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Technological Diffusion and Dynamic Gains from Trade

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Abstract

We consider a technologically backward country and analyse the implications on competitiveness and long-run growth of the quality content of traded goods. We build an endogenous growth model where quality improvements stem from research activity taking place in the R&D sector, and where the relative quality content of goods matter for export and import demand functions. We show that the possibility of an optimal growth with a balanced current account and no adverse terms-of-trade effects is closely related to the evolution of the country's technological distance with respect to the trade partner: with an unfavourable quality-dynamics the country cannot engage successfully in "non-price" competition. Thus, long-run growth is coupled with an adverse export to import ratio, and a balanced trade requires a continuous offsetting fall in relative prices, either through devaluations or wage deflations. We then allow for international knowledge spillovers that increase the productivity of labour resources devoted to research in a way which is proportional to the technological distance between the countries. We show that the greater the country's ability to absorb foreign knowledge and improve upon foreign technologies, the greater the gains in competitiveness, and the benefits to long-run growth. A numerical simulation confirms our findings.

Keywords: *Vertical innovation; Technological change and catching up; Economic growth of open economies*

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1. Introduction

One of the central questions in current policy debates, as well as in theoretical analyses and empirical investigations, is what allows lagging-behind countries to catch up with more advanced economies.

The recent development of theories of technological change has given rise to a new perspective in the analysis of the relation among trade, growth, and technological change in open economies. Grossman and Helpman (1991), Aghion and Howitt (1992), Edwards (1998), among others, emphasize the existence of a positive association between openness and growth driven by technological transfers and knowledge diffusion, the factors at the bulk of a catching-up process.

The contribution of the paper is to address the aforesaid issue with reference to technologically lagging-behind economies. At the basis of our analysis is the new-Schumpeterian perspective that countries compete not so much by varying prices and quantities, as by innovating. In this view, we contend that technologically backward countries suffer from a structural lack of competitiveness, due to their lower rate of innovation, which is reflected in the relative poor quality of the goods they produce.

As mentioned above, it is widely agreed upon that openness is among the most crucial factors for a catching-up process. The more countries are open to the rest of the world, the greater is their ability to benefit by the higher stock of knowledge capital generated in leading countries. In fact, there is wide empirical evidence that the international diffusion of knowledge is promoted, among others, by trade in goods and services, foreign direct investment, migration and business contacts,[1] and that the integration of markets favours both the spread of general scientific knowledge and the diffusion of more product-specific information.

The benefits of international knowledge spillovers accrue in terms of an increase in the productivity of labour resources allocated to research. For instance, the laggard economy can gain skills in applying to the goods it manufactures the know-how embodied in products of the advanced country; since traded goods are differentiated, entrepreneurs have the possibility to apply a given know-how to a different type of good, with respect to the one which it was originally conceived for. Hence, in the R&D sector both innovation and imitation activities are carried on.[2]

On the basis of the above considerations, we build an endogenous growth model[3] where a country trades with "the rest of the world" in a number of differentiated products. Manufactured products embody a "quality content" which reflects the stock of knowledge capital available in the country at a given time; as in quality-ladder models innovations stem from research activity taking place in the R&D sector, and lead to the quality upgrading of manufactured goods through time. We consider the case of a country that lags behind technologically, so that its products have a lower quality content than foreign products. In the model we derive analytically export and import demand functions in which the relative quality content of the goods matter. We discuss the implications on competitiveness and long-run growth of a lower rate of innovation with respect to the more advanced trade partners. We show that the possibility of an optimal growth with a balanced current account and no adverse terms-of-trade effects is closely related to the

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[1] Keller (1999, 2002); Crispolti and Marconi (2005); Léon-Ledesma (2005).

[2] Pack and Westphal (1986).

[3] In this paper we extend to a general equilibrium setting the analysis developed in Cavallaro and Mulino (2007).

evolution of the country's technological distance with respect to the trade partner: an unfavourable quality-dynamics determines the inability to engage successfully in "non-price" competition. It follows that long-run growth is coupled with an adverse export to import ratio so that a balanced trade requires a continuous offsetting fall in relative prices, either through devaluations or wage deflations.

We then allow for international knowledge spillovers. Formally, we let foreign knowledge enter the "stock of knowledge capital" available to domestic researchers, and allow for a higher efficiency of the economy in making that stock of knowledge capital useful in R&D activity. Overall, these effects determine an increase in the productivity of labour resources devoted to research, in a way which is proportional to the technological distance between the countries.[4] In the paper, we derive analytically the impact of the technological catching up on export and import demand functions and show that the greater the country's ability to absorb foreign knowledge and improve upon foreign technologies, the greater the gains in competitiveness, and the benefits to long-run growth. A numerical simulation confirms our findings.

2. The model: the basic set-up

We consider a lagging-behind economy trading with the "rest of the world" which is technologically more advanced. For the sake of simplicity, we will name the "rest of the world" as the "advanced country". As in more recent trade literature, the context is that of a *semi-small* open economy, where imported goods are purchased at given world prices, whereas the country faces a downward sloping demand schedule for its exports, because these are perceived to be imperfect substitutes for the tradable goods of other countries. In addition, we assume there are no tariffs, transportation costs or other trade barriers, and that only final goods are internationally traded.

2.1 - Consumers' behaviour

Households have a preference for diversity and therefore derive utility from the consumption of different products, which are manufactured domestically and abroad, and which substitute imperfectly for each other. We assume the set of available products to be fixed through time since we are not concerned with the issue of brand proliferation, whereas the quality content of such products varies through time: as in quality ladder models,[5] each product can potentially be produced in an unlimited number of qualities, and the pace innovation in a given country determines the quality upgrading of the products it manufactures. We assume that at each time *t* the quality difference between domestic and foreign products is reflected in households' (static) choice of consumption.

We thus describe intertemporal preferences of the representative consumer with the standard form:

$$U_t = \int_t^{\infty} e^{-\rho(\tau - t)} \log u_\tau d\tau \tag{1}$$

where ρ represents the subjective discount rate and u_t each household's instantaneous utility:

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[4] Edwards (1998); Smulders (2004).

[5] Grossman and Helpman (1991); Aghion and Howitt (1992).

$$u_t = \left\{ \left(q_t^H \sum_{i=1}^{n} C_{it}^{\frac{\theta-1}{\theta}} + q_t^F \sum_{j=1}^{m} C_{jt}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right\} \tag{2}$$

The specification of utility in equation (2) reflects the constant elasticity of substitution between products, equal to $\theta > 1$,[6] and the preference for quality: the *n* domestic and *m* foreign products enter in each consumer's preferences in relation to their quality content, that is, in proportion to q_t^H and q_t^F.

