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Inverse spin Hall effect and anomalous Hall effect in a two-dimensional electron gas

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Abstract – We study the coupled dynamics of spin and charge currents in a two-dimensional electron gas in the transport diffusive regime. For systems with inversion symmetry there are established relations between the spin Hall effect, the anomalous Hall effect and the inverse spin Hall effect. However, in two-dimensional electron gases of semiconductors like GaAs, inversion symmetry is broken so that the standard arguments do not apply. We demonstrate that in the presence of a Rashba type of spin-orbit coupling (broken structural inversion symmetry) the anomalous Hall effect, the spin Hall and inverse spin Hall effect are substantially different effects. Furthermore, we discuss the inverse spin Hall effect for a two-dimensional electron gas with Rashba and Dresselhaus spin-orbit coupling; our results agree with a recent experiment.

Despite the anomalous and spin Hall effect being closely related, their histories are rather different. The anomalous Hall effect was experimentally discovered [1] almost at the same time as the ordinary Hall effect, while the spin Hall effect, first predicted in 1971 [2,3] and several times recently [4–7], has been experimentally seen only in the last few years [8–12]. This is not surprising as the anomalous Hall effect entails the measurement of currents and voltages which is well established experimentally, whereas the spin Hall effect requires the detection of a spin current, which has to be done in an indirect way; for a review see for example [13,14] and [15,16].

Since the anomalous and spin Hall effect have the same physical origin, namely the spin-orbit interaction which couples charge and spin degrees of freedom, their dependence on various physical parameters is expected to share similar trends. Depending on whether the spin-orbit coupling is intrinsic in the band structure or appears due to coupling to impurities one speaks about intrinsic or extrinsic mechanisms. The interplay of intrinsic and extrinsic mechanisms is non-trivial. For instance the intrinsic Rashba type of spin-orbit coupling in a two-dimensional electron gas suppresses drastically the extrinsic (skew-scattering) contribution to the spin Hall conductivity [17–20].

It is the purpose of this paper to develop a similar analysis for the anomalous Hall effect and for the inverse spin Hall effect. We will start with a phenomenological discussion of charge and spin currents in a metal or semiconductor with diffusive charge carrier dynamics. We will study in detail the two-dimensional Rashba model including extrinsic skew-scattering. In the anomalous Hall conductivity we find an unexpected anomaly in the magnetic field dependence. Our analysis of the inverse spin Hall effect for a system with Rashba and Dresselhaus spin-orbit coupling is consistent with the experimental results of [21].

As a starting point we consider a system with spin-orbit coupling, where a spin-polarized current in the x-direction generates a small current δj_y into the transverse direction with

$$\delta j_{y\uparrow} = 2\gamma j_{x\uparrow}, \qquad \delta j_{y\downarrow} = -2\gamma j_{x\downarrow}. \tag{1}$$

Clearly, from these equations we can conclude that: a) a charge current generates a transverse spin current, $\delta j_{y\uparrow} - \delta j_{y\downarrow} = 2\gamma(j_{x\uparrow} + j_{x\downarrow})$ (spin Hall effect), b) a spin current generates a transverse charge current, $\delta j_{y\uparrow} + \delta j_{y\downarrow} = 2\gamma(j_{x\uparrow} - j_{x\downarrow})$. If the spin current is due to an electric field in a spin-polarized medium like a ferromagnet this is called the anomalous Hall effect. If it is instead due

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to spin-injection into a non-magnetic material we have the inverse spin Hall effect. The three Hall effects mentioned are thus very closely related, and the magnitude of all of them is determined by the dimensionless parameter γ.

