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Spin Hall Effect in a 2DEG in the Presence of Magnetic Couplings

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Abstract. It is now well established that the peculiar linear-in-momentum dependence of the Rashba (and of the Dresselhaus) spin-orbit coupling leads to the vanishing of the spin Hall conductivity in the bulk of a two-dimensional electron gas (2DEG). In this paper we discuss how generic magnetic couplings change this behaviour providing then a potential handle on the spin Hall effect. In particular we examine the influence of magnetic impurities and an in-plane magnetic field. We find that in both cases there is a finite spin Hall effect and we provide explicit expressions for the spin Hall conductivity. The results can be obtained by means of the quasiclassical Green function approach, that we have recently extended to spin-orbit coupled electron systems.

1. Introduction

In the field of spintronics, much attention has recently been paid to spin-orbit related phenomena in semiconductors. One such phenomenon is the spin Hall effect, i.e. a spin current flowing perpendicular to an applied electric field [1, 2, 3, 4]. It is now well known that for linear-in-momentum spin-orbit couplings like the Rashba or Dresselhaus ones the spin Hall current vanishes exactly in the bulk of a disordered system [5, 6, 7, 8, 9, 10, 11]. For a *magnetically* disordered two-dimensional electron gas (2DEG), or in the case of an applied in-plane magnetic field, things are however different, and a non-vanishing spin Hall conductivity is found [12, 13, 14, 15, 16].

For the calculations we rely on the Eilenberger equation for the quasiclassical Green function in the presence of spin-orbit coupling [17]. Our results are valid for any value of the disorder parameter $\alpha p_F \tau$, where α is the spin-orbit coupling constant, p_F the Fermi momentum in the absence of such coupling, and τ the elastic quasiparticle lifetime due to non-magnetic scatterers. The standard metallic regime condition is assumed, i.e. $\epsilon_F \tau \gg 1$, and contributions of order $(\alpha/v_F)^2$ are neglected throughout. We focus on intrinsic effects in the Rashba model; extrinsic ones [18], Dresselhaus terms [19] and hole gases [20] are not taken into account. Finally, weak localization corrections, which could in principle play an important role [11], are beyond the scope of our work.

2. The model

The Hamiltonian describing the 2DEG, confined to the $x-y$ plane, reads

$$H = \frac{\mathbf{p}^2}{2m} - \mathbf{b}\cdot\boldsymbol{\sigma} + V(\mathbf{x}), \tag{1}$$

where $\mathbf{b} = -\alpha\mathbf{p}\times\mathbf{e}_z + \omega_s\mathbf{e}_x$ is the effective magnetic field, containing both the external (applied) one and the internal Rashba spin-orbit field, $\boldsymbol{\sigma}$ the vector of Pauli matrices, and $V(\mathbf{x}) = V_{\rm nm}(\mathbf{x}) + V_{\rm m}(\mathbf{x})$ the disorder potential due to randomly distributed impurities. Non-magnetic scatterers give rise to $V_{\rm nm}(\mathbf{x})$, while $V_{\rm m}(\mathbf{x})$ describes magnetic s-wave disorder

$$V_{\rm nm}(\mathbf{x}) = \sum_i U(\mathbf{x}-\mathbf{R}_i), \quad V_{\rm m}(\mathbf{x}) = \sum_i \mathbf{B}\cdot\boldsymbol{\sigma}\delta(\mathbf{x}-\mathbf{R}_i). \tag{2}$$

Both potentials are treated in the Born approximation, and the standard averaging technique is applied. In order to better clarify the distinct roles of magnetic impurities and of the magnetic field, let us first separately consider a Hamiltonian H_1 - magnetic impurities but no magnetic field - and a Hamiltonian H_2 - magnetic field but no magnetic impurities

$$H_1 = \frac{\mathbf{p}^2}{2m} + \alpha\mathbf{p}\times\mathbf{e}_z\cdot\boldsymbol{\sigma} + V_{\rm nm}(\mathbf{x}) + V_{\rm m}(\mathbf{x}), \tag{3}$$

$$H_2 = \frac{\mathbf{p}^2}{2m} + (\alpha\mathbf{p}\times\mathbf{e}_z + \omega_s\mathbf{e}_x)\cdot\boldsymbol{\sigma} + V_{\rm nm}(\mathbf{x}) \tag{4}$$