Each representative household supplies labour services inelastically, earns labour income and receives profits which are paid out by firms as dividends from his share of domestic assets holdings. Moreover, each household can borrow freely on the world market at the international rate to finance the excess of expenditure over labour and non-labour incomes. The dynamic budget constraint is therefore:

$$\left[\sum_{i=1}^{n} P_{it} C_{it} + \sum_{j=1}^{m} x P_{jt} C_{jt} \right] + \frac{dS_t}{dt} = W_t L_t + r\, S_t \qquad \text{for all } t \tag{3}$$

where P_i and P_j represent the prices of good *i* and *j* produced in the domestic and foreign economy, respectively, *x* is the exchange rate, S_t is the value of the consumer's total asset holdings net of foreign liabilities, W_t is the wage rate, L_t is labour supply, r is the rate of return on net asset holdings. In order to prevent individuals to choose paths with exploding debt, we impose the transversality condition that requires each household's debt, B_t, not increase asymptotically faster than the interest rate:

$$\lim_{t \to \infty} B_t\, e^{-r\,t} = 0 \tag{4}$$

The solution to the above maximization problem leads to the following static demand functions for the domestic good *i* and for the foreign good *j*:[7]

$$C_i = \frac{\left(\frac{P_i}{q^H} \right)^{-\theta} E}{\left(q^H\right)^{\theta} \sum_{i=1}^{n} P_i^{1-\theta} + \left(q^F\right)^{\theta} \sum_{j=1}^{m} x P_j^{1-\theta}} \tag{5}$$

$$C_j = \frac{\left(\frac{x P_j}{q^F} \right)^{-\theta} E}{\left(q^H\right)^{\theta} \sum_{i=1}^{n} P_i^{1-\theta} + \left(q^F\right)^{\theta} \sum_{j=1}^{m} x P_j^{1-\theta}} \tag{6}$$

where *E* denotes the representative consumer's expenditure, as resulting from the dynamical budget constraint in equation (3). Equations (5) and (6) may be restated as:

$$C_i = \frac{P_i^{-\theta} \left(q^H\right)^{\theta} E}{P^{1-\theta} \left(q^F + q^H\right)^{\theta}} = \left(\frac{P_i}{P} \right)^{-\theta} \delta^{\theta}\, \frac{E}{P} \tag{5a}$$

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[6] To save on notation, we assume the same value of the elasticity of substitution among all goods, irrespective of their place of production.

[7] Pollack (1971).

$$C_j = \frac{xP_j^{-\theta}\left(q^F\right)^{\theta} E}{P^{1-\theta}\left(q^F + q^H\right)^{\theta}} = \left(\frac{xP_j}{P}\right)^{-\theta} \left(1-\delta\right)^{\theta} \frac{E}{P} \tag{6a}$$

where $\delta \equiv \frac{q^H}{q^F + q^H}$ represents the relative quality content of domestic goods, which should be intended as the relative amount of services provided by domestic goods, and where $P \equiv \left(\delta^{\theta}\sum_{i=1}^{n} P_i^{1-\theta} + \left(1-\delta\right)^{\theta}\sum_{j=1}^{m} xP_j^{1-\theta}\right)^{\frac{1}{1-\theta}}$ is the price index consistent with the preference specification given in equation (2).

Equations (5a) and (6a) show that the representative household's demands for good *i* and for good *j*, respectively, are functions of relative prices, with elasticity θ, real income, with unitary elasticity, and that the relative quality content of domestic and foreign goods impacts on the magnitude of the expenditure proportionality. The importance of the quality content in the structure of demand appears clearly by taking the ratio between equations (5a) and (6a):

$$\frac{C_i}{C_j} = \left(\frac{P_i / q^H}{xP_j / q^F}\right)^{-\theta} \tag{7}$$

Equation (7) shows that relative demand depends on the goods' price ratio and the know-how content ratio; to put it differently, relative demand is decreasing in quality-adjusted prices.

As standard in endogenous growth models, the individual optimal spending profile obeys the Ramsey rule: $\widetilde{E} = r - \rho$, where $\widetilde{E}$ denotes the growth rate of expenditure.[8]

Market demand for good *i* is obtained by summing over residents and non-residents' individual demands. Domestic demand for good i, C_i^H, takes exactly the same form as equation (5a), but with expenditure E^H referred to national-wide expenditure.[9] As to foreign demand, C_i^F, we make the usual assumption that consumers' preferences in the two countries are symmetric, so that:

$$C_i^F = \left(\frac{P_i}{xP^F}\right)^{-\theta} \delta^{\theta} \frac{R^F}{P^F} = \left(\frac{P_i}{P}\right)^{-\theta} \delta^{\theta} \frac{E^F}{P} \tag{8}$$

where $xP^F = P$ is the foreign price index in terms of the domestic currency,[10] and $xR^F = E^F$ is foreign expenditure in terms of the domestic currency.

The overall demand for good *i* at each time *t* takes the following form:

$$D_i = C_i^H + C_i^F = \left(\frac{P_i}{P}\right)^{-\theta} \delta^{\theta} \frac{E^H}{P} + \left(\frac{P_i}{P}\right)^{-\theta} \delta^{\theta} \frac{E^F}{P} \tag{9}$$

2.2 - Firms' behavior

The supply side of the economy is characterized by the sector that manufactures final goods, and the sector that manufactures intermediates; in the latter firms undertake two

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[8] In the paper, the symbol ~ over a variable denotes its rate of growth.

[9] In the aggregate the economy's expenditure is given by total wages and profits earned in the different sectors of the economy plus the flow of external borrowing net of interest payments on debt.

[10] This follows from the assumption that consumers' preferences in the two countries are symmetric.

activities: they create blueprints by engaging in R&D activity, and they manufacture innovative intermediates. The firms that succeed in up-front research have the ability to gain industry leadership for the innovative product and to capture the monopoly profits accruing from the production of the intermediate until next research success.