Often, however, eq. (1) is not sufficient for the theoretical description and, in the following, we will achieve the necessary generalization of the equations. Let us write the current in the x-direction as

$$j_{x\uparrow} = \sigma_\uparrow E_{x\uparrow}, \qquad j_{x\downarrow} = \sigma_\downarrow E_{x\downarrow}, \tag{2}$$

where $\sigma_{\uparrow,\downarrow}$ and $E_{x\uparrow,\downarrow}$ are the spin-dependent conductivity and electric field in the x-direction, respectively. In order to allow later arbitrary directions of the spin-polarization, we find it convenient to introduce here the charge and spin components for the field and the current, $E_x \pm \frac{1}{2}E_x^z = E_{x\uparrow,\downarrow}$ and $\frac{1}{2}j_x \pm j_x^z = j_{x\uparrow,\downarrow}$. Equation (2) can now be rewritten as

$$j_x = \sigma E_x + \sigma_{0z} E_x^z, \tag{3}$$

$$j_x^z = \frac{1}{4}\sigma E_x^z + \sigma_{z0} E_x, \tag{4}$$

where $\sigma = \sigma_\uparrow + \sigma_\downarrow = \mu\rho$ is the Drude conductivity, ρ and μ being the charge density and the mobility, respectively. The conductivities $\sigma_{0z} = \sigma_{z0} = \frac{1}{2}(\sigma_\uparrow - \sigma_\downarrow) = \mu s_z$, with s_z the spin density, mix spin and charge currents and appear due to the fact that electrons carry both degrees of freedom; the Onsager relations require the symmetry $\sigma_{0z}(s_z) = -\sigma_{z0}(-s_z)$. Notice that the charge and spin currents (as well as charge and spin density) as defined here have equal units. The transverse currents are given by

$$\delta j_y = 4\gamma j_x^z + \gamma_{0z} j_x, \tag{5}$$

$$\delta j_y^z = \gamma j_x + \gamma_{0z} j_x^z, \tag{6}$$

with $\gamma = \frac{1}{2}(\gamma_\uparrow + \gamma_\downarrow)$ and $\gamma_{0z} = (\gamma_\uparrow - \gamma_\downarrow)$ when we allow different γ's for spin up an down.

In the next step diffusive currents are considered too. This is achieved be replacing the electric fields by

$$\sigma E_x \to \sigma \mathcal{E}_x = -D\partial_x \rho + \sigma E_x, \tag{7}$$

$$\frac{1}{4}\sigma E_x^z \to \frac{1}{4}\sigma \mathcal{E}_x^z = -D\partial_x s_z + \frac{1}{4}\sigma E_x^z. \tag{8}$$

The diffusion coefficient, D, is related to the conductivity via the relation $\sigma = 2e^2 D N_0$, where N_0 is the single-particle density of states at the Fermi energy. Allowing now an arbitrary direction of the fields and the spin polarization we obtain the set of equations

$$j_l = \sigma \mathcal{E}_l + \sigma_{0a}\mathcal{E}_l^a + \delta j_l, \tag{9}$$

$$\delta j_l = -4\gamma\epsilon_{lab}\left[\frac{1}{4}\sigma\mathcal{E}_a^b + \sigma_{b0}\mathcal{E}_a\right] - \epsilon_{lab}\gamma_{0b} j_a, \tag{10}$$

$$j_l^a = \frac{1}{4}\sigma\mathcal{E}_l^a + \sigma_{a0}\mathcal{E}_l + \delta j_l^a, \tag{11}$$

$$\delta j_l^a = \gamma\epsilon_{lab}\left[\sigma\mathcal{E}_b + \sigma_{0c}\mathcal{E}_b^c\right] + \epsilon_{lab}\gamma_{0c} j_b^c. \tag{12}$$

The structure of eqs. (9)–(12) is similar to the equations given in ref. [22], the difference being the terms $\sigma_{0a}\mathcal{E}_l^a$ and $\gamma_{0b} j_a$ in the charge current and $\gamma_{0c} j_b^c$ in the spin one, which do not appear in [22]. The last term in eq. (12) will however be of no importance in the present article and, as such, will be ignored in the following.