The two Hamiltonians lead to the following continuity equations for the s_y spin component [13, 14, 16]

$$H_1 \ : \ \partial_t s_y + \partial_{\mathbf{x}}\cdot\mathbf{j}_{s_y} = -2m\alpha j^y_{s_z} - \frac{4}{3\tau_{sf}}s_y \tag{5}$$

$$H_2 \ : \ \partial_t s_y + \partial_{\mathbf{x}}\cdot\mathbf{j}_{s_y} = -2m\alpha j^y_{s_z} + 2\omega_s s_z. \tag{6}$$

The second term on the r.h.s. of Eq.(5) is due to magnetic impurities, τ_{sf} being the spin-flip time which stems from the potential $V_m(\mathbf{x})$. Under stationary and uniform conditions the above equations imply

$$j^y_{s_z} = -\frac{2}{3m\alpha\tau_{sf}}s_y \tag{7}$$

$$j^y_{s_z} = \frac{\omega_s}{m\alpha}s_z. \tag{8}$$

That is, the spin current vanishes unless either magnetic scattering or a magnetic field is present.

We now present some selected results, while the interested reader should refer to [17, 13, 14, 16] for all details regarding the formalism.

3. Magnetic impurities, no magnetic field

From the expression for the in-plane spin polarization s_y one can, through Eq.(5), obtain the one for the frequency dependent spin Hall conductivity $\sigma_{sH}(\omega)$

$$\sigma_{sH}(\omega) = \frac{|e|}{4\pi}\frac{\left(\frac{4}{3\tau_{sf}} - i\omega\right)\, 2(\alpha p_F)^2}{\left[\left(\frac{1}{\tau_{tr}} - i\omega\right)\left(\frac{1}{\tau_E} - i\omega\right)\left(\frac{4}{3\tau_{sf}} - i\omega\right) + 2(\alpha p_F)^2\left(\frac{1}{\tau_E} + \frac{4}{3\tau_{sf}} - 2i\omega\right)\right]}. \tag{9}$$

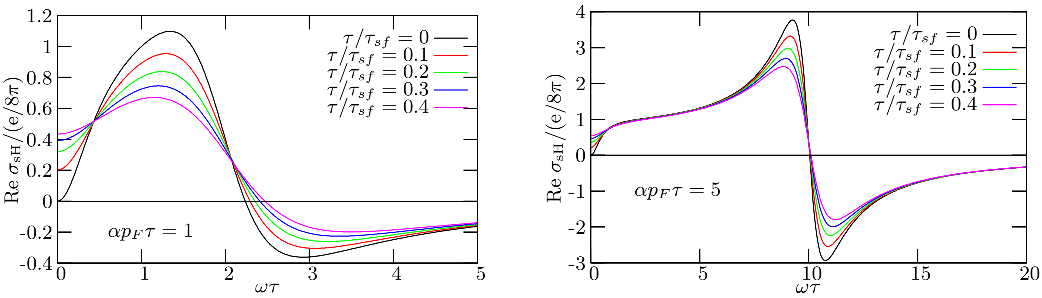

Figure 1. Real part of the frequency dependent spin Hall conductivity in units of the universal value $|e|/8\pi$, plotted for various values of the ratio τ/τ_{sf} and for $\alpha p_F \tau = 1$ (left) and $\alpha p_F \tau = 5$ (right).