In the final sector there is a fixed number of firms, each producing a differentiated final product with a constant-return-to-scale technology where labour and intermediates goods are the two inputs:

$$Y_i = F_i\, L_{Y_i}^{\alpha} A_i^{1-\alpha} \tag{10}$$

where F_i is an arbitrary constant reflecting the choice of units, L_{Y_i} is employment of labour and A_i the amount of intermediates in the manufacture of product *i*. We follow Grossman-Helpman (1991) in assuming that firms employ a fixed assortment of intermediate inputs which are vertically differentiated, that is, each input can be produced in various qualities. We shall thus employ the following index of intermediate inputs:

$$log\, A = \int_0^1 log\left(\sum_{\tau} q_{\tau} Z_{\tau h} \right) dh \tag{11}$$

where $Z_{\tau h}$ represents the component h in the index whose quality is q_{τ}. The innovation process is such that each new intermediate provides γ additional services with respect to the good of the previous generation, that is $q_{\tau} = \gamma\, q_{\tau-1}$, where q_{τ} denotes the quality of the τ^{th} generation good. The intermediate index (11) has the property that vertically differentiated inputs in a given industry *h* substitute perfectly for one other when quality differences are appropriately accounted for. Moreover, each intermediate *h* enters the index symmetrically, and therefore enters symmetrically in the production of final goods, too.

We assume that each firm producing the final product is a monopolistic competitor in the world market, so that the behaviour of firm *i* is described by a standard profit maximization problem, given the technology and the demand constraint (equations (10) and (9), respectively). The optimal price rule followed by producers in the final sector implies a fixed margin over the marginal costs of production.[11]

[11] The analytical set-up is that of monopolistic competition, as in Blanchard – Kiyotaki (1987). In our model each firm solves the following maximization programme: $\max_{P_i} \frac{P_i}{P} Y_i - \frac{W}{P} L_{Y_i} - \frac{P_A}{P} A_i$, subject to the technology constraint: $Y_i - F_i\, L_{Y_i}^{\alpha} A_i^{1-\alpha} = 0$ and to the demand constraint: $Y_i = C_i^H + C_i^F = \left(\frac{P_i}{P}\right)^{-\theta} \delta^{\theta} \frac{E^H + E^F}{P}$, where P_A is the price of the intermediate index A. Each firm takes the wage rate and the price of the intermediate index as given. Moreover, due to the assumption of a large number of firms in the world market, each firm takes the price index P as given, too. The optimal price rule implies a fixed margin over the marginal costs of production: $P_i = \frac{\theta}{\theta - 1} \frac{W\, L_{Y_i}}{\alpha\, Y_i}$ and $P_i = \frac{\theta}{\theta - 1} \frac{P_A\, A_i}{(1-\alpha) Y_i}$. The above conditions lead to the following demand functions for labour and for the aggregate intermediate: $L_{Y_i} = \frac{\theta - 1}{\theta} \alpha \frac{P_i Y_i}{W}$ and

As to innovation activity, we follow closely Grossmann-Helpman (1991). Recall that each firm that manufactures an intermediate product undertakes also research activity: each firm may enter freely into R&D and issue equity to finance innovation costs. The successful research effort leads to industry leadership for the manufacture of the intermediate, and to the stream of monopoly profits until the next research success. We assume that an entrepreneur engages labour resources in R&D at intensity ι for a time interval of length dt.[12] The probability of gaining success in lab activity is proportional to the resources devoted: to achieve a research intensity of ι, it is necessary to invest $L_R = \ell \iota$ units of labour services per unit of time, where ℓ is a parameter reflecting the productivity of labour in research. The entrepreneur will invest labour in research activity up to the amount for which the cost of R&D activity, $W\ell\iota$, equals the expected revenues $\upsilon\iota$. It follows that:

$$W\ell \geq \upsilon \quad , \quad \text{with equality whenever } \iota > 0 \tag{12}$$

The above equation (12) may be interpreted as an arbitrage condition between the stock market value of the innovating firm υ and the expected cost of quality upgrading.

As to the innovative intermediates, each component Z_h is manufactured with a constant-return-to-scale technology where labour is the only input. For the sake of simplicity, we choose units so that one unit of the intermediate input requires one unit of labour input. Given monopolistic competition in the industry and limit pricing outcome all intermediates bear the same price, that is $P_{Z_h} = P_Z$. Such a price is determined on the basis of a mark-up γ over unit costs, where γ is the increase in quality embodied in the superior, state-of-the-art intermediate, that is: $P_Z = \gamma W$. Since better quality inputs are more productive, producers of the final goods buy only the state-of-the-art varieties; and since all demanded components Z_h are employed in equal quantities, the aggregate intermediate *A* can be expressed as $A = q^H Z$ where *Z* denotes the aggregate volume of intermediates and q^H is an index of productivity of intermediates. The above assumptions imply also that $P_A A = P_Z Z$, so that $P_A = \frac{P_Z}{q^H}$. As to the productivity index q^H, it reflects the country's state of knowledge embodied in the final products, at a given time *t*, and it is proportional to the total "number" of R&D successes.[13]

The pricing strategy in the innovative sector leads to a flow of profits per firm equal to:

$A_i = \frac{\theta - 1}{\theta}(1-\alpha)\frac{P_i Y_i}{P_A}$. The optimal pricing strategy leads to a flow of monopoly profits at each time *t* equal to $\Pi_{Y_i} = \frac{1}{\theta} P_i Y_i$.

[12] We are thus assuming that success in innovation and imitation arises randomly, following a Poisson processes, with ι denoting the parameter of the density function.

[13] From equation (11) we have $log\, A = \int_0^1 log\, \overline{q}_h dh + log\, Z$, where $\overline{q}_h$ represents the quality of the state-of-the-art brand of intermediate *h*. Hence, the index q^H is $q^H = \gamma^{I_t}$, where $I_t \equiv \int_0^T \iota(t)dt$ represents the total "number" of R&D successes from time $t = 0$ up to $t = T$.

$$\Pi_{Z_h} = \left(\frac{\theta - 1}{\theta}\right)\left(1 - \frac{1}{\gamma}\right)(1 - \alpha)\sum_{i=1}^{n} P_i Y_i \tag{13}$$

The above flow of profits, together with the expected capital gains or losses, constitute the overall return to the owner of the innovative asset. In a perfect-foresight equilibrium, arbitrage in the capital market requires that the overall return to capital be equal to the return on a riskless loan r. We then have:

$$\Pi_{Z_h} + \dot{\upsilon} - \iota\,\upsilon = r\upsilon \tag{14}$$

where $\dot{\upsilon} - \iota\,\upsilon$ is the expected capital gains or losses on the value of the equity claims.[14]