We proceed by calculating the parameters entering eqs. (9)–(11) from a microscopic model. We consider a disordered two-dimensional electron gas (2DEG) with Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} - \frac{\mathbf{A}\cdot\mathbf{p}}{m} + V(\mathbf{x}) - \frac{1}{\hbar}\frac{\lambda_0^2}{4}\boldsymbol{\sigma}\times\partial_{\mathbf{x}}V(\mathbf{x})\cdot\mathbf{p} - \frac{1}{2}\mathbf{b}\cdot\boldsymbol{\sigma}. \tag{13}$$

In this Hamiltonian we have both intrinsic and extrinsic spin-orbit coupling. The intrinsic spin-orbit interaction modifies the band structure and enters in the form of a spin-dependent vector potential [20,23–26], which for the Rashba model is given by

$$\mathbf{A} = \frac{m\alpha}{\hbar}\,\boldsymbol{\sigma}\times\hat{\mathbf{e}}_z \equiv \frac{1}{2}\sum_a (A_x^a, A_y^a, A_z^a)\sigma_a, \tag{14}$$

with the only non-zero components $A_x^y = -A_y^x = 2m\alpha/\hbar$. The inclusion of the linear-in-momentum Dresselhaus term for a (001) quantum well is achieved by adding components $A_y^y = -A_x^x = 2m\beta/\hbar$, where β is the corresponding spin-orbit coupling parameter. $V(\mathbf{x})$ is the scalar potential due to the scattering from impurities and gives rise to the extrinsic spin-orbit coupling with strength characterized by the length λ_0. Both spin-orbit couplings are assumed to be weak, *i.e.* $\mathbf{A}\cdot\mathbf{p}_F/m \ll \epsilon_F$ and $\lambda_0 p_F \ll \hbar$. The Zeeman field $\mathbf{b}$ may be due to an external magnetic field or may arise due to the exchange field of a ferromagnet. In the following, for the sake of simplicity, we take units such that $\hbar = 1$.

For our microscopic model the density of states is $N_0 = m/2\pi$ and the diffusion constant is $D = \frac{1}{2}v_F^2\tau$, with τ the elastic scattering time. The latter is determined from the disorder potential and in the Born approximation, assuming $\langle V(\mathbf{x})V(\mathbf{x}')\rangle = \delta(\mathbf{x}-\mathbf{x}')/(2\pi N_0\tau)$. The parameter γ has, in principle, contributions from the skew-scattering, side-jump, and the intrinsic mechanism. In this paper, motivated by the fact that in 2DEGs the skew-scattering is typically considerably stronger than the side-jump, we limit our discussion to the interplay of skew-scattering and the intrinsic mechanism. We then write the parameter γ as

$$\gamma = \gamma_{\text{skew}} + \gamma_{\text{intr}}, \tag{15}$$

with

$$\gamma_{\text{skew}} = -\frac{\lambda_0^2 p_F^2}{16}(2\pi N_0 v_0), \tag{16}$$

$$\gamma_{\text{intr}} = -m\alpha^2\tau. \tag{17}$$

For an explicit derivation, one may see [20,27]. In eq. (16) v_0 is the scattering amplitude from the impurity potential. For the parameter γ_{0z} we find in our model only a skew-scattering contribution, explicitly $\gamma_{0z} = 4\gamma_{\text{skew}}\sigma_{0z}/\sigma$.

It is stressed in ref. [22] that eqs. (9)–(12) are only valid in systems with inversion center. In the absence of the inversion symmetry —which is the case in the situation we consider here— extra terms appear. However for our model Hamiltonian (13) these extra terms are conveniently taken into account by a redefinition of the field $\mathcal{E}_l^a$, which is now given by

$$\frac{1}{4}\sigma\mathcal{E}_l^a = -D\partial_l(s_a - s_a^{eq}) - D\epsilon_{abc}A_l^b(s_c - s_c^{eq}), \quad (18)$$

where $\mathbf{s}^{eq} = (-e)\frac{1}{2}N_0\mathbf{b}$. For example the fields $\mathcal{E}_y^z$ and $\mathcal{E}_x^z$ are in the Rashba model given by

$$\frac{1}{4}\sigma\mathcal{E}_y^z = -D\partial_y(s_z - s_z^{eq}) + D(2m\alpha)(s_y - s_y^{eq}), \quad (19)$$

$$\frac{1}{4}\sigma\mathcal{E}_x^z = -D\partial_x(s_z - s_z^{eq}) + D(2m\alpha)(s_x - s_x^{eq}). \quad (20)$$