Its real part is plotted in the Figure, while the various time scales appearing above are defined as

$$\frac{1}{\tau_{tr}} \equiv \sum_{\mathbf{p}'} W(\mathbf{p},\mathbf{p}')(1-\cos(\vartheta_{\mathbf{pp}'})) + \frac{1}{\tau_{sf}}, \quad \frac{1}{\tau_E} \equiv \sum_{\mathbf{p}'} W(\mathbf{p},\mathbf{p}')(1-\cos(2\vartheta_{\mathbf{pp}'})) + \frac{1}{\tau_{sf}}, \tag{10}$$

$W(\mathbf{p},\mathbf{p}')$ being the angle dependent scattering probability (the subscript in τ_E stands for "Edelstein" [14]). As one can see, the static limit of $\sigma_{sH}(\omega)$ approaches values of the order of $e/8\pi$ when the ratio τ/τ_{sf} grows. This could be especially relevant in II-VI heterostructures, in which high electron mobilities and strong magnetic scattering are simultaneously present [21, 22, 23]. In the diffusive regime, $\omega\tau_{tr} \ll 1, \alpha p_F \tau_{tr} \ll 1$, and assuming $\tau_{tr}/\tau_{sf} \ll 1, \tau_E/\tau_{sf} \ll 1$, one obtains the following Bloch equations

$$\partial_t s_x = -\left(\frac{1}{\tau_s} + \frac{4}{3\tau_{sf}}\right) s_x \tag{11}$$

$$\partial_t s_y = -\left(\frac{1}{\tau_s} + \frac{4}{3\tau_{sf}}\right) s_y + \alpha N_0 |e| \mathcal{E} \frac{\tau_E}{\tau_s} \tag{12}$$

$$\partial_t s_z = -\left(\frac{2}{\tau_s} + \frac{4}{3\tau_{sf}}\right) s_z, \tag{13}$$

where $(2\alpha p_F \tau_{tr})^2/2\tau_{tr} \equiv 1/\tau_s$ is the Dyakonov-Perel spin relaxation rate, and $N_0 = m/2\pi$ is the density of states in the absence of spin-orbit. From the above the sensitivity of the in-plane spin polarization to spin-flip scattering is apparent, as this leaves the source unchanged, whereas it enhances the relaxation rate so that in the end s_y is reduced.

4. Magnetic field, no magnetic impurities

From the expression for the in-plane spin polarization s_z, using Eq.(8), one obtains the (static) spin Hall conductivity to leading order in the external magnetic field

$$\sigma_{sH} = -\frac{|e|}{4\pi}\left(\frac{\omega_s}{\alpha p_F}\right)^2 \frac{\tau_{tr} - \tau_E}{\tau_{tr}}. \tag{14}$$

The Bloch equations in this case are

$$\partial_t s_x = -\frac{1}{\tau_s}(s_x - s_x^{eq}) \tag{15}$$

$$\partial_t s_y = -\frac{1}{\tau_s}\left(s_y + \alpha N_0 |e| \tau_E \mathcal{E}\right) + 2\omega_s s_z \tag{16}$$

$$\partial_t s_z = -\frac{1}{2\tau_s} s_z - 2\omega_s \left(s_y + \alpha N_0 |e| \tau_{tr} \mathcal{E}\right), \tag{17}$$

which means that

$$s_z = \alpha N_0 |e| (\tau_E - \tau_{tr}) \mathcal{E} \frac{\omega_s \tau_s}{1 + 2(\omega_s \tau_s)^2}, \tag{18}$$

a result in agreement with what experimentally observed in [24], although the sample studied is not strictly a 2DEG and the present analysis may not be applied directly. It is important to notice that in the diffusion equations for s_y and s_z angle dependent scattering makes for the appearance of two different time scales, respectively τ_E and τ_{tr}. This is a priori not obvious, but also fundamental in order to obtain a non-vanishing stationary s_z polarization - and thus a non-vanishing spin Hall conductivity.

5. Conclusions

We have shown how the interplay of non-magnetic (angle dependent) and magnetic (short range) scattering and an in-plane magnetic field non-trivially affects the spin-charge dynamics of a 2DEG with Rashba spin-orbit coupling.

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