3. Long-run growth and the external balance

We now turn to the macrodynamical behaviour of the model in a general equilibrium context. Given that the demand functions for each domestic product are identical, and that the production functions of each firm are identical, too, we have symmetry among firms. Hence, in a general equilibrium the prices set by firms in the final- good sector are identical, that is $P_i = P^H$. The equilibrium condition between aggregate production $\sum_{i=1}^{n} \frac{P_i Y_i}{P} \equiv \frac{P^H Y^H}{P}$ and aggregate demand $\sum_{i=1}^{n} \frac{P_i D_i}{P} \equiv \frac{P^H D}{P}$, at each time *t*, may be stated by recalling equation (9):

$$\frac{P^H Y^H}{P} = \left(\frac{P^H}{P}\right)^{1-\theta} \delta^{\theta}\, \frac{E^H + E^F}{P} \tag{15}$$

We now consider the market clearing conditions for the labour and the intermediate good markets. As to the aggregate intermediate, it has only a single use and its supply is equal to the derived demand by final good producers; as to labour, it is used in R&D, in the manufacture of the consumption products and of intermediates. Therefore, the market clearing condition , at each time *t*, is obtained by equating the fixed labour supply, $\overline{L}$, and the derived demands for labour in research, and in the intermediate and final sectors:

$$\iota\,\ell + \frac{\theta - 1}{\theta}(1 - \alpha)\frac{P^H Y^H}{\gamma W} + \frac{\theta - 1}{\theta}\alpha\,\frac{P^H Y^H}{W} = \overline{L} \tag{16}$$

where now ι denotes the aggregate intensity of research targeted at a state-of-the-art product.[15] We may thus reformulate the arbitrage condition as:

$$\Pi_Z V - \frac{\dot{V}}{V} - \iota = \rho \tag{17}$$

where Π_Z is the aggregate profit in the innovative sector and $V = 1/\upsilon$ is the inverse of the aggregate value of the stock market. The dynamical behaviour of the economy through time is described by the above arbitrage condition, together with the labour market equilibrium condition. We concentrate on the steady-state properties, when the value of the stock market is constant, that is, $\dot{V} = 0$.

[14] With efficient markets υ_t is equal to the discounted value of the flow of future profits, so that no bubbles arise.

[15] The specification of the intermediate goods sector implies that the total measure of profit making firms equals 1, and therefore the aggregate value of the stock market is υ.

Equations (16) and (17), together with the side equations (12) and (15), determine the steady-state rate of innovation:

$$\iota = (1-\xi)\frac{\overline{L}}{\ell} - \xi\rho, \qquad 0 < \xi \equiv \frac{1-\alpha}{\gamma} + \alpha < 1 \tag{18}$$

It appears that the economy's pace of innovation is positively influenced by the degree of monopoly power enjoyed by the innovative sector – larger values of γ, as well as by a relatively high share of intermediates in the technology for the production of the final good – lower values of α.

As in Grossman-Helpman (1991), in the long run the pace of economic expansion depends on the restless process of productivity growth that takes place in the innovative sector. In fact, given the firm's technology in equation (10), growth accountancy relationships imply that in a symmetric equilibrium the growth rate of aggregate output is proportional to the growth rate of the aggregate intermediates A:

$$\widetilde{Y}^{H} = (1-\alpha)\widetilde{A} \tag{19}$$

Since $A = q^{H}Z$, and given that each new generation intermediates provide γ additional services with respect to the previous generation goods with probability ι, so that $\widetilde{q}^{H} = \iota \log \gamma$, equation (19) may be reformulated as

$$\widetilde{Y}^{H} = (1-\alpha)\,\iota \log \gamma \tag{20}$$

In steady state growth will be coupled with external balance. Yet, we do not rule out the possibility for an economy to absorb above its level of production for some period of time, by spending in imported products above the receipts from exports, and thus o accumulate external debt. In fact, this appears clearly from the economy's dynamical budget constraint as obtained by aggregating equation (3) over all households. National income accounting identities imply that the excess of consumption expenditure over distributed income, net of domestic asset accumulation, equals the change in the economy's stock of foreign debt, B_t:

$$\frac{dB_t}{dt} = -NX_t + rB_t \tag{21}$$

where NX_t is the trade balance. Yet, the decision of an economy to consume early in time at a rate above the level of output, net of domestic asset accumulation, is reflected in the requirement of a trade surplus in a steady-state equilibrium, in order to offset interest payments on debt.[16]

Given that the economy's long-run growth is characterised by an external equilibrium, we may as well focus on the current account equilibrium condition:

$$\sum_{i=1}^{n} P_i C_i^{F} = \sum_{j=1}^{m} x P_j C_j^{H} + rB \tag{22}$$

[16] By integrating the economy's budget constraint from time 0 to some time T, and by taking into account the transversality condition we get the standard condition that the present value of consumption is equal to total wealth at time 0, where the latter is given by aggregate non-human wealth S_0 and the present value of labour incomes. As usual, in equilibrium the present value of consumption is also equal to the present discounted value of output - net of domestic asset accumulation - minus the initial level of debt.

where the LHS is total export receipts and the RHS is total payments for imports and interest obligations on outstanding debt. By aggregating the export and import demand individual functions we get:

$$\sum_{i=1}^{n} P_i \left(\frac{P_i}{P} \right)^{-\theta} \delta^{\theta} \frac{E^F}{P} = \sum_{j=1}^{m} xP_j \left(\frac{xP_j}{P} \right)^{-\theta} (1-\delta)^{\theta} \frac{E^H}{P} + rB \tag{23}$$

and by recalling that $\delta \equiv \dfrac{q^H}{q^F + q^H}$ and $P \equiv \left(\delta^{\theta} \sum_{i=1}^{n} P_i^{1-\theta} + (1-\delta)^{\theta} \sum_{j=1}^{m} xP_j^{1-\theta} \right)^{\frac{1}{1-\theta}}$, we reformulate the current account equilibrium condition as:

$$\sum_{i=1}^{n} P_i \left(\frac{P_i}{q^H} \right)^{-\theta} E^F = \sum_{j=1}^{m} xP_j \left(\frac{xP_j}{q^F} \right)^{-\theta} E^H + rB \tag{24}$$

Since $P_i = P^H$, $P_j = P^F$, and $\widetilde{E} = \widetilde{P} + \widetilde{Y}$ both home and abroad, the above condition may be restated in dynamical terms as:

$$\widetilde{Y}^H - \widetilde{Y}^F = -\theta \left(\widetilde{P}^H - \widetilde{x} - \widetilde{P}^F \right) + \theta \left(\widetilde{q}^H - \widetilde{q}^F \right) \tag{25}$$

It appears clearly that the fulfilment over time of condition (25) will depend on the evolution of both the pure price term and the quality differential term. If the economy starts with an external deficit, given the rate of change of foreign variables (that is, income, prices and quality), an acceleration in its pace of innovation can enhance foreign demand while keeping prices constant. In such a case, an optimal growth path in our model implies a non-deflationary equilibrium trajectory, with derived factor demands proportional to aggregate output, constant productivity-adjusted input prices and constant relative factor prices. On the contrary, with an unfavourable dynamics of quality upgrading the country faces a structural lack of competitiveness so that continuous changes in relative prices - either exchange rate devaluations or wage deflations – are needed to ensure the external balance. It is clear that the latter scenario is one of immiserizing growth where the economy faces a deterioration in its terms of trade, since, over time, an increasing amount of its exports - with a given quality content – has to be exchanged for a given amount of imports - with a higher quality content - in order for a current account equilibrium to be achieved.