Again we refer to the literature for microscopic derivations. For example in [28] the expressions for the spin and charge currents in the case $\lambda_0 = 0$ were obtained by exploiting an $SU(2)$ symmetry of the Rashba model.

How does our approach compare with other studies of the diffusive dynamics of spin and charge? Combining the current density (9) and (11) with the continuity equations for spin and charge one finds coupled diffusion equations. Such diffusion equations have been derived for the Rashba model, *e.g.*, in [29,30] and for the system with both a Rashba and a linear Dresselhaus term in [31]. Our analysis extends these works: whereas the cited papers concentrate on the intrinsic spin-orbit coupling we include also the experimentally relevant skew-scattering. Furthermore the spin-charge coupling conductivities σ_{0a} and σ_{a0} are neglected in [29–31]. In the following we will apply the formalism to the various Hall effects.

Anomalous Hall effect and spin Hall effect. – The anomalous Hall effect describes a contribution to the Hall conductivity due to the spontaneous magnetization in a ferromagnet, the Hall current being perpendicular to both the magnetization and the electric field. The spin Hall effect consists instead in the appearance of a spin current orthogonal to an applied electric field in a non-magnetic material. Let us assume homogeneous conditions, take the electric field along the x-axis and the magnetization along the z-axis, and write down the charge and spin currents along the y-axis. To linear order in the electric field eqs. (9) and (11) become

$$j_y = \sigma_{0z}\mathcal{E}_y^z + \gamma\sigma\mathcal{E}_x^z + 4(\gamma + \gamma_{\text{skew}})\sigma_{z0}E_x, \quad (21)$$

$$j_y^z = \frac{1}{4}\sigma\mathcal{E}_y^z + \gamma\sigma E_x + \gamma\sigma_{0z}\mathcal{E}_x^z. \quad (22)$$

Let us at first examine the anomalous Hall current, eq. (21), in the pure Rashba model ($\lambda_0 = 0$). It is known that in the presence of spin-orbit coupling an electric field induces a spin polarization [32]. In the Rashba model this lies in-plane, and for our geometry along y [33], which implies that $\mathcal{E}_x^z = 0$, but $\mathcal{E}_y^z \neq 0$. From eqs. (17) and (19) we get

$$j_y = \sigma_{0z}\left[\frac{4D(2m\alpha)}{\sigma}s_y - 4\,m\alpha^2\tau E_x\right]. \quad (23)$$

For the spin polarization one has [33]

$$s_y = e^2N_0\alpha\tau E_x, \quad (24)$$

and thus the anomalous Hall effect in the pure Rashba model vanishes in agreement with explicit diagrammatic calculations. Notice that in the diagrammatic calculations a finite anomalous Hall effect is found from a skew-scattering-like contribution which appears at a higher order in the magnetic field and in the presence of magnetic impurities [34–37]. Such a contribution we do not consider here.

The disappearance of the anomalous Hall effect is related to the vanishing of the spin Hall effect in the pure Rashba model. To see this, let us consider the spin Hall current (22). Since $\mathcal{E}_x^z = 0$, we find by comparing eqs. (21) and (22) the relation

$$j_y = 4\frac{\sigma_{0z}}{\sigma}j_y^z, \quad (25)$$

so that a vanishing spin Hall current implies a vanishing charge Hall current.

