx's economics should not be interpreted as some kind of a causal precedence. Prices and values represent two interrelated (dual, as Morishima calls them) valuation system in Marx's economic theory. We do not think, therefore, that one would depart too far from Marx making this relationship explicit in defining labour values for joint products.

To illustrate this solution let us come back to Steedman's example again. Suppose we accept the Sraffian equilibrium prices ($p_1 = 1/3$, $p_2 = 1$) as the basis of cost division between products. Aggregating the total inputs according to the produced commodities we arrive at the following input data:

$$Q^* = \frac{1}{39} \begin{bmatrix} 650 & 325 \\ 30 & 360 \end{bmatrix} \quad w^* = \frac{1}{39} [133 \quad 101]$$

The total production is 33 and 17 units of the two commodities, respectively. From these data one can already determine the labour values. The result is $p_1 \approx 0.24$ and $p_2 \approx 0.59$.

It is also interesting to note that by evaluating the original processes at these values we will find that extra-surplusses emerge (defined as $pZ - pQ - w$). In the case of the first process this is negative (≈ -0.884 , which is about 9 percent of the total surplus), whereas it is of the same order of magnitude, but, of course, positive in the case of the second one. This is clearly what we expected, since we have seen that the first process is less efficient than the second.

We may conclude from the above analysis that the proposal based on a properly justifiable division of inputs among joint products is probably a less elegant and appealing concept than Morishima's true values, but it may save more of Marx's original concepts.

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СОПРЯЖЕННЫЕ ПРОДУКТЫ И ТРУДОВАЯ СТОИМОСТЬ

Э. ЗАЛАИ

В последнее время во всем мире оживился интерес к марксовому понятию стоимости и величине стоимости. В данной статье рассматривается проблематика сопряженных совместно производимых продуктов и величины стоимости, в первую очередь с целью уточнения понятийной стороны проблемы и критической оценки различных точек зрения. В то же время делается попытка распространить определение стоимости в соответствии с марксовыми положениями и анализом на сопряженные продукты.

В случае совместно производимых изделий основная трудность состоит в том, что мы не можем «естественным» образом распределить затраты по той или иной деятельности между производимыми продуктами. Поэтому нельзя непосредственно применить три альтернативных определения величины стоимости Маркса, вернее, их применение привело бы к бессмысленным решениям. Например, Стивдэн на этом основании делает вывод об ограниченности понятия стоимости Маркса и ставит под сомнение ее теоретическое значение. А Моришима и другие правильно, по мнению автора, видят в этом только подтверждение необходимости обобщения определения стоимости. В то же время автор указывает и на то, что нет такой возможности обобщения, которая бы не противоречила тому или иному существенному положению Маркса, то есть у Маркса нет решения вопроса, мы можем говорить в лучшем случае лишь о распространении понятия в духе Маркса.

Затем автор суммирует критерии, играющие особо важную роль в анализе стоимости Маркса, и которые могут служить мерой распространения понятия (однозначность, положительное значение, аддитивность, принцип средней, симметричность принципа средней, однозначность вновь созданной стоимости, взаимопредполагаемость прибавочной стоимости и прибавочного продукта, независимость стоимостей от цен). Автор показывает, что принятое почти всюду в международной литературе обобщение Моришимы нарушает подавляющую часть вышеуказанных критериев. В противоположность этому автор считает более приемлемым решение, при котором сначала затраты распределяются между продуктами в соответствии с традиционной практикой и тем самым определение стоимости сводят к привычной форме. Такое решение не удовлетворяло бы лишь последнему критерию, который, вообще, несостоятелен.