The results obtained follow directly from our assumptions on households' preferences, that lead to export and import demand functions where the relative quality content of products impacts on the expenditure proportionality. If growth is coupled with an unfavourable relative quality dynamics, the export to import ratio will move adversely, and a balanced trade will require a persistent offsetting fall in relative prices.

Our model shows that demand conditions are endogenous, since they are closely related to a country's pace of innovation. In this sense, it may provide an answer to the Keynesian structuralist approach to growth, where exogenously given demand conditions constraint long-run growth. As known, the so-called balance-of-payments constraint approach[17] emphasises that terms of trade changes are ineffective in adjusting external deficits because most trade occurs in highly differentiated goods. Yet, in that approach a country's exports are assumed to depend on income elasticities of exports which are taken as given, whereas in our analysis we obtain "apparent" income elasticities which vary

[17] Thirlwall (1979); McCombie, Thirlwall (1994); McCombie (1998).

with the relative quality content of traded goods, and hence with the country's pace of innovation.[18]

Another feature of our model is that the extent to which growth can be immiserizing will ultimately depend on the country's ability to compete in quality dominated markets. The economy with a lower pace of innovation that is unable to engage in "quality competition" is forced to engage in "price competition", and sell an increasing amount of production at falling relative prices. This implies that growth is coupled with favourable "apparent" income elasticities only to the extent that the country is able to close its technological gap with respect to the most advanced countries. In fact, in our model an increase in production will generate an equivalent demand with no changes in relative prices, only if the quality-content of manufactured products is "competitive".[19] If the country is unable to catch-up technologically domestic prices will have to fall persistently so as satisfy the long-run equilibrium condition stated in equation (25).

4. International knowledge spillovers, technological catching up and growth

The analysis developed above points to the fact that a technologically backward country is unfavoured in trade because the goods it produces lag behind as to their know-how content. To see this, recall that with a research intensity of ι devoted to obtain the next generation good, the growth rate of quality in the technologically backward economy is:

$$\tilde{q}^{H} = \iota \log \gamma \tag{26}$$

or equivalently:

$$\frac{dq_t^H}{dt} = q_t^H \iota \log \gamma \tag{27}$$

The above specification makes explicit the public good nature of the stock of existing domestic knowledge in the quality upgrading process. In fact, the productivity of the resources devoted to research is positively influenced by the quality level at a given time t, q_t^H, that is, by the state-of-the-art know-how embodied in the quality of manufactured goods.

Analogously, for the foreign country we assume the following:

$$\tilde{q}^{F} = \varepsilon \log \gamma \tag{28}$$

where ε is the rate of innovation in the advanced country. Given that a lagging-behind country is typically characterised by a lower rate of R&D activity, i.e., $\varepsilon > \iota$, we get an unfavourable dynamics of the quality differential term in equation (25).

Yet, if we allow for knowledge spillovers to be international in scope, we may understand why openness may favour growth. Indeed, openness is a crucial factor in promoting the diffusion of knowledge. As emphasised in recent contributions of the literature on innovation and endogenous growth,[20] lagging-behind countries benefit by the

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[18] Within this analytical perspective Cavallaro and Mulino (2008) provide an empirical assessment of the importance of quality upgrading in the performance of market shares for some EU new member states.

[19] In Krugman (1989) the economy is characterised by increasing product differentiation and households' love for variety, so that the expansion of production will bring about an increase in demand at given prices. Differently, in our model an increase in production will generate an equivalent demand with no changes in relative prices only if the quality-content of manufactured products is "competitive".

[20] See, for instance, Connolly and Valderrama (2004); Smulders (2004); Léon-Ledesma (2005).

higher level of scientific and technological knowledge of advanced countries, and the knowledge disadvantage turns to be at the basis of a process of technological catching up.

In fact, with the integration of markets both the spread of general scientific knowledge and the diffusion of more product-specific information take place. As to the latter, the backward economy can gain skills in applying to the goods it manufactures the know-how embodied in products of the advanced country; moreover, since traded goods are differentiated, entrepreneurs have the possibility to "invent around the patent", i.e., to apply a given know-how to a different type of good, with respect to the one which it was originally conceived for. Hence, we assume that in the R&D sector both innovation and imitation activities are carried on. We treat analytically the two activities in a similar way, as imitation requires labour resources much like the other type of research. In fact, it has been stressed that in lagging-behind economies imitation requires "investment in technological capability" which implies "effort to apply existing knowledge in new circumstances."[21]

Building on the above considerations we modify the specification of the technology for innovation in equations (26) and (27) and assume that, thanks to increased openness to international trade, firms take advantage in their R&D activity of a broader stock of knowledge capital, which includes the know-how of the advanced country. It follows that the productivity of labour resources in research now increases with domestic knowledge as well as with foreign knowledge that flows into the economy, so that quality changes over time take place according to the following law:

$$\frac{dq_t^H}{dt} = \varphi\left[\left(q_t^H\right)^{\omega}\left(q_t^F\right)^{1-\omega}\right]\frac{L_R}{\ell}\log\gamma \tag{29}$$

where the term in square brackets is a measure of the stock of knowledge capital useful in research. By comparing equations (27) and (29), it appears that the process of quality upgrading benefits by the international diffusion of knowledge. In fact, now the productivity of labour resources allocated to research depends both on domestic structural factors and on the aforesaid international spillovers. The coefficient $(1-\omega)$ is the degree to which foreign knowledge flows into the economy and is incorporated into the overall stock of knowledge capital; hence it reflects the relevance of the stock of foreign knowledge capital for the specific research activity undertaken in the lagging economy, and depends, among others, on the degree of markets integration. The parameter φ reflects the efficiency with which the overall stock of knowledge capital is converted into R&D activity; we assume it to be above unity, in order to take into account that with increased market integration quality upgrading in most lagging-behind economies is enhanced by both innovation and imitation activities. In fact, a given stock of knowledge capital will have a greater impact on quality upgrading because it is exploited not only to produce "general knowledge", but also to generate quality development by imitation and reverse engineering.