This relation is no longer true in the presence of both intrinsic ($\alpha \neq 0$) and extrinsic spin-orbit coupling ($\lambda_0 \neq 0$). In this case, the combination of an out-of-plane magnetic field or exchange field together with an in-plane electric field (in the x-direction) generates a component of the spin-polarization in the x-direction so that the field $\mathcal{E}_x^z$ no longer vanishes.

To calculate the spin polarization we borrow from [20] the equations

$$\dot{\mathbf{s}} = -\hat{\Gamma}(\mathbf{s} - \mathbf{s}_{eq}) - \mathbf{b}_{\text{eff}} \times \mathbf{s} + \mathbf{S}_E. \quad (26)$$

Here $\hat{\Gamma}$ is the spin relaxation matrix which in the case of pure Dyakonov-Perel spin relaxation reads

$$\hat{\Gamma} = \frac{1}{\tau_{DP}}\text{diag}(1,1,2), \quad 1/\tau_{DP} = D(2\,m\alpha)^2. \quad (27)$$

The spins relax towards the equilibrium density $\mathbf{s}_{eq} = -e\frac{1}{2}N_0\mathbf{b}$ and precess in an effective magnetic field,

$$\mathbf{b}_{\text{eff}} = \mathbf{b} + 2\,m\alpha\mu\mathbf{e}_z \times \mathbf{E}, \quad (28)$$

whereas $\mathbf{S}_E$ is an electric-field–dependent source term

$$\mathbf{S}_E = 2\,m\alpha\sigma(\gamma_{\text{skew}} + \gamma_{\text{intr}})\mathbf{e}_z \times \mathbf{E}. \quad (29)$$

Solving these equations in the static limit and ignoring possible non-linearities in the electric field, the spin polarization is determined as

$$s_x = -\frac{b_z\tau_{DP}^2}{1 + (b_z\tau_{DP})^2}2\,m\alpha\sigma\gamma_{\text{skew}}E_x, \quad (30)$$

$$s_y = e^2 N_0 \alpha \tau E_x - \frac{\tau_{DP} 2\, m \alpha \sigma}{1+(b_z \tau_{DP})^2} \gamma_{\text{skew}} E_x, \tag{31}$$

$$s_z = -e \frac{1}{2} N_0 b_z. \tag{32}$$

Knowing the spin polarization we can now calculate the Hall and spin Hall current, using eqs. (21) and (22). In the weak magnetic field limit ($b_z \tau_{DP} \ll 1$) the result for the Hall current is

$$j_y = \left(1 + \frac{1}{2}\frac{\gamma_{\text{skew}}}{\gamma_{\text{intr}}}\right) 8 \gamma_{\text{skew}} \sigma_{0z} E_x, \tag{33}$$

which means that a weak Rashba term ($\gamma_{\text{skew}} \gg \gamma_{\text{intr}}$) may considerably enhance the anomalous Hall effect. The spin Hall current, however, vanishes [20,38],

$$j_y^z = 0, \tag{34}$$

in the presence of the Rashba coupling. The term $\gamma_{\text{skew}}/\gamma_{\text{intr}}$ on the right-hand side of eq. (33) appears to be singular when the Rashba coupling goes to zero. This is because we have assumed in eq. (27) that the Elliott-Yafet spin relaxation rate, $1/\tau_s$, can be neglected when compared to the Dyakonov-Perel one, *i.e.*, $\tau_s \gg \tau_{DP}$. The same assumption leads to the vanishing spin Hall current in eq. (34). In ref. [20] it was shown that in order to correctly reproduce the $\alpha \to 0$ limit, where the spin Hall conductivity due to the extrinsic mechanism is finite, one must explicitly take into account the Elliott-Yafet spin-relaxation rate. In order to include the Elliott-Yafet relaxation due to the extrinsic spin-orbit interaction, we have to modify eq. (27) in the following way [20]:

$$\hat{\Gamma} = \frac{1}{\tau_{DP}} \text{diag}(1,1,2) + \frac{1}{\tau_s} \text{diag}(1,1,0). \tag{35}$$

As a consequence, in eqs. (30), (31) we must operate the replacement $\tau_{DP}^{-1} \to \tau_{DP}^{-1} + \tau_s^{-1}$, which guarantees the correct $\alpha \to 0$ limit.