By rearranging equation (29), we can express the growth rate of quality as:

$$\tilde{q}_t^H = \varphi\left(\frac{q_t^H}{q_t^F}\right)^{\omega-1}\frac{L_R}{\ell}\log\gamma \tag{30}$$

[21] Pack and Westphal (1976), p. 105.

where $\left(\frac{q_t^H}{q_t^F}\right)$ is the measure of the country's technological disadvantage. Equation (30) shows that when knowledge spillovers are international in scope the productivity of labour resources in research depends positively on the technological gap, given the values of φ and ω. Since $\varphi\left(\frac{q_t^H}{q_t^F}\right)^{\omega-1} > 1$, the productivity of labour resources in research is greater in equation (29), with international spillovers, than in equation (27).

We can now address the issue of the impact of the technological catching up on competitiveness and growth. By inspection of equation (30) it appears that a country's relative technological disadvantage can be a positive determinant of a catching-up process, provided that the country is able to fully capture the benefits of the international dissemination of knowledge. Indeed, the possibility of benefiting by the international diffusion of knowledge is related to the country's ability to absorb foreign technology, that is, to master and eventually improve upon technologies conceived in other countries. The factors which make a country receptive of the technology embodied in foreign goods are closely linked, among others, to the country's degree of openness, that is, to trade and financial liberalisation, as well as to an adequate institutional setting. As well known from earlier contributions in development literature,[22] the lagging-behind economy should have acquired the so-called "social capabilities" in order for the benefits of the international dissemination of technical knowledge to be fully captured. In our specification of the technology for quality upgrading this occurs for $(1-\omega)$ and φ sufficiently high.

It turns out that the country which is able to capture the knowledge embodied in foreign goods and technologies[23] will engage successfully in a reduction in the quality gap over time. This implies that the term $\left(\tilde{q}_t^H - \tilde{q}_t^F\right)$ in equation (25) will move favourably, that is, the country will improve its competitiveness. In the Appendix we study the possible patterns of "conditional" quality convergence for different values of the parameters φ and ω. We derive formally the behaviour of the ratio $\left(\frac{q_t^H}{q_t^F}\right)$ and find that the ratio increases over time and tends to a constant value, $\frac{q_T^H}{q_T^F}$, for T large enough, that is:[24]

$$\lim_{T\to\infty} \frac{q_T^H}{q_T^F} = \left(\varphi \frac{\iota}{\varepsilon}\right)^{\frac{1}{1-\omega}} \tag{31}$$

Hence, the lagging country's pace of innovation and its level of quality convergence depend on its structural parameters: the smaller ω, and the greater φ, the better the

[22] Abramovitz (1986), and Baumol *et al.* (1989) consider important factors in determining a country's "social capabilities" the availability of human capital, the structure and flexibility of trading and financial institutions, the degree of openness to international trade and investment in R&D.

[23] Given the small open economy assumption, the rate of innovation of the advanced country ε is independent from the innovation taking place in the backward country.

[24] See Appendix, condition [A.6].

country's performance. As argued above, such conditions are likely to be met with an increasing integration of markets.

The presence of international spillovers allows a technologically lagging country to increase the productivity of labour resources devoted to research, so that at each time *t* the pace of quality upgrading is determined on the basis of equation (29). In a steady-state the economy has a higher long-run growth rate of aggregate output, proportional to the higher growth rate of quality resulting from equation (30). This can be verified by calculating the long-run rate of innovation in the presence of international spillovers. When the technology for innovation is the one described in equation (30) the productivity of labour resources devoted to research is $Q_t = \varphi \left(\frac{q_t^H}{q_t^F} \right)^{\omega-1}$, so that the equations that determine the long-run rate of innovation, that is, equations (16) and (17), together with the side equations (12) and (15), lead to the following steady-state rate of innovation in the presence of international spillovers:

$$\iota_S = (1-\xi)\overline{L}\frac{Q}{\ell} - \xi\rho \ , \qquad 0 < \xi \equiv \frac{1-\alpha}{\gamma} + \alpha < 1 \qquad (32)$$

where Q is the steady-state value of Q_t, that is, the productivity of labour resources devoted to research when the benefits of the international diffusion of knowledge are all exploited so that equation (31) is verified.

5. A numerical illustration

We first derive some restrictions on the parameters that ensure "conditional" quality convergence, on the reasonable assumption that the quality level of the advanced country is never reached by the lagging country. Recalling equation (31), this implies that $\left(\varphi \frac{\iota}{\varepsilon} \right)^{\frac{1}{1-\omega}} < 1$, and hence[25]

$$\varphi \in \left(\left(\frac{q_0^H}{q_0^F} \right)^{1-\omega} \frac{\varepsilon}{\iota}, \frac{\varepsilon}{\iota} \right) \qquad (33)$$

In *figure 1* we represent the set of points corresponding to condition (33).

[25] See Appendix, condition [A.9].

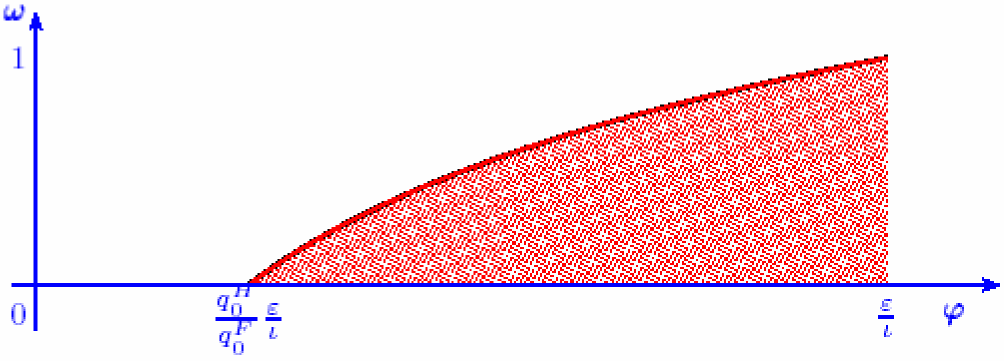

Figure 1 – Admissible range for parameters φ *and* ω

We now perform some numerical simulations in order to analyse the patterns of "conditional" quality convergence when international knowledge diffusion takes place. We consider the following values: $\frac{q_0^H}{q_0^F} = 0.16$; $\frac{\varepsilon}{\iota} = 5$; $\gamma = 1.03$, and compare the behaviour of the quality ratio $\frac{q_t^H}{q_t^F}$ through time for different values of the efficiency parameter φ, and eventually for different values of parameter ω - the weight of foreign capital in the stock of available capital available to domestic researchers. We first carry out the simulation with $\varphi_1 = 4.5$ and $\varphi_2 = 2.5$, when $\omega = 0.2$; we then perform the same exercise with $\omega_1 = 0.2$ and $\omega_1 = 0.6$, when $\varphi = 3.5$.