In the strong magnetic field limit ($b_z \tau_{DP} \gg 1$), and again assuming $\tau_s \gg \tau_{DP}$, the anomalous Hall current is given by

$$j_y = 8 \gamma_{\text{skew}} \sigma_{0z} E_x, \tag{36}$$

which is identical to the result in the absence of Rashba spin-orbit coupling.

The Hall angle, j_y/j_x, as function of the magnetic field is shown in fig. 1, for different values of the mobility and the Rashba term. In the absence of Rashba spin-orbit coupling eq. (36) implies that $j_y/j_x = 2\gamma_{\text{skew}} b/\epsilon_F$, *i.e.* the Hall angle as a function of the magnetic field is a structureless line. The Rashba term causes an anomaly in weak magnetic fields. The width of this anomaly is set by the Dyakonov-Perel relaxation rate, and therefore depends strongly on the value of the Rashba coupling but also on the mobility.

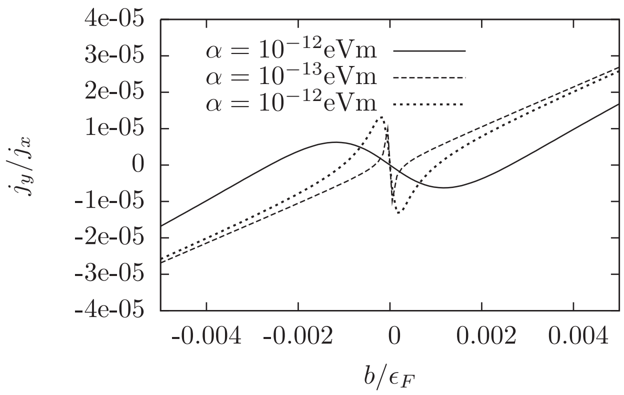

Fig. 1: The Hall angle as a function of the magnetic field. Since we concentrate on the anomalous Hall effect the contribution due to the Lorentz force is ignored. We estimate the Hall angle using parameters valid for GaAs with a carrier density of $10^{12}/\text{cm}^2$. In the absence of Rashba spin-orbit coupling the Hall angle is determined from skew-scattering, with $\gamma_{\text{skew}} \approx 2.7 \times 10^{-3}$ assuming positively charged impurities ($N_0 v_0 < 0$). We obtained the full line assuming a mobility of $\mu = 10^4\,\text{cm}^2/\text{V s}$ and a spin-orbit coupling constant $\alpha = 10^{-12}$ eV m. The dashed and dotted lines correspond, respectively, to $\mu = 10^4\,\text{cm}^2/\text{V s}$, $\alpha = 10^{-13}$ eV m and $\mu = 10^3\,\text{cm}^2/\text{V s}$, $\alpha = 10^{-12}$ eV m.

Inverse spin Hall effect. – In the inverse spin Hall effect (ISHE) spin-polarized carriers are injected into a non-magnetic material. In ref. [10], for instance, the injection of the spin current was achieved via a ferromagnet contacting the spin-orbit active material, while in ref. [21] spins were injected by applying an optical technique. The spin-current generates a transverse charge current which in the end is detected via a standard voltage measurement. We consider the situation where a spin current is injected in the x-direction and generates a charge current in the y-direction. We analyze the ISHE via eq. (9) assuming j_y is linear in the driving force. Then, the expression for the current considerably simplifies and reads

$$j_y = 4\gamma j_x^z. \tag{37}$$

From this relation one can directly read off a Hall angle. If a fully polarized current is injected into the system, then $j_x^z = \frac{1}{2} j_x$ at the injection point and the Hall angle is given by

$$\alpha_H \approx j_y/j_x = 2\gamma. \tag{38}$$

For $\alpha = 0$, *i.e.* without intrinsic spin-orbit coupling, the spin density decays exponentially with the distance from the injection point, $s_z(x) = s_z(0)\exp(-x/L_s)$, the spin current is proportional to the derivative of the spin density, $j_x^z = -D\partial_x s_z$, and therefore the Hall angle drops exponentially too. In [21] where both intrinsic and extrinsic spin-orbit coupling is present, a spin profile of the type $s_z(x) \approx s_z(0)\cos(Qx)$ is expected, where the constant Q depends on the strength of the intrinsic spin-orbit coupling. Surprisingly the measured Hall data is consistent with the assumption of j_y being proportional to the spin density instead of its derivative. In the following we analyze the experiment in more detail in order to understand this point. The experiment was designed such that the linear Rashba and Dresselhaus spin-orbit coupling in the 2DEG are of similar size. For simplicity we assume that both terms are equal, so that the Hamiltonian is given by ($\alpha = -\beta$)

$$H = \frac{\mathbf{p}^2}{2m} + \alpha(p_y - p_x)(\sigma_x + \sigma_y). \tag{39}$$

It is useful to formulate the theory in a rotated frame with unit vectors $\mathbf{e}_+ = (\mathbf{e}_x + \mathbf{e}_y)/\sqrt{2}$ and $\mathbf{e}_- = (\mathbf{e}_x - \mathbf{e}_y)/\sqrt{2}$ so that the spin-orbit coupling reads $\alpha(p_y - p_x)(\sigma_x + \sigma_y) = -2\alpha p_- \sigma_+$. Solving the spin diffusion equation with the boundary condition $\mathbf{s}(0) = s_z(0)\mathbf{e}_z$ we find a spin-spiral of the form

$$\begin{pmatrix} s_+ \\ s_- \\ s_z \end{pmatrix} = s_z(0) e^{-x_-/L_s} \begin{pmatrix} 0 \\ -\sin(4m\alpha x_-) \\ \cos(4m\alpha x_-) \end{pmatrix}, \tag{40}$$

the persistent spin helix of [31,39]. Here the spin-relaxation length L_s was introduced by hand but can be justified microscopically by any spin-relaxation mechanism like, *e.g.*, the Elliott-Yafet one. In the latter case, one finds explicitly $L_s = \sqrt{2D\tau_s}$ if $\tau_s \gg \tau_{DP}$. In [21] the 2DEG channel is patterned along the $[1\bar{1}0]$-direction, the direction of the spin-helix propagation. The Hall current is then proportional to the spin current flowing in the x_--direction. After modifying eq. (19) to include both a Rashba and a Dresselhaus term (with $\alpha = -\beta$) we find

$$j_y = 4\gamma j^z_{x_-} = 4\gamma \left[-D\partial_{x_-} s_z + D(4m\alpha) s_- \right] \tag{41}$$

$$= 4\gamma \frac{D}{L_s} s_z(0) \cos(4m\alpha x_-) \exp(-x_-/L_s). \tag{42}$$

The Hall current indeed follows the spin polarization in the z-direction with periodic changes of the sign with increasing distance from the spin-injection point, in agreement with the experimental finding. Also the absolute value of the Hall angle, which is of the order of some 10^{-3} is consistent with realistic estimates of the parameters.

Summary. – We presented equations describing the coupled dynamics of spin and charge currents in a two-dimensional electron gas. Unlike in inversion symmetric systems, where the spin Hall effect, the anomalous Hall effect and the inverse spin Hall effect are essentially the same thing no general relation between the three effects can be given. For example in the pure Rashba model we find a vanishing spin Hall and anomalous Hall effect, but a finite inverse spin Hall effect. We analyzed the inverse spin Hall effect for a system where Rashba and Dresselhaus spin-orbit coupling have equal strength; our results compare well with a recent experiment.

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