In *figure* 2 we show the results of the first simulation, where $\omega = 0.2$. Recalling that the quality growth rate in the advanced country is constant (equation 28), the difference observed in the ratio is all explained by the dynamics of q_t^H through time. By choosing $\varphi_1 = 4.5$, a value which is close to the maximum admissible value $\frac{\varepsilon}{\iota}$ (equation 31), we get the higher curve corresponding to a faster dynamics of quality. Asymptotically, the curve tends to a limit value, marked by the dotted line, which lies always below unity. The lower curve corresponds to $\varphi_2 = 2.5$, which is an intermediate value in the range of the admissible ones. The simulation shows that, other things equal, the more the lagging country is efficient in making the overall stock of knowledge capital useful for its R&D activity, the higher and the faster will be its convergence to the quality level of the advanced economy.

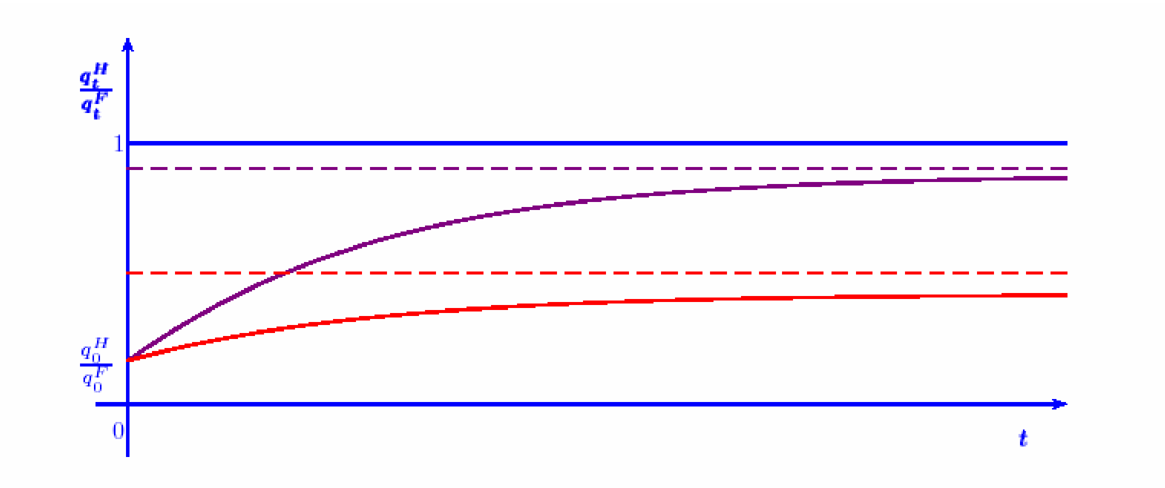

Figure 2 – Quality ratio dynamics for different values of parameter φ

In *figure 3* we represent two alternative scenarios corresponding to distinct values of parameter ω, when $\varphi = 3.5$ The higher curve is relative to a higher weight of foreign capital in the overall stock of knowledge capital, that is, $\omega_1 = 0.2$. The lower curve represents the dynamics of the technology gap when $\omega_1 = 0.6$. It appears that the more foreign knowledge is incorporated into the innovation activity of the lagging country, the better will be the country's performance, and, consequently, the lower its technological distance with respect to the advanced country in steady state.

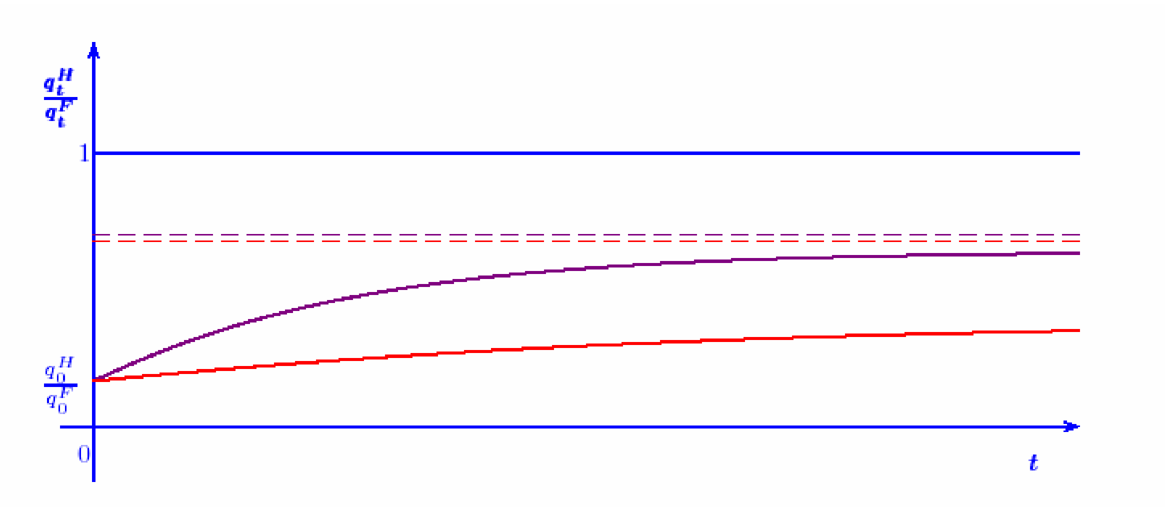

Figure 3 – Quality ratio dynamics for different values of parameter ω

6. Some final remarks

The contribution of the paper is to address the issue of trade and growth for a country that technologically lags behind. We focus on vertical innovation, and follow new-Schumpeterian models in assuming that the quality content of the goods manufactured in a given country reflect the available stock of knowledge capital, and the country efficiency in converting that capital into innovation. In the model we derive analytically export and import demand functions where we make explicit the role of goods' quality content, and show the implications for a country's competitiveness and long-run growth of a lower pace of innovation with respect to its trade partners. In line with the neo-Schumpeterian view that firms compete not so much by varying price and quantity as by innovating, we show that the possibility of long-run growth with no adverse terms of trade effects depends on the ability of the laggard country to catch-up technologically and to compete in quality-dominated markets. Hence, the more a country is able to absorb foreign knowledge and to improve upon technologies conceived in other countries, the greater the gains in competitiveness, and the benefits to long-run growth.

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Mathematical Appendix

1. Explicit solutions for q_t^F *and* q_t^H

The quality levels in the advanced and in the lagging-behind countries at time *t* are the solutions to the differential equations (28) and (30), respectively. We thus obtain for the advanced country:

$$q_t^F = q_0^F \gamma^{\int_0^t \varepsilon(\tau)d\tau} \qquad \text{[A.1]}$$

where q_0^F is the initial quality level.

As to the lagging country, we first reformulate equation (31) as:

$$\frac{dq_t^H/dt}{\left(q_t^H\right)^{\omega}} = \varphi\left(q_t^F\right)^{1-\omega}\iota(t)\ln\gamma$$

and then integrate in the time interval between 0 and *t*:

$$\int_{q_0^H}^{q_t^H}\frac{dq_s^H}{\left(q_s^H\right)^{\omega}} = \int_0^t \varphi\left(q_s^F\right)^{1-\omega}\iota(s)\ln\gamma\, ds$$

We thus obtain:

$$\frac{1}{1-\omega}\left[\left(q_t^H\right)^{1-\omega} - \left(q_0^H\right)^{1-\omega}\right] = \varphi\ln\gamma\int_0^t \iota(s)\left(q_s^F\right)^{1-\omega} ds$$

where q_0^H is the initial quality level of the lagging country. By further simplifying, we get:

$$q_t^H = \left[\left(q_0^H\right)^{1-\omega} + \varphi(1-\omega)\ln\gamma\left(q_0^F\right)^{1-\omega}\int_0^t \iota(s)\gamma^{(1-\omega)\int_0^s \varepsilon(\tau)d\tau} ds\right]^{\frac{1}{1-\omega}} \qquad \text{[A.2]}$$

2. Convergence relationships

In what follows we derive the conditions for quality convergence among the two countries, by looking at the behaviour of the ratio $\left(\frac{q_t^H}{q_t^F}\right)$ through time. Given that

$$\int_0^T \varepsilon(\tau)d\tau = \varepsilon T$$

which implies that on average the number of jumps on the quality ladder is εT, when T is large enough, the limit of equation [A.1] for $t \to T$ is:

$$q_T^F = \lim_{t \to T} q_t^F = q_0^F \gamma^{\varepsilon T} \qquad [A.3]$$

Analogously, we calculate the limit of equation [A.2] for $t \to T$:

$$q_T^H = \lim_{t \to T} q_t^H = \left[\left(q_0^H\right)^{1-\omega} + \varphi(1-\omega) \ln\gamma \left(q_0^F\right)^{1-\omega} \int_0^T \iota(s)\gamma^{(1-\omega)\varepsilon s} ds\right]^{\frac{1}{1-\omega}}$$

$$= \left[\left(q_0^H\right)^{1-\omega} + \varphi(1-\omega) \ln\gamma \left(q_0^F\right)^{1-\omega} \iota \frac{\gamma^{(1-\omega)\varepsilon T} - 1}{\ln\gamma^{(1-\omega)\varepsilon}}\right]^{\frac{1}{1-\omega}}$$

By rearranging the above formula, we get:

$$q_T^H = \left[\left(q_0^H\right)^{1-\omega} - \varphi\frac{\iota}{\varepsilon}\left(q_0^F\right)^{1-\omega} + \varphi\frac{\iota}{\varepsilon}\left(q_0^F\right)^{1-\omega}\gamma^{(1-\omega)\varepsilon T}\right]^{\frac{1}{1-\omega}} \qquad [A.4]$$

Turning to the ratio $\left(\frac{q_T^H}{q_T^F}\right)$, as resulting from equations [A.3] and [A.4], with some algebra we get:

$$\frac{q_T^H}{q_T^F} = \left\{\left[\left(\frac{q_0^H}{q_0^F}\right)^{1-\omega} - \varphi\frac{\iota}{\varepsilon}\right]\gamma^{-(1-\omega)\varepsilon T} + \varphi\frac{\iota}{\varepsilon}\right\}^{\frac{1}{1-\omega}} \qquad [A.5]$$

and, by taking the limit of the above ratio for $T \to \infty$

$$\lim_{T\to\infty}\frac{q_T^H}{q_T^F} = \left(\varphi\frac{\iota}{\varepsilon}\right)^{\frac{1}{1-\omega}} \qquad [A.6]$$

since the term $\gamma^{-(1-\omega)\varepsilon T}$ in equation [A.5] tends to vanish with $\omega \in (0,1)$.

On the assumption that the quality level of the advanced country is never reached by the lagging-behind country, we have the following condition:

$$\lim_{T\to\infty}\frac{q_T^H}{q_T^F} = \left(\varphi\frac{\iota}{\varepsilon}\right)^{\frac{1}{1-\omega}} < 1 \qquad [A.7]$$

which implies $\varphi\frac{\iota}{\varepsilon} < 1$, or equivalently:

$$\varphi \in \left(1, \frac{\varepsilon}{\iota}\right) \qquad [A.8]$$

Moreover, from equation [A.5] it appears that when the sign of the term in square brackets is negative we have monotonic convergence, and hence any value q_T^H rests always below the limit $\left(\varphi \frac{\iota}{\varepsilon}\right)^{\frac{1}{1-\omega}} q_T^F$. We thus obtain the following conditions for the parameters:

$$\varphi > \left(\frac{q_t^H}{q_t^F}\right)^{1-\omega} \frac{\varepsilon}{\iota}; \qquad \omega < 1 - \frac{\ln \frac{\varepsilon}{\varphi \iota}}{\ln q_0^F - \ln q_0^H}$$

By taking into account condition [A.8], and the assumptions $\omega \in (0,1)$, $\varepsilon > \iota$, $q_0^F > q_0^H$ we get the two equivalent conditions

$$\varphi \in \left(\left(\frac{q_t^H}{q_t^F}\right)^{1-\omega} \frac{\varepsilon}{\iota}, \frac{\varepsilon}{\iota}\right); \omega \in \left(0, 1 - \frac{\ln \frac{\varepsilon}{\varphi \iota}}{\ln q_0^F - \ln q_0^H}\right) \qquad \text{[A.9]}$$

which are depicted in *figure 1* in the text.

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